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RESEARCH ARTICLE

On identification of interconnected systems

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Objectives. Interconnected control systems are widely used in various technical contexts, generally involving multichannel systems. However, due to the complexity of their description, the problem of identifying interconnected systems has received insufficient attention. As a result, simplified models are commonly used, which do not always reflect the specifics of the object. Thus, the synthesis of mathematical models for the description of interconnected control systems becomes a relevant endeavor. The paper sets out to develop an approach to obtaining models under conditions of incomplete a priori information. A mathematical model is developed on the example of two-channel systems (TCSs) having cross-connections and identical channels. The case of asymmetric cross-connections is considered, along with estimates of their influence on the quality of the adaptive identification system. The problem of estimating the identifiability of the parameters of a TCS is formulated on the basis of available experimental information and subsequent synthesis of the adaptive system. The proposed approach is then generalized to the case of an interconnected system.

Methods. The adaptive system identification and Lyapunov vector function methods are used along with implicit identification representation for the model.

Results. The influence of excitation constancy on estimates of the TCS parameters is demonstrated on the basis of the proposed approach for estimating the identifiability of TCS with cross-connections. The synthesis of adaptive algorithms of parameter estimation for TCSs with cross-connections based on input-output data is generalized to the case of interconnected systems. The results are applied to building models of tracking system and two-channel corrector for automatic control systems.

Conclusions. The features of adaptive identification of TCSs with identical channels, cross-connections and feedbacks are considered. The conditions for the TCS identifiability are obtained. Adaptive algorithms for estimating TCS parameters are synthesized. The proposed approach is generalized to the case of nonidentical channels and multi-connected systems. The exponential dissipativity of the adaptive identification system is verified. The proposed methods can be used in the development of systems for identification and control of complex dynamic systems.

Keywords: adaptive identification, identifiability, stability, two-channel system, Lyapunov vector function, multiconnected system, excitation constant

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НАУЧНАЯ СТАТЬЯ

Об идентификации взаимосвязанных систем

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Резюме

Цели. Проблеме идентификации взаимосвязанных систем до настоящего времени уделялось недостаточно внимания. Взаимосвязанные системы управления широко применяются в различных технических системах. Как правило, применяются многоканальные системы. Из-за сложности их описания применяют упрощенные модели, которые не всегда отражают специфику объекта. Поэтому задача синтеза математических моделей является актуальной. Целью настоящей работы является разработка подхода к получению моделей в условиях неполной априорной информации. Для решения задачи применяется адаптивный подход. На примере двухканальных систем (ДС) с перекрестными связями и идентичными каналами разрабатывается метод получения математической модели. Рассматривается случай асимметричных перекрестных связей, и получены оценки их влияния на качество работы адаптивной системы идентификации. В рамках предлагаемой постановки ставится задача оценки идентифицируемости параметров двухканальной системы на основе имеющейся экспериментальной информации и последующем синтезе адаптивной системы. Дается обобщение предлагаемого подхода на случай многосвязной системы.

Методы. Применяются метод адаптивной идентификации системы, неявное идентификационное представление для модели, метод векторных функций Ляпунова.

Результаты. Предложен подход к оценке идентифицируемости двухканальных систем с перекрестными связями. Показано влияние постоянства возбуждения на оценки параметров двухканальной системы. Предложен метод синтеза адаптивных алгоритмов оценки параметров для двухканальных систем с перекрестными связями по данным «вход-выход». Дано обобщение подхода на случай взаимосвязанных систем. Результаты применены для построения моделей системы слежения и двухканального корректора для систем автоматического регулирования.

Выводы. Рассмотрены особенности адаптивной идентификации двухканальных систем с идентичными каналами, перекрестными и обратными связями. Получены условия идентифицируемости ДС. Синтезированы адаптивные алгоритмы оценивания параметров ДС. Дано обобщение предлагаемого подхода на случай неидентичных каналов и многосвязных систем. Доказана экспоненциальная диссипативность адаптивной системы идентификации. Предлагаемые методы могут использоваться при разработке систем идентификации и управления сложными динамическими системами.

Ключевые слова: адаптивная идентификация, идентифицируемость, устойчивость, двухканальная система, векторная функция Ляпунова, многосвязная система, постоянство возбуждения

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INTRODUCTION

Interconnected systems (ICS) are widely used in control systems [1–5]. They are most commonly used to control robot and manipulator drives [6, 7] as well as forming the basis for various technical systems [8, 9].

In [10], the possibility of applying adaptive identification to ICS is considered. Here, the application of iterative-probabilistic method is proposed. In [11], a black-box model identification method for interconnected nonstationary dynamic objects with uncertainty is proposed. The development of adaptive algorithms for decentralized robust control with a reference model for ICS with time delay is discussed in [12]. Here, the asymptotic stability of the system is justified. The identifiability of a closed-loop interconnected stochastic system is considered in [13] along with the proposed decomposition of the system into subsystems. The identifiability both of separate elements of the system and separate closed loops without simultaneous identification of other elements and of loops of the system are considered. Sufficient conditions for almost certain convergence estimates of likelihood parameters are determined. The high-modular normalized adaptive lattice algorithm for multichannel filtering proposed in [14] is based on the least squares method.

In [15], an algorithm is presented for model identification using a neural network in the form of transfer functions for two-dimensional spatial ICS, which are causal for both open and closed loop. In [16], decentralized robust adaptive stabilization with output feedback is considered. The synthesis of control laws is based on adaptive nonlinear damping as well as the application of robust adaptive state observer and Lyapunov functions (LF). Similar results are obtained in [17, 18]. In [19], a method for identifying the ICS in rational form using input-output data is presented along with a Rasser-shaped model.

The adaptive control over a class of large-scale systems consisting of an arbitrary number of interacting subsystems with unknown parameters, nonlinearities, and bounded disturbances is considered in [20] using the reference model method. In [21], topological structural identification of large-scale subsystems having sparse flows of interconnected dynamical systems due to a small amount of data is considered. In [22], an approach to blind identification of a two-channel system (TCS) with finite impulse response from a limited number of output measurements in the presence of additive white noise is described. The proposed approach, which is based on data in the frequency domain, allows frequency-based estimation. In [23], fundamental problems of blind multichannel identification are considered on the basis of an analysis of some modern adaptive algorithms.

Review [24] covers some modern approaches based on decomposition of the problem of identifying systems with multiple inputs/single output. Proposed identification procedures combine low-dimensional solutions with the iterative version of the Wiener filter. The identification of stationary linear systems of this class are considered in [13, 25, 26]. In [25], a method for identifying multidimensional systems in the frequency domain using the correlation approach is proposed. A parametric model in the form of a vector-difference equation was further transformed into a corresponding frequency domain model. The identifiability of multidimensional discrete dynamical systems is considered in [26]. A priori estimates for the identifiability of coefficients are proposed on the basis of numerical characteristics of the asymptotic variance lower bound of coefficient estimates.

Thus, models in the form of transfer functions and in state space are used to study processes in ICS. Adaptive procedures are used to synthesize control algorithms. Adaptive control algorithms have been developed in the presence of unmodulated dynamics and disturbances. This condition can be explained by insufficient information about the state and parameters of the system or object, as well as the difficulty of accounting for interconnections in the system. The identification of multidimensional system parameters is based on applying statistical procedures, frequency methods, and neural network technologies. A number of studies deal with the application of adaptive methods, which are mainly used for adjusting (identifying) control device parameters. The resulting parametric uncertainties are compensated by selecting appropriate control algorithms. However, few publications cover the problem of TCS identification.

The present paper proposes a measurement-based approach to the ICS adaptive identification. The proposed approach to the identifiability of TCS having cross-connections involves various assumptions regarding the parameters of cross-connections. The stability of the adaptive system is demonstrated on the basis of obtained identifiability estimates to inform the considered approach to ICS identification.

