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REVIEW ARTICLE

Ontology of functional synergetics in virtual cognitive-semiotic design of information processes and systems

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Abstract
Objectives. In view of the currently expanding practice of applying methods for configuring ontologies of designing information processes and systems, information technology (IT) specialists need to disclose the definition of such a new concept as functional synergetics. This concept is actualized by the course of development of virtual information technologies, forming the system foundation for methods of design ontologies and indicating attributes for the correct formation of design ontologies of relevant dynamic information processes and systems. In the present work, this is done by means of analytical review.
Methods. The analytical review is based on the authors' vision of deepening the concept of synergetics as applied to a variety of modern information processes and systems. This context concessionally conjugates the combined use and consideration of such approaches as the method of ontologies (the major component of the resulting concession of methods), methods of cognitive semiotics, autopoiesis, and other manifestations of synergetics with related methods and techniques of emergent evaluation of the role and effectiveness of any occurring systemic changes. It is this combined dynamics of information processes and systems that allows the authors to put forward a developing treatment of the methods of synergetics as functional.
Results. The concept of functional synergetics was clarified and deepened based on the method of ontologies. In the theory of information processes and systems, it is manifested in updating ontologies accompanying scientific and engineering projects that use synergetics as an initial methodological basis. It turns out that the functional features of synergetics in the context of assessments and functional ordering of modern IT devices give it new opportunities in highlighting the significant indicators of system changes: properties, attributes, and manifestations of functional-synergistic nature. It is these three concepts, revealed by synergetics as a functional eyepiece, that distinguish functional synergetics from the generally accepted definition of synergetics as such. Matching form to content, autopoiesis, development, and transformations occurring with informational processes and systems with their emergent consequences are the essence and feature of functional synergetics. Tracing what is happening in dynamics, in the inseparability of assessments and regulations of the set of properties, attributes, and manifestations of the analyzed processes to an even greater extent clarifies the essence and role of the concept introduced by the authors in the general classical theory of information processes and systems. In the related analysis, virtual reality, augmented reality, mixed reality, expanded reality, composite reality, coupled reality, geoinformation systems, multidimensional computer graphics, fractal graphics, holographic graphics, computer teletype games, X-reality, etc., which have essential dynamic characteristics and properties, are included here as objects of research and design.
Conclusions. Improvement of the theory and practice of creating and using information processes and systems from the position of recognizingthe accelerating speed and dynamics of how their properties and indicators are modified leads the synergistic methodology of assessments and control mechanisms to the emergence of a clarifying concept of functional synergy. This is a complex and dynamic concept, which includes its interpretation of the positions of

classical synergetics and related paradigms of cognitive semiotics, etc. The method of ontologies, which is gaining more and more popularity, is the main means and tool of such unification. When combined with cognitive semiotics, functional synergetics becomes a powerful science-intensive tool for further development of the theory and practice of modern multimedia intensified information systems and their information fields described by both imperative and convivial paradigms.

Keywords: functional synergetics, spatial synergetics, confluence, emergence, autopoiesis

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ОБЗОРНАЯ СТАТЬЯ

Онтология функциональной синергетики в виртуальном когнитивно-семиотическом конструировании информационных процессов и систем

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Резюме
Цель. В связи с расширением в настоящее время практики применения методов онтологий проектирования информационных процессов и систем в кругу специалистов ИТ обозначилась потребность в том, чтобы дать и раскрыть определение такого нового, актуализированного самим ходом развития виртуальных информационных технологий понятия, как функциональная синергетика, образующего системные начала методов онтологий проектирования, обозначив в результате этого ее признаки для корректного формирования онтологий проектирования соответствующих динамичных информационных процессов и систем.
Методы. Аналитический обзор строится на обозначении авторского видения углубления синергетики как синергетики функциональной в применении к разнообразию современных информационных процессов и систем. В указанном контексте концессионно сопрягается совокупное использование и рассмотрение таких методов, как метод онтологий (генералитетная составляющая образуемой концессии методов), методы когнитивной семиотики, аутопоэзис и иные проявления синергетики со связанными с ними методами и методиками эмерджентной оценочности роли и эффективности любых происходящих системных изменений. Именно такая совокупная динамика информационных процессов и систем позволяет авторам выдвинуть развивающую трактовку методов синергетики, как синергетики функциональной.
Результаты. Уточняется и углубляется понятийная сторона нововведения – функциональной синергетики (с опорой на метод онтологий) – в теорию информационных процессов и систем, что проявляется обновлением онтологий, сопровождающих научные и инженерные проекты, использующие синергетику

как исходный методологический базис. При этом выясняется, что функциональные черты синергетики в контексте оценок и упорядочения функционала современных устройств информационных технологий придают ей новые возможности в выделении значимых показателей системных изменений: свойств, признаков и проявлений функционально-синергетического характера. Именно эти три понятия, раскрываемые синергетикой как функциональным окуляром, отличают функциональную синергетику от общепринятого определения синергетики как таковой. Соответствие формы содержанию, аутопоэзис, развитие и преобразования, происходящие с информационными процессами и системами с их эмерджентными последствиями, являются сутью и особенностью синергетики, трактуемой как синергетика функциональная. Прослеживание происходящего в динамике, в неразрывности оценок и регулирований совокупности свойств, признаков и проявлений анализируемых процессов в еще большей степени уточняет сущность и роль вводимого авторами понятия в общую классическую теорию информационных процессов и систем. В связанный с этим анализ в качестве объектов исследования и проектирования здесь включены обладающие существенно динамическими характеристиками и свойствами виртуальная, дополненная, смешанная, расширенная, составная, сопряженная реальности, геоинформационные системы, многомерная компьютерная графика, фрактальная, голографическая графика, компьютерные телетайповые игры, дополненная реальность и т.п.

Выводы. Совершенствование теории и практики создания и применения информационных процессов и систем с позиций учета все более существенно возрастающих скоростей, динамики видоизменений их свойств и показателей приводит синергетическую методологию применения оценок и механизмов управления ими к появлению уточняющего понятия функциональной синергетики как понятия комплексного и динамического, вбирающего в свое толкование как позиции самой классической синергетики, так и сопряженных с ней парадигм когнитивной семиотики и т.п. Базисным генералитетным средством и инструментом такого рода объединения является набирающий все большую известность метод онтологий. В соединении с когнитивной семиотикой функциональная синергетика становится мощным наукоемким инструментом дальнейшего развития теории и практики современных разноплановых интенсифицированных информационных систем мультимедиа и их информационных полей, описываемых как императивной, так и конвенциональной парадигмой.

Ключевые слова: функциональная синергетика, пространственная синергетика, конфлюэнтность, эмерджентность, аутопоэзис

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INTRODUCTION

The study aimed to give and reveal the definition of such a new concept, actualized by the very course of development of virtual information technologies, as *functional synergetics*, and identify its signs for the correct formation of design ontologies of the corresponding dynamic information processes and systems. The expediency of introducing this somewhat updated concept of synergetics into the basic ontology of the modern theory of information processes and systems is prompted by two important interrelated circumstances.

First, the variety of interpretations of the terms of *synergy*, *synergetics*, *synergetic effect*, and other similar concepts requires clarification [1–9]. In other words, it is essential to determine how these terms are understood in relation to the tasks and procedures for constructing design ontologies in the information technology (IT) sphere and its information fields. The fundamental provisions of synergetics serve as the basis for the new term *functional synergetics*. The innovation creates effective prerequisites and opportunities for the analysis and ordering of complex information processes in the information fields of modern dynamic IT [10–14].

Secondly, synergetics in any of its representations and interpretations presupposes the possibility of both a positive synergistic effect with a plus sign (“+1”), and a negative one with a minus sign (“−1”), as a rule, without fixing intermediate values in the range from −1 to +1 and, moreover, not illustrating the dynamics of the sliding of synergy within the limits indicated here.

The algorithm of actions for the implementation of the task is transparent:

Step one: outlining at least an approximate list of types of virtualization, to which the definition of the concept of *functional synergetics* and its ontological interpretation are correlated here.

Step two: determination of the boundaries and parameters of processes, phenomena and properties of the entire list of virtualizations, to which the modified term belongs, or, which is the same, the determination of the continual area, continual boundaries in information fields, which are subject to the declared paradigms, properties, and manifestations of those or other processes and phenomena considered from the standpoint of synergetics.

Step three: clarifying the formulation of the signs of continuity in line with the functionality of synergetics, thereby giving a clear idea of the semantic meaning and setting capabilities of the introduced concept in the theory and practice of synthesis of ontologies of certain dynamic information processes and systems.

Step four: assessment of the possibility and prospects of applying the updated approach in the vast field of diversity of modern information processes and systems, including for the prospect of their expected development from a technological perspective.

In the most pronounced form, this is manifested in the tasks of research, creation, design, and maintenance of a wide range of IT multimedia related to a wide range of virtualization of multimedia objects of high dynamics, which, for example, are virtualization devices as part of IT multimedia, such as virtual reality devices and their various modifications, geoinformation systems, systems of modern highly dynamic multidimensional computer graphics, holographic and fractal graphics, computer teletype games, X-reality, etc.

The formation of basic ontologies of this kind of IT arrangements based on the emerging positions and views of *functional synergetics* is fragmentarily presented below using examples of multimedia virtualization objects as a starting point for development, by analogy with an extensive list of concepts of the ontology of IT *functional synergetics*, intended to serve as an ontology for the design of information processes and systems.

OBJECTS OF VIRTUALIZATION AS A PART OF MULTIMEDIA IN THE VISION OF THE PARADIGMATICS OF *FUNCTIONAL SYNERGETICS*

Virtual reality is an imagery created by technical means, transmitted to the user through his sensations and perceptions by the senses and his own cognitive processing. Virtual reality simulates both an exposure and responses to the exposure. To create a convincing complex of sensations, close to true reality, the computer synthesis of properties and reactions of virtual reality is performed in real time[1, 2] [15]. The user can act on these objects in accordance with the principles, procedures and technological features of state and control indications embedded in the information system (IS) of virtual reality. In this variety, as in all subsequent components of this ontological assembly, there are clearly expressed functional synergistic features, traced in the properties, manifestations, and signs of information processes and in the system arrangement of very dynamic (in the most general case) virtual reality systems, in high continuous dynamics conjugation of forms and the content of procedural and displayed signs and manifestations, including those related to the phenomena of spatial synergetics and autopoiesis.

Augmented reality is the result of the introduction of various sensory data into the field of perception and cognitive processing of the IS of augmented reality in order to supplement information about the environment and improve the perception of information[3]. The most common element of augmenting virtual reality that converts it into augmented reality is the input commentary text.

Mixed reality is functionally complicated augmented reality, reflecting the explicitness of the concept of the virtual continuum and the application of the taxonomy of this classification to the means of displaying reality[4, 5] [16].

Extended reality, often referred to as a hybrid reality, represents the synclide of augmented reality and augmented virtuality, and is a consequence of combining the real and virtual worlds to create new environments and visualizations that have a synergistic effect in relation to the components, that is, the effect of the appearance of new properties,

[1] Isaac J. Step into a new world – Virtual Reality (VR). Basic Concepts of Virtual Reality along with Research Challenges explained in simple words. 2016. URL: https://www.completegate.com/2016070154/blog/virtual-reality-explained. Accessed August 13, 2021.

[2] Astonishing innovations of VR. URL: https://web.archive.org/web/20200112191904/https://caersidi.net/blog/vr-astonishing-innovations. Accessed August 13, 2021.

[3] What is augmented reality? URL: https://www.fi.edu/what-is-augmented-reality. Accessed August 13, 2021.

[4] What is mixed reality? URL: https://docs.microsoft.com/ru-ru/windows/mixed-reality/discover/mixed-reality. Accessed August 16, 2021 (in Russ.).

[5] A Taxonomy of Mixed Reality Visual Displays. URL: https://search.ieice.org/bin/summary.php?id=e77-d_12_1321. Accessed August 16, 2021.

signs, and manifestations that are invisible in the components. It is a highly merged virtualization implementation[6].

Composite reality, in the author's vision, is an extended reality with pronounced properties of confluence, as a result of which the specified variety can be considered as a set of layers into which it is dismembered, but it is protected from this via the principles and mechanisms of confluence within the life cycle of an IS composite reality.

Conjugate reality, in the author's vision, is a composite, augmented reality that has, throughout the life cycle, general features of the interconnection of IS layers, nonseparability of the virtuality continuum and mediality continuum of serviced objects and their virtualizations [16].

Geographic IS (GIS) is a system for collecting, storing, analyzing, and graphically visualizing the spatial (geographical) data and related information about the required objects [17–19]. The popularity and wide variety of GIS, methods and technologies for working with them exclude the possibility of introducing any detailed description of them into this thematic list. Here, something else is essential. There is a synergistic nature of the arising effects, which is quite traceable by an observer, both during the synthesis, processing of GIS, and when they are perceived by users. These technologies are dynamic, multicolored, and multidimensional, and have pronounced features and scalability, and therefore should be considered primarily from a synergistic standpoint.

Multidimensional computer graphics does not need additional definitions, as a well-known visualization tool, which is primarily an important part of data analysis, allowing you to combine several dimensions in one model representation[7] [20]. The dynamic properties of the processes associated with it and, especially, the multidimensionality that goes beyond the boundaries of the conceptual perception of images (computer graphics models with a large number of dimensions) are the most difficult, science-intensive, and promising task for the further development of the theory and practice of graphics virtualization. An example of virtual mastering of a multidimensional space is shown in the illustration of a six-dimensional tutorial[7]. This is an elegant but private solution. According to the authors, the way of creating and putting into practice a universal model of virtual adapters, transformers and clones of multidimensional N-graphics ISs with an unlimited number of measures should rely on methods and mathematical descriptions of increasing or decreasing the complexity measures of infologies and morphologies of the original virtual layered structure possessing a certain set of confluent properties and features that is stable and unchanged in the process of variation.

Fractal graphics (fractal reality). The fractal picture itself can be positioned as an image, graphics. At the same time, the contours, outlines, and color gamut (RGB) of a fractal image are significantly dependent and changeable depending both on the image processing editor and on the technological means of its screen or print display. Thus, it is proposed to understand, that computer fractal graphics directly is a graphic formation, which momentarily in this particular implementation has the corresponding particular manifestations of contours and color gamut. Any editorial influences and/or changes in the technological package of image processing inevitably lead to changes in the informative essence of fractal images. At the same time, there is a synergistic effect of a negative property in assessing the status of emergence. The measure and the very possibility of mixing certain informative features in the graphics of fractal images can vary widely and most of all depends on the set of scaling factors and relationships between the virtualization layers, and the consequences and results are determined via the confluence function of the transformations taking place. The dynamics and implicit discreteness of the ongoing processes bring to the fore their evaluativeness from the standpoint of *functional synergetics*, where the mechanisms and methods of synergetics are described by appropriate models.

Holographic graphics, both flat and volumetric, manipulates the synthesis, perception, and transformations of incorporeal images similar to reality, that is, with holograms and holography. Holography (ancient Greek: ὅλος is whole and γράφω is graph) is a set of technologies for accurate recording, reproduction, and reformulation of optical electromagnetic radiation wave fields. Holography is a special photographic method in which laser images of three-dimensional objects, extremely similar to real ones, are recorded and then restored. A hologram is the acquisition of images using wavefront reconstruction. All of the above regarding fractal graphics and multidimensional computer graphics can be attributed to holographic graphics.

[6] What is mixed reality? URL: https://docs.microsoft.com/ru-ru/windows/mixed-reality/discover/mixed-reality. Accessed August 16, 2021 (in Russ.).

[7] Multidimensional graphs in Python – from 3-D to 6-D. URL: https://habr.com/ru/post/456282/. Accessed August 18, 2021. (in Russ.).

Here, removing each layer from the system (part of the hologram) to a certain physical limitation within a wide range of scaling preserves the integrity of the images, but with a possible decrease in quality indicators. This phenomenon is the best illustration of the presence of a functional synergistic effect in what is happening. Evaluating and using the essence of this effect in the synthesis procedures for design ontologies of information processes and holographic graphics systems (and in other similar cases) elevates the traditional ideas about synergetics as a descriptive entity to the level of a certain function of analyzing and managing information processes and systems.

This definition is well illustrated by the following specific case. Windows Mixed Reality is a mixed reality platform, presented as part of the Windows 10 operating system, and provides a holographic embodiment of objects in mixed reality in technical implementations with appropriate helmets (Windows Holographic)[8, 9].

Computer teletype games (intellectualized computer games) are implementations of functional connections with game partners or a substitution of a partner by means of IS intellectualization, optionally implemented on computer displays or spatially, in holographic performance, in augmented and other realities based on the initiation of specially created resource-intensive computer programs for multivector execution of transactions that serve to organize the game process (gameplay). The latter refers to the most popular and promising area of the development of computer games—to intellectualized games. It features game artificial intelligence, i.e., a set of software techniques that are used in computer games to create the illusion of intelligence in the behavior of computer-controlled characters. This development is especially promising in the actualization of interactive, Bell–Lancaster, and distance learning and self-study. There is no need to re-enumerate everything that is referred above to the abundance of variety of virtualizations from the standpoint of synergetics. All these signs and properties are not only present, but to a large extent exacerbated by a large number of development trajectories of modern games, and therefore, by the synergistic effect arising for each option, each "move" of a player, or the IS itself. In addition, everything that happens in computer games is carried out in the conditions of their so inflated dynamics that these effects overlap one another. This leads to the emergence of new, even more unexpected emergent consequences. There are signs of *functional synergetics* with its emergent dynamic manifestations, that is, derivatives of certain current values of emergencies [21−23].

X-reality is a set of orientational images and their models adding the development of all the varieties of virtualizations indicated above. X-reality is a kind of generalization of both already known and practiced design solutions, and those whose appearance in the near future can only be predicted, and even then, very generally.

Let us define *a priori*: X-reality has the highest indicators of dynamic properties in transformations of synergetics, the power and emergence of synergies from the entire list above, which predetermines the description of the ontologies of X-realities as a kind of universe of basic ontology (core of ontologies) of the entire direction of IT multimedia development discussed here.

Thus, here is an assembly of updated and systematized concepts of design ontologies of modern information processes and systems in the field of virtualized highly dynamic multimedia from the standpoint of synergetics in its somewhat updated interpretation—*functional synergetics*. This concept, apparently, needs clarification of its interpretation and additional disclosure of properties, signs, and manifestations, which is carried out further in this article in relation to multimedia information fields.

DEFINITIONS AND PROPERTIES OF *FUNCTIONAL SYNERGETICS* IN THE METHODOLOGY OF VIRTUALIZATION OF MULTIMEDIA FIELDS

The first thing that needs to be done in the formulation of the preparation of this section of the article is to define the updated concept of synergetics. There are many different interpretations of this capacious concept in details. Common transcription is as followes: synergetics (from the Greek συν- is a prefix with the meaning of compatibility and ἔργον is energies) is the interdisciplinary field of science that studies the general laws of phenomena, processes, and development in complex nonequilibrium systems (including ISs) based on inherent them self-organization principles. The main concept of synergetics is the definition of a structure as a

[8] A Taxonomy of Mixed Reality Visual Displays. URL: https://search.ieice.org/bin/summary.php?id=e77-d_12_1321. Accessed August 16, 2021.

[9] Multidimensional graphs in Python – from 3-D to 6-D. URL: https://habr.com/ru/post/456282/. Accessed August 18, 2021. (in Russ.).

state that arises as a result of the multivariate and ambiguous behavior of such multielement structures or multifactorial media that do not degrade to thermodynamic type averaging which is standard for closed systems, but develop due to openness, energy inflow from the outside, nonlinearity of internal processes, and the appearance of special modes with exacerbation and the presence of more than one stable state. The conceptual circle of synergetics also includes the idea of conformity (measure of conformity) between form and content.

It is to be considered what is true here for the spectrum of virtualized multimedia systems described above, and what requires additional rethinking and, perhaps, rebuilding. Let us list:

- *Interdisciplinarity*—the feature and properties are obvious from the very list of virtualized multimedia systems.
- *Signs of complexity and disequilibrium* take place in an expressed form, since everything opposite in the theory of information processes and systems is only partial simplified cases.
- *Regularities of phenomena, processes and development* are observed in synergistic descriptions of approaches to the list given here, but they require quantitative evaluations and the construction of appropriate mechanisms and controls for ongoing or expected information processes. This position requires development and most obviously fits into the concept of *functional synergetics*.
- *Variability and self-development under the influence of internal factors and external influences* is characteristic of the multimedia spectrum discussed here.
- *Conformity and measure of conformity of the form to the content*. This is essential, must be uniquely, and quantitatively, be determined and ordered.
- *Determination of the structure as a state.* Exactly like that, but along with this, within the life cycle of an IS, their state changes repeatedly and continuously, while the structure, architecture, and infology at some intervals of the life cycle trend can remain unchanged, but most likely, are subject to either drift or abrupt changes up to the occurrence of collapses. In this view, the question also fits into the conceptual nature of synergetics as *functional synergetics*.

All positions listed here assume adherence to ergodic principles. Among the most important principles are the calculability of processes and phenomena, the repetition of the results of calculations and their at least conditional predictability, which should become a supporting position for the formation of ideas about synergetics as *functional synergetics*. The components of the vision of synergetics named in this way are responsible, therefore, for the identification, declaration, model (including mathematical) description, and application in the procedures of analysis, modeling, design, and maintenance of all varieties (separately and jointly) of quantitative indicators and regulators of all of the above the synergistic features, properties, and manifestations inherent in the statics and dynamics of information processes and systems.

Three obvious additional investments in the conceptual apparatus of *functional synergetics* of highly dynamic multimedia information processes and systems follow from this capacious universe.

First. *Functional synergetics* as a tool for analyzing and influencing these processes and systems works in interrelated areas **signs, properties, and manifestations** in the functioning of processes and systems and the means of their creation and maintenance.

Second. Correspondences of forms to contents from the standpoint of majority (minority), confluence, and autopoiesis are included in the evaluative characteristics and tools for ordering synergetic properties, manifestations, and features in a quantitatively calculated and simulated form, thereby representing the functional essence of synergetics as a tool for creation and management. Let us clarify: **content** here is predetermined by the actual component of virtualization and its object; **form** is a consequence, the observed virtual, synthesized, artificial, and drawn image of an object that displays content. This is the connection between content and image in a synergistic view of multimedia virtualized systems. Functionally, virtualization refers to a real primordial object as a mirror or pseudo-mirror image of it, which has a mandatory feature **chiral purity** in relation to the source of creation. At the same time, virtualization must meet a number of additional attributes and properties assigned to it, which are not necessarily present in a real source, namely: meeting the requirements of cognitive semiotics; ensuring a rationalized balance of harmonization and standardization of content (especially educational and scientific content initiating basic subject ontologies of information processes and multimedia systems). It is also necessary to analyze the totality of technical, aesthetic, ergonomic, psychophysical requirements, etc. The emergence and manifestation of all these properties and features in virtualizations, new in relation to the object, is nothing more than a polynomial of emergent bursts as a result of virtualization, the dynamics of which gives rise to the concept of derivatives of emergent contributions that form the specified polynomial.

Third. The generalizing idea of the dynamics and complexity of the changes occurring in the balance of these signs (architecture, infology, morphology, etc.) inherent in this approach requires the introduction of components into the tools of *functional synergetics* of the IS that reflect, along with traditionally used for this purpose, emergent estimates derived from the aggregate emergencies, which, based on the use of systems of differential equations or some other mathematical apparatus (for example, matrices of a planned experiment, Monte Carlo method, etc.), allows one to evaluate, improve, and optimize the indicated synergetic indicators both from the standpoint of achieving conformity of the form to the content, and in parts related to the inextricable triumvirate of signs, properties, and manifestations, including in dynamics.

CONCLUSIONS

Functional synergetics used in evaluative and regulatory actions in virtualized multimedia systems and fields of a wide variety of composition is designed to display and provide the ability to influence the signs, properties, and manifestations of an IS in models of quantitative measures throughout their entire life cycle, as well as to assess and organize the necessary conformity of the form to the content of the ongoing systemic processes and transformations.

The functional synergy of information processes and systems, including in relation to multimedia, manipulates the triple set of its main **categories: signs, properties, and manifestations** coupled with ensuring adequate **conformity of form and content.** Moreover, all this plot has quantifiable meanings in the models and methods synthesized for this.

The concepts and functionality of *functional synergetics* disclosed in the study can be an extension of the core of the basic design ontology of information processes and systems in the multimedia section.

In combination with cognitive semiotics, functional synergetics becomes a powerful science-intensive tool for the further development of the theory and practice of modern multi-faceted intensified multimedia ISs and their information fields, described by both the imperative and conventional paradigms.

Authors' contribution

R.G. Bolbakov—systematic construction of the nature and results of the research displayed in the article, editing, participation in writing the article.

V.A. Mordvinov—ideomatics, analysis of sources, participation in writing the article, information management of the research shown in the article.

P.V. Berezkin—formalization of semantic and synergetic features based on existing synergetic classifiers of information processes and systems, editing, participation in writing the article.

I.I. Sivitsky—systematic structuring of the sources introduced in the article, analysis and collection of materials, editing, participation in writing the article.

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RESEARCH ARTICLE

Equivalence of the schemes of programs based on the algebraic approach to setting the semantics of programming languages

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Abstract
Objectives. The paper deals with the equivalence of program schemes. According to A.A. Lyapunov and Yu.I. Yanov, the founders of this theory, a program scheme is understood as a program model wherein abstraction from contensive values of operators and expressions is performed. In this case, the program structure containing symbolic notation of operators and expressions remains unchanged while maintaining their execution sequence. The programming language model presented in the paper contains basic constructs of sequential languages and is the core of the existing sequential programming languages. The paper aimed at developing an effective algorithm for studying equivalence (nonequivalence) of program schemes of sequential programming languages.
Methods. An algebraic approach to specifying semantics of programming languages was used for studying the equivalence of program schemes.
Results. A process semantics being the new algebraic approach to specifying the formal semantics of sequential programming languages was proposed. The process semantics was specified by matching programs (program schemes) with a set of computation sequences. The computation sequence was understood as the execution sequence of actions (commands and tests) of the program. Two types of concatenation operations (test–command and command–command) and the merge operation, which properties are given by axiomatic systems, were defined in the introduced semantic domain. The finiteness of the semantic value representation in the form of systems of recursive equations was proved. The algorithm for proving the equivalence (nonequivalence) of systems of recursive equations characterizing semantic values for a pair of program schemes was proposed, which implies the equivalence (nonequivalence) of programs in the strong sense.
Conclusions. The paper demonstrates the efficient use of the proposed algorithm for proving the equivalence of sequential program schemes excluding side effects when calculating expressions, i.e., sequential computation of the expression more than once does not change anything. The example of proving the equivalence of program schemes by two methods—the well-known de Bakker–Scott fixed-point induction method and the method proposed by the author—is given. Comparison of the above methods testifies not only to the new method's effectiveness but also to its significant simplicity, proved in practice by students who performed corresponding tasks when studying the Semantics of Programming Languages at the Institute of Information and Computing Technologies at the National Research University Moscow Power Engineering Institute (Moscow, Russia).

