Russian Technological Journal. 2024;12(5):77–89 ISSN 2500-316X (Online)

Mathematical modeling

Математическое моделирование

UDC 621.372.8
https://doi.org/10.32362/2500-316X-2024-12-5-77-89
EDN GTXQII

RESEARCH ARTICLE

Models of symmetric three-layer waveguide structures with graded-index core and nonlinear optical liners

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Abstract
Objectives. Determining the patterns of dispersion properties of waveguide modes of the optical range in layered media with distributed optical properties is a both a pressing and significant matter for study. It has fundamental and applied importance in nonlinear optics and optoelectronics. The combination of a nonlinear response and graded-index distributions of the optical properties of adjacent layers of a layered structure enables the desired values of the output characteristics using a wide range of control parameters to be selected easily. This renders such waveguides the most promising from the point of view of possible technical applications. The aim of this paper is to develop the theory of three-layer planar waveguide structures with a graded-index core and nonlinear optical liners with arbitrary profiles. By doing so it may be possible to find exact analytical solutions to nonlinear stationary wave equations describing explicitly the transverse electric field distribution of waveguide modes.
Methods. The analytical methods of mathematical physics and the theory of special functions applied to nonlinear and waveguide optics are used herein.
Results. The study provides a theoretical description of transverse stationary waves propagating along a symmetrical three-layer planar waveguide structure consisting of the inner graded-index layer sandwiched between nonlinear optical plates. It assumes an arbitrary spatial profile of the interlayer dielectric constant and the nature of the nonlinear response of the liner medium. The mathematical model of this waveguide structure formulated herein is based on nonlinear equations with distributed coefficients. The solutions obtained describe in general terms the transverse distribution of the amplitude of the electric field envelope. The transverse symmetry of the three-layer waveguide structure enables even and odd stationary modes corresponding to symmetric and antisymmetric transverse field profiles to be excited in it. A method was developed for constructing even (symmetric) and odd (antisymmetric) solutions which exist at certain discrete values of the effective refractive index/propagation constant. These discrete spectra were obtained in layers with graded-index linear, parabolic, and exponential profiles. The symmetrical three-layer waveguide structure with inner graded-index layer characterized by parabolic spatial profile and outer liners as Kerr nonlinear optical media is analyzed in detail, as an example of the application of the formulated theory. Analysis of the resulting exact analytical solution indicates that the electric field strength for the fundamental and first-order modes increases with increasing parabolic profile parameter, characterizing the relative change of the dielectric constant in the interlayer, while decreasing for higher order modes.
Conclusions. The theory developed in this paper supports the unambiguous description of the transverse distributions of the stationary electric field in planar symmetrical three-layer waveguides in an explicit analytical form. The results extend the understanding of the physical properties of nonlinear waves and the localization patterns of light beams in distributed media, and may be useful in the design of various optical waveguide devices.

Keywords: layered structure, layered waveguide, optical waveguide, nonlinear optics, optical nonlinearity, graded-index layer, nonlinear waves, Kerr nonlinear optical media, guided waves, waveguide mode

• **Submitted:** 01.03.2024 • **Revised:** 29.03.2024 • **Accepted:** 05.08.2024

For citation: Savotchenko S.E. Models of symmetric three-layer waveguide structures with graded-index core and nonlinear optical liners. *Russ. Technol. J.* 2024;12(5):77–89. https://doi.org/10.32362/2500-316X-2024-12-5-77-89

Financial disclosure: The author has no financial or property interest in any material or method mentioned.

The author declares no conflicts of interest.

НАУЧНАЯ СТАТЬЯ

Модели симметричных трехслойных волноводных структур с градиентной сердцевиной и нелинейно-оптическими обкладками

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Резюме

Цели. Выявление закономерностей дисперсионных свойств волноводных мод оптического диапазона в слоистых средах с распределенными оптическими характеристиками представляет собой актуальную и важную задачу, имеющую фундаментальное и прикладное значение в нелинейной оптике и оптоэлектронике. Сочетание нелинейного отклика и градиентных распределений оптических свойств соседних слоев слоистой структуры дает возможность легко подобрать требуемые значения выходных характеристик с помощью широкого ряда управляющих параметров, что делает такие волноводы наиболее перспективными с точки зрения возможных технических приложений. Цель работы – развитие теории трехслойных плоских волноводных структур с градиентной сердцевиной и нелинейно-оптическими обкладками с произвольными профилями, в рамках которой представляется возможным нахождение точных аналитических решений нелинейных стационарных волновых уравнений, описывающих в явном виде поперечное распределение электрического поля волноводных мод.

Методы. Использованы аналитические методы математической физики и теории специальных функций применительно к нелинейной и волноводной оптике.

Результаты. Проведено теоретическое описание поперечных стационарных волн, распространяющихся вдоль плоской симметричной трехслойной волноводной структуры, состоящей из внутреннего градиентного слоя, зажатого между нелинейно-оптическими обкладками, причем пространственный профиль диэлектрической проницаемости прослойки и вид нелинейного отклика среды обкладок предполагаются произвольными. Сформулирована математическая модель такой волноводной структуры на основе нелинейных уравнений с распределенными коэффициентами. Получены решения, описывающие в общем виде поперечное распределение амплитуды огибающей электрического поля. В силу поперечной симметрии трехслойной волноводной структуры в ней могут возбуждаться четные и нечетные стационарные моды, соответствующие симметричным и антисимметричным поперечным профилям поля. Разработан метод построения четных (симметричных) и нечетных (антисимметричных) решений, существующих при определенных дискретных значениях эффективного показателя преломления / константы распространения. Такие дискретные спектры получены в слоях с градиентными линейным, параболическим и экспоненциальным профилями. В качестве примера применения сформулированной теории детально проанализирован случай симметричной трехслойной волноводной структуры, внутренний градиентный слой которой характеризуется параболическим пространственным профилем, а внешние обкладки представляют собой керровские нелинейно-оптические среды. На основе анализа полученного точного аналитического решения установлено, что напряженность электрического поля для основной моды и моды первого порядка увеличивается с ростом параметра параболического профиля, характеризующего относительное изменение диэлектрической проницаемости в прослойке, однако уменьшается для мод более высоких порядков.