1. TCSs

1.1. Problem statement

We consider the TCS with cross-connection. The links in the channels are considered identical. Transfer functions are used to analyze these systems [27]. For identification tasks, describing TCSs in the state space is more convenient. Let the system contain n sequentially interconnected vertical layers:

1) first layer

$$\begin{cases} \dot{\mathbf{X}}_{11} = \mathbf{A}_1 \mathbf{X}_{11} + \mathbf{B}_1 \mathbf{v}_{11}, \\ y_{11} = \mathbf{C}_1^T \mathbf{X}_{11}, \\ \dot{\mathbf{X}}_{21} = \mathbf{A}_1 \mathbf{X}_{21} + \mathbf{B}_1 \mathbf{v}_{21}, \\ y_{21} = \mathbf{C}_1^T \mathbf{X}_{21}, \end{cases} \quad (1)$$

2) k th layer ($1 < k \leq n$)

$$\begin{cases} \dot{\mathbf{X}}_{1,k} = \mathbf{A}_k \mathbf{X}_{1,k} + \mathbf{B}_k u_{1,k}, \\ y_{1,k} = \mathbf{C}_k^T \mathbf{X}_{1,k}, \\ u_{1,k} = y_{1,k-1} + v_{1,k-1}, \\ v_{1,k-1} = d_{k-1}(y_{2,k-1} + d_{k-2}v_{2,k-1}), \\ \dot{\mathbf{X}}_{2,k} = \mathbf{A}_k \mathbf{X}_{2,k} + \mathbf{B}_k u_{2,k}, \\ y_{2,k} = \mathbf{C}_k^T \mathbf{X}_{2,k}, \\ u_{2,k} = y_{2,k-1} + v_{2,k-1}, \\ v_{2,k-1} = d_{k-1}(y_{1,k-1} + d_{k-2}v_{1,k-1}), \end{cases} \quad (2)$$

where $v_{11} = g_1 - y_{1,n}$, $v_{21} = g_2 - y_{2,n}$, $v_{i,k-1}$ is the output of cross-connection of the i th channel, $i = 1, 2$; $\mathbf{X}_{i,k} \in \mathbb{R}^q$ is the state vector of the k th layer of the i th channel of the TCS; $\mathbf{A}_k \in \mathbb{R}^{q_k \times q_k}$; $\mathbf{C}_k \in \mathbb{R}^{q_k}$; $\mathbf{B}_k \in \mathbb{R}^{q_k}$; $y_{i,k} \in \mathbb{R}$ is the output of k th layer of i th channel; $v_{i,k-1}$ is the output of cross-connection, and $g_i(t) \in \mathbb{R}$ is the system input (master control). Matrix $\mathbf{A}_k$ is a Hurwitz matrix.

Parameter d_{k-1} represents an operator. Depending on the problems solved by the TCS, d_{k-1} can be a constant, nonlinear function, or differential operator.

The information structure for system (1), (2) has the following form:

$$\mathbb{I}_0 = \{g_i(t), y_{i,k}(t) \mid (i=1,2) \& (k=\overline{1,n}), t \in J = [t_0, t_c]\}, \quad (3)$$

where J is the data capture interval; t is time; and t_0 and t_c are the beginning and the end of time interval.

The system is identified using a model with a structure similar to (1) and (2) and with outputs $\hat{y}_{i,k} \in \mathbb{R}$, where $i = 1, 2$; $k = \overline{1,n}$.

The problem is reduced to selecting algorithms for adjusting model parameters in such a way that

$$\lim_{t \rightarrow \infty} |\hat{y}_{i,k} - y_{i,k}| \leq \delta_{i,k}, \quad (4)$$

where $\delta_{i,k} \geq 0$ is the specified value.

1.2. On structural aspects of the system

The identification of systems (1) and (2) depends on the possibility of estimating its parameters significantly. We introduce a model for TCS (1):

$$\begin{cases} \dot{\hat{\mathbf{X}}}_{1,1} = \mathbf{K}_1(\hat{\mathbf{X}}_{1,1} - \mathbf{X}_{1,1}) + \hat{\mathbf{A}}_1 \mathbf{X}_{1,1} + \hat{\mathbf{B}}_1 \mathbf{v}_{1,1}, \\ \hat{y}_{1,1} = \mathbf{C}_1^T \hat{\mathbf{X}}_{1,1}, \\ \dot{\hat{\mathbf{X}}}_{2,1} = \mathbf{K}_1(\hat{\mathbf{X}}_{2,1} - \mathbf{X}_{2,1}) + \hat{\mathbf{A}}_1 \mathbf{X}_{2,1} + \hat{\mathbf{B}}_1 \mathbf{v}_{2,1}, \\ \hat{y}_{2,1} = \mathbf{C}_1^T \mathbf{X}_{2,1}, \end{cases} \quad (5)$$

where $\mathbf{K}_1 \in \mathbb{R}^{q_1 \times q_1}$ is the known stable matrix (reference model); $\hat{\mathbf{A}}_1 \in \mathbb{R}^{q_1 \times q_1}$, $\hat{\mathbf{B}}_1 \in \mathbb{R}^{q_1}$ is the matrices of model (5), and $\hat{\mathbf{X}}_{i,1}$ is the state vector.

We denote $\mathbf{E}_{1,1} \triangleq \hat{\mathbf{X}}_{1,1} - \mathbf{X}_{1,1}$, $\mathbf{E}_{2,1} \triangleq \hat{\mathbf{X}}_{2,1} - \mathbf{X}_{2,1}$. Then for the first layer

$$\begin{aligned} \dot{\mathbf{E}}_{1,1} &= \mathbf{K}_1 \mathbf{E}_{1,1} + \Delta \mathbf{A}_1 \mathbf{X}_{1,1} + \Delta \mathbf{B}_1 \mathbf{v}_{1,1}, \\ \dot{\mathbf{E}}_{2,1} &= \mathbf{K}_1 \mathbf{E}_{2,1} + \Delta \hat{\mathbf{A}}_1 \mathbf{X}_{2,1} + \Delta \mathbf{B}_1 \mathbf{v}_{2,1}, \end{aligned} \quad (6)$$

where $\Delta \mathbf{A}_1 \triangleq \hat{\mathbf{A}}_1 - \mathbf{A}_1$, $\Delta \mathbf{B}_1 \triangleq \hat{\mathbf{B}}_1 - \mathbf{B}_1$ are matrices of parametric mismatches.

Similarly, error equations for the remaining layers of TCSs are obtained:

$$\begin{aligned} \dot{\mathbf{E}}_{1,k} &= \mathbf{K}_k \mathbf{E}_{1,k} + \Delta \mathbf{A}_k \mathbf{X}_{1,k} + \Delta \mathbf{B}_k u_{1,k}, \\ \dot{\mathbf{E}}_{2,k} &= \mathbf{K}_k \mathbf{E}_{2,k} + \Delta \mathbf{A}_k \mathbf{X}_{2,k} + \Delta \mathbf{B}_k u_{2,k}. \end{aligned} \quad (7)$$

Let input $g_i(t)$, $i = 1, 2$ satisfy the excitation constancy (EC) condition:

$$\mathcal{E}_{\underline{\alpha}_i, \bar{\alpha}_i} : \underline{\alpha}_i \leq g_i^2(t) \leq \bar{\alpha}_i \quad \forall t \in [t_0, t_0 + T], \quad (8)$$

where $\underline{\alpha}_i$ and $\bar{\alpha}_i$ are positive numbers, $T > 0$. Further, condition (8) is written as $g_i(t) \in \mathcal{E}_{\underline{\alpha}_i, \bar{\alpha}_i}$. If $g_i(t)$ does not have an EC property, then it is written as $g_i(t) \notin \mathcal{E}_{\underline{\alpha}_i, \bar{\alpha}_i}$.