Keywords: program scheme, semantic domains, process semantics, equational characterization of the semantic meanings of programs, equivalence of program schemes

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НАУЧНАЯ СТАТЬЯ

Эквивалентность схем программ на основе алгебраического подхода к заданию семантики языков программирования

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Резюме
Цели. Статья посвящена вопросам эквивалентности схем программ. В соответствии с работами А.А. Ляпунова и Ю.И. Янова – основоположников данной теории, под схемой программы понимается ее модель, в которой осуществляется абстрагирование от содержательных значений операторов и выражений. При этом неизменной остается структура программы, включающая символические обозначения операторов и выражений при сохранении последовательности их выполнения. Представленная в статье модель языка программирования содержит основные конструкции последовательных языков и является ядром имеющихся языков последовательного программирования. Цель работы – разработка эффективного алгоритма исследования вопросов эквивалентности (неэквивалентности) схем программ последовательных языков программирования.
Методы. Используется алгебраический подход к заданию семантики языков программирования для исследования вопросов эквивалентности схем программ.
Результаты. Предложен новый алгебраический подход к заданию формальной семантики языков последовательного программирования – процессная семантика. Процессная семантика задается посредством сопоставления программам (схемам программ) множества вычислительных последовательностей. Под вычислительной последовательностью понимается последовательность выполнения действий (команд и тестов) программы. На введенной семантической области (множестве вычислительных последовательностей) определены операции конкатенации двух видов (тест-команда и команда-команда) и операция объединения, свойства которых заданы системами аксиом. Доказана конечность представления семантических значений в виде систем рекурсивных уравнений. Предложен алгоритм доказательства эквивалентности (неэквивалентности) систем рекурсивных уравнений, характеризующих семантические значения для пары схем программ, откуда вытекает эквивалентность (неэквивалентность) программ в сильном смысле.
Выводы. Показана эффективность применения предложенного алгоритма для доказательства эквивалентности схем последовательных программ, в которых отсутствует побочный эффект при вычислении выражений, т.е. последовательное вычисление выражения более, чем один раз, ничего не меняет. В статье приведен демонстрационный пример доказательства эквивалентности схем программ двумя методами: известным методом индукции фиксированной точки де Баккера – Скотта и предложенным в статье методом. Сравнение приведенных методов свидетельствует не только об эффективности нового метода, но и его существенной простоте, что было подтверждено на практике при выполнении соответствующих заданий студентами специальности «Прикладная математика и информатика» Национального исследовательского университета МЭИ в процессе изучения дисциплины «Семантика языков программирования».

Ключевые слова: схема программы, семантические области, процессная семантика, эквациональная характеризация семантических значений программ, эквивалентность схем программ

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INTRODUCTION

Program equivalence is an extremely important aspect of the theory and practice of programming languages underlying such problems as program correctness [1–10], program completeness (incompleteness), and equivalent transformations of programs for optimizing them [11–13] in one way or another. It is clear that to solve these problems, appropriate formal methods for specifying semantics of programming languages should be developed. Propositional logic-based methods have been among the first to be developed, the most known being Floyd's inductive assertion method [5] and Hoare's axiomatic method [6]. The use of these methods allows proving partial correctness and completeness (incompleteness) of a sufficiently large class of programs limited in size. The denotational approach [7] allows more opportunities for solving the problem of program equivalence using methods based on fixed point properties, in particular, the de Bakker–Scott fixed-point induction method [4, 7]. In the paper, the algebraic method for specifying the process semantics of programs, matching programs with a set of computation sequences (execution paths) of a program as semantic values, is proposed. The algorithm for analyzing the equivalence of program schemes based on the idea of representing semantic values in the form of finite systems of recursive equations with further analysis for equivalence (non-equivalence) of the obtained systems of recursive equations developed by the author in his thesis[1] and [14] is proposed. In contrast to the method proposed for proving the equivalence of program schemes[1], the recursive equations obtained in the systems have a more complex form and require more detailed analysis. The effectiveness of this method for proving the program scheme equivalence is shown.

1. FORMAL MODEL OF PROGRAMMING LANGUAGE

We shall specify the syntax and semantics of language L used as a programming language model.

[1] Korablin Yu.P. Semantic methods of analysis of distributed systems. Dr. Thesis (Eng.). Moscow: MEI; 1994. 40 p. (in Russ.).

1.1. Syntax of Language L

The alphabet of the language consists of the following:

- the set of elementary commands Com with typical element a;
- the set of Boolean expressions Exp with typical element b.

Typical elements of sets may be indexed. In addition, the following three constants may be defined: skip is empty command; tt and ff are identically true and identically false Boolean values, respectively.

The set of Cmd commands with a typical element c may be defined as follows:

c ::= skip | a | [gc] | ⋆ [gc] ,

where gc is a typical element of the set of protected Gcom commands with the following syntax:

gc ::= g → c | gc { □ gc },
g ::= tt | ff | b

Parentheses are used to denote zero or more iterations of the construct enclosed in brackets. A typical element of the set of protections G is denoted by g.

The symbol □ connects alternative program components, i.e., such constructs are analogous to switch branching statement in programming languages.

The set of programs with typical element pr may be defined as follows:

pr ::= c | pr; c.

1.2. Algebraic semantics: mathematical foundations and semantic equalities

The semantic function matches the program written in L language with a set of computation sequences (CS) that may be used for executing that program.

To specify the program semantics, the principle of constructing semantic value of the entire program based on semantic values of that program components is proposed.

Assuming that any program may be, in turn, a component of another program, *a priori* semantics of the entire program may always be obtained by composing semantic values of program components.

1.2.1. Semantic domains

1. The set of values of elementary commands ACom with typical element A.
2. The set of values of Boolean expressions Bexp with typical element B.
3. The Const set containing the constants τ (identity transformation), T (identically true value), and F (identically false value).

Typical elements of semantic domains may be indexed.

We shall define the set of computation sequences CPath with typical element cp.

Two additional sets being the Test set with typical element β and the Action set with typical element α may be predefined, as follows:

$\beta ::= T \mid F \mid B,$
$\alpha ::= \tau \mid A,$
$cp ::= \alpha \mid \beta \wedge cp \mid cp1 \circ cp2,$

where cp∗ is finite CS while cpω is infinite CS. We shall introduce the set SP = $\mathcal{P}$(CPath) with typical element sp, i.e., SP is the power set defined as the set of all subsets of the CPath set.

Then, operations of the set–theoretic union (sp1 + sp2), sequential composition (sp1 ∘ sp2), and the least fixed point (sp^+) are defined in the SP set.

Before determining the value for sp^+, two auxiliary definitions should be introduced.

Definition 1. CS cp is called empty and denoted by ε if:

1) cp ≡ τ;
2) cp = T ^ ε.

Infinite sequence of the form εω is denoted by LOOP (looping).

Definition 2. ε ∈ sp if:

sp ≡ ε;

sp = sp1 + sp2 and ε belongs to at least one of the sets sp1 or sp2;

sp = sp1 ∘ sp2 and ε belongs to both sp1 and sp2 simultaneously.

We shall define sp^+ as the least fixed point of the operator F(sp), i.e., $sp^+ = \mu F(sp)$, where F(sp) may be written in the following form:

$F(sp) = \lambda q.\ sp \circ q + \nu$, where $\nu = \tau$.

1.2.2. Semantics of language L

The semantic function may be defined as follows:

$\mathbb{C}[\![skip]\!] = \tau,$
$\mathbb{C}[\![a]\!] = A,$
$\mathbb{C}[\![tt \to pr]\!] = \mathbb{E}[\![tt]\!] \wedge \mathbb{C}[\![pr]\!],$
$\mathbb{C}[\![ff \to pr]\!] = \mathbb{E}[\![ff]\!] \wedge \mathbb{C}[\![pr]\!],$
$\mathbb{C}[\![b \to pr]\!] = \mathbb{E}[\![b]\!] \wedge \mathbb{C}[\![pr]\!],$

where function $\mathbb{E}$: Exp → TEST defines semantic values of the expressions.

$\mathbb{C}[\![*\{gc\}]\!] = (\mathbb{C}[\![gc]\!])^+,$
$\mathbb{C}[\![gc1 \ \square\ gc2]\!] = \mathbb{C}[\![gc1]\!] + \mathbb{C}[\![gc2]\!],$
$\mathbb{C}[\![pr1;\ pr2]\!] = \mathbb{C}[\![pr1]\!] \circ \mathbb{C}[\![pr2]\!].$

Then, the semantic function for the following expressions may be defined:

$\mathbb{E}$: Exp → TEST,
$\mathbb{E}[\![tt]\!] = T,$
$\mathbb{E}[\![ff]\!] = F,$
$\mathbb{E}[\![b]\!] = B.$

1.3. Axiomatization of semantic domain properties

The properties of operations on elements of the semantic domain SPσ = $\mathcal{P}$(CPathσ) may be described using an axiomatic system (axiom schemes) and inference rules. The axiomatic system may be defined as a set of blocks, each describing certain properties of semantic objects.

The ACTION ∪ TEST set with typical element d′ may be denoted by D′. Metavariables X, Y, and Z may be used for denoting elements of the semantic domain.

Axioms defining basic properties of operations "∘", "^" and "+" may be written as follows:

(A1) $X + X = X,$
(A2) $X + Y = Y + X,$
(A3) $X + (Y + Z) = (X + Y) + Z,$
(A4) $(X + Y) \circ Z = X \circ Z + Y \circ Z,$
(A5) $(X \circ Y) \circ Z = X \circ (Y \circ Z),$
(A6) $\tau \circ X = X,$
(A7) $X \circ \tau = X,$
(A8) $LOOP \circ X = LOOP,$
(A9) $\varnothing \circ X = \varnothing,$
(A10) $X + \varnothing = X,$
(A11) $(\beta \wedge X) \circ Y = \beta \wedge (X \circ Y),$
(A12) $F \wedge X = \varnothing,$
(A13) $\varnothing \wedge X = \varnothing.$

2. EQUATIONAL CHARACTERIZATION OF *A PRIORI* SEMANTIC VALUES

In this section, the possibility of representing the program semantic values in the form of finite systems of recursive equations is described.

Definition 3. Let α ∈ ACT = ACTION \ {τ, Ø} and (β ∈ TES = TEST \ {F, Ø} and PREF = ACT ∪ TES.

The partial functions prefix: SP → PREF and suffix: SP → SP may be defined using Table 1:

Table 1. Defining prefixes and suffixes of computation sequences (sp)

sp	prefix (sp)	suffix (sp)
α ∘ X	α	X
τ ∘ X	prefix (X)	suffix (X)
β ^ X	β	X

It should be noted that prefix and suffix may be not defined for the expression sp ≡ F ^ X, since F ^ X ≡ Ø (axiom A12).

Definition 4. The CS set of P ∈ SPσ may be equationally characterized if there exists the finite set P1, P2, ..., Pn ∈ SP such that P ≡ P1 for any i ($1 \le i \le n$):

$$P_i = \sum_{j\in N} \alpha_{ij} \circ P_{ij} + \sum_{k\in N} \beta_{ik} \wedge P_{ik} + \delta(P_i), \qquad (*)$$

where $N = \{1, 2, \ldots, n\}$, α_{ij} ∈ ACT, β_{ik} ∈ TES,

∀i δ(Pi) ⊂ 𝒫(ACT′ ∪ {LOOP}), where ACT′ = ACTION \ {τ},

∀i ∀j ∃l ∈ N, that Pij = Pl,

∀i ∀k ∃r ∈ N, that Pik = Pr.

Theorem 1. Every SP set matched as a semantic value with a program may be equationally characterized using a finite system of equations of the form (*).

Proof. The proof is by induction method for structure P.

Basis. The equational characterization for p ≡ d′, where d′ ∈ D′ follows trivially.

Inductive step. Let P1 = SP and P2 = SP be equationally characterizable. Then it is necessary to prove that P1 ∘ P2, β ^ P1, P1 + P2, and P1$^+$ are equationally characterizable.

We shall prove the expression P = P1 ∘ P2.

By the induction hypothesis, there exist sets P11, P12, ..., P1n and P21, P22, ... P2m such that P1 ≡ P11 and P2 ≡ P21, and

$$\mathrm{P1}i = \sum_{j\in N} \alpha_{ij} \circ \mathrm{P1}_{ij} + \sum_{k\in N} \beta_{ik} \wedge \mathrm{P1}_{ik} + \delta(\mathrm{P1}i),\ i = \overline{1,n}, \qquad (**)$$

and

$$\mathrm{P2}i = \sum_{r\in N} \alpha_{ir} \circ \mathrm{P2}_{ir} + \sum_{p\in N} \beta_{ip} \wedge \mathrm{P2}_{ip} + \delta(\mathrm{P2}i),\ i = \overline{1,m}, \qquad (***)$$

We shall denote

$$\eta(u, v1, \ldots, vr) = \mathrm{P1}_u \circ \mathrm{P2} + \mathrm{P2}_{v_i} + \ldots + \mathrm{P2}_{v_r}, \quad (u = \overline{0,n},\ r \ge 0,\ 1 \le vi \le m,\ i = \overline{1,r}). \qquad (1)$$

We shall write $\mathrm{P10} \circ \mathrm{P2} + \mathrm{P2}_{v_i} + \ldots + \mathrm{P2}_{v_r}$ instead of $\mathrm{P2}_{v_i} + \ldots + \mathrm{P2}_{v_r}$.

The number of expressions (1) is finite. By the inductive hypothesis, the following may be written:

$$\eta(u, v1, \ldots, vr) = \left(\sum_{j\in N} \alpha_{uj} \circ \mathrm{P1}_{uj} + \sum_{k\in N} \beta_{uk} \wedge \mathrm{P1}_{uk} + \delta(\mathrm{P1}u)\right) \circ \mathrm{P2} + \sum_{j\in N} \alpha_{v_1 j} \circ \mathrm{P2}_{v_1 j} + \sum_{k\in N} \beta_{v_1 k} \wedge \mathrm{P2}_{v_1 k} + \delta(\mathrm{P2}v_1) + \ldots + \sum_{j\in N} \alpha_{v_r j} \circ \mathrm{P2}_{v_r j} + \sum_{k\in N} \beta_{v_r k} \wedge \mathrm{P2}_{v_r k} + \delta(\mathrm{P2}v_r).$$

Then, applying axioms A1, A2, A3, and A5, the following may be written:

$$\eta(u, v1, \ldots, vr) = \sum_{j\in N} \alpha_{uj} \circ \mathrm{P1}_{uj} \circ \mathrm{P2} + \sum_{k\in N} \beta_{uk} \wedge (\mathrm{P1}_{uk} \circ \mathrm{P2}) + \sum_{j\in N} \alpha_{v_1 j} \circ \mathrm{P2}_{v_1 j} + \sum_{k\in N} \beta_{v_1 k} \wedge \mathrm{P2}_{v_1 k} + \ldots + \sum_{j\in N} \alpha_{v_r j} \circ \mathrm{P2}_{v_r j} + \sum_{k\in N} \beta_{v_r k} \wedge \mathrm{P2}_{v_r k} + \delta(\mathrm{P1}u) \circ \mathrm{P2} + \delta(\mathrm{P2}v_1) + \ldots + \delta(\mathrm{P2}v_r).$$

If $\delta(\mathrm{P1}u) = \sum_{j\in N} \delta_{ui}$, where $\delta ui \in \mathrm{D}'$, then axiom A4 is applied again replacing summand δ(P1u) ∘ P2 with expression $\sum_{j\in N} \delta_{ui} \circ \mathrm{P2}$, in η($u$, v1, ..., vr), and if $\delta_{ui} = \tau$, then P2 is replaced by its representation for P21 from (***).

Then, applying axioms A1, A2, A3, and A5, the following may be written:

$$\eta(u, v1, \ldots, vr) = \sum_{j\in N} \alpha_{uj} \circ \mathrm{P}_{uj} + \sum_{k\in N} \beta_{uk} \wedge \mathrm{P}_{uk} + \sum_{q\in N}\sum_{j\in N} \alpha_{qj} \circ \mathrm{P}_{qj} + \sum_{q\in N}\sum_{k\in N} \beta_{qk} \wedge \mathrm{P}_{qk} + \delta(u, v1, \ldots, vr),$$

where all expressions $\mathrm{P}_{uj}, \mathrm{P}_{uk}, \mathrm{P}_{qj}$, and P_{qk} are included in (1). Since $\eta(1,1) \equiv$ P1 ∘ P2, then P1 ∘ P2 is equationally characterizable.

P = β ^ P1. This is a trivial case. In this case, only one equation P = P1 = β ^ P11 is added to the finite system of equations for P1 = P11, which implies the equational characterizability of the expression β ^ P1.

Proving the equational characterization of the expression P = P1 + P2 is analogous, where the following set is used as a finite set:

$$\xi(u,v) \equiv \mathrm{P1}u + \mathrm{P2}v,\ u = \overline{0,n};\ v = \overline{0,m}. \quad (2)$$

For proving the equational characterization of the expression $\mathrm{P} = \mathrm{P1}^{+}$, the set

$$\zeta(u1,...,ur) \equiv (\mathrm{P1}u_1 + ... + \mathrm{P1}u_r) \circ \mathrm{P1}^{+},$$
$$r \geq 0,\ 1 \leq ui \leq n,\ i = \overline{1,r}. \quad (3)$$

may be used.

The number of expressions (3) is finite.

Thus, the set of execution paths for a program, i.e., process semantics allowing studying various program properties, may be defined as the semantic value of a program. In particular, as described below, this approach allows studying the equivalence of program schemes in a strong sense.

3. COMPARISON OF PROMRAM SCHEMES IN LANGUAGE L

We shall define a method for comparing systems of recursive equations of the form (*), thus allowing obtaining a formal method for comparing program schemes in language L.

The significant factor of the considered method is the uniqueness of prefixes of any recursive equation being achieved through applying the axiom of the form X ∘ (Y + Z) = X ∘ Y + X ∘ Z. The result of applying this axiom is that any recursive equation is reduced to the form where all α_{ij} are pairwise distinct.

Thus, proving the equivalence (or non-equivalence) of two systems of recursive equations in each step is reduced to proving the equivalence of expression pairs having the same prefixes in the considered recursive equations.

The comparison process ends when new expression pairs stop appearing, or there is a mismatch of prefixes or expression sets δ for a certain expression pair in a certain step.

The first case of completing the comparison process implies the equivalence of two systems of recursive equations, while the second case implies their noncomparability, and therefore non-equivalence of two systems of recursive equations.

Let there be two systems of recursive equations of the form (*) to be tested for equivalence. These systems contain recursive equations for expressions P1, P2, …, P*n* и P1′, P2′, ..., P*m*′, respectively. We shall denote the set of expressions {P1, P2, ..., P*n*} by P, {P1′, P2′, ..., P*m*′} by P′, and the sets of all subsets of P and P′ by $\mathcal{P}$(P) and $\mathcal{P}$(P′), respectively.

We shall consider the equations for P1 and P1′. Here, the following cases are possible:

a) δ(P1) = δ(P1′) and $\forall j \in N\ \exists k \in N$ are such that $\alpha 1j \equiv \alpha 1k'$ and *vice versa*, and also $\forall k \in N\ \exists p \in N$ is such that $\beta 1k \equiv \beta 1p'$ and *vice versa*. In this case, the process of writing out equations for all pairs $(\mathrm{P}_{1j}, \mathrm{P}_{1k})$ having the same prefixes of type α as well as for all pairs $(\mathrm{P}_{1k}, \mathrm{P}_{1p})$ having the same prefixes of type β should be continued.

b) At least one of the conditions given in paragraph a) is not satisfied. This is the case of noncomparability of the prefix set or that of noncomparability of absolute terms of equations (δ(P1) ≠ δ(P1′)), whence it follows that computation sequence sets (CSSs) given by systems of recursive equations are noncomparable.

The above process of writing equations should be proceeded for the resulting pairs until obtaining one of the following results:

- two systems of recursive equations equivalent up to notations (with only case a) occurred in each step) are set up. In this case, CSSs given by systems of recursive equations are comparable, and thus, the program schemes matched with these expressions are equivalent;
- the condition given in paragraph a) is not satisfied. In this case, CSSs given by systems of recursive equations are not equivalent, and thus, the program schemes matched with these expressions are not equivalent.

We shall analyze the effectiveness of the proposed method for proving the equivalence of program schemes. In particular, an example of proving the following statement applying the extensively used method of fixed-point induction [4] may be firstly considered:

ℂ ⟦while B do C1 od; while B do C2 od⟧ = ℂ ⟦while B do C1 od⟧.

Solution. We shall introduce the following notations: 𝔼 ⟦E⟧=ω; ℂ ⟦C1⟧ = γ1; ℂ ⟦C2⟧ = γ2.

ℂ ⟦while E do C1 od;⟧ = fix(λγ.λσ.ωσ → γ·γ1σ,σ) = = fix H1 = γ1′,

where fix H1 stands for taking the least fixed point of the operator H1= λγ.λσ.ωσ → γ·γ1σ,σ.

ℂ ⟦while E do C2 od;⟧ = fix(λγ.λσ.ωσ → γ·γ2σ,σ) = = fix H2 = γ2′,

where H2 = λγ.λσ.ωσ → γ·γ2σ,σ.

The left-hand side (LHS) of the equation is given as follows:

LHS = ℂ ⟦while E do C2 od;⟧ · ℂ ⟦while E do C1 od;⟧ = = $\gamma 2' \cdot \gamma 1'$.

The right-hand side (RHS) of the equation is, respectively, of the following form:

RHS = $\gamma 1'$.

Thus, it is required to prove that: $\gamma 2' \cdot \gamma 1' = \gamma 1'$.

For proving, the method of fixed-point induction is applied.

Let $q(x) \equiv \gamma 2' \cdot x = x$.

1. $q(\bot) \equiv \gamma 2' \cdot \bot = \bot$
2. Let $q(x) \equiv \gamma 2' \cdot x = x$ is valid.

We shall show that $q(H1(x)) \equiv \gamma 2' \cdot H1(x) = H1(x)$ is valid.

LHS = $\gamma 2' \cdot H1(x) = \gamma 2' \cdot (\lambda\sigma.\omega\sigma \to x \cdot \gamma 1\sigma, \sigma)$ = (left factoring)

= $\lambda\sigma.\omega\sigma \to \gamma 2' \cdot x \cdot \gamma 1\sigma, \gamma 2'\sigma$ = (assuming validity of $q(x)$)

= $\lambda\sigma.\omega\sigma \to x \cdot \gamma 1\sigma, \gamma 2'\sigma$ = (by the fixed point property)

= $\lambda\sigma.\omega\sigma \to x \cdot \gamma 1\sigma, H2(\gamma 2')\sigma$ =

= $\lambda\sigma.\omega\sigma \to x \cdot \gamma 1\sigma, (\lambda\sigma.\omega\sigma \to \gamma 2' \cdot \gamma 2\sigma, \sigma)\sigma$ =

= $\lambda\sigma.\omega\sigma \to x \cdot \gamma 1\sigma, (\lambda\sigma.\omega\sigma \to \gamma 2' \cdot \gamma 2\sigma, \sigma)$ = (by the conditional operator property)

= $\lambda\sigma.\omega\sigma \to x \cdot \gamma 1\sigma, \sigma$.

RHS = $H1(x) = \lambda\sigma.\omega\sigma \to x \cdot \gamma 1\sigma, \sigma$

It follows from paragraphs 1 and 2 that $q(\text{fix } H1)$ = = $q(\gamma 1')$ is valid, and thus,

$\gamma 2' \cdot \gamma 1' = \gamma 1'$, as required.

This statement way be written using the proposed notation, as follows:

*[b → c1]; *[b → c2] = *[b → c1].

Using the above algorithm, the validity of the following statement may be proved:

ℂ ⟦ *[b → c1]; *[b → c2] ⟧ = ℂ ⟦ *[b → c1] ⟧.

We shall denote ℂ ⟦ *[b → c1]; *[b → c2] ⟧ by P1, and ℂ ⟦ *[b → c1] ⟧ by P2.

P1 $= (B \wedge C1)^+ \circ (B \wedge C2)^+$,
P2 $= (B \wedge C1)^+$.

Then, P1 and P2 may be represented by systems of recursive equations according to the program scheme comparison algorithm given above. Let P11 ≡ P1 and P21 ≡ P2. Then:

$P11 = \tau \circ (B \wedge C2)^+ + (B \wedge C1) \circ (B \wedge C1)^+ \circ (B \wedge C2)^+ =$
$= \tau + (B \wedge C2) \circ (B \wedge C2)^+ + (B \wedge C1) \circ (B \wedge C1)^+ \circ$
$\circ (B \wedge C2)^+ =$
$= \tau + B \wedge \{C2 \circ (B \wedge C2)^+ + C1 \circ (B \wedge C1)^+ \circ (B \wedge C2)^+\} =$
$= \tau + B \wedge P12.$

$P21 = \tau + (B \wedge C1) \circ (B \wedge C1)^+ = \tau + B \wedge \{C1 \circ$
$\circ (B \wedge C1)^+\} = \tau + B \wedge P22.$

It can be easily seen that the set P12 contains CSs starting at C2 while the set P22 does not contain CSs starting at C2, whence it follows that P12 ≠ P22.

At first glance, this result seems contradicting what obtained above. However, the crux of the problem is that obtaining this result, the lack of side effects while calculating expressions (conditions for execution of protected commands) has been assumed. It follows from this assumption directly that recalculation of the same expression would always give the same result. In our case, however, recalculating the same expression may give, in general, different results. This may be illustrated by the following example. Let be a program enabling side effects, i.e., value changes in variables when calculating expressions,

x: = 1; *[x: = 2 × x = 4 → c1]; [x: = 2 × x = 4 → c2].

The initial calculation of the expression x: = 2 × x = 4 results in a false value, and thus in breaking out of the loop *[x: = 2 × x = 4 → c1] and in proceeding to calculations of the next instruction. In this case, the value of variable x before executing is equal to 2. Recalculating the expression x: = 2 × x = 4 results in the true value. Obviously, the presence of side effects is usually an undesirable situation. Therefore, it would be advisable to create a formalism allowing analyzing programs excluding side effects. To that end, a number of notions introduced previously should be redefined, as well as axioms characterizing their properties should be added.

We shall first expand the test set by adding negation and difficult tests.

β ::= . . . | ⌉β | β1 ∗ β 2 , where the ellipsis stands for the predefined test sets.

The new test set is characterized by the following block of axioms:

(G1) β1 ∗ β2 = β2 ∗ β1,
(G2) β ∗ ⌉β = F,
(G3) F ∗ β = F,
(G4) Ø ∗ β = Ø,

(G5) $T * \beta = \beta$,
(G6) $\beta * \beta = \beta$,
(G7) $\beta 1 * (\beta 2 * \beta 3) = (\beta 1 * \beta 2) * \beta 3$,
(G8) $\beta 1 \wedge (\beta 2 \wedge C) = (\beta 1 * \beta 2) \wedge C$.

The expansion of multiple tests results in the need to modify the predefined notions of prefix and suffix.

Definition 5. The partial functions prefix: SP → PREF and suffix: SP → SP are defined by Table 2 being the extension of Table 1:

Table 2. Defining prefixes and suffixes of computation sequences (sp)

SP	prefix (SP)	suffix (SP)
$\alpha \circ X$	α	X
$\tau \circ X$	prefix (X)	suffix (X)
$\beta 1 \circ X1$, where $X1 \neq \beta 2 \wedge X2$, and $X1 \neq \tau \circ X$, and $X1 \neq X2^+$	$\beta 1$	X1
$\beta 1 \wedge (\beta 2 \wedge X)$	prefix $((\beta 1 * \beta 2) \wedge X)$	suffix $((\beta 1 * \beta 2) \wedge X)$
$\beta \wedge (\tau \circ X)$	prefix $(\beta \wedge X)$	suffix $(\beta \wedge X)$

It should be noted that when sp ≡ $\beta 1 \wedge X1$ and $X1 \equiv X2^+$, it would be essential to use first the property of the least fixed point up to the appearance of one of the constructs which the suffix and prefix defined for in Table 2.