Выводы. Развитая в данной работе теория позволяет наглядно описать в явном аналитическом виде поперечные распределения стационарного электрического поля в плоских симметричных трехслойных волноводах. Полученные результаты расширяют представления о физических свойствах нелинейных волн и закономерностях локализации световых пучков в распределенных средах и могут быть полезными для разработки различных оптических волноводных устройств.

Ключевые слова: слоистая структура, слоистый волновод, оптический волновод, нелинейная оптика, оптическая нелинейность, градиентный слой, нелинейные волны, керровские нелинейно-оптические среды, управляемые волны, волноводная мода

• **Поступила:** 01.03.2024 • **Доработана:** 29.03.2024 • **Принята к опубликованию:** 05.08.2024

Для цитирования: Савотченко С.Е. Модели симметричных трехслойных волноводных структур с градиентной сердцевиной и нелинейно-оптическими обкладками. *Russ. Technol. J.* 2024;12(5):77−89. https://doi.org/10.32362/2500-316X-2024-12-5-77-89

Прозрачность финансовой деятельности: Автор не имеет финансовой заинтересованности в представленных материалах или методах.

Автор заявляет об отсутствии конфликта интересов.

INTRODUCTION

The development of optical waveguides with desired characteristics is an important applied problem in the area of nonlinear optics [1–3]. The successful resolution of this issue requires theoretical modeling of the designed systems which will enable their properties to be described, characteristics to be predicted and the development process optimized. Thus, much attention is paid in scientific literature to the development of theoretical foundations for modeling the processes of excitation, propagation, and localization of electromagnetic waves in a variety of optical media [4, 5].

The requisite and often unique characteristics of fields in waveguide structures can be obtained most effectively in a combination of media with different optical properties [6, 7]. Classes of media where optical properties depend significantly on the spatial distribution of the refractive index (or dielectric permittivity) [8] or can be characterized by a nonlinear optical response in which the dielectric permittivity depends on the electric field intensity [9] are in particular considered promising and possessed of a wide variety of properties. The first group of media is called graded-index [10] while the second group is called nonlinear [11].

The dependence of optical characteristics on the quantities mentioned above may differ, and may be determined by the physical properties of real crystals. In particular, the most common form of nonlinear response is the linear dependence of dielectric permittivity on the square of amplitude (intensity) of the electric field (light), referred to as Kerr nonlinearity [12]. Waves and other localized disturbances in such media have been quite well studied in various modifications [13, 14]. This includes analytical methods [15, 16] which assume that exact solutions be obtained for the nonlinear wave equation used in various models [17, 18].

In order to describe the experimentally dependencies of the spatial distribution of optical characteristics as described above, a variety of functions (profiles) are used to model the change of the refractive index with distance from the optical media interface [19]. Certain profiles, such as linear [20], parabolic [21], exponential [22], and others [23, 24], allow exact analytical solutions to be established.

The theoretical study of waveguide properties of interfaces between graded-index and nonlinear media has in recent times intensified [25, 26]. In particular, solutions have been developed which describe the localization of light along the interface between the nonlinear Kerr medium and medium with linear [27, 28] and exponential refractive index profiles [29].

In terms of technical application, the studies on waveguide properties of multilayer media [30] including three-layer structures [31] are of great importance. Nonlinear waves in three-layer structures have been a focus of theoretical study for many years [32, 33], including in layered graded-index media [34]. In recent times, analytical solutions have been obtained for symmetric three-layer structures in which the inner layer is described by a symmetric linear graded-index profile. The outer layers are characterized by photorefractive nonlinear response [35], Kerr nonlinearity [36], and step nonlinearity [37]. The symmetric structure with a parabolic graded-index inner layer placed between media with Kerr nonlinearity has also been considered.

Due to the emerging variety of possible combinations of nonlinearities and graded-index layer profiles, it would be useful to construct a generalized model of a symmetric waveguide structure. This paper proposes a generalization of the model for the three-layer symmetric planar structure, in which the inner layer and the outer layers are characterized by an arbitrary graded-index profile and a nonlinear optical response, respectively. Substituting the particular type of dielectric permittivity profiles and the shape of nonlinear response into model equations allows analytical solutions to be obtained which describe the amplitude spatial distribution of the envelope perpendicular to structure layers. The resulting analytical expressions, in turn, allow localization patterns of light beams to be determined in layered waveguide structures.

1. THEORETICAL MODELING OF THREE-LAYER WAVEGUIDE STRUCTURE

1.1. Model formulation

We consider a three-layer planar structure which is symmetric about the center. It is made of nonmagnetic materials with optically homogeneous properties in the longitudinal direction. The interfaces between layers are assumed to be planar. We place the origin of the coordinates in the middle of the inner layer (core or interlayer), in the *yz* plane; with the *x*-axis perpendicular to the planes of interfaces and the *z*-axis along the layers in the direction of wave propagation. Let layer interfaces be located in planes $x = \pm a$ (then the thickness of the layer is assumed to be $2a$). The media in all layers are considered with no allowance for dielectric losses.

In the model considered herein, the inner layer is characterized by spatial inhomogeneity of optical properties in the direction transverse to the plane of the layers (graded-index layer), while the outer adjacent layers (liners) are characterized by optical nonlinearity (nonlinear layers), i.e., by the dependence of the refractive index (or dielectric permittivity) on the light intensity. In this case, the interlayer thickness is considered to be much less than the thicknesses of the outer liners. Therefore, when studying the distribution of the electric field localized near the core, the liners can be considered to be semi-limited media, neglecting the influence of boundaries located at a further distance when compared to the value of a. This consideration is acceptable provided that the field rapidly decreases at a distance from interfaces and becomes negligible before reaching the outer boundaries of thick liners.