Theorem 1. Let the following conditions be satisfied:

1) $g_i(t) \in \mathcal{E}_{\underline{\alpha}_i, \bar{\alpha}_i}$, where $(\underline{\alpha}_i, \bar{\alpha}_i) > 0$; 2) system (5) is stable and detectable; 3) matrix $\mathbf{K}_1 \in \mathbb{R}^{q_1 \times q_1}$ is a Hurwitz matrix; 3) outputs $v_{i1}(t) \in \mathcal{E}_{\underline{\sigma}_{i1}, \bar{\sigma}_{i1}}$, where $\underline{\sigma}_{i1} > 0$, $\bar{\sigma}_{i1} > 0$, $i = 1, 2$; 4) $\mathbf{X}_{i,1}(t) \in \mathcal{E}_{\underline{\alpha}_{\mathbf{X}_{i,1}}, \bar{\alpha}_{\mathbf{X}_{i,1}}}$, where $(\underline{\alpha}_{\mathbf{X}_{i,1}}, \bar{\alpha}_{\mathbf{X}_{i,1}}) > 0$. Then system (5) is identifiable if

$$\pi_{12} \|\Delta \mathbf{A}_1\|^2 + 0.5 v_{12} \|\Delta \mathbf{B}_1\|^2 \leq (\lambda - 0.5) V_1, \quad (9)$$

where $\lambda_1 > 0.5$, $\pi_{12} = 2 \max(\bar{\alpha}_{X_{11}}, \bar{\alpha}_{X_{12}})$, $\nu_{12} = \bar{\sigma}_{11} + \bar{\sigma}_{21}$, $\|\Delta \mathbf{A}_1\|^2 = \text{tr}(\Delta \mathbf{A}_1^T \Delta \mathbf{A}_1)$, tr is a matrix trace, and $V_1(t) = 0.5 \mathbf{E}_{11}^T(t) \mathbf{R}_1 \mathbf{E}_{11}(t) + 0.5 \mathbf{E}_{21}^T(t) \mathbf{R}_1 \mathbf{E}_{21}(t)$, $\mathbf{R}_1 = \mathbf{R}_1^T > 0$ is a symmetric matrix.

When condition (9) is satisfied, system (5) is called parametrically $\mathcal{E}_{\mathcal{A},X}$ -identifiable on the set of state variables. The identifiability of subsystem layer (1) depends on the properties of the TCS output.

We consider system (7) and introduce LF:

$$V_k(t) = 0.5 \mathbf{E}_{1,k}^T(t) \mathbf{R}_k \mathbf{E}_{1,k}(t) + 0.5 \mathbf{E}_{2,k}^T(t) \mathbf{R}_k \mathbf{E}_{2,k}(t). \quad (10)$$

Theorem 2. Let the following conditions be satisfied: 1) matrix $\mathbf{K}_k \in \mathbb{R}^{q_k \times q_k}$ is a Hurwitz matrix; 2) $\|\mathbf{X}_{1,k}\|^2 \in \mathcal{E}_{\underline{\alpha}_{\mathbf{X}_{1,k}}, \bar{\alpha}_{\mathbf{X}_{1,k}}}$, $\|\mathbf{X}_{2,k}\|^2 \in \mathcal{E}_{\underline{\alpha}_{\mathbf{X}_{2,k}}, \bar{\alpha}_{\mathbf{X}_{2,k}}}$; $\pi_k = \max(\bar{\alpha}_{\mathbf{X}_{1,k}}, \bar{\alpha}_{\mathbf{X}_{2,k}})$, $y_{2,k-1} \in \mathcal{E}_{\underline{\alpha}_{y_{2,k-1}}, \bar{\alpha}_{y_{2,k-1}}}$; 3) system (5) is $\mathcal{E}_{\mathcal{A},X}$ -identifiable; 4) system (7) is stable and detectable; 5) $v_{2,k-1}^2 \in \mathcal{E}_{\underline{\alpha}_{v_{2,k-1}}, \bar{\alpha}_{v_{2,k-1}}}$; 6) operator d_{k-1} is constant: $d_k \leq \omega_k \leq \omega$, where ω is some number; 7) $\lambda_k \geq 0.5$. Then system (7) is $\mathcal{E}_{\mathcal{A},X}$ -identifiable, if

$$0.5 \pi_{k,i} \|\Delta \mathbf{A}_k\|^2 + 0.5 \|\Delta \mathbf{B}_k\|^2 \times \\ \times (\tilde{\alpha}_k + 2\omega^2(\tilde{\alpha}_k + 2\omega^2\beta_k)) \leq (\lambda_k - 0.5)V_k, \quad (11)$$

where $\pi_{k,i} = 2 \max(\bar{\alpha}_{X_{1,k}}, \bar{\alpha}_{X_{2,k}})$, $\tilde{\alpha}_k = 2 \max(\bar{\alpha}_{y_{2,k-1}}, \bar{\alpha}_{y_{1,k-1}})$, $\beta_k = \max_i(\bar{\alpha}_{y_{i,k-1}} + \omega^2 \bar{\alpha}_{v_{i,k-1}})$.

Note 1: Conditions $\|\mathbf{X}_{i,k}\|^2 \in \mathcal{E}_{\underline{\alpha}_{\mathbf{X}_{i,k}}, \bar{\alpha}_{\mathbf{X}_{i,k}}}$, $i = 1, 2$ follow from $v_{i1}(t) \in \mathcal{E}_{\underline{\alpha}_{v_{i1}}, \bar{\alpha}_{v_{i1}}}$.

Note 2: The identifiability of k th layer (7) depends on the properties of previous layers of systems (1) and (2) and cross-connections. The parameters of cross-connections should be selected so that condition (11) is satisfied.

We consider the case when operator d_k is differentiable.

Theorem 3. Let the conditions of Theorem 2 be satisfied and: 1) operator d_{k-1} is differentiable, i.e., $v_{1,k-1} = d(y_{2,k-1} + d_{k-2}v_{2,k-1})/dt$; 2) systems (1) and (2) are stable, detectable, and recoverable. Then system (7) is $\mathcal{E}_{\mathcal{A},X}$ -identifiable, if

$$0.5 \pi_{k,i} \|\Delta \mathbf{A}_k\|^2 + 0.5 \|\Delta \mathbf{B}_k\|^2 \times \\ \times (\tilde{\alpha}_{k,y} + 2(\bar{\alpha}_{\dot{y}} + \tilde{\alpha}_{\dot{v}})) \leq (\lambda_k - 0.5)V_k, \quad (12)$$

where $\tilde{\alpha}_{k,y} = 2 \max_i \bar{\alpha}_{\dot{y}_{2,k-1}}$, $\tilde{\alpha}_{k,v} = 2 \max_i \bar{\alpha}_{\dot{v}_{i,k-1}}$, $\pi_{k,i} = 2 \max(\bar{\alpha}_{\mathbf{X}_{1,k}}, \bar{\alpha}_{\mathbf{X}_{2,k}})$, $V_k(t)$ is of the form (10).

Thus, the findings allow estimating the identifiability of systems (1) and (2) under measurability of the state vector of elements of all TCS layers. Most often, only set (3) is available for observation. In this case, it is necessary to operate with the available information to estimate the TCS identifiability.

We transform the TCS to a form convenient for using set $\mathbb{I}_0$ [28, 29] and consider the system (1). Let $\mathbf{A}_1$ be the Frobenius matrix with vector of parameters $\mathbf{A}_{1,s} \in \mathbb{R}^{q_1}$, $\mathbf{A}_{1,s} = [a_{1,s,1}, a_{1,s,2}, \dots, a_{1,s,q_1}]^T$, $\mathbf{C}_1 = [1, 0, \dots, 0]^T$, $\mathbf{B}_1 = [0, \dots, 0, b_{1,s}]^T$. In space (v_{11}, y_{11}) , system (1) corresponds to the following representation:

$$\dot{y}_{1,1} = \bar{\mathbf{A}}_{1,1}^T \mathbf{P}_{1,1}, \quad (13)$$

where $\bar{\mathbf{A}}_{1,1}^T = [-a_{1,1,1}, a_{1,1,2}, \dots, a_{1,1,q_1}; b_{1,s}, b_{1,2}, \dots, b_{1,q_1}]$, $\bar{\mathbf{A}}_{1,1} \in \mathbb{R}^{2q_1}$, $\mathbf{P}_{1,1} \in \mathbb{R}^{2q_1}$.