Then, the following statement given above should be proved:

$$\mathbb{C} [\![*[b \to c1]; *[b \to c2]]\!] = \mathbb{C} [\![*[b \to c1]]\!].$$

We shall show that $(B \wedge C1)^+ \circ (B \wedge C2)^+ = (B \wedge C1)^+$. Denoting, as before, the left-hand side of this statement by P1 while the right-hand side by P2, we shall set up systems of equations using the new formalism.

Then, the value **ν** should be modified. Taking into account new definition of tests, the value **ν** for expression of the form $(B1 \wedge C1 + B2 \wedge C2 + \ldots + Bn \wedge Cn)^+$ may be defined as $\nu = \beta \wedge \tau$, where $\beta = \neg B1 * \neg B2 * \ldots * \neg Bn$.

Thus, the systems of equations may be written in the following form:

$P11 = P1 = (B \wedge C1) \circ (B \wedge C1)^+ \circ (B \wedge C2)^+ + (\neg B \wedge \tau) \circ$
$\circ (B \wedge C2)^+ =$
$= B \wedge C1 \circ (B \wedge C1)^+ \circ (B \wedge C2)^+ + (\neg B \wedge \tau) \circ (B \wedge C2) \circ$
$\circ (B \wedge C2)^+ + (\neg B \wedge \tau) \circ (\neg B \wedge \tau) =$
$= B \wedge P12 + \neg B \wedge (B \wedge C2) \circ (B \wedge C2)^+ + \neg B \wedge \tau =$
$= B \wedge P12 + (\neg B * B) \wedge C2 \circ (B \wedge C2)^+ + \neg B \wedge \tau =$
$= B \wedge P12 + F \wedge C2 \circ (B \wedge C2)^+ + \neg B \wedge P13 =$
$= B \wedge P12 + \varnothing + \neg B \wedge P13 = B \wedge P12 + \neg B \wedge P13$,
$P12 = C1 \circ P11$,
$P13 = \tau$,
$P21 = P2 = (B \wedge C1) \circ (B \wedge C1) + \neg B \wedge \tau =$
$= B \wedge C1 \circ (B \wedge C1)^+ + \neg B \wedge \tau =$
$= B \wedge P22 + \neg B \wedge P23$,
$P22 = C1 \circ P21$,
$P23 = \tau$.

The following two systems of recursive equations may be written:

$P11 = B \wedge P12 + \neg B \wedge P13$,
$P12 = C1 \circ P11$,
$P13 = \tau$,

and

$P21 = B \wedge P22 + \neg B \wedge P23$,
$P22 = C1 \circ P21$,
$P23 = \tau$.

The resulting systems of equations coincide with the accuracy of variable denoting, which implies the equivalence of expressions P1 and P2.

We shall consider another example of applying the proposed method to proving another statement. Suppose we need to prove the following statement:

$\mathbb{C}$ ⟦while E do C1 od; while E do C2; while E do C2 od od ⟧ =
$\mathbb{C}$ ⟦if E then C1; while E do C1 od; while E do C2 od else e fi⟧.

In our formalism, the above statement may be written in the following form:

$*[b \to c1];*[b \to c2;*[b \to c2]] = (b \to [c1;*[b \to c1];* *[b \to c2]) \,\square\, \bar{b} \to skip]$.

Let: $B = E[b]\rho$, $\neg B = E[\bar{b}]\rho$, $C1 = C[c1]\rho$, $C2 = C[c2]\rho$.

$P1 = C[LHS]\rho = P11 = (B \wedge C1)^+ \circ (B \wedge (C2 \circ$
$\circ (B \wedge C2)^+))^+ = ((B \wedge C1) \circ (B \wedge C1)^+ + \neg B \wedge \tau) \circ (B \wedge$
$\wedge (C2 \circ (B \wedge C2)^+))^+ = B \wedge P12 + \varnothing + \neg B \wedge P13 = B \wedge P12 +$
$+ \neg B \wedge P13$,
$P12 = C1 \circ (B \wedge C1)^+ \circ (B \wedge (C2 \circ (B \wedge C2)^+))^+ =$
$= C1 \circ P11$,
$P13 = \tau$.

Thus, the following may be written:

$P11 = B \wedge P12 + \neg B \wedge P13$,
$P12 = C1 \circ P11$,
$P13 = \tau$,

P2 = C[RHS]ρ = P21 = B ^ (C1 ∘ (B ^ C1)$^+$ ∘ ∘ (B ^ C2)$^+$) + ⌉B ^ τ = B ^ P22 + ⌉B ^ P23,

P22 = C1 ∘ (B ^ C1)$^+$ ∘ (B ^ C2)$^+$ = C1 ∘ P11,

P23 = τ.

Thus, the following may be written:

P21 = B ^ P22 + ⌉B ^ P23,

P22 = C1 ∘ P21,

P23 = τ.

The resulting systems of equations coincide with the accuracy of variable denoting, which implies the equivalence of expressions P1 and P2.

CONCLUSIONS

In the paper, the algebraic method for proving the equivalence of sequential program schemes was developed. To solve the problem, the algebraic model of semantic domain representing the set of all computation sequences of a program was proposed. The axiomatic system describing properties of operations over this semantic domain was given, as well as the presentability of axiomatic values for programs (program schemes) in the form of finite systems of recursive equations was proved. The algorithm for proving the equivalence of resulting systems of equations was proposed. Demonstration examples of proving the equivalence (non-equivalence) of program schemes applying the proposed method were given to illustrate its efficiency. This approach is not inferior to the well-known de Bakker–Scott fixed-point induction method in its capabilities, but as seen from the given examples, it simplifies proving the equivalence of program schemes significantly.

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RESEARCH ARTICLE

Data backup methods for mission-critical information systems

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Abstract
Objectives. Digitalization of the economy has led to a situation in which organizations accumulate huge amounts of digital data, the loss or damage of which leads to irreparable damage to the organizations. The issue of increasing data safety is relevant. One of the ways to improve the safety of data is backing them up. The study aimed to develop an effective strategy for backing up data for critical enterprise information systems.
Methods. The method for solving the problem was to create backup copies of enterprise information systems using a flexible architecture based on the backup—as a service in external cloud structures—in combination with technical resources of the organization.
Results. This article discussed backup solutions and tools, which include: backup volumes and schedule, target point and recovery time. The strategies and mechanisms of data backup were analyzed. The most common backup mechanisms are removable media, backups, external hard drive, hardware, backup software, cloud backup services. To create backups on a network, a large external hard drive is created and archival software is used to save changes to local files on that hard drive. This article covered: backup strategy, concept of backup storage, different types of backup storage methods, including network storage, external hard drives, and cloud storage. The main provisions and rules for backing up critical information systems were described. The rules for copying servers were given.
Conclusions. This article discusses a data backup architecture for mission-critical enterprise information systems. The authors believe that there should be at least three backups, two of which are located in the "cloud." The 3–2–1 strategy developed by the authors gives quite satisfactory results for the safety of critical data.

Keywords: backup, data recovery, network storage, cloud technologies, banking sector, hybrid clouds

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НАУЧНАЯ СТАТЬЯ

Методы резервирования данных для критически важных информационных систем предприятия

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Резюме
Цели. Цифровизация экономики привела к тому, что любая организация накапливает огромное количество цифровых данных, потеря или порча которых приводит к невосполнимому ущербу для организации. Актуальным является вопрос повышения сохранности данных. Одним из способов повышения сохранности данных является резервное копирование. Целью статьи является разработка эффективной стратегии резервирования данных для критически важных информационных систем предприятия.
Методы. Методом решения задачи является создание резервных копий информационных систем предприятия на основе применения гибкой архитектуры, основанной на сочетании резервного копирования как услуги во внешних облачных структурах с техническими средствами, находящимися в распоряжении организации.
Результаты. В статье обсуждаются решения и инструменты для резервного копирования, среди которых: объемы и расписание резервного копирования, целевая точка и время восстановления данных. Рассмотрены стратегии и механизмы резервного копирования данных. Наиболее распространенными механизмами резервного копирования являются съемные носители, резервирование, внешний жесткий диск, аппаратные средства, программное обеспечение для резервного копирования, услуги резервного копирования в «облако». Для создания резервных копий в своей сети создается внешний жесткий диск большого объема и используется архивное программное обеспечение для сохранения изменений в локальных файлах на этом жестком диске. В статье рассмотрены: стратегия резервного копирования, концепция хранилища резервных копий, различные типы методов хранения резервных копий, включая сетевое хранилище, внешние жесткие диски и облачное хранилище. Описаны основные положения и правила резервного копирования критически важных информационных систем. Приведены правила копирования серверов.
Выводы. В статье обсуждается архитектура резервного копирования данных для критически важных информационных систем предприятия. Авторы считают, что резервных копий должно быть не менее трех, две из которых размещаются в «облаке». Разработанная авторами стратегия 3–2–1 дает вполне удовлетворительные результаты по сохранности критически важных данных.

Ключевые слова: резервное копирование, восстановление данных, сетевое хранилище, облачные технологии, банковский сектор, гибридные «облака»

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Авторы заявляют об отсутствии конфликта интересов.

INTRODUCTION

In the digital economy [1], backing up data [2–4] is vital for the functioning of an organization [5]. Data can be stolen, corrupted, or lost. Data backup is a practice that combines methods and solutions for efficient and economical data storage with the process of parallel computing [6, 7], that leads to a synergistic effect [8, 9]. Corporate data are copied to one or more locations at predetermined frequency and varying capacities. A flexible backup operation can be set up using either the company's own architecture or the available Backup as a Service (BaaS) solutions, mixing them with local storage [10–12]. Today there are many technical solutions for corporate storage [13], which allow the company to protect data [14–16], avoid their loss, prevent leakage, and calculate costs [17–19].

Data backup is the practice of copying information from a primary to a secondary location to protect it in the event of a disaster, accident, or malicious act. Data is a vital resource of modern organizations, and its loss can cause serious damage to security [20, 21]. Therefore, backups are critical for all businesses.

The most common causes of data loss are hardware and system failure (31%), human error (29%), viruses and ransomware (29%) [14, 22, 23].

METHODS FOR SOLVING THE PROBLEM

Typically, backup data is essential information for workloads running on a corporate server. These can include documents, media files, configuration files, computer images, operating systems, and registry files. Basically, any data that needs to be saved can be backed up.

Backing up data includes several important concepts.

Backup solutions and tools. While it is possible to manually back up the data, in order to ensure that this is done regularly and consistently, most organizations use technology-based solutions to back up their data.

Backup administrator. Each organization should designate a backup administrator. This person verifies that the backup systems are configured correctly, tests them periodically, and ensures that critical data is actually backed up.

The volume and schedule of backups. An organization should choose a backup policy, specifying which files and systems are important enough to back up and how often to back it up.

Recovery Point Objective (*RPO*). This is the amount of data that an organization is prepared to lose in the event of a disaster, and it is determined by the frequency of backups. If the systems are backed up once a day, the RPO is 24 h. The lower the RPO, the more storage, compute, and network resources are required for frequent backups.

Recovery Time Objective (*RTO*). This is the time it takes for an organization to restore data or systems from a backup and resume normal operation. For large amounts of data and/or backups stored off-site, copying data and restoring systems can take a significant amount of time. Reliable technical solutions are required to ensure low RTO values.

DATA BACKUP MECHANISMS

There are many ways to back up a file [3, 24, 25]. Choosing the right option can help an organization to create the best backup plan. The most common backup mechanisms are: removable media, backups, external hard drive, hardware, backup software, cloud backup services.

An easy option is to back up the files to removable media such as CDs, DVDs, Blu-Ray discs, or USB drives.

An additional hard drive can be configured, which will be a copy of the drive of critical information systems of the enterprise. It is possible set up a large external hard drive on your network and use archive software to save changes to local files on that hard drive. The archiving software allows you to recover files from external hardware in just a few minutes. However, as data volumes grow, a single external drive will not be enough.

Many vendors provide off-the-shelf backup devices, typically 19″ rack mountable. Backup devices come with large storage capacities and pre-integrated software. Backup agents are installed on the systems for which you want to run the backup, a schedule and backup policy are defined, and data are transferred to the backup device. As with the other options, you can place the backup device isolated from the local network and, if possible, in a remote location.

Backup software solutions are more difficult to set up and configure than hardware appliances, but they provide more flexibility. They allow you to determine which systems and data will be backed up, place the backups on the selected storage device, and automatically manage this process.

Cloud providers offer BaaS solutions where you can send on-premises data to the public or private cloud, and in the event of a disaster, recover the data back from the cloud. BaaS solutions are easy to use; their great advantage is that the data are stored remotely. However, when using the public cloud, you must comply with applicable regulations and standards, and bear in mind that over time, the cost of storing data in the cloud will be much higher than the cost of setting up similar storage locally.

STRATEGY 3–2–1 BACKUP FOR CRITICAL CORPORATE INFORMATION SYSTEMS

The authors proposed a 3–2–1 backup strategy for critical corporate information systems. This strategy is to ensure that systems are adequately backed up and recovered reliably. The essence of the strategy is that three copies of important information systems of an organization are created on at least two different media, and at least one copy is stored remotely.

Three copies of the data include the original data and two duplicates. This ensures that a lost backup or damaged media will not affect your ability to recover.

Two different storage types reduce the risk of media-specific failures by using two different technologies. Common options include internal and external hard drives, removable media, or cloud storage

In this case, one copy is stored directly in the organization. Incremental or differential backups are performed daily. There are two backups in different remote locations ("cloud"). Once a week, a full copy of the computer disks is made and sent to "cloud" No. 1. Additionally, once a month, another full copy is made, the results of which are sent to "cloud" No. 2. The composition of the daily and weekly set is constant. Thus, compared to simple rotation, the archive contains only monthly copies and the latest weekly and daily copies.

One remote copy eliminates the risk of a single point of failure. External copies are essential for robust disaster recovery and data backup strategies, and can provide failover during local failures when needed.

BACKUP OF CRITICAL IT SYSTEMS

The easiest way to back up a server is the solutions proposed in [10, 11]. This is a full server backup. It can be performed weekly, monthly, or quarterly and performed using compression techniques. These solutions are usually designed to help back up server data to another on-premises server, cloud server, or hybrid system. In particular, backups to hybrid systems are becoming increasingly popular. This is because such systems can optimize resources, support simple redundancy across multiple regions, and can provide faster recovery and failover.

Typically, server backup solutions should include the following functions.

Support for various file types. All kinds of files must be supported. In particular, solutions must support documents, spreadsheets, media files, and configuration files.

Backup location. It should be possible to specify the backup locations. The solution should support backups to multiple locations and media, including internal and external resources.

Scheduling and automation. In addition to be able to perform manual backups, solutions must support the automation of backups through scheduling. This ensures that the user always has the most recent backup and that the backups are created in a consistent manner.

Backup management. You need to be able to manage the lifecycle of your backups, including the number and length of storage. Ideally, solutions also make it easy to export backups for transfer to external resources.

Section selection. Partitions are isolated segments of a storage resource and are often used to separate data on a system. Solutions should allow users to self-back up and recover partitions.

Compression of data. To minimize the amount of storage required for multiple backups, solutions must compress the backup data. This compression must be lossless and maintain the integrity of all data.

Selecting the type of backup. It should be possible to create various types of backups, including full, differential and incremental. Differential backups contain a copy of the changes since the last full backup, while incremental backups contain a record of the changes since the last incremental backup. This can help reduce the size of backups and speed up backup times.

Scaling. Backup capabilities should not be limited by the amount of data on users' servers. Solutions must be scaled as well as data and support backups of any size.

BACKUP TECHNOLOGY

Backup storage is the physical location or device for storing copies of data for recovery in the event of a disaster or data loss. Backup storage systems typically include both hardware and software for backup and recovery management. This includes everything from simple flash storage to hybrid local physical storage and remote cloud storage.

Whichever method is used for backup, in the end, the data must be stored somewhere [12–14]. The technology used to store the backup data is very important. The more economical the technology, the more data can be stored and the faster that data will be retrieved. The more reliable the storage technology, the more reliable will be the safety of backups.

Network resources and NAS. You can configure centralized storage such as Network Attached Storage (NAS), Storage Area Network (SAN), or regular hard drives connected as a network share using the Network File System (NFS) protocol. This is a convenient option for providing local devices with a large amount of backup storage. However, it is susceptible to threats such as fire, flooding and other threats of destruction affecting the entire data center, as well as cyber attacks.

Cloud storage of objects. When using cloud providers, there is access to a variety of storage services. There are tools that allow you to automatically create backups of data, both from the "cloud" and from local machines. The most popular are:

AMANDA (Advanced Maryland Automatic Network Disk Archiver) is a system for backing up and archiving information.

Bacula is a client/server software that allows you to manage backup, recover and validate data over the network for computers and operating systems of various types.

Duplicity is backs up encrypted volumes in tar-format locally or to a remote host.

CONCLUSIONS

This article discusses a data backup architecture for mission-critical enterprise information systems. The authors believe that there should be at least three backups, two of which are located in the "cloud." The 3–2–1 strategy developed by the authors gives quite satisfactory results for the safety of critical data. It protects against both accidents and malicious threats such as ransomware and provides reliable data backup and recovery.

Authors' contribution. All authors equally contributed to the research work.

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RESEARCH ARTICLE

The operational readiness factor of satellite communication networks

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Abstract
Objectives. The most important distinguishing feature of satellite communication networks (SCNs) is topology, which consolidates the scheme for combining nodes and communication channels into a single structure and largely determines the main characteristics of communication systems. The following topologies are used in SCNs: fully connected, tree-like, ring-shaped, and radial (“star” type). The topology can be changed depending on the tasks being solved; for example, to ensure high reliability rates. The most frequently used indicator characterizing the reliability of communication networks is the readiness factor. Considering the SCN as a complex recoverable system, it is advisable to analyze the operational readiness factor along with the readiness factor. This paper investigates the influence of the network topology on the reliability of the SCN.
Methods. Queuing theory was used to analyze the flow of events, that is, the flow of failures and recoveries.
Results. Assuming that the exponential Mean Time Between Failures (MTBF) model can be used for a central node with a radial network topology, the time dependences of the operational readiness factor were obtained. The reliability of networks with ring and radial topology was compared in terms of the operational readiness factor.
Conclusions. To achieve a higher reliability, it is necessary to use an SCN with a radial structure. For example, on a time interval of 12000 h, the operational readiness factor of a two-node SCN with a radial structure is 0.9, and for an SCN with a ring topology with the number of nodes 2, 3, 4, it is 0.7, 0.59, and 0.5, respectively. The study also showed that radial topology is more efficient even with less reliable nodes, that is, with higher failure rates. The advantage of a radial network topology increases as the number of nodes increases. However, in an SCN with a radial topology, failure of the central unit leads to complete degradation of the entire system.

Keywords: reliability, satellite communication, communication network, network topology, operational readiness factor

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НАУЧНАЯ СТАТЬЯ

Коэффициент оперативной готовности спутниковых сетей связи

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Резюме
Цели. Важнейшим отличительным признаком спутниковых сетей связи (ССС) является топология, характеризующая схему объединения узлов и каналов связи в единую структуру и во многом определяющая основные характеристики систем связи. В ССС используются следующие топологии: полносвязная, древовидная, кольцевая и радиальная (типа «звезда»). Топологию можно изменять в зависимости от решаемых задач, например, для обеспечения высоких показателей надежности. Общим, наиболее часто применяемым показателем, характеризующим надежность сетей связи, является коэффициент готовности. Рассматривая ССС как сложную восстанавливаемую систему, целесообразно наряду с коэффициентом готовности анализировать коэффициент оперативной готовности. В работе исследуется влияние топологии сети на надежность ССС.
Методы. Используется теория массового обслуживания для анализа потока событий, то есть потока отказов и восстановлений.
Результаты. Полагая, что для центрального узла при радиальной топологии сети можно использовать экспоненциальную модель наработки до отказа, получены зависимости коэффициента оперативной готовности от времени. Выполнено сравнение надежности сетей с кольцевой и радиальной топологией по коэффициенту оперативной готовности.
Выводы. Показано, что для достижения более высокой надежности необходимо использовать ССС с радиальной структурой. Например, на интервале времени 12000 часов коэффициент оперативной готовности двухузловой ССС с радиальной структурой равен 0.9, а для ССС с кольцевой топологией при количестве узлов 2, 3, 4 соответственно равен 0.7, 0.59, 0.5. Исследование также показало, что радиальная топология эффективнее даже при менее надежных узлах, то есть при увеличении интенсивности отказов. Преимущество радиальной топологии сети возрастает по мере увеличения числа узлов. Однако в ССС с радиальной топологией выход из строя центрального узла приводит к полной деградации всей системы.

Ключевые слова: надежность, спутниковая связь, сеть связи, топология сети, коэффициент оперативной готовности

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Авторы заявляют об отсутствии конфликта интересов.

INTRODUCTION

It is extremely difficult to increase the reliability of satellite communication networks (SCNs) during operation. This is due to the fact that expected reliability is mainly based on the design of networks and the manufacture of equipment. During operation, reliability only decreases [1, 2]. The rate of decrease in reliability depends on the operating methods, the qualifications of the maintenance personnel, and the operating conditions.

There are effective methods to improve the reliability of communication networks:

- redundancy [3, 4],
- simplification of the system,
- use of the most reliable elements,
- standardization and unification of elements and assemblies,
- built-in checks,
- automation of checks.

The effectiveness of these methods lies in that they allow building reliable systems from unreliable elements. These techniques can reduce system failure rates, decrease mean time to recovery, and increase the system's duration of continuous operation. The issues of assessing the reliability of communication networks and analyzing the effectiveness of methods for increasing reliability are relevant for INFOCOM communication networks [5–9].

The general indicator characterizing the reliability of any equipment (including communication networks) is the readiness factor—the limit of the instantaneous readiness factor when the considered moment of time tends to infinity. The instantaneous readiness factor (readiness function) is the probability that an object will be in a working state at a given time[1]. The readiness factor is one of the main reliability indicators of the communication network [10]. When formulating the requirements for the reliability indicators of communication networks, the readiness factor is usually standardized [11].

Considering the SCN as a complex recoverable system, it is advisable, along with the readiness factor, to analyze the operational readiness factor—the probability that the object will be in a working state at an arbitrary point in time, except for the scheduled periods during which the use of the object for its intended purpose is not provided, and, starting from this moment in time, will work flawlessly for a specified time interval[1].

The operational readiness factor K_{or} is determined by the formula [1]:

$$K_{or}(t) = K_r P(t), \quad (1)$$

where K_r is the readiness factor; $P(t)$ is the probability of failure-free operation of the system at time t, provided that the system is ready for operation at an arbitrary moment in time.

The most important distinguishing feature of a communication network is the network topology[2], the properties of which can be changed during the operation depending on the tasks being solved, for example, ensuring high reliability indicators [12]. The following network topologies are used in the SCN [13].

Fully coupled topology provides the highest reliability of the SCN functioning, since in case of a dedicated communication channel failure, information can be transmitted along bypass paths through intermediate nodes. However, the implementation of such technology requires the organization of direct communication among all nodes, which leads to an increase in the cost of SCN.

The tree topology is characterized by low reliability, since the failure of even one of the communication channels can lead to separation of the network into two isolated subnets. Therefore, the most commonly used topologies are the ring and radial topologies.

The ring topology (Fig. 1a), like the tree topology, has low reliability, since the failure of any one channel leads to a network failure. However, redundancy can improve the reliability of a ring topology by using multiple rings.

The radial topology ("star" type) (Fig. 1b) has a fairly high reliability compared to other topologies, since the peripheral nodes operate independently of each other. The failure of the central node leads to the failure of the entire network. Therefore, it is necessary to use the redundancy of this node.

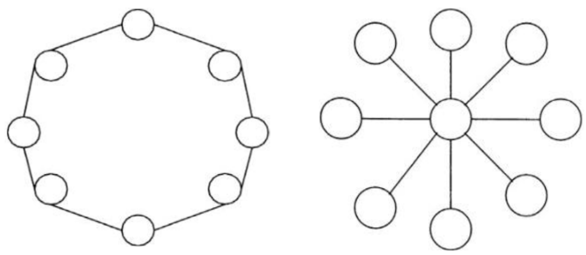

Fig. 1. Main SCN topologies: (a) ring, (b) radial ("star")

If a node or communication channel fails, the system should automatically recover in a set (usually very short) time. In this case, one of the backup routes of data delivery from the sender to the recipient should be used. Redundancy of communication channels allows you to minimize delays in data transmission, while at the same time significantly increase the values of the system's MTBF parameters.

Let us compare the reliability of the ring and radial topologies in terms of the operational readiness factor.

DETERMINATION OF THE OPERATIONAL READINESS FACTOR FOR THE "STAR" STRUCTURE

An SCN with radial topology is operational if at least one peripheral node is working while the central node is working. If all nodes or the central node fail, the network is inoperative.

As an example of a star structure SCN, consider a network consisting of a central node and 2 peripheral nodes. Then the operational readiness factor is determined according to the basic structural diagram of reliability by the following expression [1]:

[1] GOST 27.002–2015. *Nadezhnost' v tekhnike. Terminy i opredeleniya* (*Dependability in technics. Terms and definitions*). Moscow: Standartinform; 2016. 30 p. (in Russ.).

[2] Gol'dshtein B.S., Sokolov N.A., Yanovskii G.G. *Communication networks:* textbook for universities. St. Petersburg: BHV-Petersburg; 2010. 400 p. (in Russ.). ISBN: 978-5-9775-0474-4

$$K_{\text{or star}}(t) = \frac{2T_{\text{f}}}{T_{\text{r}} + 2T_{\text{f}}} P_{\text{cn}}(t)[2e^{-\lambda t} - e^{-2\lambda t}], \quad (2)$$

where $T_{\text{f}} = \frac{1}{\lambda}$ is the mean time between failures of the peripheral unit; $T_{\text{r}} = \frac{1}{\mu}$ is the average recovery time of the peripheral node; $P_{\text{cn}}(t)$ is the probability of failure-free operation of the central node; λ is the rate of failure of the peripheral node; μ is the rate of recovery of the peripheral node.

Assuming that an exponential model of MTBF can be used for the central node, we will transform expression (2) to the following form:

$$K_{\text{or star}}(t) = \frac{2T_{\text{f}}}{T_{\text{r}} + 2T_{\text{f}}} e^{-\lambda_{\text{cn}} t}[2e^{-\lambda t} - e^{-2\lambda t}], \quad (3)$$

where λ_{cn} is the failure rate of the central node. Usually, the central node is more reliable than the peripheral nodes, so we set $\lambda_{\text{cn}} = \lambda/2$.

The dependencies of $K_{\text{star}}(t)$ for μ = 0.04 and different values of λ ($1.5 \cdot 10^{-5}$, $2.5 \cdot 10^{-5}$, and $5 \cdot 10^{-5}$) are shown in Fig. 2.