Let a transverse electric wave (TE wave) propagate along interfaces of a three-layer waveguide structure whose electric field strength component can be written in the following form:

$$E_y(x,z) = \psi(x)e^{i(\beta z - \omega t)}, \qquad (1)$$

wherein $\psi(x)$ is spatial distribution of electric field strength in the transverse layer direction (envelope amplitude), ω is frequency, $\beta = kn$ is the propagation constant, $n = ck/\omega$ is effective refractive index, c is the speed of light in vacuum, $k = 2\pi/\lambda$ is wave number, λ is wavelength, and t is time.

It is known [4, 5] that field $\psi(x)$ is defined as the solution for the stationary equation (magnetic permeability is equal to unity):

$$\frac{d^2\psi(x)}{dx^2} + \{\varepsilon(x,I) - n^2\}k^2\psi(x) = 0, \qquad (2)$$

wherein dielectric permittivity of the three-layer waveguide structure can be written as follows:

$$\varepsilon(x,I) = \begin{cases} \varepsilon_G(x), & |x| < a, \\ \varepsilon_N(I), & |x| > a, \end{cases} \qquad (3)$$

wherein function $\varepsilon_G(x)$ defines the dependence of the dielectric permittivity on the spatial coordinate in the direction perpendicular to layers (dielectric permittivity of the graded-index layer), while function $\varepsilon_N(I)$ defines the dependence of the dielectric permittivity on the light intensity $I = |E|^2$, wherein E stands for the amplitude of the electric field strength (dielectric permittivity of the nonlinear layers).

We can represent the transverse field distribution in the following form:

$$\psi(x) = \begin{cases} \psi_N^{(-)}(x), & x < -a, \\ \psi_G(x), & |x| < a, \\ \psi_N^{(+)}(x), & x > a, \end{cases} \qquad (4)$$

Then the following equations may be derived from (2):

$$\frac{d^2\psi_N^{(-)}(x)}{dx^2} + \{\varepsilon_N(I) - n^2\}k^2\psi_N^{(-)}(x) = 0, \quad x < -a, \qquad (5)$$

$$\frac{d^2\psi_G(x)}{dx^2} + \{\varepsilon_G(x) - n^2\}k^2\psi_G(x) = 0, \quad |x| < a, \qquad (6)$$

$$\frac{d^2\psi_N^{(+)}(x)}{dx^2} + \{\varepsilon_N(I) - n^2\}k^2\psi_N^{(+)}(x) = 0, \quad x > a, \qquad (7)$$

which are supplemented by boundary conditions corresponding to the requirements of continuity of field components at layer interfaces:

$$\psi_N(\pm a \pm 0) = \psi_G^{(\pm)}(\pm a \pm 0),$$
$$\frac{d\psi_N(\pm a \pm 0)}{dx} = \frac{d\psi_G^{(\pm)}(\pm a \pm 0)}{dx}, \qquad (8)$$

as well as to the field vanishing condition at infinity: $|\psi_N^{(\pm)}(x)| \to 0, |x| \to \infty$.

In terms of physics, the requirement of limitation of the solution should be considered an obvious supplement. Thus, the set of Eqs. (5)–(7) and boundary conditions (8) represent a mathematical formulation of the proposed model for the three-layer waveguide structure with a dielectric permittivity profile described by means of the distributed expression (3).

1.2. The dispersion equation in the general case

In the inner layer, the solution to Eq. (6) can be represented as follows:

$$\psi_G(x) = C_1F_1(x) + C_2F_2(x), \tag{9}$$

wherein $C_{1,2}$ are the values depending on optical and geometrical parameters of the system and determined by boundary conditions (8). $F_{1,2}(x)$ are special functions which are linearly independent solutions to Eq. (6) at the given dielectric permittivity profile $\varepsilon_G(x)$. Since (6) is a linear homogeneous differential equation of the second order with a coefficient depending on variable x, its solutions are often expressed in the most general form through hypergeometric functions. In certain types of dielectric permittivity profiles, the solutions can be expressed through other pairs of linearly independent special functions, such as Bessel functions, Airy functions, and others. The main requirements for $F_{1,2}(x)$ are continuity and limitation on the interval $-a < x < a$ of these functions, as well as their derivatives $F'_{1,2}(x)$.

In the outer liners, the solutions to Eqs. (5) and (7) can be represented as follows:

$$\psi_N^{(\pm)}(x) = \psi_a^{(\pm)} \frac{\Psi_N^{(\pm)}(x)}{\Psi_N^{(\pm)}(\pm a)}, \tag{10}$$

wherein $\psi_a^{(\pm)}$ is field amplitudes at layer interfaces in planes $x = \pm a$. $\Psi_N^{(\pm)}(x)$ are solutions to nonlinear Eqs. (5) and (7) are limited in regions $x < -a$ and $x > a$, respectively, while satisfying requirement $|\Psi_N^{(\pm)}(x)| \to 0, |x| \to \infty$. The explicit form of functions $\Psi_N^{(\pm)}(x)$ is determined by the type of nonlinearity model of the outer liner medium. For example, for the most common Kerr nonlinearity, $\Psi_N^{(\pm)}(x)$ are expressed through hyperbolic functions depending on the nonlinear response sign.