Considering (13), we transform the system (1) in the input-output space to the following form:

$$\begin{cases} \dot{y}_{1,1} = \bar{\mathbf{A}}_{1,1}^T \mathbf{P}_{1,1}, \\ \dot{y}_{2,1} = \bar{\mathbf{A}}_{1,1}^T \mathbf{P}_{2,1}. \end{cases} \quad (14)$$

In order to evaluate the identifiability of system (14) by the output ($\mathcal{E}_{\mathcal{A},y}$ -identifiability), we consider the following model:

$$\begin{cases} \dot{\hat{y}}_{1,1} = -\chi_1 e_{1,1} + \hat{\mathbf{A}}_{1,1}^T \mathbf{P}_{1,1}, \\ \dot{\hat{y}}_{2,1} = -\chi_2 e_{2,1} + \hat{\mathbf{A}}_{1,1}^T \mathbf{P}_{2,1}, \end{cases} \quad (15)$$

where $\chi_1 > 0$, $e_{i,1} = \hat{y}_{i,1} - y_{i,1}$ is the prediction error of the $y_{i,1}$ th output, $i = 1, 2$. The following equation is derived for $e_{1,1}$:

$$\dot{e}_{1,1} = -\chi_1 e_{1,1} + \Delta \bar{\mathbf{A}}_{1,1}^T \mathbf{P}_{1,1}, \quad \Delta \bar{\mathbf{A}}_{1,1} = \hat{\mathbf{A}}_{1,1} - \bar{\mathbf{A}}_{1,1}. \quad (16)$$

We consider LF $V_{1,2}(e_{1,1}, e_{2,1}) = 0.5(e_{1,1}^2 + e_{2,1}^2)$. For $\dot{V}_{1,2}$, we derive:

$$\begin{aligned} \dot{V}_{1,2} &= -2\chi_1 V_{1,2} + \Delta \bar{\mathbf{A}}_{1,1}^T (\mathbf{P}_{1,1} e_{1,1} + \mathbf{P}_{2,1} e_{2,1}) \leq \\ &\leq -\frac{\chi_1}{2} V_{1,2} + \frac{1}{2\chi_1} \Delta \bar{\mathbf{A}}_{1,1}^T \mathbf{P}_{1,1} \mathbf{P}_{1,1}^T \Delta \bar{\mathbf{A}}_{1,1}, \end{aligned}$$

where $\mathbf{P}_1 \mathbf{P}_1^T = \mathbf{P}_{1,1} \mathbf{P}_{1,1}^T + \mathbf{P}_{2,1} \mathbf{P}_{2,1}^T$.

It follows from (16) that $\Delta\bar{\mathbf{A}}_{1,1} = 0$, if vector $\mathbf{P}_i(t)$ is extremely nondegenerate, i.e., $\mathbf{P}_i(t) \in \mathcal{E}_{\bar{\mathbf{u}}_{\mathbf{P}_i}, \bar{\alpha}_{\mathbf{P}_i}}$ ($i = 1, 2$) and subsystem (1) is parametrically $\mathcal{E}_{\bar{\mathbf{u}}_{\mathbf{P}_i}, \bar{\alpha}_{\mathbf{P}_i}}$ -identifiable by the output. The $\mathcal{E}_{\bar{\mathbf{u}}_{\mathbf{P}_i}, \bar{\alpha}_{\mathbf{P}_i}}$ -identifiability of subsystem (2) is justified similarly.

For the k th layer (system (2)), the following representation similar to (13) can be obtained:

$$\begin{cases} \dot{y}_{1,k} = \bar{\mathbf{A}}_{1,k}^T \mathbf{P}_{1,k}, \\ \dot{y}_{2,k} = \bar{\mathbf{A}}_{1,k}^T \mathbf{P}_{2,k} \end{cases} \quad (17)$$

where $\mathbf{P}_{i,k} \in \mathbb{R}^{2q_k}$ is the generalized input. For convenience, representations (13)–(15) and (17) are called noncanonical identification representations.

When identifying TCS and incomplete a priori information, the task of determining the type of cross-links may arise. In this case, the following approach can be used. We consider k th layer of system (2) with identical cross-connections and $d_{k-1} = \text{const}$ and construct structure $\mathbf{S}_{u_{i,k}, y_{i,k}}$ described by function $f_{u_{i,k}, y_{i,k}} : u_{i,k} \rightarrow y_{i,k}$, $i = 1, 2$ for both channels of the k th layer. Structure $\mathbf{S}_{u_{i,k}, y_{i,k}}$ represents the TCS input-output state. We define secants for $\mathbf{S}_{u_{i,k}, y_{i,k}}$:

$$\xi_{u_{i,k}, y_{i,k}} = a_{\xi_{0,i,k}} + a_{\xi_{1,i,k}} u_{i,k}, \quad (18)$$

where $a_{\xi_{0,i,k}}$ and $a_{\xi_{1,i,k}}$ are parameters determined by the least squares method.

Since the cross-connection is rigid, the angle between secants $\xi_{u_{i,k}, y_{i,k}}$ would not exceed a certain value: $|a_{\xi_{1,1,k}} - a_{\xi_{1,2,k}}| \leq \delta_{\xi_{i,k}}$. Hence, cross-connections are positive. If cross-connections are asymmetric, then $|a_{\xi_{1,1,k}} - a_{\xi_{1,2,k}}| > \delta_{\xi_{i,k}}$. In this case, signal $v_{1,k-1}$ operates out of phase with output $y_{1,k-1}$ of the previous layer of the first channel.

Note 3: If the channels of each layer are nonidentical, the identifiability of the system layers can be easily derived from the above theorems.

1.3. Adaptive identification of TCSs

We consider representations (13)–(15) and (17) for TCS. For subsystem (17), the following model is introduced:

$$\begin{cases} \dot{\hat{y}}_{1,k} = -\chi_k e_{1,k} + \hat{\mathbf{A}}_{1,k}^T \mathbf{P}_{1,k}, \\ \dot{\hat{y}}_{2,k} = -\chi_k e_{2,k} + \hat{\mathbf{A}}_{1,k}^T \mathbf{P}_{2,k}, \end{cases} \quad (19)$$

where $\chi_k > 0$, $e_{i,k} = \hat{y}_{i,k} - y_{i,k}$ is the prediction error for output $y_{i,k}$ ($k = 1, 2$, i is the i th element in the k th layer). For $e_{i,k}$, the following equation is derived:

$$\dot{e}_{i,k} = -\chi_k e_{i,k} + \Delta\bar{\mathbf{A}}_{i,k}^T \mathbf{P}_{i,k}, \quad \Delta\bar{\mathbf{A}}_{1,k} = \hat{\mathbf{A}}_{1,k} - \bar{\mathbf{A}}_{1,k}. \quad (20)$$

The adaptive algorithm for adjusting parameters of model (19) has the following form:

$$\Delta\dot{\bar{\mathbf{A}}}_{1,k} = \dot{\hat{\mathbf{A}}}_{1,k} = -\mathbf{\Gamma}_k e_{i,k} \mathbf{P}_{1,k}, \quad (21)$$

where $\mathbf{\Gamma}_k \in \mathbb{R}^{q_k \times q_k}$ is the diagonal matrix with $\gamma_{k,j} > 0$.

We consider subsystem (14)–(16) and LF $\tilde{V}_1(e_{1,1}) = 0.5e_{1,1}^2$. From condition $\dot{\tilde{V}}_1 \leq 0$, the adaptive algorithm for adjusting vector $\hat{\mathbf{A}}_{1,1}$ is derived, as follows:

$$\Delta\dot{\bar{\mathbf{A}}}_{1,1} = \dot{\hat{\mathbf{A}}}_{1,1} = -\mathbf{\Gamma}_1 e_{1,1} \mathbf{P}_{1,1}, \quad (22)$$

where $\mathbf{\Gamma}_1 \in \mathbb{R}^{q_1 \times q_1}$ is the diagonal matrix with $\gamma_{1,j} > 0$, j is the diagonal element number.

Note 4: Since TCS has identical channels, only one of them is adjusted.

Due to feedback, some parameters of system (14)–(16), (22) may be unidentifiable. Considering this, Eq. (16) may be written in the following form:

$$\dot{e}_{1,1} = -\chi_1 e_{1,1} + \Delta\bar{\mathbf{A}}_{1,1}^T \mathbf{P}_{1,1} + f(\bar{\mathbf{A}}_{1,1}, p_{v_{1,1}}), \quad (23)$$

where $f(\cdot)$ is uncertainty resulting from the nonfulfillment of the EC condition.