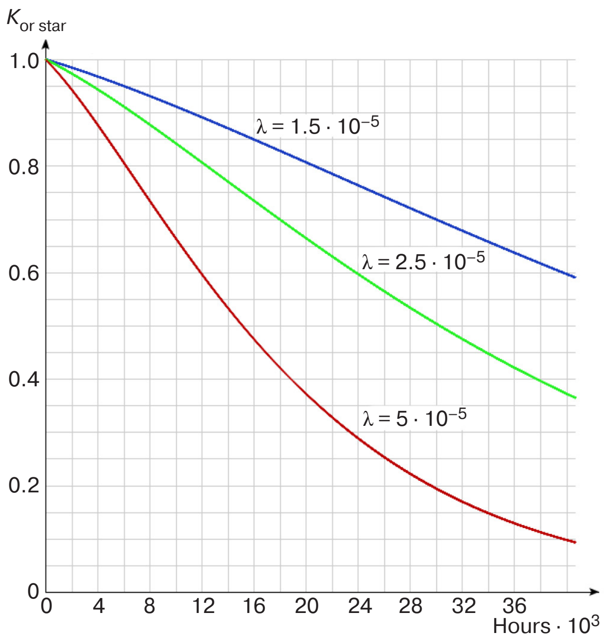

Fig. 2. Operational readiness factor of an SCN with radial topology

DETERMINATION OF THE OPERATIONAL READINESS FACTOR WITH A RING TOPOLOGY

An SCN with a ring topology is operational if all nodes are operational, which corresponds to a sequential reliability model. If one of the nodes fails, the network is inoperative.

The operational readiness factor of the SCN with the "ring" structure is determined by the following expression [1]:

$$K_{\text{or ring}}(t) = \frac{T_{\text{f}}}{T_{\text{r}} + T_{\text{f}}} e^{-n\lambda t}, \quad (4)$$

where n is the number of nodes.

The dependencies $K_{\text{or ring}}(t)$ for $\lambda = 5 \cdot 10^{-5}$, $\mu = 0.04$ and the number of nodes n = 2, 3, 4 are shown in Fig. 3.

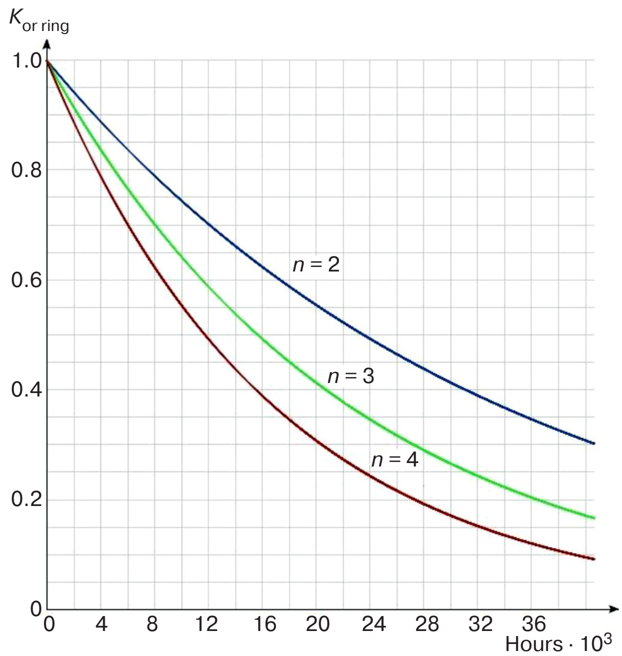

Fig. 3. Operational readiness factor of an SCN with ring topology

Figures 2 and 3 show that to achieve higher reliability, it is necessary to use an SCN with a "star" topology. For example, at t = 12000 h, the operational readiness factor of a network with a radial topology is 0.9, and for an SCN with a ring topology at n = 2, 3, and 4, it is 0.7, 0.59, and 0.5, respectively.

CONCLUSIONS

The operational readiness factor determines the probability that the network is in operational state during the given time interval starting from a certain moment in time. The comparison of the ring and radial SCN topologies made in terms of the operational readiness factor showed that the radial topology, even with less reliable nodes, features for any time interval higher reliability than that of the ring topology. The advantage of a radial network topology increases as the number of nodes increases.

Authors' contribution. All authors equally contributed to the research work.

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RESEARCH ARTICLE

Optimal nonlinear filtering of MPSK signals in the presence of a Doppler frequency shift

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Abstract
Objectives. Phase-shift keyed (PSK) signals are widely used in many telecommunication, communication, and cellular information transmission systems. Phase-shift keying provides a higher noise immunity than amplitude and frequency modulations do. An increase in the modulation order of such a signal leads not only to an increase in its spectral efficiency, but also to a certain decrease in the noise immunity of reception. To ensure a high noise immunity of reception of multiple phase-shift keyed (MPSK) signals, a demodulator should provide the coherence of the reference oscillation with the carrier. Ignorance of the frequency and phase of the received signal leads to significant energy losses. The purpose of this work was to synthesize and analyze algorithms for receiving MPSK signals with phase fluctuations caused by changes in the carrier frequency due to the Doppler effect against the background of white Gaussian noise.
Methods. The problem was solved using the apparatus of optimal nonlinear filtering and methods of statistical radio engineering.
Results. A demodulator was synthesized, which includes two interconnected units. One of them is a discrete symbol estimation unit, at the output of which a decision on the received symbol is issued, and the other is a phase-lock circuit. Analytical expressions were derived to estimate the characteristics of the receiver noise immunity as functions of the signal-to-noise ratio and fluctuation parameters. It was shown that the synthesized quasi-coherent algorithm compensates well for the MPSK signal phase fluctuations caused by the instability of the master oscillator and the Doppler effect.
Conclusions. Comparison of the results of this work with results obtained in the case of the absence of fluctuations in the initial phase showed that, at a high relative speed of the transmitter and the receiver (satellite radio channel), the energy loss is no more than 1 dB, and at lower speeds of the objects, it is about 0.2 dB and less.

Keywords: multiple phase-shift keying, Doppler effect, optimal nonlinear filtering, noise immunity, bit error probability

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НАУЧНАЯ СТАТЬЯ

Оптимальная нелинейная фильтрация сигналов М-ФМ при наличии доплеровского смещения частоты

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Резюме

Цели. Сигналы с фазовой манипуляцией (ФМ) широко применяются во многих телекоммуникационных, связных и сотовых системах передачи информации. По сравнению с амплитудной и частотной манипуляцией применение ФМ обеспечивает более высокую помехоустойчивость. Увеличение позиционности такого сигнала приводит к повышению его спектральной эффективности, но в то же время к некоторому снижению помехоустойчивости приема. Для обеспечения высокой помехоустойчивости при приеме многопозиционных сигналов (М-ФМ) в демодуляторе требуется обеспечить когерентность опорного колебания с несущей. Незнание частоты и фазы принимаемого сигнала приводит к существенным энергетическим потерям. Цель работы – синтез и анализ алгоритмов приема сигналов М-ФМ с флуктуациями фазы, вызванными изменениями его несущей частоты, связанными с эффектом Доплера, на фоне белого гауссовского шума.

Методы. Задача решается с применением аппарата оптимальной нелинейной фильтрации и методов статистической радиотехники.

Результаты. Синтезирован демодулятор, включающий в себя взаимосвязанные блок оценки дискретного символа, на выходе которого выдается решение о принимаемом символе, и блок фазовой автоподстройки частоты. Получены аналитические выражения, позволяющие оценить характеристики помехоустойчивости приемника в зависимости от отношения сигнал/шум и параметров флуктуаций. Показано, что синтезированный квазикогерентный алгоритм хорошо компенсирует флуктуации фазы сигнала М-ФМ, вызванные нестабильностью задающего генератора и доплеровским эффектом.

Выводы. Сравнение полученных результатов с результатами в случае отсутствия флуктуации начальной фазы, показывает, что энергетический проигрыш при большой относительной скорости движения передатчика и приемника (спутниковый радиоканал) составляет не более 1 дБ, при меньших скоростях движения объектов – около 0.2 дБ и менее.

Ключевые слова: многопозиционная фазовая манипуляция, эффект Доплера, оптимальная нелинейная фильтрация, помехоустойчивость, вероятность битовой ошибки

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INTRODUCTION

One of the most known classes of complex signals used widely in digital information transmission systems are multiple phase-shift keyed (MPSK) signals [1, 2]. In so modulated signals, the phase of the carrier changes stepwise, depending on the transmitted signal. The maximum gain in noise immunity in reception of such signals is reached by using a coherent receiver, in which the reference oscillation is frequency- and phase-locked with the carrier frequency oscillation [3]. The absence of information on the phase of the received signal inevitably leads to losses of noise immunity [4–9]. This can be caused by the instability of the frequency of the basic oscillator, the Doppler shift of the signal carrier frequency, and others. In these cases, the problem of signal reception is solved using the apparatus of optimal nonlinear filtering [10–16].

The purpose of this work was to develop an algorithm of reception of MPSK signals based on optimal nonlinear filtering theory in the presence of a Doppler frequency shift and to estimate the noise immunity of the synthesized quasi-coherent receiver.

SYNTHESIS OF AN ALGORITHM OF THE OPTIMAL NONLINEAR FILTERING OF MPSK SIGNALS

Let the received signal be the sum of two components:

$$x(t) = s_{\sum}(\mathbf{C}_k, t, \varphi) + n(t);\ t \in (0, kT];$$
$$\mathbf{C}_k = \{C_1, \ldots, C_k\}, \quad (1)$$

where $s_{\sum}(\mathbf{C}_k, t, \varphi)$ is an MPSK signal with random initial phase φ that, in the kth digit-time slot, has the form

$$s_k(C_k = i, t, \varphi) = A_0 \cos(\omega_0 t + \varphi_i + \varphi),$$
$$\varphi_i = \frac{i2\pi}{M},\ t \in \left((k-1)T, kT\right],\ i = \overline{0, M-1}; \quad (2)$$

and $n(t)$ is a noise interference with parameters

$$\langle n(t)\rangle = 0; \langle n(t_1)n(t_2)\rangle = \frac{N_0}{2}\delta(t_2 - t_1). \quad (3)$$

We assume that the random initial phase φ of the signal is determined by the instability $\varphi_{mo}(t)$ of the basic oscillator and the Doppler shift of the signal carrier frequency [10]:

$$\varphi(t) = \int_0^t (\omega(\tau) - \omega_0)d\tau + \varphi_{mo}(t).$$

The processes ω(t) and φ(t) can be described by the system of the stochastic differential equations [11]

$$\left.\begin{aligned} \dot{\varphi} &= \omega - \omega_0 + n_{\varphi_{mo}}(t), \\ \dot{\omega} &= -\alpha_\omega(\omega - \omega_0) + n_\omega(t), \end{aligned}\right\}, \quad (4)$$

where α_ω is the Doppler frequency fluctuation spectrum band; and $n_{\varphi_{mo}}(t)$ and $n_\omega(t)$ are mutually independent processes with delta-function correlation, zero means, and single-sided spectral densities $N_{\varphi_{mo}}$ and N_ω, respectively.

The mixed *a posteriori* probability density of $s_{\sum}(\mathbf{C}_k, t, \varphi)$ is

$$p_{ps}(t, \mathbf{C}_k, \varphi) = w_{ps}(t, \varphi)\, p_{ps}(t, \mathbf{C}_k|\varphi)$$

where $w_{ps}(t, \varphi)$ is the *a posteriori* probability density of the parameter φ, and $p_{ps}(t, \mathbf{C}_k|\varphi)$ is the conditional (on φ) *a posteriori* probability of the vector $\mathbf{C}_k$.

The conditional *a posteriori* probability of $p_{ps}(t, \mathbf{C}_k|\varphi)$ is found from the equation [12]

$$\dot{p}_{ps}(t, \mathbf{C}_k|\varphi) =$$
$$= p_{ps}(t, \mathbf{C}_k|\varphi)\left\{F(t, \mathbf{C}_k, \varphi) - \langle F(t, \varphi)\rangle\right\}, \quad (5)$$

where

$$F(t, \mathbf{C}_k, \varphi) = \sum_{j=1}^{k} F_j(t, \mathbf{C}_j, \varphi),$$
$$F_j(t, \mathbf{C}_j, \varphi) = \frac{2}{N_0}x(t)s_{\Sigma j}(\mathbf{C}_j, t, \varphi), \quad (6)$$
$$\langle F(t, \varphi)\rangle = \sum_{C_1=0}^{M-1}\sum_{C_2=0}^{M-1}\ldots\sum_{C_k=0}^{M-1} F(t, \mathbf{C}_k, \varphi)p_{ps}(t, \mathbf{C}_k|\varphi).$$

We assume that the *a priori* probabilities of the values of the channel symbols for MPSK are equal and are $1/M$. If it is also taken into account that the probabilities of the transition of a symbol from one state to another are also equal, then the solution of Eq. (5) at time $t = kT$ can be represented as

$$p_{ps}(kT, \mathbf{C}_k|\varphi) =$$
$$= \frac{\exp[\sum_{j=1}^{k}\int_{(j-1)T}^{jT} F_j(t, C_j, \mathbf{C}_{j-1}, \varphi)dt]}{\sum_{\mathbf{C}_k}\exp[\sum_{j=1}^{k}\int_{(j-1)T}^{jT} F_j(t, C_j, \mathbf{C}_{j-1}, \varphi)dt]}. \quad (7)$$

The *a posteriori* probability of the value of the symbol C_k is determined by averaging relation (7) over M possible values $C_1, C_2, \ldots, C_{k-1}$ under the assumption of a good quality of the signal reception. As a result, the symbols received before the time $t = (k-1)T$ are replaced by their estimated values, and the terms independent of C_k can be combined into coefficient K. This coefficient will contain all the time history, which allows one to simplify relation (7):

$$p_{ps}(T, C_k|\varphi) =$$
$$= K\frac{\exp[\int_0^T F_k(t, C_k, \varphi)dt]}{\sum_{C_k=0}^{M-1}\exp[\int_0^T F_k(t, C_k, \varphi)dt]}.$$

A channel symbol estimation rule is found from the condition of maximum of the *a posteriori* probability at time $t = T$:

$$(C_k = i) \Rightarrow \max\{p_{ps}(T, C_k \mid \varphi)\}. \quad (8)$$

Let us introduce the notation

$$J_0 = \int_0^T F_k(t, C_k = 0, \varphi)dt,$$

$$\ldots \qquad (9)$$

$$J_{M-1} = \int_0^T F_k(t, C_k = M-1, \varphi)dt,$$

where the integrands are determined from expressions (6).

Rule (8) can be written differently, using the monotonicity of the exponential function:

$$(C_k = i) \Rightarrow \max(J_i). \qquad (10)$$

Transformation of integrals (9) using expressions (1)–(3) gives

$$J_0 = \frac{2}{N_0}\int_0^T x(t)s_k(t, C_k = 0, \varphi)dt;$$

$$\ldots \qquad (11)$$

$$J_{M-1} = \frac{2}{N_0}\int_0^T x(t)s_k(t, C_k = M-1, \varphi)dt.$$

The structure of channel symbol estimation of a receiver that implements algorithm (10) is classical for MPSK signals and is characteristic of reception against the background of white Gaussian noise [1, 2]. This receiver is a multichannel correlator, which determines the degree of similarity between the received process $x(t)$ and the reference signals corresponding to all the possible values of the channel symbol C_k.

Equations of the optimal nonlinear filtering of the random quantities φ and ω are derived by the Gaussian approximation of their *a posteriori* probability densities [13, 14]. Equations for the expected values (marked with asterisks *) in the steady state form the system

$$\left.\begin{aligned} \dot{\varphi}^* &= \omega^* - \omega_0 + \overline{K_{\varphi\varphi}} < F_k >_{\varphi}, \\ \dot{\omega}^* &= -\alpha_{\omega}(\omega^* - \omega_0) + \overline{K_{\omega\varphi}} < F_k >_{\varphi}, \end{aligned}\right\} \qquad (12)$$

and the *a posteriori* variances of the approximating distributions can be found from the relations

$$\left.\begin{aligned} &\frac{1}{2}N_{\varphi_{mo}} + 2\overline{K_{\varphi\omega}} + \left(\overline{K_{\varphi\varphi}}\right)^2 \cdot \overline{< F_k >_{\varphi\varphi}} = 0, \\ &\frac{1}{2}N_{\omega} + \left(\overline{K_{\varphi\omega}}\right)^2 \cdot \overline{< F_k >_{\varphi\varphi}} - 2\alpha_{\omega} \cdot \overline{K_{\omega\omega}} = 0, \\ &\overline{K_{\omega\omega}} - \alpha_{\omega} \cdot \overline{K_{\varphi\omega}} + \overline{K_{\varphi\varphi}} \cdot \overline{K_{\varphi\omega}} \cdot \overline{< F_k >_{\varphi\varphi}} = 0, \end{aligned}\right\} \qquad (13)$$

where $\left\langle F_k \right\rangle_{\varphi} = \frac{\partial \left\langle F_k(t, \varphi^*) \right\rangle}{\partial \varphi^*}$, $\left\langle F_k \right\rangle_{\varphi\varphi} = \frac{\partial^2 \left\langle F_k(t, \varphi^*) \right\rangle}{\partial \varphi^{*2}}$, and

$$< F_k(t, \varphi^*) > = F_k(t, C_k = 0, \varphi)\frac{\exp J_0}{\sum_{i=0}^{M-1}\exp J_i} + $$

$$+ F_k(t, C_k = 1, \varphi)\frac{\exp J_1}{\sum_{i=0}^{M-1}\exp J_i} + F_k(t, C_k = 2, \varphi)\frac{\exp J_2}{\sum_{i=0}^{M-1}\exp J_i} + $$

$$+ \ldots + F_k(t, C_k = M-1, \varphi)\frac{\exp J_{M-1}}{\sum_{i=0}^{M-1}\exp J_i}.$$

In view of expressions (6),

$$< F_k >_{\varphi} = \frac{2A_0}{N_0}x(t)\sum_{i=0}^{M-1} s_k^h(t, C_k = i, \varphi^*)\frac{\exp J_i}{\sum_{l=0}^{M-1}\exp J_l},$$

here,

$$s_k^h(t, C_k = i, \varphi^*) = \frac{ds_k(t, C_k = i, \varphi^*)}{d\varphi^*} =$$

$$= -A_0\sin(\omega_0 t + \varphi_i + \varphi^*).$$

At a high signal-to-noise ratio, the latter expressions can be simplified. If the parity and symmetry of the constellation diagram of the MPSK signal are also taken into account, then,

$$< F_k >_{\varphi} \approx \frac{2A_0}{N_0}x(t)\sum_{i=0}^{M/2-1} s_k^h(t, C_k = i, \varphi^*)\,\mathrm{th}\, J_i.$$

Making statistical averaging and assuming that, in the steady-state mode, at small filtering error,

$$\cos(\varphi - \varphi^*) \approx 1,$$

we obtain

$$\overline{< F_k >_{\varphi\varphi}} \approx -\frac{A_0{}^2}{2N_0}\left(1 + \mathrm{th}\frac{A_0{}^2 T}{2N_0}\right). \qquad (14)$$

Solving the system of Eqs. (13) gives [11]

$$\left.\begin{aligned} \overline{K_{\varphi\varphi}} &= -\frac{\alpha_{\omega}\left(\sqrt{1+2G+L}-1\right)}{\overline{< F_k >_{\varphi\varphi}}}, \\ \overline{K_{\varphi\omega}} &= -\frac{\alpha_{\omega}{}^2\left(1+G-\sqrt{1+2G+L}\right)}{\overline{< F_k >_{\varphi\varphi}}}, \\ \overline{K_{\omega\omega}} &= -\frac{\alpha_{\omega}{}^3\sqrt{1+2G+L}\left(1+G-\sqrt{1+2G+L}\right)}{\overline{< F_k >_{\varphi\varphi}}}, \end{aligned}\right\} \qquad (15)$$

where

$$L = -\frac{N_{\varphi_{mo}}}{2\alpha_\omega^2}\overline{<F_k>_{\varphi\varphi}} = \frac{A_0^2 N_{\varphi_{mo}}}{4N_0\alpha_\omega^2}\left(1+\operatorname{th}\frac{A_0^2 T}{2N_0}\right),$$

$$G = \sqrt{-\frac{N_\omega + \alpha_\omega^2 N_{\varphi_{mo}}}{2\alpha_\omega^4}\overline{<F_k>_{\varphi\varphi}}} = \sqrt{\frac{A_0^2(N_\omega + \alpha_\omega^2 N_{\varphi_{mo}})}{4N_0\alpha_\omega^4}\left(1+\operatorname{th}\frac{A_0^2 T}{2N_0}\right)}.$$

Thus, algorithms (12) of filtrating of the random phase of the signal in the presence of a Doppler frequency shift have the form

$$\overline{\varphi}^* = SA_0\left(K_1 + \frac{K_2}{1+T_\omega D}\right)x(t)\times \times \sum_{i=0}^{M/2-1} s_k^h(t, C_k = i, \varphi^*)\operatorname{th} J_i, \quad (16)$$

where S is the transconductance of the control element (CE) in the phase-lock channel, $K_1 = \frac{2\overline{K_{\varphi\varphi}}}{N_0 S}$, $K_2 = \frac{2\overline{K_{\varphi\omega}}}{N_0 S\alpha_\omega}$, $T_\omega = \frac{1}{\alpha_\omega}$, and $D = \frac{d}{dt}$.

Figure 1 presents the circuit diagram of the receiver in which interrelated algorithms (10), (11), and (16) are implemented. The synthesized receiver contains two main units: a channel symbol estimator, a phase-lock unit of reference oscillator O, and cross couplings between them. The phase-lock unit includes a proportionally integrating filter with transfer function $K_1 + \frac{K_2}{1+T_\omega D}$. The linear branch of the filter tracks phase fluctuations. The second (integrating) component of the filter tracks frequency fluctuations. The integration time constant should be chosen depending on the average rate of frequency change due to the Doppler effect.

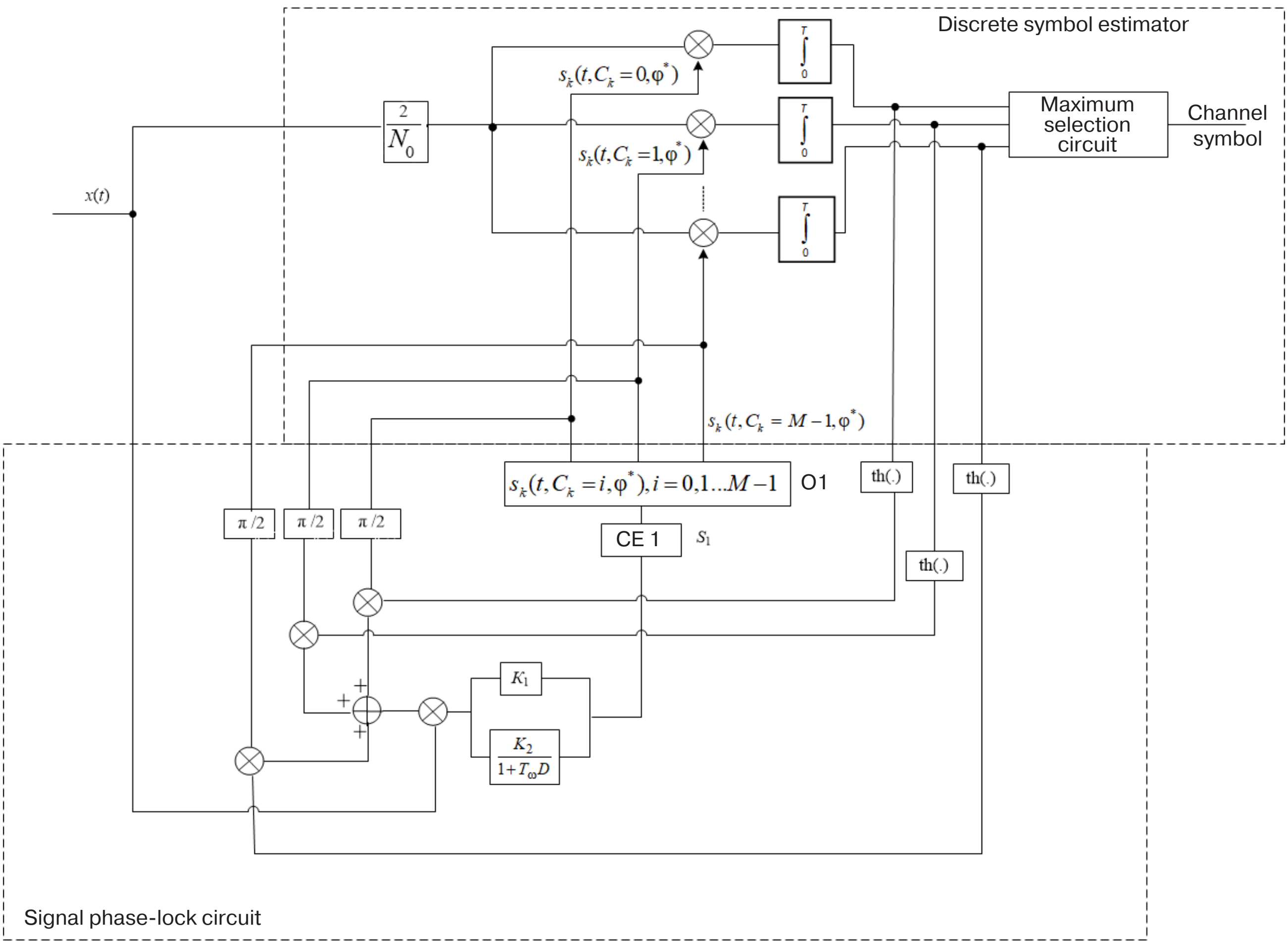

Fig. 1. Circuit diagram of a quasi-coherent receiver of an MPSK signal in a presence of a Doppler frequency shift