Substituting solutions (9) and (10) into boundary conditions (8) results in the following system of algebraic equations for values $C_{1,2}$ and amplitudes $\psi_a^{(\pm)}$:

$$\begin{cases} \psi_a^{(\pm)} = C_1F_1(\pm a) + C_2F_2(\pm a), \\ \psi_a^{(\pm)}\varepsilon_{\text{Neff}}^{(\pm)} = C_1F_1'(\pm a) + C_2F_2'(\pm a), \end{cases} \tag{11}$$

where we denote

$$\varepsilon_{\text{Neff}}^{(\pm)} = \frac{1}{\Psi_N^{(\pm)}(\pm a)} \cdot \frac{d\Psi_N^{(\pm)}(\pm a)}{dx}. \tag{12}$$

The solvability condition of the system (11) allows the dispersion equation to be obtained which determined the values of the propagation constant for waveguide modes of the considered three-layer structure in the general case:

$$\Delta_1^{(+)}\Delta_2^{(-)} = \Delta_1^{(-)}\Delta_2^{(+)}, \tag{13}$$

wherein

$$\Delta_{1,2}^{(\pm)} = F_{1,2}(\pm a)\varepsilon_{\text{Neff}}^{(\pm)} - F'_{1,2}(\pm a). \tag{14}$$

This dispersion equation defines the relationship between propagation constant β, wave number k, the optical characteristics of the layers (unperturbed values of dialectic constants and parameters of dielectric permittivity dependencies in graded-index and nonlinear layers as determined by a certain type of model). The geometric parameter of the three-layer structure is considered the half-width of interlayer a.

The amplitude at one interface can be chosen as an independent characteristic, through which other parameters of solutions (9) and (10) can be expressed. In particular, the following ratio of amplitudes at the left and right interlayer boundaries may be derived from (11):

$$\frac{\psi_a^{(+)}}{\psi_a^{(-)}} = \frac{\Delta_1^{(-)}\Delta^{(+)}}{\Delta_1^{(+)}\Delta^{(-)}}, \tag{15}$$

wherein

$$\Delta^{(\pm)} = F_1(\pm a)F_2'(\pm a) - F_2(\pm a)F_1'(\pm a). \tag{16}$$

Then values $C_{1,2}$ can be written in the following form:

$$C_{1,2} = \psi_a^{(+)} \frac{\Delta_{1,2}^{(+)}}{\Delta^{(+)}} = \psi_a^{(-)} \frac{\Delta_{1,2}^{(-)}}{\Delta^{(-)}}. \tag{17}$$

Taking into account (15), field distribution in the inner layer can be rewritten in the following form:

$$\psi_{\rm G}(x) = \frac{\psi_a^{(\pm)}}{\Delta^{(\pm)}}\{\Delta_1^{(\pm)}F_1(x) + \Delta_2^{(\pm)}F_2(x)\}. \qquad (18)$$

Thus, the resulting expressions (10) and (18) determine the field distribution in the transverse layer direction. The parameters thereof are determined by expressions (12), (14), and (16) while the propagation constant is determined by dispersion Eq. (13).

1.3. Constructing waveguide modes of a given symmetry and discrete spectrum

Due to the symmetry of the considered three-layer waveguide structure, even and odd modes should clearly exist therein. They are described by symmetric and antisymmetric field distributions in the transverse direction, respectively. The solution to problem (5)–(8) can then be searched for on semiaxis $x > 0$. We continue it in the even or odd direction for symmetric or antisymmetric modes, respectively.

For symmetric distribution, the solutions should be even functions: $\psi_{\rm N}^{(+)}(-x) = \psi_{\rm N}^{(-)}(x)$ and $\psi_{\rm G}(-x) = \psi_{\rm G}(x)$, while for antisymmetric distribution, they should be odd: $\psi_{\rm N}^{(+)}(-x) = -\psi_{\rm N}^{(-)}(x)$ and $\psi_{\rm G}(-x) = -\psi_{\rm G}(x)$. The upper indices (±) can be omitted due to the given symmetry chosen.

The mode with given symmetry can be described by the following solution:

$$\psi_{\rm G}(x) = \psi_a \frac{F_{\rm G}(g(x))}{F_{\rm G}(g(a))}, \qquad (19)$$

where $F_{\rm G}(g)$ is the special function selected in a certain manner to resolve Eq. (6) on semiaxis $x > 0$, with internal argument $g(x)$. The explicit form thereof is related to the spatial dependence profile type of the inner layer dielectric permittivity. The argument g contains propagation constant β, as well as optical and geometric parameters of the waveguide system.

For the symmetric mode, function $F_{\rm G}$ should have an extremum at the symmetry center of the three-layer waveguide structure (at $x = 0$). This implies that the derivative of function $F_{\rm G}$ should go to zero at $x = 0$. For the antisymmetric mode, function $F_{\rm G}$ should go to zero itself. Due to the necessity for symmetry of the desired field profile, these requirements result in the spectrum of propagation constant values (or effective refractive index) becoming discrete.

In particular, it should be as follows for the symmetric mode:

$$\left.\frac{dF_{\rm G}(g(x))}{dx}\right|_{x=0} = F'_{\rm G}(g(0))g'(0) = 0. \qquad (20)$$

Thus, if $g'(0) \neq 0$, then

$$g(0) = \xi_j \ (j = 1, 2, \ldots), \qquad (21)$$

wherein ξ_j are zeros of the derivative of special function $F_{\rm G}$. Since argument g contains propagation constant β, resolving Eq. (21) with respect to it allows a discrete spectrum of its values $\beta = \beta(\xi_j)$ to be obtained. This is determined by the sequence of zeros of the derivative of special function $F_{\rm G}$ solving Eq. (6).

Similarly, it should be as follows for antisymmetric mode:

$$F_{\rm G}(g(0)) = 0. \qquad (22)$$

Hence,

$$g(0) = \zeta_j \ (j = 1, 2, \ldots), \qquad (23)$$

wherein ζ_j are zeros of special function $F_{\rm G}$. Solving Eq. (23) with respect to the propagation constant allows a discrete spectrum of its values $\beta = \beta(\zeta_j)$ to be obtained. This is determined by the sequence of zeros of special function $F_{\rm G}$ solving Eq. (6).