We consider LF $\tilde{V}_{1,e}(e_{1,1}) = 0.5e_{1,1}^2$, $\tilde{V}_{1,\Delta}(\Delta\bar{\mathbf{A}}_{1,1}) = 0.5\Delta\bar{\mathbf{A}}_{1,1}^T \mathbf{\Gamma}_1^{-1} \Delta\bar{\mathbf{A}}_{1,1}$.

Theorem 4: Let the following conditions be satisfied: 1) $g_i(t) \in \mathcal{E}_{\bar{\mathbf{u}}_i, \bar{\alpha}_i}$, $\mathbf{P}_{1,1} \notin \mathcal{E}_{\bar{\mathbf{u}}_{\mathbf{P}_{1,1}}, \bar{\alpha}_{\mathbf{P}_{1,1}}}$; 2) $y_{i,1} \notin \mathcal{E}_{\bar{\mathbf{u}}_{y_{i,1}}, \bar{\alpha}_{y_{i,1}}}$, $i = 1, 2$; 3) $|f|^2 \leq \alpha_f$ where $\alpha_f \geq 0$; 4) there exists such $v > 0$ that $e_{1,1} \Delta\bar{\mathbf{A}}_{1,1}^T \mathbf{P}_{1,1} = v(e_{1,1}^2 + \Delta\bar{\mathbf{A}}_{1,1}^T \mathbf{P}_{1,1} \mathbf{P}_{1,1}^T \Delta\bar{\mathbf{A}}_{1,1})$ is valid at $t \gg t_0$; 5) $\lambda_{\max}^{-1}(\mathbf{\Gamma}_1) \|\Delta\bar{\mathbf{A}}_{1,1}\|^2 \leq \tilde{V}_{1,\Delta} \leq \lambda_{\min}^{-1}(\mathbf{\Gamma}_1) \|\Delta\bar{\mathbf{A}}_{1,1}\|^2$; 6) the following system of differential inequalities is valid for system (16), (23):

$$\begin{bmatrix} \dot{\tilde{V}}_{1,e} \\ \dot{\tilde{V}}_{1,\Delta} \end{bmatrix} \leq \underbrace{\begin{bmatrix} -\frac{\chi_1}{2} & 2\frac{\pi P_{1,1}}{\chi_1} \\ 4v & -\frac{v\eta}{2} \end{bmatrix}}_{\mathbf{A}_{S_1}} \begin{bmatrix} \tilde{V}_{1,e} \\ \tilde{V}_{1,\Delta} \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{\alpha_f}{\chi_1} \\ 0 \end{bmatrix}}_{\mathbf{B}_{S_1}} \quad (24)$$

while comparison system $\dot{\mathbf{S}}_1(t) = \mathbf{A}_{S_1} \mathbf{S}_1(t) + \mathbf{B}_{S_1}$ for (24), where $\mathbf{S}_1(t) = [s_{1,e}(t), s_{1,\Delta}(t)]^T$, $s_{1,w}(t)$

($w=e, \Delta$) is majorant for $\tilde{V}_{1,w}(t)$ and $s_{1,w}(t_0) \geq \tilde{V}_{1,w}(t_0)$. Then system (14)–(16), (23) is exponentially dissipative with estimate

$$[\tilde{V}_{1,e}(t) \tilde{V}_{1,\Delta}(t)]^T \leq e^{A_{S_1}(t-t_0)} S_1(t_0) + \int_{t_0}^t e^{A_{S_1}(t-\tau)} B_{S_1} d\tau,$$

if $\chi_1^2 \eta > 89 P_{1,1}$, where $\eta = \pi_{P_{1,1}} \lambda_{\min}^2(\Gamma_1)$, $\pi_{P_{1,1}} \geq 0$, $\|P_{1,1} P_{1,1}^T\| \leq 9 P_{1,1}(\bar{\alpha} P_{1,1})$, $\lambda_{\min}(\Gamma_1)$ is minimum eigenvalue of matrix Γ_1 .

It follows from Theorem 4 that the adaptive identification system allows obtaining biased estimates of system (14)–(16) parameters.

We consider (17), (19), (21). Let d_{k-1} in (2) is constant.

We represent $P_{1,k}$ and $\Delta \bar{A}_{1,k}$ as: $P_{1,k} = [\tilde{P}_{1,k}^T, p_{j,v_{1,2,k-1}}]^T$, $\Delta \bar{A}_{1,k} = [\Delta \bar{A}_{1,k}^T, \Delta d_{j,k}]^T$, where $p_{j,v_{1,2,k-1}}$ is transformation $v_{1,2,k-1}$, j is element $\Delta \bar{A}_{1,k}$ that is the transformation of d_{k-1} . Then (20) has the following form:

$$\dot{e}_{1,k} = -\dot{e}_{1,k} e_{1,k} + \Delta \bar{A}_{1,k}^T P_{1,k} + f_{1,k}(\bar{A}_{1,k} P_{1,k}), \quad (25)$$

where $f_{1,k}(\cdot) \in \mathbb{R}$ is uncertainty resulting from the non-fulfillment of the EC condition.

We consider system (19), (20), (25), and LF $\tilde{V}_{k,e}(e_{1,k}) = 0.5 e_{1,k}^2$, $\tilde{V}_{k,\Delta}(\Delta \bar{A}_{1,k}) = 0.5 \Delta \bar{A}_{1,k}^T \Gamma_k^{-1} \Delta \bar{A}_{1,k}$.

Theorem 5. Let the Theorem 4 conditions be satisfied and: 1) $y_{1,k}(t) \notin \mathcal{E}_{\bar{u}_{y_{1,k}}, \bar{\alpha}_{y_{1,k}}}$, $P_{1,k} \notin \mathcal{E}_{\bar{u}_{P_{1,k}}, \bar{\alpha}_{P_{1,k}}}$; 2) $u_{1,k} = v_{u_{1,k}}(y_{1,k-1}, v_{1,k-1})$, $u_{1,k} \notin \mathcal{E}$; 3) $f_{1,k}^2 \leq \alpha_{f_{1,k}}$, where $\alpha_{f_{1,k}} \geq 0$; 4) estimate $\lambda_{\max}^{-1}(\Gamma_k) \|\Delta \bar{A}_{1,k}\|^2 \leq \tilde{V}_{k,\Delta} \leq \lambda_{\min}^{-1}(\Gamma_k) \|\Delta \bar{A}_{1,k}\|^2$ is valid, where $\lambda_k(\Gamma_k)$ is minimum eigenvalue of matrix Γ_k ; 5) $\|P_{1,k} P_{1,k}^T\| \leq \pi_{P_{1,k}} < \bar{\alpha}_{P_{1,k}}$; 6) there exists such $v > 0$ that at $t \gg t_0$ the following is valid:

$$e_{1,k} \Delta \bar{A}_{1,k}^T P_{1,k} = v(e_{1,k}^2 + \Delta \bar{A}_{1,k}^T P_{1,k} P_{1,k}^T \Delta \bar{A}_{1,k}); \quad (26)$$

7) the following system of differential inequalities is valid for system (21), (25):

$$\begin{bmatrix} \dot{\tilde{V}}_{k,e} \\ \dot{\tilde{V}}_{k,\Delta} \\ \dot{\tilde{V}}_k \end{bmatrix} \leq \underbrace{\begin{bmatrix} -\frac{\chi_k}{2} & 2\frac{\vartheta_{P_{1,k}}}{\chi_k} \\ 4v & -\frac{v\eta}{2} \end{bmatrix}}_{A_{S_k}} \begin{bmatrix} \tilde{V}_{k,e} \\ \tilde{V}_{k,\Delta} \end{bmatrix} + \begin{bmatrix} \frac{\alpha_{f_{1,k}}}{\chi_k} \\ 0 \end{bmatrix} B_{S_k}, \quad (27)$$

while comparison system $\dot{S}_k(t) = A_{S_k} S_k(t) + B_{S_k}$ for (27), where $S_k(t) = [s_{k,e}(t), s_{k,\Delta}(t)]^T$, $s_{k,w}(t)$ ($w=e, \Delta$) is a majorant for $\tilde{V}_{k,w}(t)$ and $s_{k,w}(t_0) \geq \tilde{V}_{k,w}(t_0)$. Then system (19), (20), (25) is exponentially dissipative with estimate

$$\tilde{V}_k(t) \leq e^{A_{S_k}(t-t_0)} S_k(t_0) + \int_{t_0}^t e^{A_{S_k}(t-\tau)} B_{S_k} d\tau,$$

if condition $\chi_k^2 \eta > 329 P_{1,k}$ is satisfied, where $\pi_{P_{1,k}} \geq 0$, $\eta = \pi_{P_{1,k}} \lambda_{\min}(\Gamma_k)$.