ANALYSIS OF THE NOISE IMMUNITY OF THE SYNTHESIZED RECEIVER

Let us analyze the noise immunity of the synthesized quasi-coherent receiver of MPSK signals using a published procedure [7]. The symbol and bit error probabilities P_{se} and P_{be}, respectively, which are conditional on the phases φ and φ* and the frequency ω, can be found by algorithm (10) using the symmetry of the constellation diagram (e.g., provided that the transmitted signal has a phase of $\varphi_i = 0$ (2)):

$$P_{se} = 1 - \prod_{i=1}^{M-1} p(J_0 - J_i > 0)\big|_0,$$

$$P_{be} = P_{se}/\log_2 M,$$

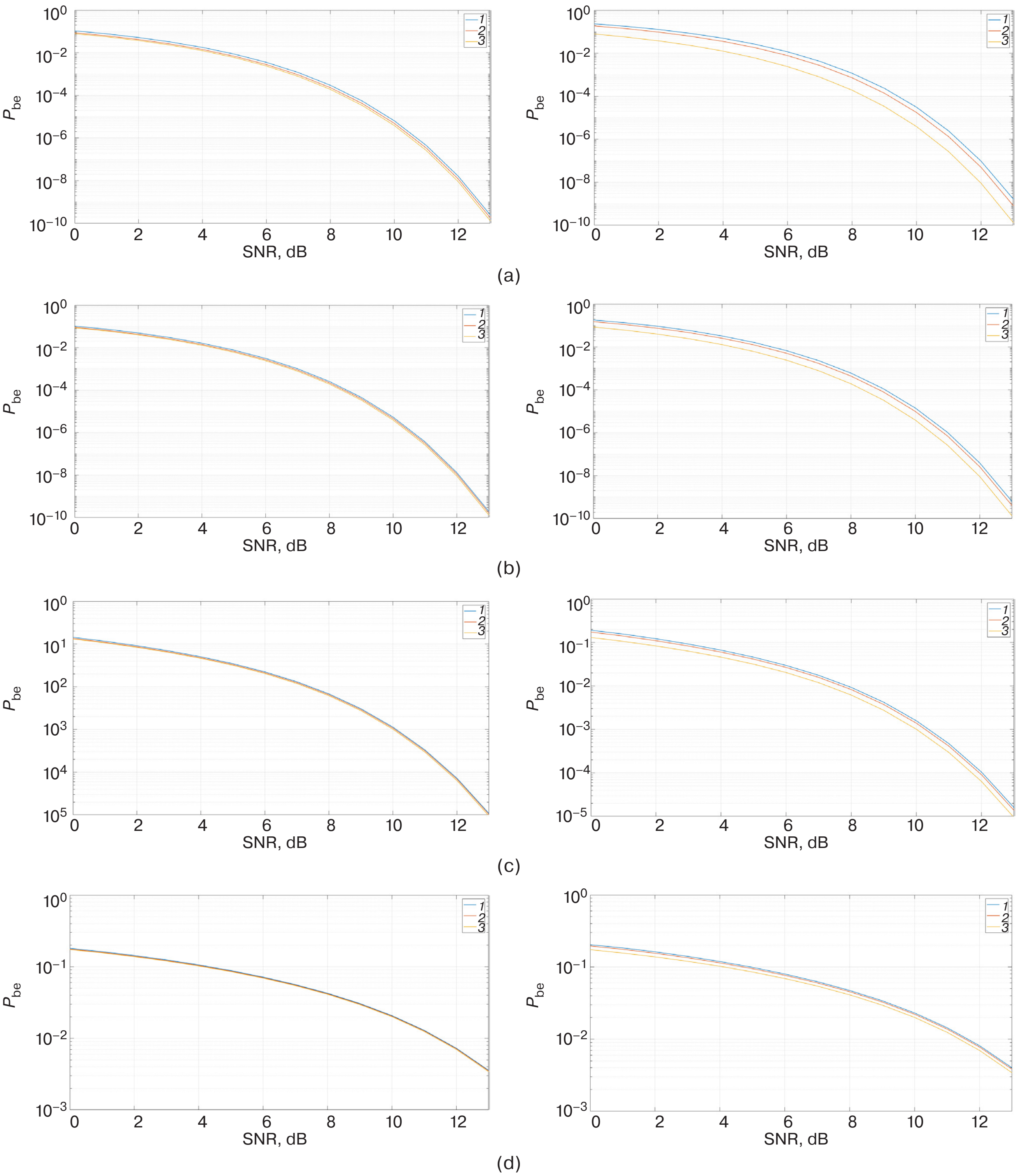

Fig. 2. Dependences of the bit error probability (P_{be}) on the signal-to-noise (SNR) ratio at M = (a) 2, (b) 4, (c) 8, and (d) 16

where

$$p(J_0 - J_i > 0)\big|_0 = 1 - \Phi\left(\frac{m_i}{\sqrt{D_i}}\right),$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \mathrm{e}^{-t^2/2} dt.$$

Using expressions (1)–(3) and (11), we obtain

$$m_i = \langle J_0 - J_i \rangle =$$

$$= \frac{2E_s}{N_0}[\cos(\varphi - \varphi^*)(1 - \cos(i2\pi/M)) -$$

$$- \sin(\varphi - \varphi^*) \sin(i2\pi/M)],$$

$$D_i = \frac{4E_s}{N_0}(1 - \cos(i2\pi/M)).$$

To obtain the unconditional probabilities, including the bit error probability, the corresponding expressions should be averaged under the assumption that the *a posteriori* probability density of the random phase φ is Gaussian:

$$w(\varphi) = \frac{1}{\sqrt{2\pi \overline{K_{\varphi\varphi}}}} \cdot \mathrm{e}^{-\frac{(\varphi - \varphi^*)^2}{2\overline{K_{\varphi\varphi}}}}.$$

In such an averaging, one can use the approximate formula [14]

$$< p(J_0 - J_i > 0) > \big|_{\text{on } \varphi} = 1 - \Phi\left(\frac{< m_i > \big|_{\text{on } \varphi}}{\sqrt{D_i}}\right),$$

where [17]

$$< m_i > = \frac{2E_s}{N_0}\left[\left(1 - \cos\frac{i2\pi}{M}\right) \cdot \exp\left(-\frac{\overline{K_{\varphi\varphi}}}{2}\right)\right].$$

The *a posteriori* phase variance $\overline{K_{\varphi\varphi}}$ is found from the first of expressions (15) and depends on both the phase variance $\sigma^2_{\varphi_{\text{mo}}} = \frac{N_{\varphi_{\text{mo}}} T}{2}$, and the frequency variance $\sigma^2_{\omega} = \frac{N_{\omega}}{4\alpha_{\omega}}$.

Figure 2 presents the curves constructed by the above formulas for the dependences of the bit error probability P_{be} on the signal-to-noise ratio E_{b}/N_0 for the quasi-coherent receiver of MPSK signals at various M in the presence of a Doppler frequency shift. The curves in the left panels of Fig. 2 were constructed at (*1*) $\sigma^2_{\varphi_{\text{mo}}} = 0.01$, $\alpha_{\omega}T = 0.1$, and $\sigma_{\omega}T = 0.25$; and (*2*) $\sigma^2_{\varphi_{\text{mo}}} = 0.01$, $\alpha_{\omega}T = 0.1$, and $\sigma_{\omega}T = 0.01$. Such parameter values are characteristic of aircraft radio communication (at a relative speed of objects to 3000 km/h). The curves in the right panels of Fig. 2 were constructed at (*1*) $\sigma^2_{\varphi_{\text{mo}}} = 0.01$, $\alpha_{\omega}T = 0.1$, and $\sigma_{\omega}T = 5$; and (*2*) $\sigma^2_{\varphi_{\text{mo}}} = 0.01$, $\alpha_{\omega}T = 0.1$, and $\sigma_{\omega}T = 2.5$. This is characteristic of satellite radio communication (at a relative speed of objects to 10 km/s). For comparison, Fig. 2 presents curves *3* constructed in the case of a deterministic signal.

The curves in Fig. 2 show that the synthesized quasi-coherent algorithm compensates well for the MPSK signal phase fluctuations caused by the instability of the basic oscillator and the Doppler effect. At a high relative speed of the transmitter and the receiver (satellite radio channel), the residual energy loss in comparison with the deterministic case is no more than 1 dB, and at lower speeds of the objects, it is about 0.2 dB and less.

CONCLUSIONS

In this work, an algorithm was synthesized for the optimal nonlinear filtering of MPSK signals in the presence of a Doppler frequency shift. The algorithm compensates well for the Doppler effect on the noise immunity of reception of discrete information at both low and high relative speeds of the transmitter and the receiver.

Authors' contribution. All authors equally contributed to the research work.

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RESEARCH ARTICLE

Simulation of subnanosecond radio pulse electro-optical repeater

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Abstract

Objectives. The study aimed to construct and analyze a computational model of an electro-optical repeater of radio pulses, capable of reconstructing a pulsed radio image with subnanosecond resolution in a single reception under conditions of additive noise.

Methods. Numerical methods of network analysis were used, which are the basis of specialized computer-aided design systems, numerical methods of statistical radio engineering.

Results. A radiophotonic radio pulse repeater scheme was constructed, which is implemented based on the principle of fractional multiplexing with delayed feedback. Software simulation of infrared repeaters in the *Simulink* environment (Trial Version Soft) was developed, which allows for analyzing and investigating the efficiency of the optical reconstruction method of radio pulses using a fractional multiplexing with delayed feedback. It is shown that repeaters schemes, implemented on the principles of fractional multiplexing with delayed feedback, are able to effectively solve the scientific and practical problems of multiple probing of objects with ultrashort pulses (USP) for obtaining a radio image of a target with reliable reproducibility. In the course of numerical simulations, it was found that the two- and four-cascade schemes of delay lines with feedback do not provide reliable reproducibility in the case of reconstruction of an ultrashort pulse with a complex time profile. At the same time, the scheme with a cascade of 8 delay lines showed good results, providing a correlation reproducibility of more than 0.9. In this case, the scheme of an electro-optical repeater with a cascade of 16 delay lines did not make a significant contribution to increasing the accuracy of the USP reconstruction if compared to the scheme with a cascade of 8 lines; therefore, the latter can be determined as an optimal solution. An electro-optical method was proposed for solving the radio engineering problem of stroboscopic registration and reconstruction of subnanosecond radio pulses, which represent the signature of the radio image of dynamic objects for active radio imaging systems.

Conclusions. It was found that an electro-optical repeater with 8 delay lines is able to recover a complex pulse reflected from a target in 30 iterations with a correlation coefficient greater than 0.9 between the reference and reconstructed pulses at a signal-to-noise ratio of at least 9 dB.

Keywords: strobe-frame sampler, ultrashort radio pulse, radio image, electro-optical repeater, feedback delayed communication with fractional multiplexing

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НАУЧНАЯ СТАТЬЯ

Моделирование электрооптического повторителя субнаносекундных радиоимпульсов

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Резюме
Цели. Целью данной работы является построение и анализ программно-численной модели электрооптического повторителя радиоимпульсов, обеспечивающего восстановление импульсного радиоизображения за однократный прием в условиях воздействия аддитивных шумов с достижением субнаносекундного разрешения.
Методы. Использованы численные методы схемотехнического анализа, лежащие в основе специализированных систем автоматизированного проектирования, численные методы статистической радиотехники.
Результаты. Построена радиофотонная схема повторителя радиоимпульсов, реализуемая на принципе дробного мультиплексирования с обратной задержанной связью. Разработаны программные модели инфраоптических повторителей в среде *Simulink* (Trial Version Soft), позволяющие анализировать и исследовать эффективность метода оптической регенерации радиоимпульсов при помощи схемы дробного мультиплексирования с задержанной обратной связью. Показано, что схемы повторителей, реализуемые на принципах дробного мультиплексирования с задержанной обратной связью, способны эффективно решить научно-практическую задачу многократного зондирования объектов сверхкороткими импульсами для получения радиоизображения цели с достоверной воспроизводимостью. В ходе численного моделирования установлено, что двух- и четырехкаскадные схемы линий обратной задержки не обеспечивают надежной воспроизводимости в случае восстановления сверхкороткого импульса (СКИ) со сложным временным профилем. В то же время схема с каскадом из 8 линий задержки справляется с поставленной задачей, обеспечивая корреляционную воспроизводимость более 0.9. При этом схема электрооптического повторителя с каскадом из 16 линий задержки не дает весомого вклада в повышение точности восстановления СКИ относительно схемы с каскадом из 8 линий, поэтому последнюю можно определить в качестве оптимального решения. Предложен электрооптический метод решения радиотехнической задачи стробоскопической регистрации и восстановления сверхкоротких радиоимпульсов субнаносекундной длительности, составляющих сигнатуру радиоизображения динамических объектов для систем активного радиовидения.
Выводы. Установлено, что электрооптический повторитель с 8 линиями задержки за 30 итераций способен восстановить сложный импульс, отраженный от цели с коэффициентом корреляции больше 0.9 между эталонным и восстановленным импульсом при отношении сигнал/шум не менее 9 дБ.

Ключевые слова. строб-фрейм дискретизатор, сверхкороткий радиоимпульс, радиоизображение, электрооптический повторитель, обратная задержанная связь с дробным мультиплексированием

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INTRODUCTION

The wide bandwidth of subnanosecond pulses makes it possible to extensively study the parameters of the irradiated media and surfaces of objects by pulse characteristics [1, 2]. Mastering the subnanosecond range opens up new possibilities in the study of radiophysical properties of materials in the area of laser optics and electrodynamics. The scientific interest in the radio wave features of the change in shape of ultrashort optical pulses in infrared range or subnanosecond radio pulses reflected from irradiated surfaces or transmitted through various media makes the search for high-speed methods of receiving ultrashort pulse (USP) signals relevant for their reconstruction [3] and further digital processing. Since a laser pulse can be easily converted into a radio-frequency one, the work is devoted to the development of new methods for the shape-time recuperation of USP in the problems of high-speed digitization of these pulses by adding an electro-optical repeater to such systems, which provides fractional cloning of USP.

The technology of strobe-frame-sampling (SFS) used in this work eliminates the disadvantages of analog reception, including high-frequency clocking of the USP signal for taking digital samples [4, 5].

THE STROBE-FRAME DISCRETIZER MODEL

The proposed SDF model was previously implemented in the *Simulink* software environment [6]. The model consists of a generator to produce pulses of a given shape, an array of elementary signal time delay lines, an array of counters, a comparator, and an output system for displaying the results.

The main disadvantage of this model is the need for stroboscopic reception to reconstruct the USP signal scattered by the signature of a dynamic object. In this case, the pulse is emitted as many times as the number of the comparator levels we need to reconstruct the USP signal [7].

An arbitrary 1 ns pulse (Fig. 1), consisting of the superposition of several bipolar Gaussian pulses (1) with different amplitudes, phases and time delays, was chosen as the probing USP signal.

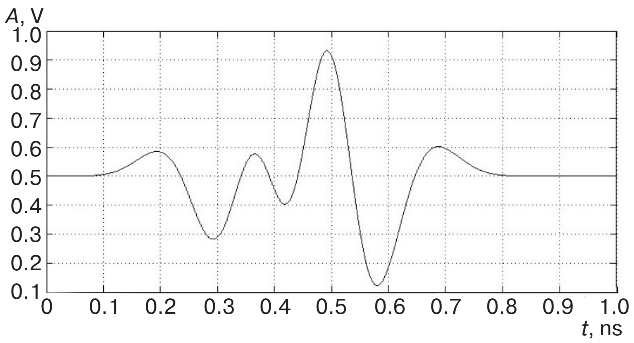

Fig. 1. Probing pulse; *A*, V, is the signal amplitude in volts, *t* is time in ns

$$A(t) = -0.33\alpha t e^{-\beta t^2} - 0.63\alpha(t-0.5\tau)e^{-\beta(t-0.5\tau)^2} + \\ + 0.33\alpha(t-0.25\tau)e^{-\beta(t-0.25\tau)^2} + \\ + 0.63\alpha(t-0.75\tau)e^{-\beta(t-0.75\tau)^2} + \\ + 0.25\alpha(t-\tau)e^{-\beta(t-\tau)^2} - \\ - 0.23\alpha(t-1.25\tau)e^{-\beta(t-1.25\tau)^2} - \\ - 0.13\alpha(t-1.5\tau)e^{-\beta(t-1.5\tau)^2} - \\ - 0.13\alpha(t-1.75\tau)e^{-\beta(t-1.75\tau)^2} + \\ + 0.33\alpha(t-2\tau)e^{-\beta(t-2\tau)^2} + \\ + 0.63\alpha(t-2.25\tau)e^{-\beta(t-2.25\tau)^2} - \\ - 0.33\alpha(t-2.5\tau)e^{-\beta(t-2.5\tau)^2} - \\ - 0.63\alpha(t-2.75\tau)e^{-\beta(t-2.75\tau)^2} - \\ - 0.25\alpha(t-3\tau)e^{-\beta(t-3\tau)^2} + \\ + 0.13\alpha(t-3.25\tau)e^{-\beta(t-3.25\tau)^2} + \\ + 0.13\alpha(t-3.75\tau)e^{-\beta(t-3.75\tau)^2} + \\ + 0.13\alpha(t-4\tau)e^{-\beta(t-4\tau)^2}, \quad (1)$$

where, $A(t)$ is the signal amplitude as a function of time, t; $\alpha = 5 \cdot 10^{10}$; $\beta = 10^{20}$; $\tau = 10^{-10}$; $e \approx 2.718$.

To evaluate the value of correlation coupling between the reference and reconstructed pulses, we use the Chaddock scale [8].

The correlation coefficient between the reference and the reconstructed pulses is calculated by the formula

$$r_{x,y} = \frac{\sum_{i-1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i-1}^{n}(x_i - \bar{x})^2 \sum_{i-1}^{n}(y_i - \bar{y})^2}}, \quad (2)$$

where $r_{x,y}$ is the correlation coefficient between the original and the reconstructed pulse, x_i is the *i*th value of the initial pulse, $\bar{x}$ is the average value of the sample of the original pulse, y_i is the *i*th value of the reconstructed pulse, and $\bar{y}$ is the average value of the sample of the reconstructed pulse.

THE MODEL OF ELECTRO-OPTICAL USP REPEATER WITH FRACTIONAL MULTIPLEXING AND DELAYED FEEDBACK

As one of the solutions of the radio engineering problem of registration and reconstruction of ultrashort radio pulses of subnanosecond duration, which represent the signature of the radio image of dynamic objects for active radio imaging systems, the authors propose a model of an electro-optical repeater (Fig. 2). This model combines the advantages of fractional multiplexing and delayed feedback schemes [9–11]. This makes it possible to more effectively solve the problem of spectral-temporal iterative degradation of the received USP signal due to the accumulation of amplifier noise as the number of reconstruction iterations increases [12, 13]. In addition, the proposed scheme architecture eliminates the main disadvantages of fractional multiplexing associated with the overall bulkiness of the scheme [8].

The model shown in Fig. 2, consists of an optical signal forming unit, an optical amplifier, a delayed feedback scheme, an optical beam divider, a cascade of optical delay lines, an optical multiplexer, and a photodetector.

The USP signal is delivered to the optical signal forming unit (OSFU), where it is amplified to the values required for its detection, and modulates the optical carrier. After that, the optical pulse is sent to the optical beam divider. Part of the pulse is split off the divider and sent to the cascade of delay lines, while the other part of the pulse is sent to the feedback loop. The chain of units consists of an amplifier that amplifies the input signal to the value required for digitizing the USP signal, and a delay line. The delay time is equal to the duration of the pulse reconstruction iteration in the scheme. In the cascade of delay lines, each copy of the pulse is delayed for a time from T_d to NT_d, where T_d is the delay time, which is equal to the duration of the frame; N is the number of delay lines. Further, all optical beams/pulses are combined by an optical multiplexer and sent to a photodiode, where the signal is demodulated and sent to a strobe-frame-discretizer.

The transfer functions $W(p)$ of the USP electro-optical repeater schemes with different delay lines cascades can be described by formulas (3.1–3.5).

$$W_{\text{wod}}(p)=\frac{1}{1-K_{\text{amp}}e^{-\tau_{\text{gld}}p}} \tag{3.1}$$

$$W_{\text{2d}}(p)=\frac{1+e^{-\tau_1 p}}{1-K_{\text{amp}}e^{-\tau_{\text{gld}}p}}, \tag{3.2}$$

$$W_{\text{4d}}(p)=\frac{1+e^{-\tau_1 p}+e^{-\tau_2 p}+e^{-\tau_3 p}}{1-K_{\text{amp}}e^{-\tau_{\text{gld}}p}}, \tag{3.3}$$

$$W_{\text{8d}}(p)=\frac{1+\sum_{i=1}^{7}e^{-\tau_i p}}{1-K_{\text{amp}}e^{-\tau_{\text{gld}}p}}, \tag{3.4}$$

$$W_{\text{16d}}(p)=\frac{1+\sum_{i=1}^{15}e^{-\tau_i p}}{1-K_{\text{amp}}e^{-\tau_{\text{gld}}p}}, \tag{3.5}$$

where $W(p)$ is the ratio of the Laplace image of the output signal to the Laplace image of the input signal at zero initial conditions; $W_{\text{wod}}(p)$ is the transfer function without delay lines; $W_{\text{2d}}(p)$, …, $W_{\text{16d}}(p)$ are the transfer functions with a particular number of delay lines; K_{amp} is the gain (for how many times the input signal was amplified); τ_i is the length of the ith delay line; τ_{gld} is the length of the global delay line.

When modeling this scheme, we will consider the frequency distortions introduced by the Mach–Zehnder modulator, optical amplifier, and photodetector to

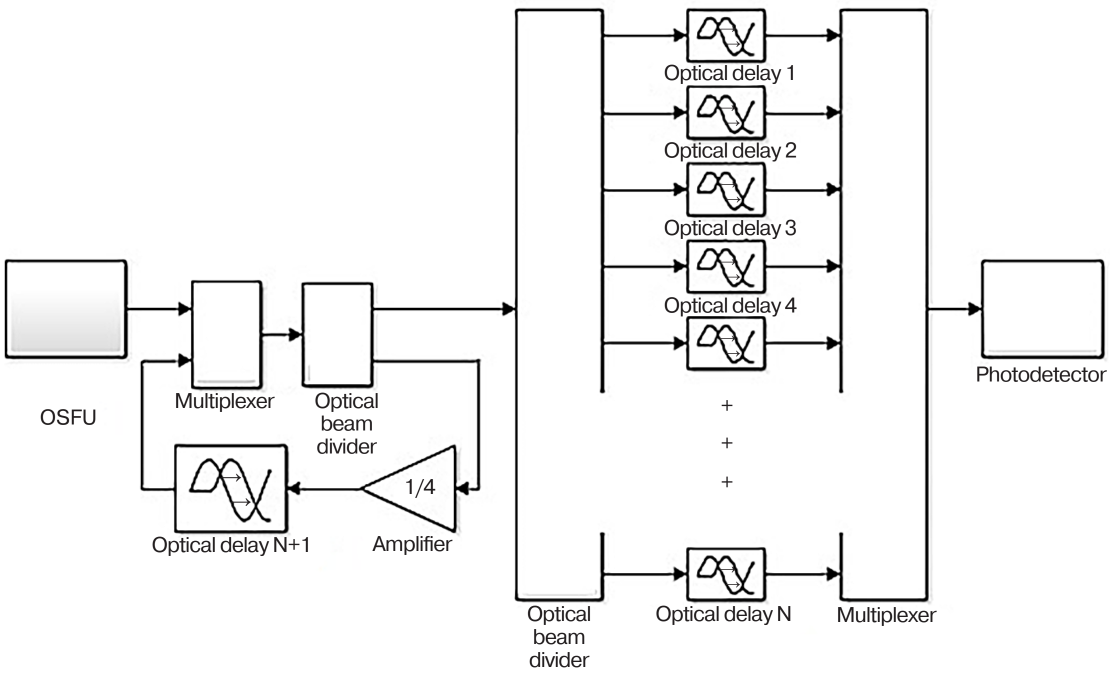

Fig. 2. The model of the USP electro-optical repeater with delayed feedback and fractional multiplexing

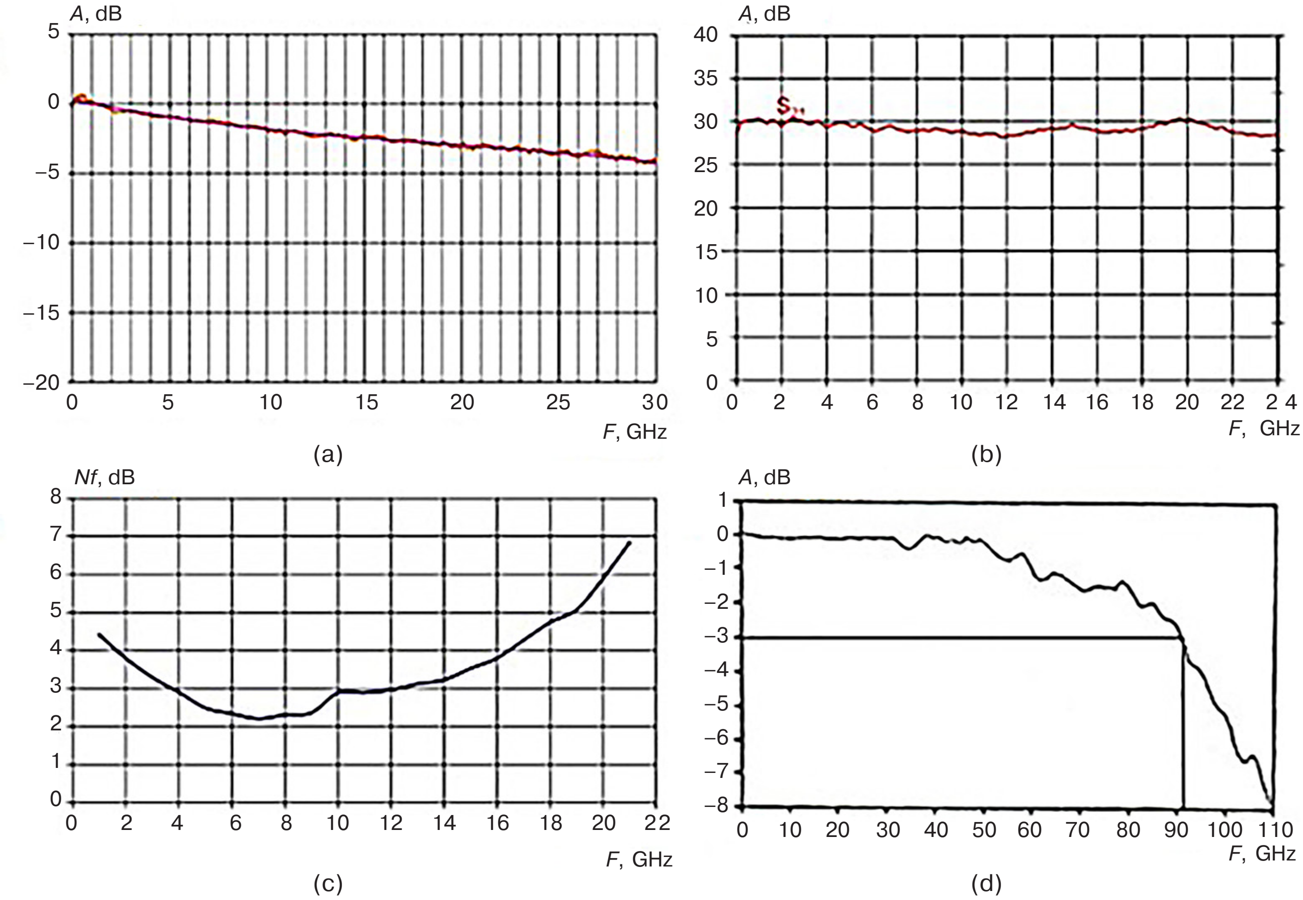

Fig. 3. Characteristics of the devices used in the model of the electro-optical repeater:
(a) frequency response of the electro-optical Mach–Zehnder modulator, (b) frequency response of the optical amplifier,
(c) *Nf* is the noise figure of the amplifier, and (d) frequency response of the photodetector

be insignificant, since they do not exceed 1 dB in the selected range. The noise figure of the amplifier is assumed to be 4 dB (Fig. 3).

MODELING AN ELECTRO-OPTICAL USP REPEATER WITH FRACTIONAL MULTIPLEXING AND DELAYED FEEDBACK OF DIFFERENT CONFIGURATIONS

In the course of modeling, different configurations of electro-optical repeaters with 2, 4, 8, and 16 delay lines were investigated, as well as a scheme with only delayed feedback as a reference (Fig. 4). Electro-optical repeaters were investigated with a different number of reconstruction cycles, which strongly affected the correlation coefficient between the original and the reconstructed pulse. The modeling was carried out with a signal-to-noise ratio (SNR) in the range from 3 to 18 dB. The graphs below show the mathematical expectations of 100 measurements for each point (Fig. 5 and Fig. 6).

The scheme, in which only the feedback was used, performed the worst as compared to other schemes. This is due to the accumulation of amplifier noise during pulse reconstruction and a complex pulse shape, which requires a large number of quantization levels to achieve the required correlation coefficient, and, accordingly, the reconstruction cycles.

The schemes with 2 and 4 delay lines (Fig. 5) demonstrated the best results relatively to the reference scheme, however, they did not provide the required threshold value of 0.9 for the correlation coefficient. Also observed in these schemes, was a negative interrelation between the correlation coefficient and the number of reconstruction cycles, therefore their use with an increase in the number of comparator levels is impossible.