The solution to nonlinear Eq. (7) in the outer layer at $x > a$ can be represented in the following form:

$$\psi_{\rm N}(x) = \psi_a \frac{\Psi_{\rm N}(q_{\rm N}(x - a - x_{\rm N}))}{\Psi_{\rm N}(q_{\rm N}x_{\rm N})}, \qquad (24)$$

where the dependence of coefficient $q_{\rm N}$ on the system parameters is known for a certain nonlinear response model, while value $x_{\rm N}$ is determined by the boundary conditions. Function $\Psi_{\rm N}$ should have parity coinciding with that of function $F_{\rm G}$.

It should be noted that the solutions chosen in the form of (19) and (24) automatically satisfy the continuity conditions at interfaces of waveguide structure layers at $x = \pm a$.

In order to meet the continuity condition of the field derivative at the interface between the graded-index and nonlinear layers, (19) and (24) should be substituted into (8) at $x = a$ whence the following equation is derived:

$$\frac{F'_{\rm G}(g(a))g'(a)}{F_{\rm G}(g(a))} = \frac{q_{\rm N}\Psi'_{\rm N}(q_{\rm N}x_{\rm N})}{\Psi_{\rm N}(q_{\rm N}x_{\rm N})}, \qquad (25)$$

which allows value $x_{\rm N}$ to be determined depending on the optical and geometrical characteristics of the layered structure.

This shows the possible existence of waveguide modes of a given symmetry in the case when the propagation constant is given by a discrete spectrum of values.

2. RESULTS AND DISCUSSION

2.1. Some analytically solvable profiles of the graded-index layer

First, we consider the types of symmetric dielectric permittivity profiles in the graded-index layer for which exact analytical solutions are known. For comparison, we also consider the case of a step structure with the inner layer characterized by the constant dielectric permittivity value, independent of spatial coordinate (Fig. 1a): $\varepsilon_G = \varepsilon_0$ – const. Then the solution to Eq. (6) is determined by trigonometric functions and has the following form for even modes:

$$\psi_G(x) = \psi_a \frac{\cos(px)}{\cos(pa)}, \tag{26}$$

while for odd modes:

$$\psi_G(x) = \psi_a \frac{\sin(px)}{\sin(pa)}, \tag{27}$$

wherein $p^2 = k^2(\varepsilon_0 - n^2)$. These modes exist for values of effective refractive index $n^2 < \varepsilon_0$.

Symmetric graded-index profiles:

1) Linear (Fig. 1b):

$$\varepsilon_G(x) = \varepsilon_0\left(1 - \Delta\frac{|x|}{a}\right), \tag{28}$$

wherein ε_0 is the dielectric permittivity at the center of the waveguide structure symmetry, and $\Delta = (\varepsilon_0 - \varepsilon_a)/\varepsilon_0$ is the change in dielectric permittivity from ε_0 to the value of dielectric permittivity at the interface of layers ε_a.

The solution to Eq. (6) on interval $0 < x < a$ with linear profile (28) can be written as [36]:

$$\psi_G(x) = \psi_a \frac{\mathrm{Ai}(x/x_G + \delta)}{\mathrm{Ai}(a/x_G + \delta)}, \tag{29}$$

wherein $F_G = \mathrm{Ai}(g)$ is the Airy function, $g(x) = x/x_G + \delta$,

$$\delta = -(\varepsilon_0 - n^2)(ak/\varepsilon_0\Delta)^{2/3}, \tag{30}$$

$$x_G = \left(\frac{a}{k^2\varepsilon_0\Delta}\right)^{1/3}. \tag{31}$$

When constructing the even solution (as noted in Section 1.3), there is a necessary requirement that function (29) has an extremum in the middle of the waveguide at $x = 0$. Therefore, $\delta = \xi_j$, $j = 1, 2, \ldots$, where for a linear profile, ξ_j are zeros of the derivative of the Airy function: $\mathrm{Ai}'(\xi_j) = 0$: $\xi_1 = -1.018792972$, $\xi_2 = -3.248197582$, $\xi_3 = -4.820099211$, … Then (30) yields the following discrete spectrum of effective refractive index values:

$$n_j^2 = \varepsilon_0 - |\xi_j|(\varepsilon_0\Delta/ak)^{2/3}. \tag{32}$$

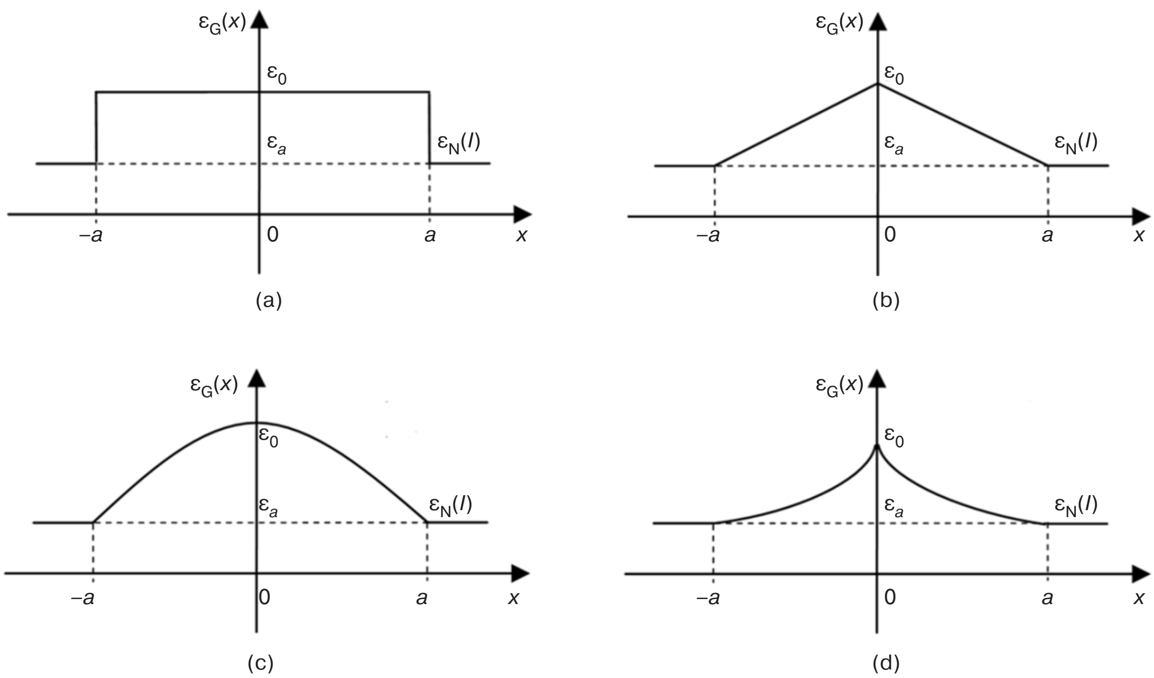

Fig. 1. Spatial symmetric profiles of dielectric permittivity of a three-layer waveguide structure: constant (a), linear (b), parabolic (c), and exponential (d)

When constructing an odd solution, there is a necessary requirement that function (29) turns to zero in the middle of the waveguide at $x = 0$. Therefore, $\delta = \zeta$ are zeros of the Airy function: $\mathrm{Ai}(\zeta_j) = 0$: $\zeta_1 = -2.338107410$, $\zeta_2 = -4.087949444$, $\zeta_3 = -5.520559828$, …

2) Parabolic (Fig. 1c):

$$\varepsilon_G(x) = \varepsilon_0\left(1 - \Delta\left(\frac{x}{a}\right)^2\right). \quad (33)$$

For parabolic profile (33), the limited solution to Eq. (6) is known to be expressed through Hermite polynomials $H_j(x)$:

$$\psi_G(x) = \psi_a \frac{H_j(x/x_0)}{H_j(a/x_0)} e^{-(x^2-a^2)/2x_0^2}, \quad (34)$$

wherein $j = 2m$ for even modes and $j = 2m + 1$ for odd modes, $m = 0, 1, 2, \ldots$, $x_0^2 = a/k\sqrt{\varepsilon_0\Delta}$, which exist at discrete values of effective refractive index [38]:

$$n_j^2 = \varepsilon_0 - (2j+1)\sqrt{\varepsilon_0\Delta}/ak. \quad (35)$$

3) Exponential (Fig. 1d):

$$\varepsilon_G(x) = \varepsilon_0\{1 - \Delta(1 - e^{-|x|/a})\}. \quad (36)$$

The solution to Eq. (6) with exponential profile (36) can be written as follows:

$$\psi_G(x) = \psi_a \frac{J_{2w}(2ve^{-|x|/2a})}{J_{2w}(2ve^{-1/2})}, \quad (37)$$

wherein $F_G = J_{2w}(g)$ is the first order Bessel function of order $w = ak(n^2 - \varepsilon_a)^{1/2}$, $g = 2ve^{-|x|/2a}$, $v = ak(\Delta\varepsilon_0)^{1/2}$ [4].

Even modes are defined by the dispersion equation $J'_{2w}(2v) = 0$. With the roots of the equation $J'_{\xi j}(2v) = 0$ denoted by ξ_j, the following discrete spectrum of effective refractive index values is obtained:

$$n_j^2 = \varepsilon_a + (\xi_j/2ak)^2. \quad (38)$$

Odd modes are defined by the dispersion equation $J_{2w}(2v) = 0$. With the roots of the equation $J_{\zeta j}(2v) = 0$ denoted be ζ_j, the following discrete spectrum of effective refractive index values is obtained:

$$n_j^2 = \varepsilon_a + (\zeta_j/2ak)^2. \quad (39)$$

The analysis of roots of such equations and corresponding spectra is given in [39].

It should be noted that there are exact analytical Eqs. (6) for other spatial profiles of dielectric permittivity. These include a smooth step described by a hyperbolic tangent [40], symmetric Epstein profile (inverted symmetric Pöschl–Teller potential) described by a hyperbolic cosine [41]. Such rather complex solutions are expressed through a hypergeometric function. They cause difficulties for simple analysis, so they have not been considered here.

Thus, the solutions quoted above are the exact analytical solutions which describe the field distributions for three different dielectric permittivity spatial profiles.

2.2. Some analytically solvable models of outer liner nonlinearity

Now we consider some types of nonlinear medium models of the outer liners in which the dielectric constant depends on the electric field strength and for which exact analytical solutions are known. For comparison, we also consider the linear medium in which outer layers are characterized by a constant value of dielectric permittivity, independent of the field strength: $\varepsilon_N = \varepsilon_{0N}$ – const. Then the solution to Eq. (7) limited on semiaxis $x > a$ may be written as follows form:

$$\psi_N(x) = \psi_a e^{-q_N(x-a)}, \quad (40)$$

wherein $q_N^2 = k^2(n^2 - \varepsilon_{0N})$. On the negative semiaxis, such a solution obviously continues in an even or odd way to describe symmetric and antisymmetric modes, respectively. Using conditions (8) for functions (19) and (40), the analog of dispersion Eq. (25) for a waveguide with linear liners $\gamma_G + q_N = 0$ may be derived. Here $\tilde{a}_G = F'_G(g(a))g'(a)/F_G(g(a))$, from which the spectrum of the effective refractive index values is obtained as follows:

$$n^2 = \varepsilon_{0N} + (\gamma_G/k)^2. \quad (41)$$

By relating (41) to the discrete spectrum obtained for a particular graded-index profile of the inner layer, constraints on the mode orders excited in the interlayer of a given thickness can be obtained.