It follows from (25) that the properties of the k th layer adaptive identification system depend on cross-connections.

Thus, the TCS identifiability is proved by the state and output of the TCS k th layer. The results confirming the convergence of the obtained estimates for system parameters are obtained. The adaptive identification system properties depend on parameters of the system cross-connections and information properties of signals in the TCS.

Note 5: If TCS contains nonidentical channels, models (17) should be applied to each k th layer. The same is true for cross-connections.

2. ICS

We consider system S_{ICS}

$$\begin{aligned} \dot{X}(t) &= AX(t) + DF_1(X, t) + BU(t), \\ LY(t) &= CX(t) + F_2(X, t), \end{aligned} \quad (28)$$

where $X \in \mathbb{R}^m$ is the state vector, $A \in \mathbb{R}^{m \times m}$ is the state matrix, $D \in \mathbb{R}^{m \times q}$, $F_1(X, t): \mathbb{R}^m \rightarrow \mathbb{R}^q$ is the nonlinear vector function, $U \in \mathbb{R}^k$ is the input (control) vector, $B \in \mathbb{R}^{m \times k}$, $Y \in \mathbb{R}^n$ is the output vector, $C \in \mathbb{R}^{n \times m}$, $F_2(X, t): \mathbb{R}^m \rightarrow \mathbb{R}^n$ is the disturbance (measurement errors) vector, and L is the operator determining the way vector Y is formed. In some cases, L can be the differential operator representing dynamic properties of the measuring system or the way of mutual interaction between subsystems. Matrices A, D, B are block matrices representing the state of certain subsystems. Vector $F_2(X, t)$ can be either a disturbance (measurement error) or a variable representing the influence of certain subsystems.

The data set has the following form:

$$\mathbb{I}_0 = \{Y(t), U(t), t \in [t_0, t_N]\}, t_N < \infty. \quad (29)$$

Assumption 1: Elements $\varphi_{1,i}(x_j) \in F_1$, $\varphi_{2,i}(x_j) \in F_2$ are smooth single-valued functions.

For estimating parameters of matrices $\mathbf{A}$, $\mathbf{D}$, $\mathbf{B}$, $\mathbf{C}$, the following model is used:

$$\begin{cases} \dot{\hat{\mathbf{X}}}(t) = \hat{\mathbf{A}}(t)\hat{\mathbf{X}}(t) + \hat{\mathbf{D}}(t)\mathbf{F}_1(\mathbf{X}, t) + \hat{\mathbf{B}}(t)\mathbf{U}(t), \\ \mathbf{L}\hat{\mathbf{Y}}(t) = \mathbf{C}\hat{\mathbf{X}}(t) + \mathbf{F}_2(\hat{\mathbf{X}}, t), \end{cases} \quad (30)$$

where $\hat{\mathbf{A}}(t)$, $\hat{\mathbf{D}}(t)$, $\hat{\mathbf{B}}(t)$ are matrices with adjustable parameters.

Problem. For system (27) satisfying Assumption 1, to develop model (30) on the basis of analysis $\mathbb{I}_0$ and find such rules for adjusting parameters of matrices $\hat{\mathbf{A}}(t)$, $\hat{\mathbf{D}}(t)$, and $\hat{\mathbf{B}}(t)$ so that

$$\lim_{t \rightarrow \infty} \|\hat{\mathbf{Y}}(t) - \mathbf{Y}(t)\| \leq \delta_y, \quad \delta_y \geq 0,$$

where $\|\cdot\|$ is the Euclidean norm.

Note 6: For some class of systems, Eq. (28) can be considered as an equation describing connections between subsystems. In this case, the type of vector $\mathbf{F}_2(\mathbf{X}, t)$ should be estimated.

For the synthesis of adaptive identification algorithms, the approach described in Section 1.3 can be used.

Note 7: If system S_{ICS} contains nonlinear subsystems, then in order to decide on the class of nonlinearity under uncertainty, the interconnection graph should be plotted [30] from which the type of nonlinearity can be decided. The same is valid when the system contains several nonlinearities in one subsystem.

The adaptation models and algorithms match the equations derived in the previous section. For evaluating the quality of the adaptive identification subsystem, the theorems from Section 1.2 can be applied.

3. EXAMPLES

Example 1. We consider the two-channel target angle tracking system with identical azimuth and elevation channels and asymmetric cross-connections [31]:

$$\dot{x} = -a_x x + b_x (g - y), \quad (31)$$

$$\ddot{y} = -a_y \ddot{y} + b_y (x - (g_1 - y_1) - k(g_1 - y_1)'), \quad (32)$$

$$\dot{x}_1 = -a_x x_1 + b_x (g_1 - y_1), \quad (33)$$

$$\ddot{y}_1 = -a_y \ddot{y}_1 + b_y (x_1 + (g - y) + k(g - y)'), \quad (34)$$

where $a_x = T_x^{-1}$, $b_x = T_x^{-1}k_x$; T_x , k_x are the time constant and amplifier gain, respectively; k is the cross-connection parameter; $a_y = T_y^{-1}$, $b_y = T_y^{-1}k_y$ are

servomotor parameters; g , g_1 are input influences. The cross-connection is shown in parentheses as a differentiating link with parameter k , $(g_1 - y_1)' = d(g_1 - y_1)/dt$.

The information about link outputs $x_i(t)$, $y_i(t)$, and outputs $g_i(t)$ at some time interval is available for measurement. The parameters of systems (31) and (32) should be estimated.

We use transformation [28] to obtain a model for $y(t)$ and consider the system of filters (transformation):

$$\begin{aligned} \dot{p}_y &= -\mu_1 p_y + y, \\ \dot{p}_g &= -\mu_1 p_g + g, \\ \dot{p}_v &= -\mu_1 p_v + v, \end{aligned} \quad (35)$$

where $g \triangleq x - g_1 + y_1$, $v \triangleq d(g_1 - y_1)/dt$, $p_i(0) = 0$, $i = y, g, v$, $\mu_1 > 0$ is a number that does not match the roots of the characteristic equation for the second equation in (31). Then the model for (31) has the following form:

$$\dot{\hat{x}} = -\chi_x e_x + \hat{a}_x x + \hat{b}_x (g - y), \quad (36.1)$$

$$\dot{\hat{y}} = -\chi_y e_y + \hat{a}_y y + \hat{a}_{p_y} p_y + \hat{a}_g p_g + \hat{a}_v p_v, \quad (36.2)$$

where $e_x = \hat{x} - x$, $e_y = \hat{y} - y$, χ_x, χ_y are positive numbers (reference model). Algorithms for adjusting parameters of system (36) are the following:

$$\begin{aligned} \dot{\hat{a}}_x &= -\gamma_{a_x} e_x x, & \dot{\hat{b}}_x &= -\gamma_{b_x} e_x (g - y), \\ \dot{\hat{a}}_y &= -\gamma_{a_y} e_y y, & \dot{\hat{a}}_{p_y} &= -\gamma_{a_{p_y}} e_y p_y, \\ \dot{\hat{a}}_g &= -\gamma_{a_g} e_y p_g, & \dot{\hat{a}}_v &= -\gamma_{a_v} e_y p_v \end{aligned} \quad (37)$$

where $\gamma_{a_i}, \gamma_{b_i}$ ($i = x, y, a_y, p_y, g, v$) are positive numbers ensuring convergence of (36).

System (31), (32) is modelled with parameters: $a_x = 1.2$, $b_x = 2$, $a_y = 5.95$, $b_y = 1$, $k = 5$. The inputs are $g(t) = 1.5 \sin(0.1\pi t)$, $g_1(t) = 1.5 \sin(0.1\pi t)$.