Schemes with cascades of 8 and 16 delay lines (Fig. 6) showed better results in comparison with other configurations of USP electro-optical repeaters. The correlation coefficient between the reconstructed and reference USP signals was more than 0.9 at SNR of 9 dB. The best configuration was a scheme with 8 delay lines and 30 pulse reconstruction cycles. A further increase in the number of reconstruction cycles does not lead to a significant increase in the correlation coefficient between the original and restored pulses. The 16-line delay scheme also does not offer significant advantages over the 8-line delay scheme. At the same time, it has disadvantages associated with greater complexity and the pulse attenuation.

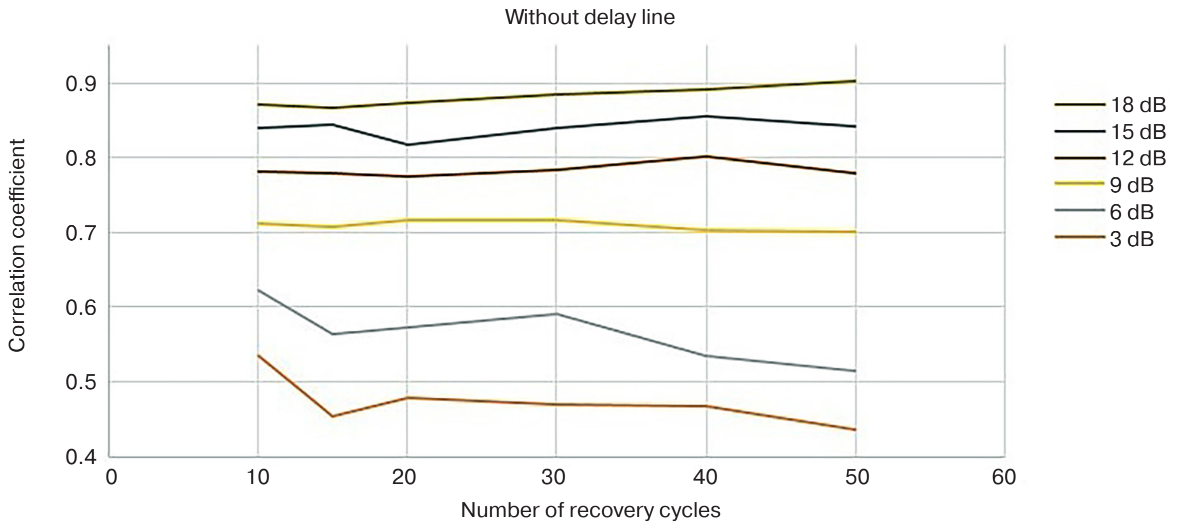

Fig. 4. Correlation coefficient between the original and reconstructed USP signals obtained for a scheme without a cascade of delay lines

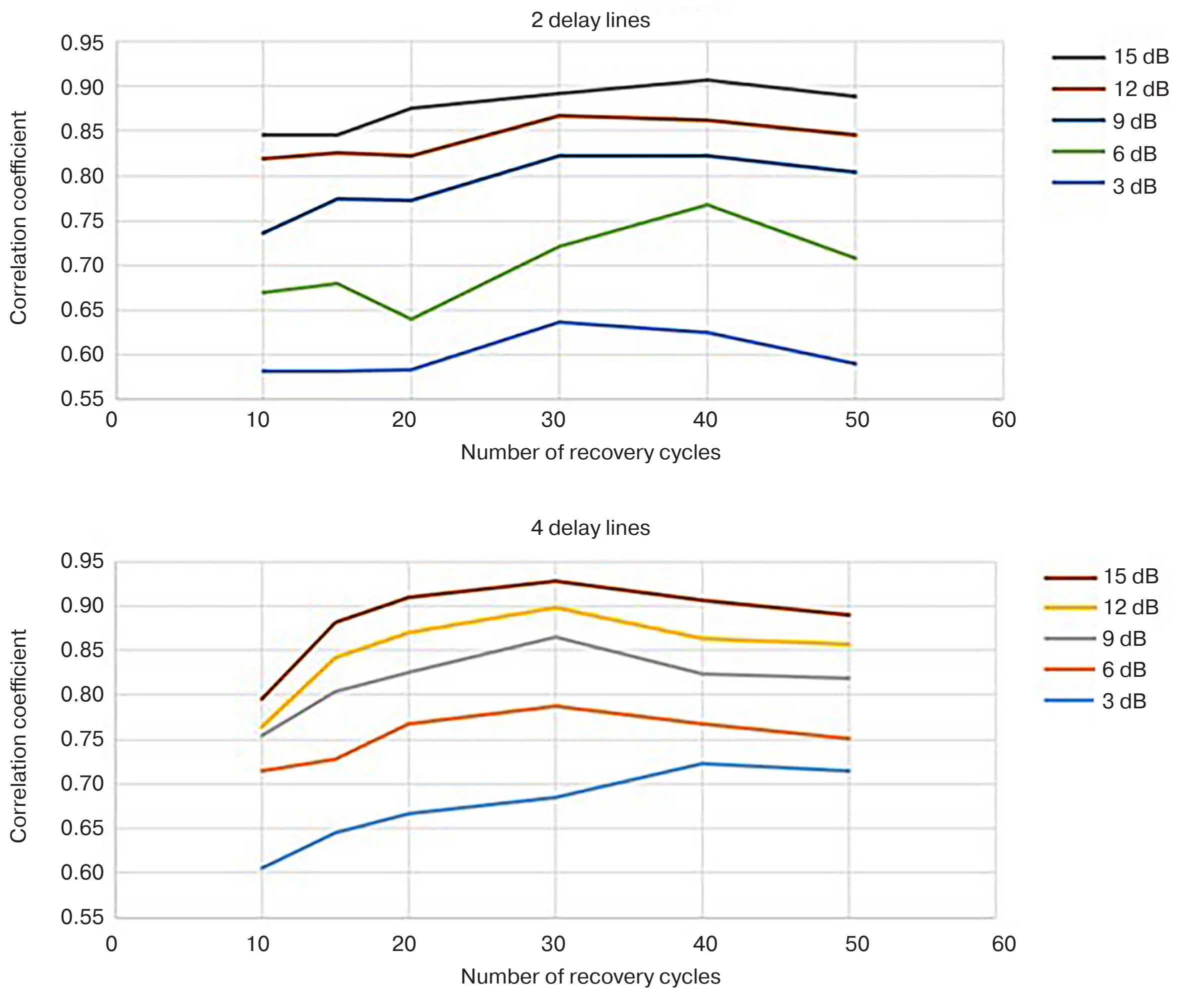

Fig. 5. Correlation coefficient between the original and reconstructed USP signals obtained for schemes with 2 and 4 delay lines as a function of the number of reconstruction cycles at different SNRs

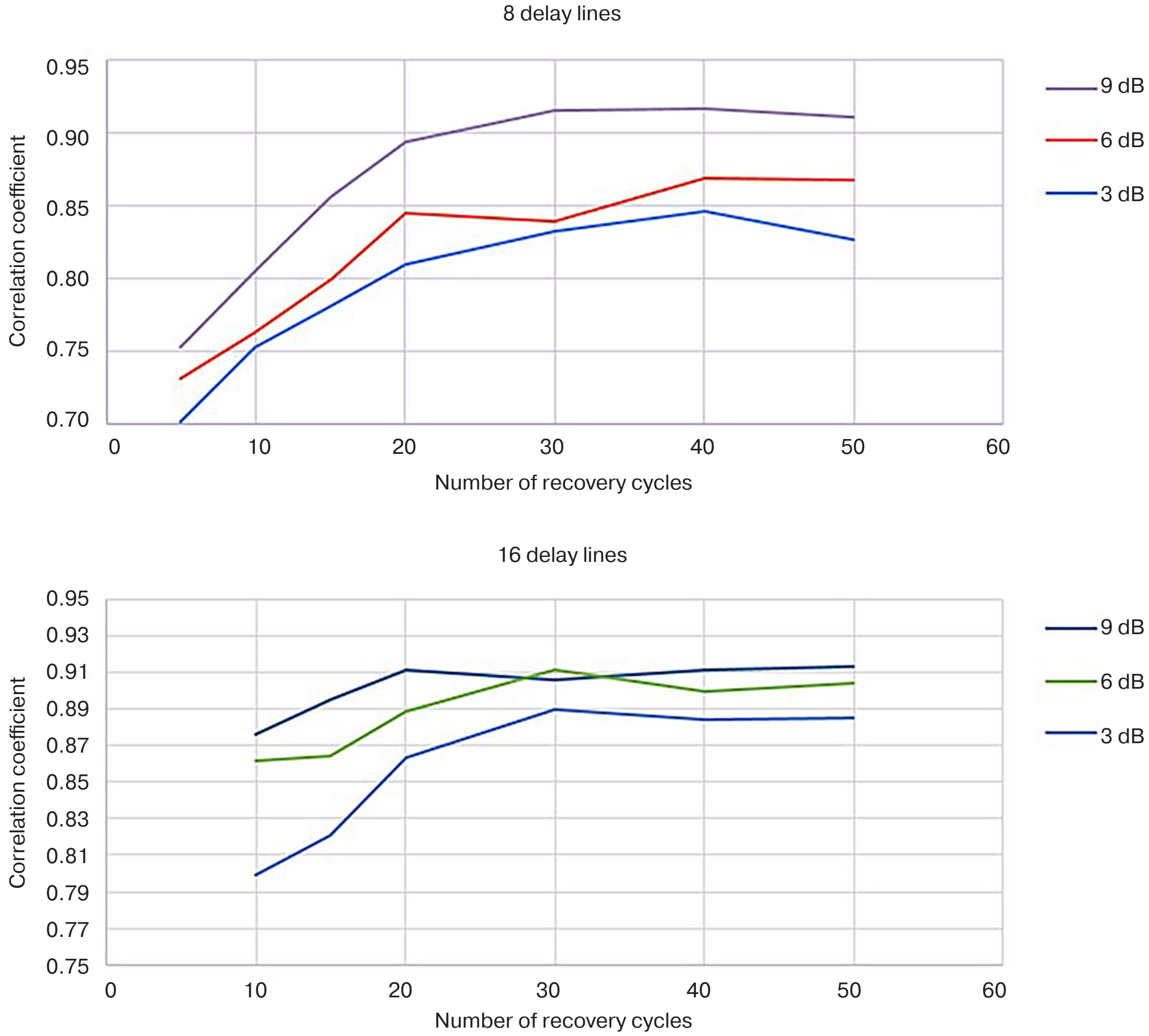

Fig. 6. Correlation coefficient between the original and reconstructed USP signals obtained for schemes with 8 and 16 delay lines as a function of the number of reconstruction cycles at different SNRs

Figure 7 shows the dependence of the correlation coefficient between the reconstructed and reference pulses on the SNR of the reconstructed pulse with confidence intervals for each point. The configuration includes a cascade of 8 delay lines and requires 30 iterations to reconstruct the pulse because of 30 comparator levels. We can see that at the point corresponding to the SNR of 9 dB, the mathematical expectation of the correlation coefficient exceeds 0.9, which meets the condition of strong correlation[1]. So, with a probability of 0.95, all values of the correlation coefficient lie in the confidence interval, the lower value of which is greater than 0.9. This configuration of an electro-optical repeater of subnanosecond pulses is the most advantageous in terms of the reliability of the reconstruction of a noisy pulse reflected from the target, if we take into account such parameters as the complexity of the scheme, its power consumption, and operating time.

CONCLUSIONS

The schemes of electro-optical repeaters of subnanosecond pulses considered in this work make it possible to reconstruct an USP in a single reception. The analysis of the schemes of electro-optical repeaters including 2, 4, 8, and 16 delay lines showed the following results:

- the schemes with 2 and 4 delay lines do not provide reliable reproducibility in the reconstruction of an USP radio image with a complex time profile;
- the scheme with 8 delay lines was good in modeling, providing a correlation reproducibility of greater than 0.9;

[1] Alibekov I.Yu. *Numerical Methods*: tutorial. Moscow: MGIY; 2008. 220 p. (in Russ.).

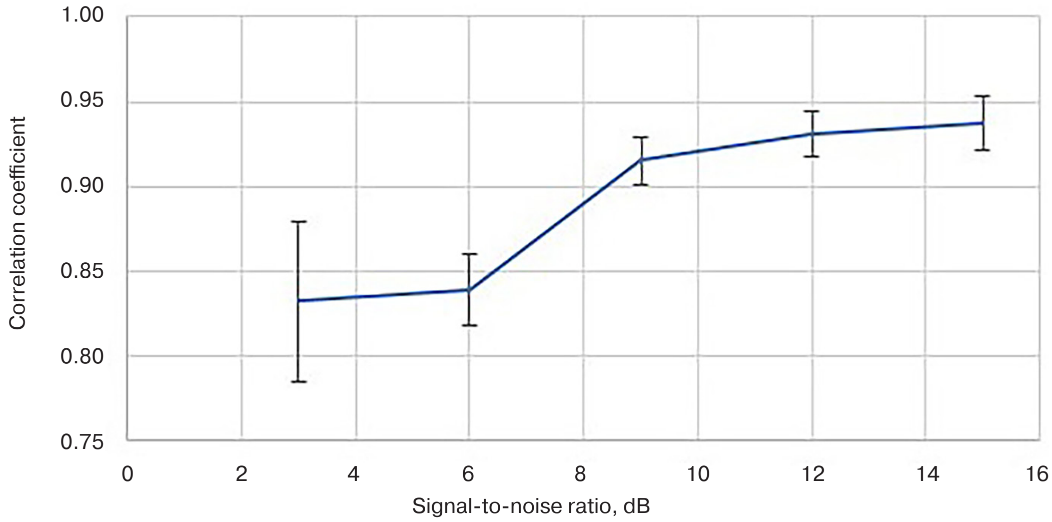

Fig. 7. Experimental dependence of correlation coefficient between the reference and reconstructed pulses on an SNR at the receiver input (with confidence intervals shown by brackets)

- the scheme with 16 delay lines does not offer significant advantages over the scheme with 8 delay lines, while it has disadvantages associated with the greater complexity and attenuation of the registered USP.

The results of modeling obtained in this work solve the problem associated with the necessity of multiple radiating an ultrashort pulse for the reconstruction of reflected by a target radio image by methods of stroboscopic registration.

Authors' contribution. All authors equally contributed to the research work.

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Mathematical modeling

Математическое моделирование

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RESEARCH ARTICLE

The modeling of two-dimensional vortex flows in a cylindrical channel using parallel calculations on a supercomputer

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Abstract
Objectives. The study aimed to examine vortex structures formed during the interaction of incident and reflected shock waves in a cylindrical channel. The shock wave was described by the Hugoniot relations, which make it possible to determine the parameters of the gas behind the shock front by a given Mach number and the values of the gas-dynamic parameters ahead of the pressure jump. The propagation of a strong shock wave (Mach number was 20) in argon was simulated.
Methods. The methods of mathematical modeling were used. A parallel algorithm for solving two-dimensional equations of gas dynamics in cylindrical coordinates (r, z, t) was developed and a new version of the *NUTCY_ps* program created. The calculations were performed on an MVS-100K supercomputer.
Results. Two methods of parallelization when solving a system of equations were considered. Using a specific task as an example, a comparison of the effectiveness of these methods was conducted. A parallel algorithm was developed and a program was upgraded for solving two-dimensional equations of gas dynamics in cylindrical coordinates (r, z are spatial coordinates, t is time). Numerical calculations were performed to simulate: 1) the shock wave incidence to and reflection from a metal screen; 2) the propagation of the shock wave through a hole in the screen; 3) the propagation of the shock wave through a cylindrical channel and its reflection from the bottom of the channel and interaction with the incident wave.
The results obtained by the parallel supercomputer with different numbers of processors are presented. It is shown that using 16 processors, it is possible to reduce the computation time for getting a solution for the test problem by approximately 12 times.
Conclusions. It is shown that the interaction of incident shock wave and the one reflected at an angle leads to the formation of regions with low and high gas densities, as well as vortex flows. The vortex interaction area (turbulence zone) gets a complex shape. The article discusses the possibility of carrying out full-scale experiments in shock tubes or using a laser shock tube. Such studies would make it possible to compare experimental data with the results of numerical simulation and, on their basis, to develop more advanced models of turbulent motions.

Keywords: numerical simulation, supersonic flows, vortices, two-dimensional gas dynamics in cylindrical coordinates

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НАУЧНАЯ СТАТЬЯ

Моделирование двумерных вихревых течений в цилиндрическом канале с помощью параллельных вычислений на суперкомпьютере

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Резюме
Цели. Изучается эволюция вихревых структур, формирующихся при взаимодействии падающей и отраженной под углом ударных волн в цилиндрическом канале. Сама ударная волна задается с помощью соотношений Гюгонио, позволяющих определить параметры газа за фронтом ударной волны по заданному числу Маха и значениям газодинамических параметров перед скачком давления. Моделировалось распространение сильной ударной волны (число Маха равнялось 20) в инертном газе аргоне.
Методы. Используются методы математического моделирования. Разработан параллельный алгоритм решения двумерных уравнений газовой динамики в цилиндрических координатах (r, z, t) и создана новая версия программы *NUTCY_ps*. Расчеты выполнены на суперкомпьютере МВС-100К.
Результаты. Рассмотрены две методики распараллеливания процессов при решении системы уравнений. На примере конкретной задачи проведено сравнение эффективности этих методик. Развит параллельный алгоритм и модернизирована программа для решения двумерных уравнений газовой динамики в цилиндрических координатах (r, z – пространственные координаты, t – время). Проведены численные расчеты, моделирующие: 1) падение и отражение ударной волны от металлического экрана; 2) прохождение ударной волны через отверстие в экране; 3) прохождение ударной волны через цилиндрический канал и ее отражение от дна канала, взаимодействие с падающей волной. Представлены результаты тестовых решений на параллельном суперкомпьютере с использованием различного числа процессоров. Показано, что при использовании 16 процессоров удается приблизительно в 12 раз сократить время расчета тестовой задачи.
Выводы. Показано, что при взаимодействии падающей и отраженной под углом ударными волнами формируются области с пониженной и повышенной плотностью газа, а также вихревые течения. Область взаимодействия вихрей (зона турбулентности) приобретает сложную форму. В статье обсуждается возможность проведения натурных экспериментов на ударных трубах или с помощью лазерной ударной трубы. Такие исследования позволили бы сравнить экспериментальные данные с результатами численных расчетов и на их основе развить более совершенные модели турбулентных движений.

Ключевые слова: численное моделирование, сверхзвуковые течения, вихри, двумерная газовая динамика в цилиндрических координатах

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Авторы заявляют об отсутствии конфликта интересов.

INTRODUCTION

The study of complex turbulence in supersonic gas flows is a demanding problem of modern gas dynamics and high energy density physics. Such phenomena occur when flying vehicles enter the Earth's atmosphere, in astrophysics, when analyzing the consequences of collisions of our planet with space objects, and in researches in the field of laser thermonuclear fusion.

Methods of mathematical modeling are used for the numerical solution of such problems [1–4].

In [5], the peculiarities of the interaction of two vortices in a gaseous medium were studied. To simulate this phenomenon, a parallel algorithm and a 2D program *NUT_2D* were developed to solve the equations of dynamics of a two-component gas in Cartesian coordinate system on MVS-100K supercomputer at the Joint Supercomputer Center of the Russian Academy of Sciences (JSCC RAS)[1]. To solve the equations, TVD-difference schemes of a higher order of approximation are used [6, 7].

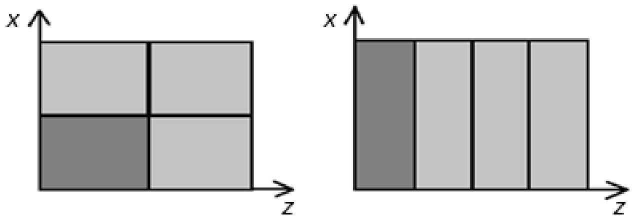
The next step is the development of a 2D program for solving the equations of gas dynamics in cylindrical coordinates and an algorithm for solving problems using a parallel supercomputer MVS-100K.

In this article we present the results of calculations performed using the advanced 2D *NUTCY* program in cylindrical coordinates (r, z are spatial coordinates, t is time) [8]. The mathematical model of the program is based on the equations of gas dynamics along with the Hugoniot conditions, which make it possible to set the parameters of the gas behind the shock front [9].

An algorithm for parallel computations was developed and a new version of the *NUTCY_ps* program was created to find the solution to two-dimensional equations of gas dynamics in cylindrical coordinates. The calculations were conducted on the MVS-100K complex using a large number of processors.

There are two main types of parallelization: algorithmic (through control) and geometric (through data). Due to significant potential of the *NUTCY* program in data parallelization, a geometric method was chosen to solve the problem: 1) to be computed region in the form of a cylinder in the section orthogonal to the cylinder's axis was divided by planes perpendicular to the axis into subregions of equal linear dimensions; 2) the division was conducted in two directions—along the Oz axis and into cylindrical layers of equal thickness along the Or[1] axis (x = r, Fig. 1). The boundaries and locations of the subregions computed by the processor were calculated using their indexes, which were assigned by the MPI communicator during initialization. Each processor stores in memory the values of only those cells that are included in its subregion; the control processor has additional buffers for data input and output. At each computational node, the calculation was conducted according to the same scheme. In the course of the calculations, we used both variants of partitioning the region: along two axes (Fig. 1a) and along one axis (Fig. 1b).

[1] www.jscc.ru



(a) (b)

Fig. 1. Schemes of parallelization in the new version of *NUTCY_ps*: (a) along two axes, (b) along one axis. In Fig. 1, $x = r$

To keep the uniformity of the computational scheme for each new subregion, two layers of fictitious cells were introduced on each of the four (two in the case of a one-dimensional distribution) boundaries. The values in these cells were determined in two ways, depending on the position in the grid:

1) based on the boundary conditions (if the boundary of the new subregion coincided with the boundary of the entire computational region);
2) data were transferred to the boundary of the next subdivision from the processor that processes the previous subregion (for internal subregions).

A special procedure has been written for data exchange. Each processor maintains indexes of its "neighbour processors." The procedure implements the transfer of the values of the gas-dynamic values of the boundary cells to the "processors–neighbours" and the receipt of the values of the fictitious cells from the neighbours. If we consider the "left" boundary of the subregion, then the values of cells with indices 1 and 2 will be transferred to the neighbour "on its left," the values of cells 0 and −1 will be received from the latter. For non-uniform operations (for example, data input and output) the processor with index zero acts as the manager and exchanges data with the rest of the processors. All data transfer operations between computational nodes are implemented using MPI tools.

TASK STATEMENT

The *NUTCY* program enables to numerically solve the equation of gas dynamics in a 2D cylindrical configuration in the Euler coordinate system:

$$\begin{aligned}
&\frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial r\rho u}{\partial r}+\frac{\partial r\rho\omega}{\partial z}=0,\\
&\frac{\partial\rho u}{\partial t}+\frac{1}{r}\frac{\partial r\rho u^2}{\partial r}+\frac{\partial\rho u\omega}{\partial z}+\frac{\partial\rho}{\partial r}=0,\\
&\frac{\partial\rho\omega}{\partial t}+\frac{1}{r}\frac{\partial ru\omega}{\partial r}+\frac{\partial\rho u\omega}{\partial z}+\frac{\partial\rho}{\partial z}=0,\\
&\frac{\partial e}{\partial t}+\frac{1}{r}\frac{\partial ru(e+p)}{\partial r}+\frac{\partial(e+p)\omega}{\partial z}=0.
\end{aligned}\qquad(1)$$

Here ρ is the density; p is the pressure; $V_r = u$, $V_z = \omega$ are the components of the velocity vector, **V**; $e=\rho\left(\varepsilon+\frac{V^2}{2}\right)$ is total energy. In addition, the following equations are added to (1):

1) gas law:

$$p=(\gamma-1)\rho\varepsilon,$$

where, ε is specific internal energy, γ is adiabatic gas constant.

2) the continuity equation for each gas component (total number of components is n):

$$\frac{\partial\rho_i}{\partial t}=\operatorname{div}\rho_i V=0,\ \text{где}\ i=\overline{1,n-1}.\qquad(2)$$

If the gas contains two components, then it is convenient to solve the continuity equation for the mixture (the upper equation of system (1)) and the equation for one of the components (2). Next, the concentration of the first component C is determined, and the concentration of the second component is found as $1 - C$.

The shock wave (SW) was given by the Hugoniot relations:

$$\begin{aligned}
&P_1=P_0\frac{2\gamma M_x^2-(\gamma-1)}{\gamma+1},\ ZP=\frac{P_1}{P_0},\\
&C_s^2=\gamma\frac{P_0}{\rho_0},\quad D=M_xC_s,\\
&\rho_1=\rho_0\frac{\gamma+1+(\gamma-1)/ZP}{\gamma-1+(\gamma+1)/ZP},\\
&u_1^2=\left(\gamma\frac{P_1}{\rho_1}\right)\frac{\gamma-1+(\gamma+1)/ZP}{2\gamma}.
\end{aligned}\qquad(3)$$

Here $P_1(0)$, $\rho_1(0)$ are the pressure and density behind the shock front (ahead of the shock front) of the SW; C_s is the speed of sound; D is the velocity of the shock front in the laboratory frame of reference. It is sufficiently to set the Mach number (M_x) and the thermodynamic parameters of the gas ahead of the shock front to determine the corresponding parameters behind the shock front. The gas velocity behind the shock front in the laboratory frame of reference is determined by the formula $w_1 = D - u_1$.

The propagation of a SW in a region filled with gas and its interaction with an aluminum screen (density ρ = 2.7 g/cm^3) located in this region were simulated. The computational region ($0 < r < R$, $0 < z < L_z$, where R, L_z are the sizes of the region) is filled with an inert gas—argon; adiabatic exponent γ = 5/3; initial pressure P_0 = 0.5 atm; density ρ_0 = 0.804 mg/cm^3.[2] The task statement is illustrated by Fig. 2a. The SW specified by the Hugoniot relations in the region $0 < z < b_0$ (the initial boundary of the subregion, disturbed SW, Mach number M_x = 20) propagates from the top to bottom. The gas velocity behind the shock of the incident SW is V_z = −4.817 km/s.

TASK 1: SW REFLECTION FROM THE SCREEN

In Task 1, an aluminum screen with the initial density ρ = 2.7 mg/cm^3 is placed in the computational region. In the first run of calculations, the solution to the quasi-one-dimensional problem was checked, and the propagation of the SW along the channel and its reflection from the screen were simulated. The parameters used in Task 1: R = 0.25 cm, L_z = 0.5 cm, gas velocity behind the shock front v_z = –4817 m/s.

The calculations were performed on a difference grid (250 × 500 nodes) on a personal computer. Below are the results of the calculations.

The shock wave has reached the screen and reflected off. By the time t = 0.0005 ms, the reflected wave propagates towards the upper end of the shock

[2] The *NUTCY* program uses the CGS system to operate, however, here the time scale is given in milliseconds.