The simplest models of nonlinear media of the outer liners are the following:

1) Kerr nonlinearity:

$$\varepsilon_N(I) = \varepsilon_{0N} + \alpha I, \quad (42)$$

where α is the Kerr nonlinearity coefficient, $I = |E|^2$ is field intensity. Then the even/odd solution to Eq. (7) with dielectric permittivity (42) at $|x| > a$ for self-focusing nonlinearity at $\alpha > 0$ has the following form:

$$\psi_N(x) = \pm\sqrt{\frac{2}{\alpha}} \cdot \frac{q}{k\mathrm{ch}(q(x \mp a \mp x_N))}. \quad (43)$$

Using conditions (8) for functions (19) and (43), we determine the following parameter value

$$x_N = -\frac{1}{q_N}\mathrm{Arth}\left(\frac{\gamma_G}{q_N}\right) \quad (44)$$

and the field amplitude at the boundary of layers

$$\psi_a = \sqrt{\frac{2}{\alpha}(n^2 - \varepsilon_{0N} - (\gamma/k)^2)}. \quad (45)$$

The discrete spectrum obtained for a particular graded-index profile of the inner layer should be substituted into expressions (43)–(45).

2) Step nonlinearity:

$$\varepsilon_N(I) = \begin{cases} \varepsilon_1, & I < I_s, \\ \varepsilon_2, & I > I_s, \end{cases} \quad (46)$$

wherein I_s is a threshold level of intensity (a known characteristic of the medium). When crossing this threshold, a sharp change from dielectric constant value ε_1 to ε_2 occurs [42, 43]. Thus, in the neighborhood of the layer interface in a nonlinear medium where $I > I_s$, a region (near-surface domain) of width x_s is formed with dielectric constant ε_2. Beyond it, further in the liners where $I < I_s$, the dielectric constant is ε_1. Such domains arise symmetrically on both sides of the inner layer [37]. The position of the boundaries of the near-surface domain is determined by x_s, found from additional field continuity requirements at domain boundaries, as follows:

$$\psi_N(\pm x_s + 0) = \psi_N(\pm x_s - 0) = I_s^{1/2},$$
$$\psi'_N(\pm x_s + 0) = \psi'_N(\pm x_s - 0). \quad (47)$$

In the step nonlinearity model, Eq. (7) with dielectric constant (46) decomposes into two:

$$\psi''_N(x) - (n^2 - \varepsilon_1)k^2\psi_N(x) = 0,\ I < I_s,\ |x| > x_s, \quad (48)$$

$$\psi''_N(x)(x) + (\varepsilon_2 - n^2)k^2\psi_N(x) = 0,\ I > I_s,\ a < |x| < x_s. \quad (49)$$

The solution to Eq. (48) at $n^2 > \varepsilon_1$ for even/odd modes may be written as follows:

$$\psi_N(x) = \pm I_s^{1/2} e^{\mp q_1(x \mp x_s)}, \quad (50)$$

wherein $q_1^2 = (n^2 - \varepsilon_1)k^2$, while the solution to Eq. (49) at $n^2 < \varepsilon_2$ has the following form:

$$\psi_N(x) = \pm\psi_a \cos(p_2(x \mp a) \mp \phi)/\cos(\phi), \quad (51)$$

wherein $p_2^2 = (\varepsilon_2 - n^2)k^2$ while values x_s, ϕ are determined from boundary conditions (8) and (47).

Substituting solutions (19), (50), and (51) into boundary conditions (8) and (47), parameters of even modes can be found (similar for odd modes), as follows:

$$\phi = \mathrm{arctg}\left(\frac{\gamma_G}{p_2}\right), \quad (52)$$

$$x_s = a + \frac{1}{p_2}\left\{\phi + \mathrm{arctg}\left(\frac{q_1}{p_2}\right)\right\}, \quad (53)$$

$$\psi_a = I_s^{1/2}\left(\frac{p_2^2 + q_1^2}{p_2^2 + \gamma_G^2}\right)^{1/2}. \quad (54)$$

The discrete values of effective refractive index obtained for a certain spatial profile of the inner graded-index layer should be substituted into expressions (50)–(54).

Thus, exact analytical solutions for two models of media nonlinearity are obtained.

2.3. Example of field distribution in a symmetric waveguide structure

The case when the inner graded-index layer is characterized by parabolic profile (33) and outer layers by Kerr self-focusing nonlinearity (42) may be considered as a particular example of the model of a symmetric three-layer waveguide structure.

In this structure, the spatial distribution of the electric field in the transverse layer direction is determined by expressions (34) and (43). The discrete spectrum of effective refractive index values is determined by expression (35) [38]. Limited by considering even modes for which $j = 2m$, $m = 0, 1, 2, \ldots$, the field distribution symmetric about the waveguide structure center can be written in the following form:

$$\psi(x) = \psi_0 \begin{cases} \sqrt{(n_{pm}^2 - n_{\mathrm{Geff}}^2)} e^{(a^2 - x^2)/2x_0^2} \dfrac{H_{2m}(x/x_0)}{H_{2m}(a/x_0)}, \\ |x| < a, \\ \dfrac{n_{pm}}{\cosh(kn_{pm}(x \mp a \mp x_N))}, \\ |x| > a, \end{cases} \quad (55)$$

where $n_{pm}^2 = \varepsilon_0 - \varepsilon_{0N} - (4m+1)\sqrt{\varepsilon_0 \Delta} / ak$ and

$$n_{\text{Geff}} = \frac{1}{kx_0}\left\{\frac{H'_{2m}(a/x_0)}{H_{2m}(a/x_0)} - \frac{a}{x_0}\right\}. \quad (56)$$

Thus, expression (55) represents an analytical solution to problem (5)–(8), when selecting symmetry, the parabolic profile of inner layer (33), and the self-focusing nonlinearity of liners (42).