The structures confirming the presence of asymmetric connections in the system are presented in Fig. 1. In the figure, the impact of cross-connections on the channel output is shown. Analyzing $a_{\xi_{0,\varepsilon_1,y}} = 0.53$,

$a_{\xi_{0,\varepsilon,y_1}} = -0.45$ of secants $\xi_{\varepsilon_1,y}$, ξ_{ε,y_1} proves the fulfillment of condition $|a_{\xi_{0,\varepsilon_1,y}} - a_{\xi_{0,\varepsilon,y_1}}| > \delta_{\xi_0}$, where $\delta_{\xi_0} = 0.2$. Hence, cross-connections are antisymmetric.

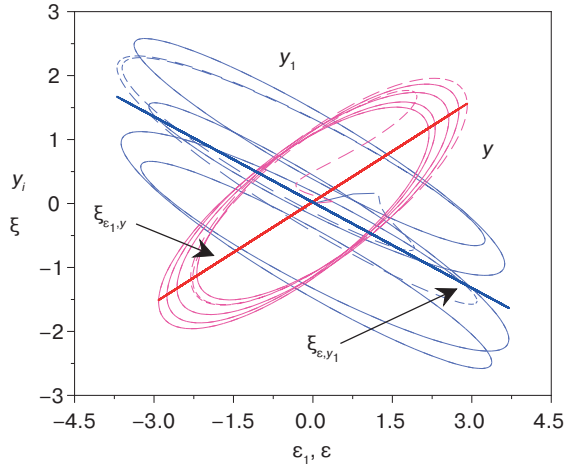


Fig. 1. Estimating the structure of cross-connections

The parameters of Eqs. (31) and (32) are estimated with respect to Note 4. The identification system parameters are: $\mu_1 = 1.5$, $\chi_x = 1.5$, $\chi_y = 2$.

The results of adaptive identification are presented in Figs. 2–6. The adaptive identification of parameters of model (36) is shown in Figs. 2 and 3, where t is hereafter the current time in relative units.

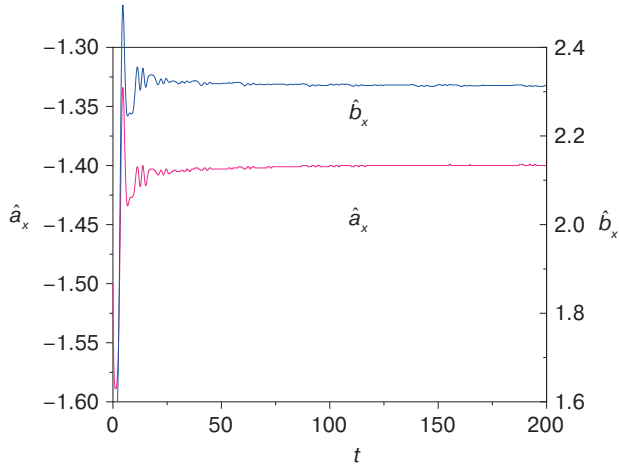


Fig. 2. Adjusting parameters of model (36.1)

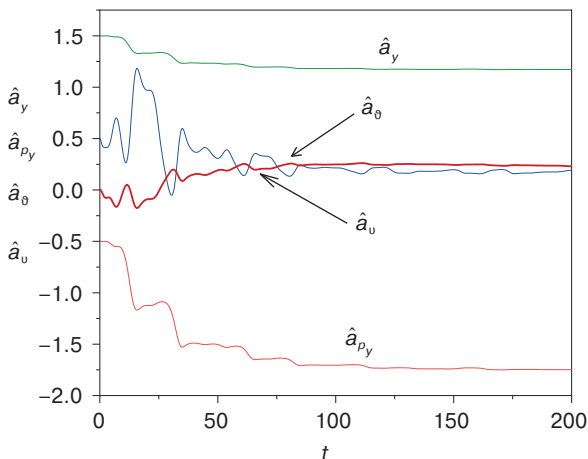


Fig. 3. Adjusting parameters of model (36.2)

The identification errors obtained using models (36) are shown in Fig. 4.

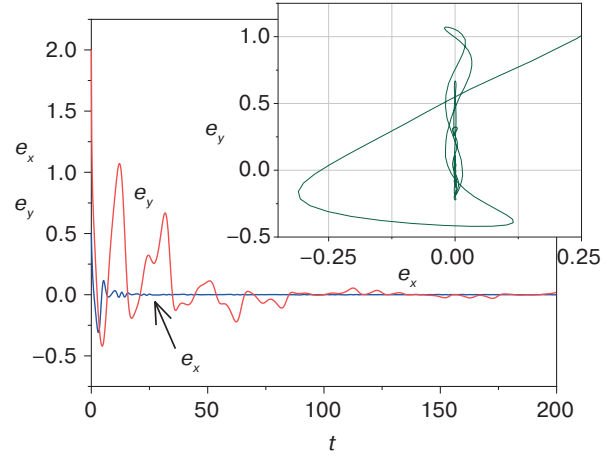


Fig. 4. A change in the prediction error

It follows from Figs. 3 and 4 that adjusting parameters of layer 2 depends on both the output of layer 1 (model (36.1)) and the properties of cross-connections. In addition, the quality of adjusting parameters of (36.2) is also dependent on the correlation between the outputs of elements (31) and (33). This correlation is transmitted through cross-connections. The conclusion drawn is supported by the structures shown in Fig. 5 as well as the statements of Theorem 5. Adjusting parameters of model (36.1) is smooth (Fig. 6).

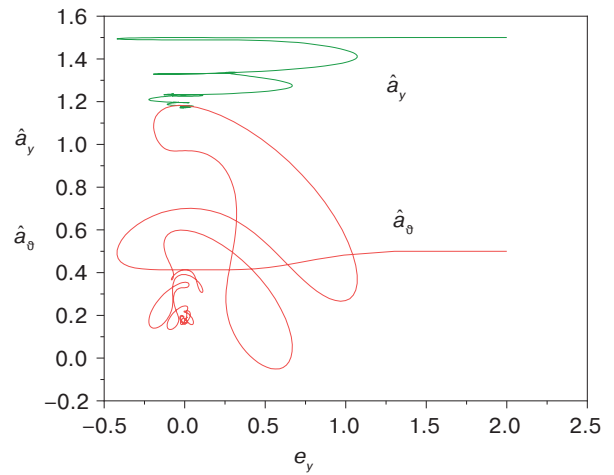


Fig. 5. Adjusting parameters of model (36.2)

Example 2. A pseudo-linear two-channel corrector is described by the system of equations in [32], as follows:

$$\begin{cases} \dot{x}_1 = -\alpha_1 x_1 + \beta_1 (g - x_6), \\ \dot{x}_2 = -\alpha_2 x_2 + \mu_2 (g - x_6)' + \beta_2 (g - x_6), \\ \dot{x}_6 = x_7, \\ \dot{x}_7 = -\alpha_{01} x_7 - \alpha_{02} x_6 + \beta_0 (x_1 \text{sign}(x_1)) (\text{sign}(x_2)), \end{cases} \quad (38)$$

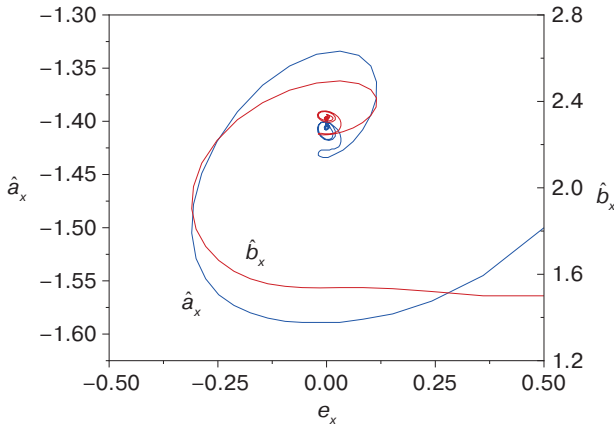


Fig. 6. Adjusting parameters of model (36.1)

where g is the master control, x_6 is the object output, $g - x_6$ is the mismatch error, x_1 is the amplitude channel output, x_2 is the phase channel output, $u = x_1 \text{sign}(x_1) \cdot \text{sign}(x_2)$ is the regulator output, and $(g - x_6)' = d(g - x_6)/dt$, $\alpha_1, \beta_1, \alpha_2, \mu_2, \beta_2, \alpha_{01}, \alpha_{02}, \beta_0$ are system parameters.