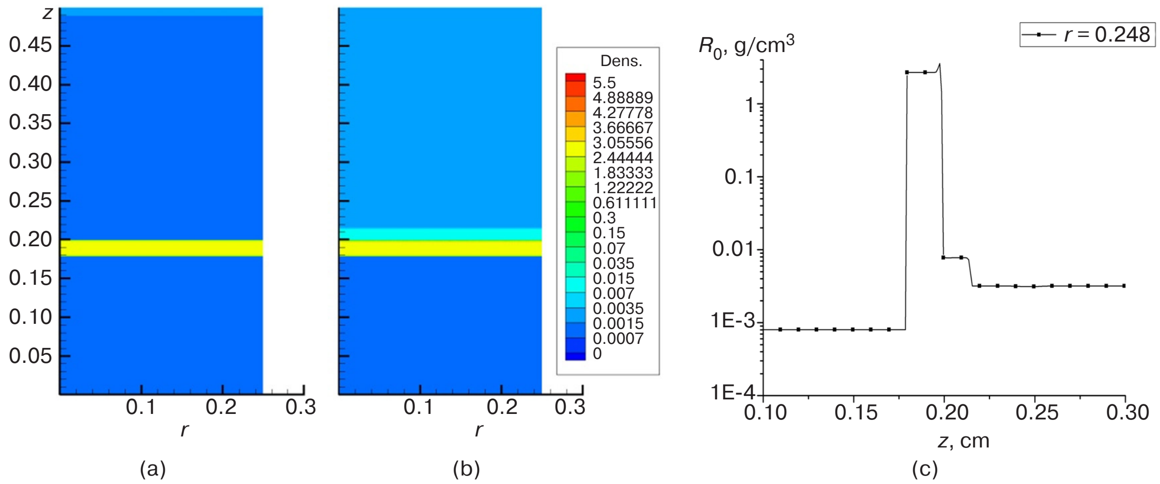


Fig. 2. Density distributions at (a) $t = 0$; (b) $t = 0.0005$ ms, (c) instantaneous density distribution along axis Oz at $t = 0.0005$ ms; $r = 0.248$ cm is the radius value for which the "density profile" is plotted along axis Oz ($R_0 = \rho(z)$ [g/cm^3])

tube. Figure 2c shows the "density profile" near the outer boundary of the cylinder $r = 0.248$ cm. The gas velocity behind the shock front of the reflected SW is $V_z = -0.198$ km/s at time $t = 0.0005$ ms, that is, the shock front of the reflected SW propagates upward towards the incident shock, and the gas keeps compressing.

TASK 2: SW PROPAGATION THROUGH A HOLE OF THE SCREEN

In the second run of calculations, a hole was added to the aluminum screen located in the computational region. Partial propagation of a SW through the hole and its diffraction at the edge of the hole were simulated. Because of complex physical phenomena involved in this process, it was necessary to use high-resolution grids. The calculations were carried out in two stages: on a personal computer with a grid of 250 × 500 nodes and on one processor of the MVS-100K complex with a grid of 1000 × 2000 nodes. The obtained data showed similar results for both cases.

A diffraction region of the SW appears in the vicinity of the "hole-screen" boundary. Refraction of the wave leads to the formation of vortices. Note that the gas velocity behind the shock front of the SW reflected from the screen (red line in Fig. 3c) is $V_z = -0.198$ km/s, that is, equal to the gas velocity in the reflected wave in Task 1.

The increased number of grid nodes allows for, in principle, to study in more detail the processes occurring in the diffraction region. This, however, significantly increases the time required for these calculations. A version of the *NUTCY* program was developed for calculations on a multiprocessor supercomputer complex

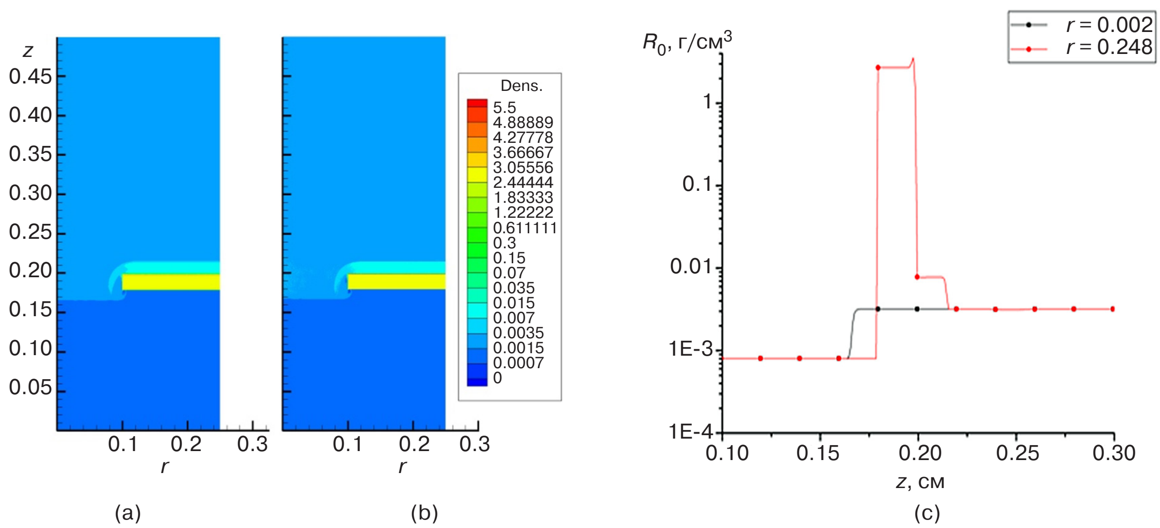


Fig. 3. Density distributions (at $t = 0.0005$ ms), obtained on (a) personal computer with a grid of 250 × 500, (b) MVS-100K (8 processors, grid 1000 × 2000). (c) density profiles at $t = 0.0005$ ms near the axis ($r = 0.002$ cm) and near outer boundary ($r = 0.248$ cm)

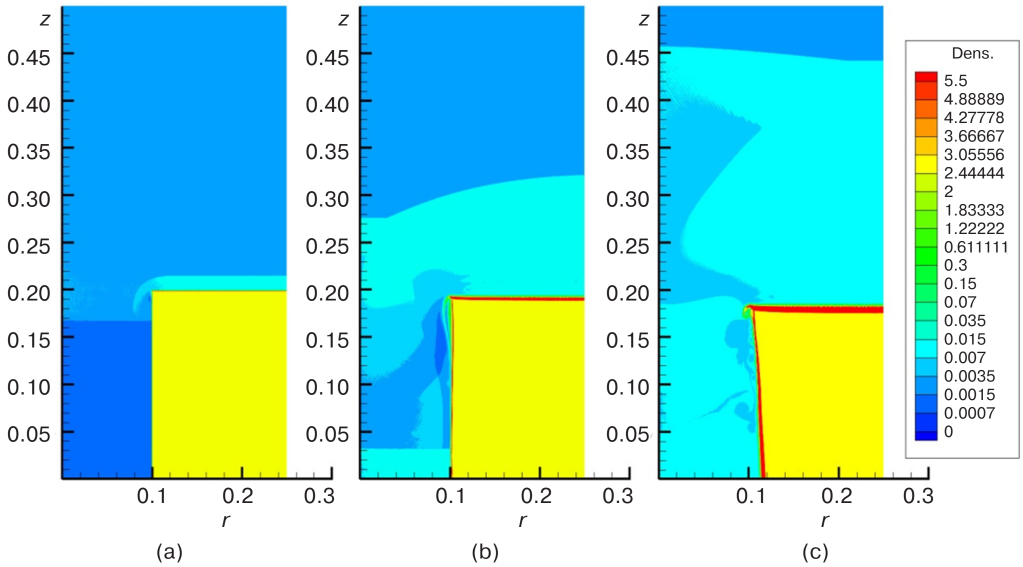


Fig. 4. Density distributions at (a) $t = 0.0005$ ms, (b) $t = 0.00085$ ms, and (c) $t = 0.00135$ ms

(*NUTCY_ps* program). Using the multiprocessor version of the program, the calculations were performed for the previous task (Task 2); the obtained results were similar to those shown in Fig. 3a (see Fig. 3b). The calculations were performed on 8 processors. This computation served as an errors-free test for the new version of the *NUTCY_ps* program.

TASK 3: SW PROPAGATING INTO A LONG CHANNEL

Calculations for the third task were conducted on a multiprocessor complex MVS-100K using the *NUTCY_ps* program.

As part of the third task, the aluminum screen was replaced by an aluminum wall with an extended channel. At its bottom end there was an absolutely elastic wall. We studied the propagation of the SW along the channel downward and reflected off—up to the time it escaped the channel. The density distribution across the computational region was studied. Due to the increased "time-consuming" calculations on the grid 1000 × 2000 nodes, the program was launched on 16 processors of the MVS-100K complex in two modes: distribution along one axis and along two axes.

The graphs illustrating the SW dynamics are shown below.

Figure 4a corresponds to the point in time when the SW enters the channel; Fig. 4b—the front of the SW reaches the bottom and is reflected off; Fig. 4c—the reflected SW escapes the channel.

The graphs show the appearance of zones of increased gas density and "bubbles" arising behind the refraction front of the SW when it enters the channel and escapes it.

The results obtained by this technique as well as the effectiveness of two approaches to geometric parallelization of computation are given in Table 1.

Table 1. Comparison of consumed time for calculating one step and 6000 steps in different modes of parallelization

Calculation mode (grid 1000 × 2000 nodes)	Average time consumed for one step, s	Total time consumed by the program when calculating 6000 steps, s
Single processor	1.982	12462
Multi-processor in a configuration 4 × 4	0.155	1175
Multi-processor in a configuration 1 × 16	0.167	1234

It turns out that with the same number of processors, two-dimensional parallelization is more effective due to reduced total number of intra-processors' exchanges. The table shows that the computation time was decreased by about 10−13 times when switching from a single-processor program to a 16-processor one. With two-dimensional parallelization, a gain of 7–8% relative to the one-dimensional one is obtained. With an increase in the number of processors, this gain can be further increased; however technical and organizational disadvantages are possible.

DISCUSSION OF RESULTS AND CONCLUSION

Three runs of calculations were conducted.

In the first run, the numerical solution to the quasi-one-dimensional problem was verified. It is shown that the results agree with the analytical solution for the propagation of a SW in gas.

In the second run, two-dimensional phenomena associated with the refraction of a SW propagating through a hole in the screen were observed. It is shown that when developing a parallel computation algorithm (*NUTCY_ps* program) and switching to grids of increased resolution, the results of calculations for Task 2 are identical to the results obtained by the calculations when a single-processor version was used.

In the third run of calculations, the propagation of a SW along an extended channel and reflection from its bottom were studied. It was observed that the interaction of the incident and reflected waves leads to the appearance of zones with increased gas density and zones of low density. In this case, the vortex interaction region gets a complex shape.

Comparison of the two methods of geometric parallelization supported the theoretical assumptions about the greater efficiency of the two-dimensional computations. In conclusion, we note that further reduction of the computation time is possible by modifying the way of operation with data input and output (multiprocessor reading and writing to a file).

At the beginning of the article, it was noted that the study of the development of hydrodynamic instability and the transition to turbulence is of great importance for researches on laser thermonuclear fusion. In Russia [10] and overseas [11−13], high-power multichannel lasers are being built to initiate thermonuclear microexplosions. Along with the study of the compression of microtargets in such installations, it will be possible to perform research on the initiation and interaction of SWs and the development of substance mixing in a laser shock tube [14]. Compared to classical shock tubes, high gas flow velocities[3] (and high Mach numbers) are easily achieved in a laser shock tube, and it is possible to simulate these processes in complex chambers filled with various media. Laser shock tubes are promising for modeling in laboratories some astrophysical phenomena associated with the protection of the Earth from collisions with space objects, as well as other problems in high energy density physics.

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Authors' contribution

I.G. Lebo—physicomathematical model and code development, problem statement, analysis of results (70%).

I.V. Obruchev—simulations and graphical treatment (30%).

[3] Velocities are comparable to ones of space objects which can enter the Earth atmosphere (~10−20 km/s).

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RESEARCH ARTICLE

New operational relations for mathematical models of local nonequilibrium heat transfer

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Abstract

Objectives. Recently, interest in studying local nonequilibrium processes has increased in the context of the development of laser technologies, the possibility of reaching ultrahigh temperatures and pressures, and the need for a mathematical description of various physical processes under extreme conditions. In simulating local nonequilibrium processes, it becomes necessary to take into account the internal structure of investigation subjects, which significantly complicates the classical transport models. An important stage here is to construct mathematical models of various physical fields in which their spatiotemporal nonlocality should be taken into account. For these purposes, hyperbolic equations are used for a wide class of phenomena and, first of all, for unsteady-state heat conduction processes based on the generalized Maxwell–Cattaneo–Luikov–Vernotte phenomenology. Mathematical models in the form of boundary value problems for hyperbolic equations are called generalized boundary value problems. These problems differ significantly in solving difficulty from the classical ones based on Fourier phenomenology. The specificity of these problems is the relative simplicity of the initial mathematical models, together with the difficulty of solving them in an analytically closed form. Hence, very little success has been achieved in finding exact analytical solutions to problems of this kind. The most acceptable approach to solving them is operational calculus. However, it gives analytical solutions in the Laplace transform space as complex functional structures, the inverse transforms of which are not available in well-known reference books on operational calculus. On this path, serious computational difficulties arise. The study aimed to analyze a set of nonstandard transforms arising from the operational solution of mathematical models of local nonequilibrium heat transfer and to obtain their inverse transforms.

Methods. Methods and theorems of operational calculus, methods of contour integration of complex transforms, and the theory of special functions were used.

Results. Operational calculus was developed for mathematical models of local nonequilibrium heat transfer in terms of the theory of unsteady-state heat conduction for hyperbolic equations (wave equations). Nonstandard operational transforms, the inverse transforms of which are unavailable in the literature, were considered. It was shown that the presented transforms are common to operational solutions of a wide class of generalized boundary value problems for hyperbolic equations in the theory of heat conduction, diffusion, hydrodynamics, vibrations, propagation of electricity, thermomechanics, and other areas of science and technology. Partially bounded and finite domains were explored. Illustrative examples were given, namely, the results of numerical experimental studies of a local nonequilibrium heat transfer process that took into account the finiteness of the heat transfer rate, which had a wave character. The latter was expressed by the presence of the Heaviside step function in the analytical solution of the problem. The physical meaning of the finiteness of the heat transfer rate was substantiated. The isochron was constructed for the temperature function in a partially bounded domain. It was shown that the temperature profile has a discontinuity on the surface of the propagating wave front. This leads to the retention of heat outflow beyond the discontinuity boundary. This is a characteristic feature of the analytical solutions of the wave equations, along with the possibility to describe the analytical solution of the problem as equivalent integral relations, which noticeably simplify numerical calculations.

Conclusions. The inverse transforms of nonstandard operational (Laplace) transforms were presented, which are contained in the operational solutions of a wide class of problems of local nonequilibrium (heat, mass, momentum) transfer processes, electrical circuits, hydrodynamics, oscillation theory, thermomechanics, and others. Illustrative examples were given, and the possibility of transition from one form of an analytical solution to another equivalent form was shown. The presented analytical solutions of hyperbolic heat transfer models in canonical domains are new in classical thermal physics.

Keywords: nonstandard operational transforms, inverse transforms, mathematical models of local nonequilibrium heat transfer, analytical solutions

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НАУЧНАЯ СТАТЬЯ

Новые операционные соотношения для математических моделей локально-неравновесного теплообмена

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Резюме
Цели. В последние годы усилился интерес к изучению локально-неравновесных процессов в связи с развитием лазерных технологий, возможностью получения сверхвысоких температур и давлений, а также ввиду необходимости математического описания различных физических процессов в экстремальных условиях. При моделировании локально-неравновесных процессов возникает необходимость учета внутренней структуры исследуемых объектов, что приводит к существенному усложнению классических моделей переноса. Важным этапом в развитии указанной области является построение математических моделей разнообразных физических полей с учетом их пространственно-временной нелокальности. Для этих целей используются уравнения гиперболического типа для широкого класса явлений и, прежде всего, для процессов нестационарной теплопроводности на основе обобщенной феноменологии Максвелла – Каттанео – Лыкова – Вернотта. Математические модели в виде краевых задач для уравнений гиперболического типа носят название краевых задач обобщенного типа. Эти задачи значительно отличаются от классических на основе феноменологии Фурье по сложности их решения. Их специфика заключается в относительной простоте исходных математических моделей и трудности решения в аналитически замкнутом виде. Отсюда весьма незначительные успехи в нахождении точных аналитических решений такого рода задач. Наиболее приемлемый метод их решения – операционный, но он приводит к аналитическим решениям в пространстве изображений по Лапласу в виде сложных функциональных конструкций, оригиналы которых не содержатся в известных справочниках по операционному исчислению. На этом пути возникают серьезные трудности вычислительного характера. Цель работы – рассмотреть серию нестандартных изображений, возникающих при операционном решении математических моделей локально-неравновесного теплообмена и получить их оригиналы.

Методы. Использованы методы и теоремы операционного исчисления, методы контурного интегрирования сложных изображений, теория специальных функций.
Результаты. Представлено развитие операционного исчисления для математических моделей локально-неравновесного теплопереноса в терминах теории нестационарной теплопроводности для уравнений гиперболического типа (волновых уравнений). Рассмотрены нестандартные операционные изображения, оригиналы которых ранее были неизвестны. Показано, что приведенные изображения являются характерными для операционных решений широкого класса обобщенных краевых задач для уравнений гиперболического типа в теории теплопроводности, диффузии, гидродинамики, колебаний, распространения электричества, термомеханики и других направлений науки и техники. Изучены частично ограниченные и конечные области. Приведены иллюстративные примеры в качестве численных экспериментов локально-неравновесного процесса теплообмена с учетом конечной скорости распространения теплоты, имеющей волновой характер. Последнее выражается наличием ступенчатой функции Хевисайда в аналитическом решении задачи. Обоснован физический смысл конечной скорости распространения теплоты; построена изохрона для температурной функции в частично ограниченной области и показано, что на поверхности фронта идущей волны температурный профиль имеет разрыв. Это приводит к задержанию оттока теплоты за границу разрыва – характерная особенность аналитических решений волновых уравнений, к которой следует добавить возможность описания аналитического решения задачи в виде эквивалентных интегральных соотношений, существенно упрощающих числовые расчеты.
Выводы. Представлены оригиналы нестандартных операционных изображений (по Лапласу), входящие в операционные решения широкого класса задач локально-неравновесных процессов переноса (теплоты, массы, импульса), электрических цепей, гидродинамики, теории колебаний, термомеханики и других областей. Приведены иллюстративные примеры и показана возможность перехода от одной формы аналитического решения к другой эквивалентной форме. Представленные аналитические решения гиперболических моделей теплопереноса в областях канонического типа являются новыми в классической теплофизике.

Ключевые слова: нестандартные операционные изображения, оригиналы, математические модели локально-неравновесного теплопереноса, аналитические решения

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INTRODUCTION

The classical models of the analytical theory of heat transfer originated from Fourier's linear gradient relation

$$\overline{q}(M,t) = -\lambda gradT(M,t),$$

presented by Fourier in his *Mémoire sur la propagation de la Chaleur dans les corps solides* (On the Propagation of Heat in Solid Bodies) in Paris in 1807. Fourier finalized his theory in 1822 in the essay *Théorie analytique de la chaleur eur* (The Analytical Theory of Heat), which Kelvin called "the great mathematical model" [1]. Together with the energy equation for isotropic solids,

$$c\rho\frac{\partial T(M,t)}{\partial t} = -\text{div}\overline{q}(M,t) + F(M,t),$$

Fourier's law leads to parabolic unsteady-state heat transfer equation

$$\frac{\partial T(M,t)}{\partial t} = a\Delta T(M,t) + \frac{1}{c\rho}F(M,t),\ M \in D,\ t > 0, \quad (1)$$

and to boundary value problems for equation (1) under initial condition

$$T(M,t)\big|_{t=0} = \Phi_0(M), M \in \overline{D} \quad (2)$$

and boundary conditions

$$\beta_1\frac{\partial T(M,t)}{\partial n} + \beta_2 T(M,t) = \beta_3\varphi(M,t),\ M \in S,\ t > 0. \quad (3)$$

Here, D is a finite of partly bounded convex domain of $M(x, y, z)$, S is a piecewise smooth surface bounding the domain D, and $\overline{n}$ is the external normal to S (a vector, continuous at points of S). The parameters of problem (1)–(3) are thermophysical characteristics of the medium, which are constant in a range of temperatures within the transition points [2]. Some paradoxes of using model concepts (1)–(3) were repeatedly noted. One of them is the absence of inertia of the process of heat conduction in Fourier's law because of the ignorance of the mechanism of heat transfer by constituent particles of matter (electrons, molecules, ionic lattices) and the relaxation time related to the mean free time of microparticles. A consequence of this is the conclusion of the infinity of the heat transfer rate, which follows from the analytical solutions of models (1)–(3). Another paradox is the singularity of the heat flow and the velocity of motion of isotherms at $x > 0$ and $t > 0$ as $x \to 0$ ($t \to 0$). Nonetheless, these circumstances do not limit the applicability range of mathematical models (1)–(3), which encompasses more and more substantive subjects of investigation and a wider and wider variety of applications [3–7].

Recently, the interest in studying local nonequilibrium processes has increased in view of the wide potential of their practical application [8–14], namely, creation of new technologies to obtain nanomaterials and coatings with unique physicochemical properties (binary and multicomponent metal alloys, polymer materials, amorphous metal semiconductors, nanoliquids, and colloidal and bio- and cryosystems); optimization of laser application conditions; intense heating and cooling of nanoelectronical and nanotechnical components; and heating, melting, and ablation of matter on exposure to ultrashort laser pulses; etc. To describe the intensification of thermal processes under these conditions, it was required to refine Fourier's hypothesis. This was done by taking into account the local nonequilibrium using relation

$$\bar{q}(M,t) = -\lambda \operatorname{grad} T(M,t) - \tau_{\mathrm{r}} \frac{\partial \bar{q}(M,t)}{\partial t}, \qquad (4)$$

which implies the conclusion of the finiteness of the heat transfer rate. Here, the time τ_{r} is a measure of the inertia of the heat flow, which is related to the heat transfer rate by the expression $v_{\mathrm{T}} = \sqrt{a/\tau_{\mathrm{r}}}$. The necessity to take into account the effect of the finiteness of the heat (mass) transfer rate was indicated by Maxwell in gas dynamics theory [15], Luikov in investigation of the heat and moisture transfer in capillary porous bodies [16], and Cattaneo [17] and Vernotte [18] in heat conduction theory. The energy equation and relation (4) lead to hyperbolic heat transfer equation

$$\begin{gathered}\frac{\partial T(M,t)}{\partial t} = a\Delta T(M,t) - \\ -\tau_{\mathrm{r}} \frac{\partial^2 T(M,t)}{\partial t^2} + \frac{\tau_{\mathrm{r}}}{c\rho}\left[\frac{\partial F(M,t)}{\partial t} + \frac{1}{\tau_{\mathrm{r}}} F(M,t)\right]\end{gathered} \qquad (5)$$

and to the corresponding generalized boundary value problems of unsteady-state heat conduction. In the mathematical formulation of these problems, the corresponding local nonequilibrium boundary conditions should be used. Using standard local equilibrium boundary conditions (3) (which is quite frequently done in publications on analytical thermal physics) can lead to physically contradictory results (e.g., to negative values of absolute temperature [19]). These issues were considered in detail previously [20]. Correct generalized boundary conditions were formulated based on relation (4) in the integral and equivalent differential forms. For example, for thermal heating (cooling) conditions, the Neumann boundary condition has the form

$$\begin{gathered}\frac{1}{\tau_r}\int_0^t \frac{\partial T(M,\tau)}{\partial n}\Big|_{M\in S} \exp\left(-\frac{t-\tau}{\tau_r}\right) d\tau = \\ = \pm(q_0/\lambda) S_+(t), t \ge 0,\end{gathered} \qquad (6)$$

and in the case of heating by the medium, one should write

$$\begin{gathered}\frac{1}{\tau_{\mathrm{r}}}\int_0^t \frac{\partial T(M,\tau)}{\partial n}\Big|_{M\in S} \exp\left(-\frac{t-\tau}{\tau_{\mathrm{r}}}\right) d\tau = \\ = h\{T(M,t)|_{M\in S} - \left[T_0 + S_+(t)(T_c - T_0)\right]\},\ t \ge 0.\end{gathered} \qquad (7)$$

Here,

$$S_+(t) = \begin{cases} 1,\ t > 0, \\ 0,\ t \le 0. \end{cases} \qquad (8)$$

As $(1/h) \to 0$ ($h \to \infty$), from relation (7), the boundary condition during thermal heating is obtained:

$$T(M,t)|_{M\in S} = T_0 + S_+(t)(T_c - T_0),\ t \ge 0. \qquad (9)$$

Note that hyperbolic equation (5) for local nonequilibrium heat and mass transfer processes was obtained for the first time by Fock [21] and Davydov [22] under the assumption of the finiteness of the velocity of energy- or mass-transferring particles. Equation (5) was also derived by Predvoditelev [23] by analyzing the velocities of isothermal surfaces using Riemann's concepts, i.e., after completely abandoning the use of relaxation formula (4).

NEW OPERATIONAL RELATIONS FOR HYPERBOLIC MATHEMATICAL MODELS

Generalized transfer problems for Eq. (5) differ significantly from classical problems (1)–(3) because analytical solutions of the former are more difficult to obtain. The specificity of these problems is the relative simplicity of the initial mathematical models together with the difficulty of solving them in an analytically closed form. Hence, very little success was achieved in finding their exact analytical solutions, which were mostly found within partially bounded domains. The main approach to solve these problems is operational calculus, which is, however, accompanied by two main challenges. Whereas solving a problem using operational calculus is not too difficult, the inverse transforms are difficult to obtain because of their absence in operational calculus tables. The formal application of operational calculus theorems to finding the inverse transforms can lead erroneous results because the sought inverse transforms should contain the Heaviside step function [24], the formal appearance of which not always can be ensured. A natural way out of this situation is to develop artificial techniques or a complex transition to the inverse transforms by contour integration of the transforms [24].

Partially bounded domain

Let us consider the operational solution of homogeneous equation (5) in the domain $\Omega = \{M(z, t): z \geq 0, t \geq 0\}$ in the dimensionless variables

$$\xi = v_p z/a,\ \tau = v_p^2 t/a,\ \beta = v_p/v_T,$$
$$W(\xi,\tau) = [T(z,t) - T_0]/(T_c - T_0),$$
$$v_T = \sqrt{a/\tau_r},$$

where $v_p = \sqrt{2G(1-\nu)/[\rho(1-2\nu]}$ is the expansion wave velocity in an elastic medium, G is the shear modulus, ν is Poisson's ratio, and ρ is the density. The problem has the form

$$\frac{\partial W}{\partial \tau} = \frac{\partial^2 W}{\partial \xi^2} - \beta^2 \frac{\partial^2 W}{\partial \tau^2},\ \xi > 0,\ \tau > 0, \qquad (10)$$

$$\left.\begin{aligned} &W(\xi,\tau)\Big|_{\tau=0} = \frac{\partial W(\xi,\tau)}{\partial \tau}\Big|_{\tau=0} = 0,\ \xi \geq 0, \\ &|W(\xi,\tau)| < \infty,\ \xi \geq 0,\ \tau \geq 0 \end{aligned}\right\}. \qquad (11)$$

Let $\overline{W}(\xi, p) = \int_0^\infty \exp(-p\tau)W(\xi,\tau)d\tau$ be the Laplace transform of the function $W(\xi, \tau)$. From Eq. (10), one can find

$$\overline{W}(\xi, p) = \bar{f}(p)\exp\left(-\xi\sqrt{\beta^2 p^2 + p}\right).$$

The transform written for $\overline{W}(\xi, p)$ determines the further purpose of this study, which is to obtain the inverse transforms of a set of transforms of the form $\bar{f}(p)\exp\left[-\xi\sqrt{(p+2\alpha)(p+2\beta)}\right] = \bar{f}(p)\exp[-\xi\bar{\mu}(p)]$, where $\bar{\mu}(p) = \sqrt{(p+2\alpha)(p+2\beta)}$, at various $\bar{f}(p)$ with the Heaviside step function.