The characteristic profiles of the solution (55) are shown in Fig. 2. It illustrates the impact of the relative change of dielectric constant in interlayer Δ on the electric field distribution at fixed values of other waveguide parameters for the main mode at $m = 0$ (Fig. 2a): even first-order mode $m = 1$ (Fig. 2b), and even second-order mode $m = 2$ (Fig. 2c).

The results show that the electric field strength increases with growing value of Δ for the main (Fig. 2a) and first (Fig. 2b) modes. However, the intensity decreases with increasing Δ for higher-order modes (Fig. 2c). Thus, the dependence of the field strength on Δ is not monotonic. Increasing the thickness of interlayer a gives the same effect.

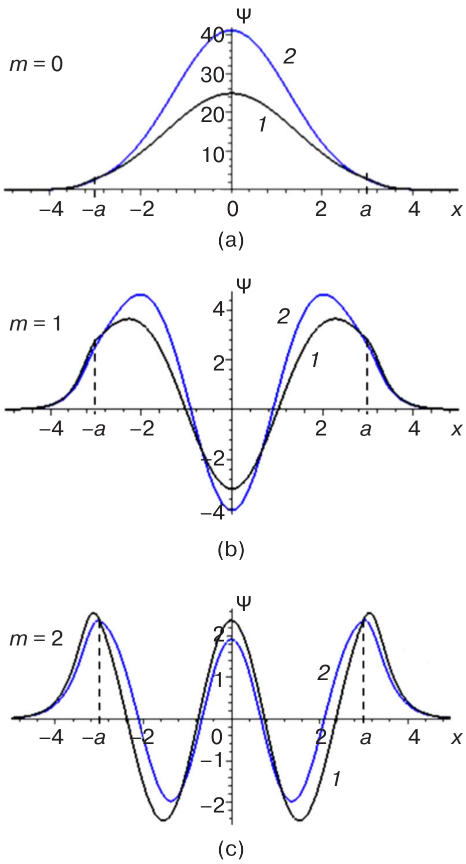

Fig. 2. Field distribution (55) at parameter values (in conventional units): $k = 0.65$, $\alpha = 6$, $\varepsilon_{0N} = 0.1$, $\varepsilon_0 = 30$, $a = 3$, and various $\Delta = 5$ (line 1), $\Delta = 8$ (line 2) for the first three even modes: $m = 0$ (a), $m = 1$ (b), $m = 2$ (c)

The existence of a discrete spectrum of effective refractive index values indicates the ability of waveguide modes to propagate at certain values of parabolic profile parameters. The propagation constant is related to the incidence angle of the beam exciting the waveguide mode. Thus, a discrete set of incidence angles should be taken into account since a waveguide mode of a certain order in the considered system can be excited only at a certain incidence angle. This depends on the dielectric permittivity at layer boundaries and the change in dielectric permittivity inside the graded-index layer.

Thus, the study obtained and analyzed the exact analytical solution which describes the field distribution in a symmetric three-layer waveguide structure where the inner graded-index layer and outer layers are characterized by parabolic profile and Kerr self-focusing nonlinearity.

CONCLUSIONS

This paper proposes a model of a symmetric three-layer planar waveguide structure with adjacent layers characterized by physically different optical properties. In particular, the inner layer is characterized by the dependence of dielectric permittivity on a spatial coordinate in the direction perpendicular to the interface plane. Outer layers are characterized by the dependence of dielectric permittivity on the electric field amplitude. In other words, the considered three-layer structure consists of the inner graded-index layer sandwiched between nonlinear optical liners. The spatial profile of the interlayer dielectric permittivity and the type of nonlinear response of the liner medium are assumed to be arbitrary.

The propagation of transverse electric waves with no losses taken into account is described from a theoretical point of view. The paper also formulates equations and boundary conditions for the transverse field distribution in a three-layer waveguide structure.

Solutions describing spatial distribution of the electric field transversely to layers are obtained in general form. The study shows that due to the transverse symmetry of the three-layer waveguide structure, even and odd stationary modes corresponding to symmetric and antisymmetric transverse field profiles can propagate along it. The paper also proposes a method of constructing even (symmetric) and odd (antisymmetric) solutions resulting in the existence of a discrete spectrum of the effective refractive index value.

Particular cases of specific spatial profiles of the inner layer dielectric permittivity with exact analytical solutions to the wave equation are considered. In particular, solutions for linear, parabolic, and exponential profiles described by corresponding special functions are given. The study also determined discrete spectra of the effective refractive index values in layers with considered graded-index profiles.

In addition, specific models of media nonlinearity, such as Kerr and step nonlinearities, are also considered. For such nonlinearities, exact analytical solutions are given to the nonlinear wave equation describing the dependencies of the stationary electric field amplitude on the distance from layer interfaces in nonlinear optical media.

The study provides a detailed analysis of symmetric three-layer waveguide structure, where the inner graded-index layer is characterized by a parabolic spatial profile and the outer liners represent Kerr nonlinear optical media. The exact analytical solution to the formulated boundary value problem describing the transverse symmetric field distribution for self-focusing nonlinearity is obtained and analyzed. The main mode intensity significantly exceeds the intensity of higher-order modes. The electric field strength grows with the increasing relative change of dielectric permittivity in the interlayer for the main mode and first-order modes. However, it decreases with an increase of its value for higher-order modes.

The results obtained can be useful in developing various optical waveguide devices. The proposed theory also expands understanding of physical properties of nonlinear waves and localization patterns of light beams in distributed media.

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[Original Russian Text: Lyakhomskaya K.D., Khadzhi P.I. Self-reflection effect in naïve model of nonlinear media. *Zhurnal Tekhnicheskoi Fiziki.* 2000;70(11):86–90 (in Russ.).]
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Translated from Russian into English by K. Nazarov
Edited for English language and spelling by Thomas A. Beavitt