Let us emphasize once again the specific features of the further calculations. The pioneering publications on the simplest hyperbolic transfer models in 1970s showed [25, 26] that the heat conduction according to the mathematical models for Eq. (5) is characterized by the wave behavior, which is expressed by the presence of the step function $\eta(\tau - \xi)$ in the analytical solution of Dirichlet boundary value problem (10)–(11) at $W(0, \tau) = 1$, $\tau > 0$. At any time, there is a thermal wake region at $\xi < \tau$ and an unperturbed region at $\xi > \tau$ (Fig. 1) (at points of the region at ξ above $\xi = \tau$, the temperature value remains unchanged and equal to the initial value). On the surface of the front of the propagating wave, we have $\xi = \tau$, and the temperature profile on the front has a discontinuity, the amplitude of which rapidly decreases with increasing heating time. It is in the region behind the front of the heat wave that there is a significant difference between the solutions of hyperbolic equation (5) and parabolic equation (1) (in the latter case, the solutions are smooth and considerably exceed the initial value). It is these features that are explained by the presence of the step function in the analytical solution of the heat problem.

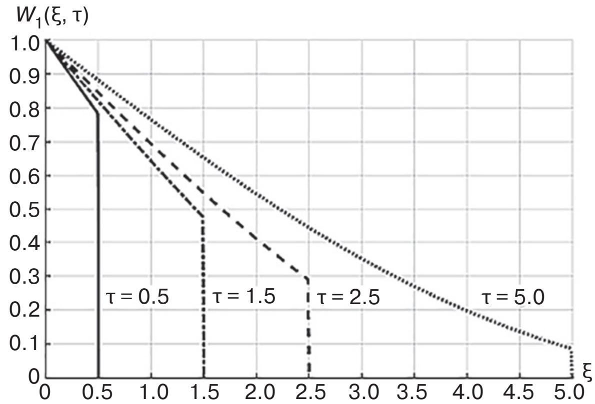

Fig. 1. Isochrone calculated by (29) (β = 1)

Below, only operational calculus relations are considered; therefore, let the time variable be denoted as t (as it is accepted).

Consider the function $\varphi(x) = x^{-m/2}I_m(x^{1/2})$, where $I_m(x^{1/2})$ is the modified Bessel function. In the theory of Bessel functions, there is relation

$$\frac{d^n}{dx^n}[\varphi(x)] = \frac{1}{2^n}x^{-\frac{m+n}{2}}I_{m+n}\left(x^{\frac{1}{2}}\right), \qquad (12)$$

which enables oner to write the Maclaurin series for $\varphi(x + x_0)$:

$$\varphi(x+x_0) = (x+x_0)^{-\frac{1}{2}m} I_m\left[(x+x_0)^{\frac{1}{2}}\right] = \sum_{n=0}^{\infty}\frac{x_0^n}{n!}\frac{1}{2^n}x^{-\frac{m+n}{2}}I_{m+n}\left(x^{\frac{1}{2}}\right). \qquad (13)$$

Set $x_0 = 2\kappa t$, $x = t^2$, and $m = 0$ in expression (13). Then, we obtain

$$\varphi(t) = I_0\left[\sqrt{t(t+2\kappa)}\right] = \sum_{n=0}^{\infty}\frac{\kappa^n}{n!}I_n(t).$$

Take the Laplace transform using the published tables [27]:

$$\bar{\varphi}(p) = \int_0^\infty \exp(-pt) I_0\left[\sqrt{t(t+2\kappa)}\right] dt =$$
$$= \frac{1}{\sqrt{p^2-1}} \sum_{n=0}^{\infty} \frac{\kappa^n}{n!}\left(p - \sqrt{p^2-1}\right)^n =$$
$$= \frac{1}{\sqrt{p^2-1}} \sum_{n=0}^{\infty} \frac{(\kappa p - \kappa\sqrt{p^2-1})^n}{n!} =$$
$$= \frac{1}{\sqrt{p^2-1}} \exp(\kappa p - \kappa\sqrt{p^2-1}).$$

Hence, it follows

$$\bar{\varphi}(p)\exp(-\kappa p) = \frac{1}{\sqrt{p^2-1}}\exp(-\kappa\sqrt{p^2-1})$$

and, further,

$$\frac{1}{\sqrt{p^2-1}}\exp(-\kappa\sqrt{p^2-1}) \leftarrow$$
$$\leftarrow \varphi(t-\kappa)\eta(t-\kappa) = \tag{14}$$
$$= I_0(\sqrt{t^2-\kappa^2})\eta(t-\kappa).$$

Let us replace κ by δ, and t by σt in relation (14) and make transformations using the similarity theorem [2]; let us then introduce the notation $\xi = \delta/\sigma$. We find

$$\frac{1}{\sqrt{p^2-\sigma^2}}\exp(-\xi\sqrt{p^2-\sigma^2}) \leftarrow$$
$$\leftarrow I_0(\sigma\sqrt{t^2-\xi^2})\eta(t-\xi). \tag{15}$$

Set $p \to p + \rho$ on the left-hand side of relation (15) and apply the shift theorem:

$$\frac{1}{\sqrt{(p+\rho+\sigma)(p+\rho-\sigma)}}\exp\left[-\xi\sqrt{(p+\rho+\sigma)(p+\rho-\sigma)}\right] \leftarrow$$
$$\leftarrow \exp(-\rho t) I_0\left(\sigma\sqrt{t^2-\xi^2}\right)\eta(t-\xi).$$

Denote $p + \sigma = 2\alpha$ and $\rho - \sigma = 2\beta$, whence $p = \alpha + \beta$ and $\sigma = \alpha - \beta$. Now, we finally arrive at the necessary result:

$$\frac{1}{\sqrt{(p+2\alpha)(p+2\beta)}}\exp\left[-\xi\sqrt{(p+2\alpha)(p+2\beta)}\right] =$$
$$= \frac{1}{\bar{\mu}(p)}\exp\left[-\xi\bar{\mu}(p)\right] \leftarrow \tag{16}$$
$$\leftarrow \exp(-\rho t) I_0(\sigma\sqrt{t^2-\xi^2})\eta(t-\xi).$$

From relation (16), we find

$$\frac{1}{p\bar{\mu}(p)}\exp\left[-\xi\bar{\mu}(p)\right] \leftarrow$$
$$\leftarrow \left[\int_\xi^t \exp(-\rho\tau) I_0(\sigma\sqrt{\tau^2-\xi^2})d\tau\right]\eta(t-\xi). \tag{17}$$

Application of the convolution theorem to relation (16) gives

$$\frac{1}{\bar{\mu}(p)}\exp[-\xi\bar{\mu}(p)]\bar{f}(p) \leftarrow$$
$$\leftarrow \left[\int_\xi^t f(t-\tau)\exp(-\rho t) I_0\left(\sigma\sqrt{t^2-\xi^2}\right)d\tau\right]\eta(t-\xi). \tag{18}$$

Differentiation of relation (18) with respect to ξ gives

$$\exp\left[-\xi\bar{\mu}(p)\right]\bar{f}(p) \leftarrow f(t-\xi)\exp(-\rho\xi) +$$
$$+ \sigma\xi\int_\xi^t f(t-\tau)\exp(-\rho\tau)\frac{I_1(\sigma\sqrt{\tau^2-\xi^2})}{\sqrt{\tau^2-\xi^2}}d\tau, \tag{19}$$
$$t > \xi, 0, t < \xi.$$

Set $\bar{f}(p) = 1$ in relation (19), whence $f(t) = \delta(t)$ is the Dirac delta function, and relation (19) takes the form

$$\exp\left[-\xi\bar{\mu}(p)\right] \leftarrow$$
$$\leftarrow \left[\exp(-\rho\xi)\delta(t-\xi) + \sigma\xi\exp(-\rho t)\frac{I_1(\sigma\sqrt{t^2-\xi^2})}{\sqrt{t^2-\xi^2}}\right] \times \tag{20}$$
$$\times\ \eta(t-\xi).$$

Let then $\bar{f}(p) = \frac{1}{p}$ in relation (19); then, $f(t) = 1$, and the inverse transform has the form

$$\frac{1}{p}\exp\left[-\xi\bar{\mu}(p)\right] \leftarrow$$
$$\leftarrow \left[\exp(-\rho\xi) + \sigma\xi\int_\xi^t \exp(-\rho\tau)\frac{I_1(\sigma\sqrt{\tau^2-\xi^2})}{\sqrt{\tau^2-\xi^2}}d\tau\right] \times \tag{21}$$
$$\times\eta(t-\xi).$$

A number of problems in thermal shock theory [2] lead to the transform

$$\frac{1}{p}\sqrt{\frac{p+2\beta}{p+2\alpha}}\exp\left[-\xi\bar{\mu}(p)\right].$$

Find the inverse transform. From relation (17), we have

$$\frac{1}{p}\frac{\exp\left[-\xi\bar{\mu}(p)\right]}{\sqrt{(p+2\alpha)(p+2\beta)}}\leftarrow$$
$$\leftarrow\int_{\xi}^{t}\exp(-\rho\tau)I_0(\sigma\sqrt{\tau^2-\xi^2})d\tau,\ t>\xi. \tag{22}$$

Multiply relation (22) by 2β and add the result term by term with relation (6) to obtain

$$\frac{1}{p}\sqrt{\frac{p+2\beta}{p+2\alpha}}\exp\left[-\xi\bar{\mu}(p)\right]\leftarrow$$
$$\leftarrow\exp(-\rho t)I_0(\sigma\sqrt{t^2-\xi^2})+ \tag{23}$$
$$+2\beta\int_{\xi}^{t}\exp(-\rho\tau)I_0(\sigma\sqrt{\tau^2-\xi^2})d\tau, t>\xi.$$

Transformation of relation (23) using the expressions $(d/dx)I_0(x) = I_1(x)$ and $I_0(0) = 1$ gives

$$\frac{1}{p}\sqrt{\frac{p+2\beta}{p+2\alpha}}\exp\left[-\xi\bar{\mu}(p)\right]\leftarrow\exp(-\rho\xi)+$$
$$+\int_{\xi}^{t}\exp(-\rho\tau)\left[\frac{\sigma\tau I_1(\sigma\sqrt{\tau^2-\xi^2})}{\sqrt{\tau^2-\xi^2}}-\sigma I_0(\sigma\sqrt{\tau^2-\xi^2})\right]d\tau, \tag{24}$$
$$t>\xi.$$

Using the above operational calculus relations and theorems, let us write the inverse, important for applications, of the more general transform:

$$\sqrt{\frac{p+2\beta}{p+2\alpha}}\exp\left[-\xi\bar{\mu}(p)\right]\bar{f}(p)\leftarrow f(t-\xi)\exp(-\rho\xi)+$$
$$+\int_{\xi}^{t}f(t-\tau)\exp(-\rho\tau)\left[\frac{\sigma\tau I_1(\sigma\sqrt{\tau^2-\xi^2})}{\sqrt{\tau^2-\xi^2}}-\sigma I_0(\sigma\sqrt{\tau^2-\xi^2})\right]d\tau,$$
$$t>\xi. \tag{25}$$

Expression (25) has a number of practically important special cases at given $f(t)\to\bar{f}(p)$.

The above operational relations have resolved the problem of finding analytical solutions of Eq. (10) under generalized boundary conditions. However, this problem has an interesting continuation, which is the possibility to represent one and the same analytical solution as different functional expressions. It is significant in this case that a certain cumbersomeness of the analytical writing of solutions can be simplified using special transformations leading to new, previously unknown, analytical solutions. Let us demonstrate this by the example of the Dirichlet boundary value problem for Eq. (10) (the boundary conditions are presented above). An analytical solution of the problem based on relation (21) has the form

$$W(\xi,\tau)=$$
$$=\left[\exp\left(-\frac{\xi}{2\beta}\right)+\frac{\xi}{2\beta}\int_{\xi}^{\tau/\beta}\exp\left(-\frac{x}{2\beta}\right)\frac{I_1\left(\frac{1}{2\beta}\sqrt{x^2-\xi^2}\right)}{\sqrt{x^2-\xi^2}}dx\right]\times \tag{26}$$
$$\times\eta(\tau-\xi\beta)=\Psi_1(\xi,\tau)\eta(\tau-\xi\beta).$$

The transform of sought function (26) has the form

$$\overline{W}(\xi,p)=(1/p)\exp\left[-\beta\xi\sqrt{p(p+1/\beta^2)}\right],$$

the inverse transform of which can be written using the Riemann–Mellin integral as follows:

$$\frac{1}{p}\exp\left[-\beta\xi\sqrt{p(p+1/\beta^2)}\right]\leftarrow$$
$$\leftarrow\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\frac{1}{p}\exp\left[p\tau-\beta\xi\sqrt{p(p+1/\beta^2)}\right]dp= \tag{27}$$
$$=\Psi_2(\xi,\tau).$$

The integrand in relation (27) satisfies the conditions of Jordan's lemma [2] and has two points of branching. Calculation of contour integral (27) gives

$$\Psi_2(\xi,\tau)=1-\frac{1}{\pi}\int_0^{1/\beta^2}\exp(-\rho\tau)\frac{\sin\xi\beta\sqrt{\rho\left(\frac{1}{\beta^2}-\rho\right)}}{\rho}d\rho. \tag{28}$$

Let us now show that the analytical solution of the Dirichlet boundary value problem in the form $W(\xi,\tau)=\Psi_1(\xi,\tau)\eta(\tau-\xi\beta)$ are $W(\xi,\tau)=\Psi_2(\xi,\tau)\eta(\tau-\xi\beta)$ equivalent; i.e., $\Psi_1(\xi,\tau)=\Psi_2(\xi,\tau)$. We have

$$\Psi_1(\xi,\tau)=\frac{\partial}{\partial\xi}\left[-\int_{\xi}^{\tau/\beta}\exp\left(-\frac{x}{2\beta}\right)I_0\left(\frac{1}{2\beta}\sqrt{x^2-\xi^2}\right)dx\right].$$

Differentiation of both sides with respect to τ gives

$$\left[\Psi_1\right]'_\tau=\frac{\partial}{\partial\xi}\left[-\frac{1}{\beta}\exp\left(-\frac{\tau}{2\beta^2}\right)I_0\left(\frac{1}{2\beta}\sqrt{\frac{\tau^2}{\beta^2}-\xi^2}\right)\right]=$$
$$=\frac{\partial}{\partial\xi}\left[-\frac{1}{\beta}\exp\left(-\frac{\tau}{2\beta^2}\right)J_0\left(\frac{1}{2\beta^2}\sqrt{(\beta\xi)^2-\tau^2}\right)\right].$$

Let us now use the (quite rare) integral

$$\int_0^a\frac{\exp(-py)}{\sqrt{ay-y^2}}\cos c\sqrt{ay-y^2}\,dy=$$
$$=\pi\exp\left(-\frac{ap}{2}\right)J_0\left(\frac{a}{2}\sqrt{c^2-p^2}\right).$$

We find

$$\left[\Psi_1(\xi,\tau)\right]'_\tau =$$

$$= \frac{\partial}{\partial\xi}\left[-\frac{1}{\beta\pi}\int_0^{1/\beta^2}\frac{\exp(-\rho\tau)}{\sqrt{\rho\left(\frac{1}{\beta^2}-\rho\right)}}\cos\left(\beta\xi\sqrt{\rho\left(\frac{1}{\beta^2}-\rho\right)}\right)d\rho\right] =$$

$$= \frac{1}{\pi}\int_0^{1/\beta^2}\exp(-\rho\tau)\sin\beta\xi\sqrt{\rho\left(\frac{1}{\beta^2}-\rho\right)}d\rho.$$

Integration with respect to τ gives

$$\Psi_1(\xi,\tau) = -\frac{1}{\pi}\int_0^{1/\beta^2}\exp(-\rho\tau)\frac{\sin\beta\xi\sqrt{\rho\left(\frac{1}{\beta^2}-\rho\right)}}{\rho}d\rho + C.$$

Under the conditions of the problem, $\Psi_1(0,\tau) = 1$; therefore, $C = 1$, and we finally obtain

$$\Psi_1(\xi,\tau) =$$

$$= 1 - \frac{1}{\pi}\int_0^{1/\beta^2}\exp(-\rho\tau)\frac{\sin\beta\xi\sqrt{\rho\left(\frac{1}{\beta^2}-\rho\right)}}{\rho}d\rho =$$

$$= \Psi_2(\xi,\tau).$$

Thus, it was shown that

$$W(\xi,\tau) = \Psi_1(\xi,\tau)\eta(\tau-\xi\beta) = \Psi_2(\xi,\tau)\eta(\tau-\xi\beta). \quad (29)$$

Figure 2 presents the curves $W(\xi, \tau)$ in the section $\xi = 2$; both curves calculated from formulas (29) virtually coincide. The above reasoning can also be extended to the Neumann and Cauchy boundary value problems under generalized boundary conditions, which emphasizes the specificity of hyperbolic transfer models.

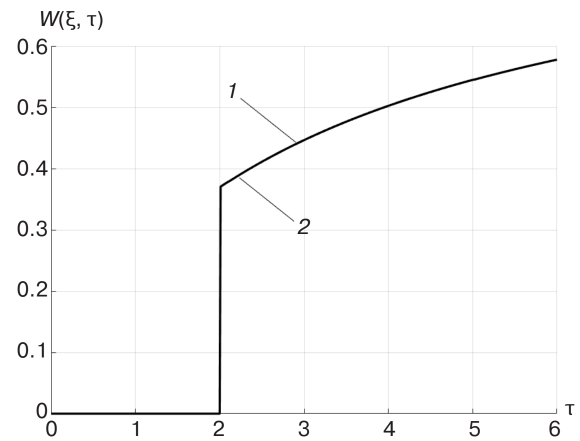

Fig. 2. Results of calculating the function $W(\xi, \tau)$ in the section $\xi = 2$ ($\beta = 1$): point 1—by (26), point 2—by (28)

Canonical finite domains

Mathematical models for Eq. (10) in the domain $\xi \in [0, \xi_0]$, $\tau \geq 0$ under generalized boundary conditions hardly constitute a necessary apparatus of operational calculus, which significantly complicates the determination of their exact analytical solutions. Let us consider a number of operational relations that are characteristic of this case. Find the inverse of the transform

$$\frac{(H-\beta p)^n}{(H+\beta p)^{n+1}}. \quad (30)$$

Use the reference formula [27]

$$\frac{(1-p)^n}{p^{n+1}} = \sum_{k=0}^{n}(-1)^k C_n^k p^{k-n-1} =$$

$$= (-1)^n\sum_{m=0}^{n}(-1)^m C_n^m p^{-(m+1)} \leftarrow$$

$$\leftarrow (-1)^n\sum_{m=0}^{n}(-1)^m C_n^m\frac{1}{m!}t^m =$$

$$= (-1)^n L_n(t),$$

where $L_n(t) = \frac{1}{n!}\exp(t)\frac{d^n}{dt^n}\left[t^n\exp(-t)\right]$ is the Laguerre polynomial [27].

Sequentially using the operational calculus theorems $(1/k)\bar{f}(pk) \leftarrow f(kt)$, $\bar{f}(p-k) \leftarrow \exp(kt)f(t)$, and $\bar{f}(p)\exp(-pt_0) \leftarrow f(t-t_0)\eta(t-t_0)$, we find the sought inverse transform

$$\frac{(H-\beta p)^n}{(H+\beta p)^{n+1}} \leftarrow \frac{(-1)^n}{\beta}\exp\left(-\frac{H}{\beta}t\right)L_n\left(2\frac{H}{\beta}t\right). \quad (31)$$

By similar reasoning, one can show that

$$\frac{(H+\beta p)^n}{(H-\beta p)^{n+1}} \leftarrow \frac{1}{\beta}\exp\left(\frac{H}{\beta}t\right)L_n^*\left(2\frac{H}{\beta}t\right), \quad (32)$$

where $L_n^*(t) = (-1)^{n+1}\sum_{m=0}^{n}C_n^m\frac{1}{m!}t^m$ is the Kartashov polynomial.

Let us find the inverse of the transform $\bar{f}(\sqrt{(p+a)^2-b^2}$, if $\bar{f}(p) \leftarrow f(t)$. Use the Efros theorem

$$\bar{f}\left[\bar{\varphi}_1(p)\right]\bar{\varphi}_2(p) \leftarrow \int_0^t f(\tau)\Psi(\tau,t)d\tau,$$

$$\Psi(\tau,t) \rightarrow \exp\left[-\tau\bar{\varphi}_1(p)\right]\bar{\varphi}_2(p). \quad (33)$$

We find

$$\bar{f}\left(\sqrt{(p+a)^2-b^2}\right)\leftarrow$$

$$\leftarrow \exp(-at)\left[f(t)+b\int_0^t yf(y)\frac{I_1\left(b\sqrt{t^2-y^2}\right)}{\sqrt{t^2-y^2}}dy\right]. \quad (34)$$

Similarly, we find

$$\frac{1}{p}\bar{f}\left(\sqrt{(p+a)^2-b^2}\right)\exp\left[-\gamma\sqrt{(p+a)^2-b^2}\right]\leftarrow$$

$$\leftarrow\int_\gamma^t f(\tau-\gamma)\left[\exp(-a\tau)+b\tau\int_\tau^t \exp(-ay)\frac{I_1\left(b\sqrt{y^2-\tau^2}\right)}{\sqrt{y^2-\tau^2}}dy\right]d\tau. \quad (35)$$

Relation (35) has a number of special cases, including the practically important expression

$$\left.\begin{aligned}&\frac{1}{p}\bar{f}\left(\sqrt{\beta^2p^2+p}\right)\exp\left(-\gamma\sqrt{\beta^2p^2+p}\right)\leftarrow\\&\leftarrow\int_0^t \exp\left(-\frac{\tau}{2\beta^2}\right)\varphi(\gamma,\tau)d\tau,\\&\varphi(\gamma,t)=f^*(t-\gamma\beta)+\\&+\frac{1}{2\beta^2}\int_0^t yf^*(y-\gamma\beta)\frac{I_1\left(\frac{1}{2\beta^2}\sqrt{t^2-y^2}\right)}{\sqrt{t^2-y^2}}dy,\\&f^*(t)=\frac{1}{\beta}f(t/\beta)\eta(t).\end{aligned}\right\}. \quad (36)$$

Let then $\bar{S}(p)=\sqrt{\beta^2p^2+p}$. From the previous relations, one can find

$$\left.\begin{aligned}&\frac{\left[H-\bar{S}(p)\right]^n}{\left[H+\bar{S}(p)\right]^{n+1}}\exp\left[-\gamma\bar{S}(p)\right]\leftarrow\\&\leftarrow\exp\left(-\frac{t}{2\beta^2}\right)\left[f(t-\gamma\beta\right]+\\&+\frac{1}{2\beta^2}\int_0^t f(\tau-\gamma\beta)\frac{\tau I_1\left(\frac{1}{2\beta^2}\sqrt{t^2-\tau^2}\right)}{\sqrt{t^2-\tau^2}}d\tau,\\&f(t)=\frac{(-1)^n}{\beta}\exp\left(-\frac{H}{\beta}t\right)L_n\left(\frac{2H}{\beta}t\right)\eta(t)\end{aligned}\right\}. \quad (37)$$

As applications of the obtained results, let us consider two model that are characteristic of thermal shock:

$$\frac{\partial W_i}{\partial\tau}=\frac{\partial^2W_i}{\partial\xi^2}-\beta^2\frac{\partial^2W_i}{\partial\tau^2},\ 0<\xi<\xi_0,\ \tau>0,\ (i=1,2), \quad (38)$$

$$W_i|_{\tau=0}=(\partial W_i/\partial\tau)_{\tau=0}=0,\ 0\le\xi\le\xi_0,$$

$$W_1|_{\xi=0}=1,\ W_1|_{\xi=\xi_0}=0,\ \tau>0, \quad (39)$$

$$W_2|_{\xi=0}=1,\ (\partial W_2/\partial\xi)_{\xi=\xi_0}=0,\ \tau>0. \quad (40)$$

In the Laplace transform space, we find

$$\bar{W}_1(\xi,p)=\frac{1}{p}\frac{\sinh\left[(\xi_0-\xi)\bar{S}(p)\right]}{\sinh\left[\xi_0\bar{S}(p)\right]},$$

$$\bar{W}_2(\xi,p)=\frac{1}{p}\frac{\cosh\left[(\xi-\xi_0)\bar{S}(p)\right]}{\cosh\left[\xi_0\bar{S}(p)\right]}.$$

Transform the hyperbolic functions in the fractions:

$$\bar{W}_1(\xi,p)=\sum_{k=0}^{\infty}\frac{1}{p}\left\{\exp\left[-\gamma_{1k}\bar{S}(p)\right]-\exp\left[-\gamma_{2k}\bar{S}(p)\right]\right\},$$

$$\bar{W}_2(\xi,p)=$$

$$=\sum_{k=0}^{\infty}\frac{(-1)^k}{p}\left\{\exp\left[-\gamma_{1k}\bar{S}(p)\right]+\exp\left[-\gamma_{2k}\bar{S}(p)\right]\right\},$$

where

$$\gamma_{1k}=2k\xi_o+\xi,\ \gamma_{2k}=$$

$$=2(k+1)\xi_0+\xi,\ \bar{S}(p)=\sqrt{\beta^2p^2+p}.$$

Calculation of the inverse transforms using the above relations gives

$$W_1(\xi,\tau)=\sum_{k=0}^{\infty}\left\{\Psi\left[\gamma_{1k}(\xi),\tau\right]-\Psi\left[\gamma_{2k}(\xi),\tau\right]\right\}, \quad (41)$$

$$W_2(\xi,\tau)=\sum_{k=0}^{\infty}(-1)^k\left\{\Psi\left[\gamma_{1k}(\xi),\tau\right]+\Psi\left[\gamma_{2k}(\xi),\tau\right]\right\}, \quad (42)$$

where

$$\Psi(\gamma_{ik},\tau)=$$

$$=\left[\exp\left(-\frac{\gamma_{ik}}{2\beta}\right)+\frac{\gamma_{ik}}{2\beta}\int_{\gamma_{ik}/2\beta}^{\tau/2\beta^2}\exp(-y)\frac{I_1\left(\sqrt{y^2-\left(\gamma_{ik}/2\beta\right)^2}\right)}{\sqrt{y^2-\left(\gamma_{ik}/2\beta\right)^2}}dy\right]\times$$

$$\times\ \eta(\tau-\gamma_{ik}\beta). \quad (43)$$

Note that solutions (41)–(43) were previously unknown in analytical thermal physics. All the nine boundary conditions for $W(\xi,\tau)$ in the domain $\xi\in[0,\xi_0]$, $\tau\ge 0$ can be analyzed similarly, and the studied problem for a finite domain can thus be considered resolved. At the time, here, as above, analytical solutions can be obtained as other functional expressions, equivalent to the presented ones. This is one of the specific features of hyperbolic transfer models. Numerical implementation of the obtained relations seems to involve no fundamental difficulties in view of the potential of the existing analytical thermal physics software.

CONCLUSIONS

The inverse transforms of nonstandard operational (Laplace) transforms are presented. These inverse transforms are encountered in operational solutions of a wide class of problems of local nonequilibrium processes of (heat, mass, momentum) transfer, electrical circuits, hydrodynamics, vibrations, thermomechanics, etc. Illustrative examples were given, and it was demonstrated that it is possible to construct analytical solutions of boundary value problems of unsteady-state heat conduction in a partially bounded domain in the form of various functional expressions, for which the equivalence was proven. The presented analytical solutions in canonical domains are new in analytical thermal physics.

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