Note 8: In [32], system parameters have been preliminary selected.

It is assumed that elements of set $\mathbb{I}_0 = \{x_1(t), x_2(t), x_6(t), t \in [0, t_k]\}$ are measured, where t_k is the known number. The parameters of system (38) are identified using the following model:

$$\dot{\hat{x}}_1 = -k_1 e_1 + \hat{\alpha}_1 x_1 + \hat{\beta}_1 (g - x_6), \quad (39)$$

$$\dot{\hat{x}}_2 = -k_2 e_2 + \hat{\alpha}_2 x_2 + \hat{\mu}_2 d(g - x_6)/dt + \hat{\beta}_2 (g - x_6), \quad (40)$$

$$\dot{\hat{x}}_6 = -k_6 e_6 + \hat{\alpha}_{01} x_6 + \hat{\alpha}_{p_{x_6}} p_{x_6} + \hat{\alpha}_{p_u} p_u, \quad (41)$$

where k_1, k_2, k_6 are known numbers (the reference model parameters); $e_i = \hat{x}_i - x_i$, $i = 1, 2, 6$; $\hat{\alpha}_i, \hat{\beta}_i$ ($i = 1, 2$), $\hat{\alpha}_{p_{x_5}}, \hat{\alpha}_{p_{x_5}}$ are adjustable parameters; $\hat{x}_i$ ($i = 1, 2, 6$) are model outputs; p_{x_6}, p_u are obtained by analogy with (33).

The adaptation algorithms are the following:

$$\begin{aligned} \dot{\hat{\alpha}}_1 &= -\gamma_{\alpha_1} e_1 x_1, & \dot{\hat{\beta}}_1 &= -\gamma_{\beta_1} e_1 (g - x_6), \\ \dot{\hat{\alpha}}_2 &= -\gamma_{\alpha_2} e_2 x_2, & \dot{\hat{\beta}}_2 &= -\gamma_{\beta_2} e_2 (g - x_6), \\ \dot{\hat{\mu}}_2 &= -\gamma_{\mu_2} e_2 (g - x_6)', \end{aligned} \quad (42)$$

$$\dot{\hat{\alpha}}_{01} = -\gamma_{\alpha_{01}} e_6 x_6, \quad \dot{\hat{\alpha}}_{p_{x_6}} = -\gamma_{p_{x_6}} e_6 p_{x_6}, \quad \dot{\hat{\alpha}}_{p_u} = -\gamma_{p_u} e_6 p_u,$$

where $\gamma_i > 0$ is the gain in the adjustment loop of the corresponding parameter.

System (38) is modelled with the following parameters: $\alpha_1 = 1.05, \beta_1 = 3.5, \alpha_2 = 2.2, \beta_2 = 2.2, \alpha_{01} = 3, \alpha_{02} = 5.03, \beta_0 = 5.2, g(t) = \sin(0.05\pi t)$. The results are shown in Fig. 7–12.

Figure 7 shows the structures (transient excluded) representing phase processes in system (38). Obviously, the system is nonlinear. The use of results [30] shows that the system is structurally identifiable. Therefore, the input is S-synchronizing and allows accounting for TCS nonlinear properties.

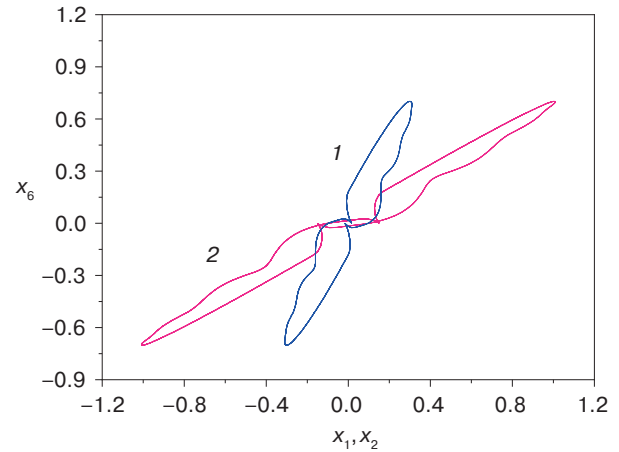


Fig. 7. Phase portraits of the system: (1) $x_6(x_2)$, (2) $x_6(x_1)$

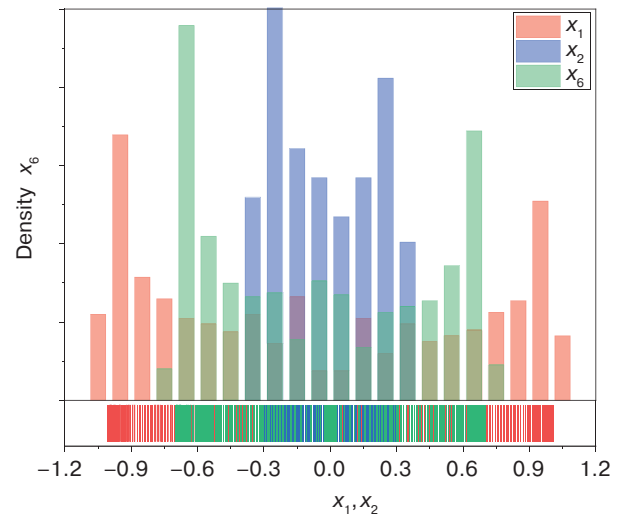


Fig. 8. Estimating the mutual influence of variables x_1, x_2 on x_6

The analysis shows the relationship between variables x_1 and x_2 (correlation coefficient is 0.94). Their impact on the TCS output is represented by the density diagram (Fig. 8). The density diagram characterizes the impact of x_1 and x_2 on the change in variable x_6 . The intervals of the impact of variables on x_6 are clearly visible. The mutual influence affects the convergence of adaptive algorithms.

The adjustment of the amplitude channel model is shown in Fig. 9 while the phase channel model is shown in Fig. 10.

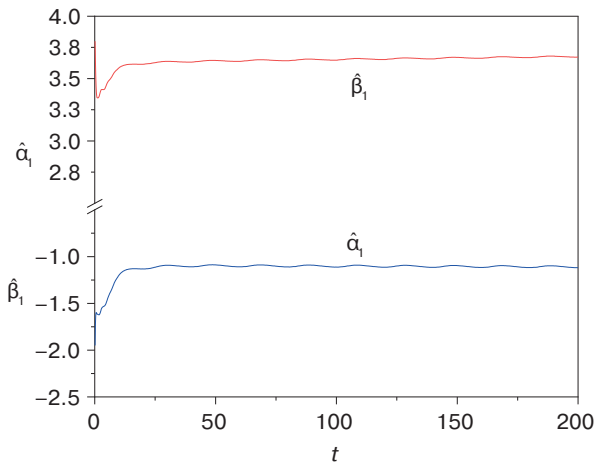


Fig. 9. Adjusting parameters of model (39)

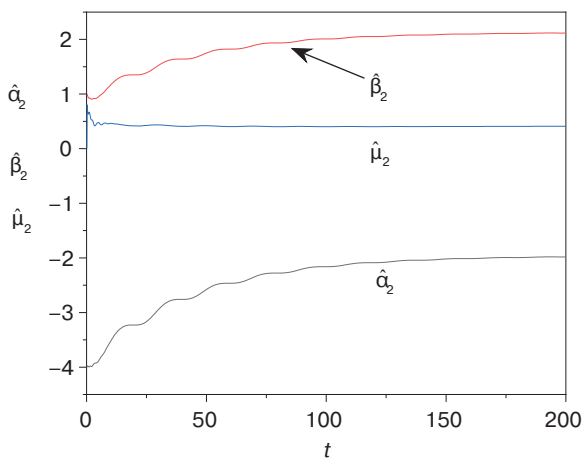


Fig. 10. Adjusting parameters of model (40)

Figure 11 shows the adjustment of the object model parameters. The adjustment is affected by the master control.

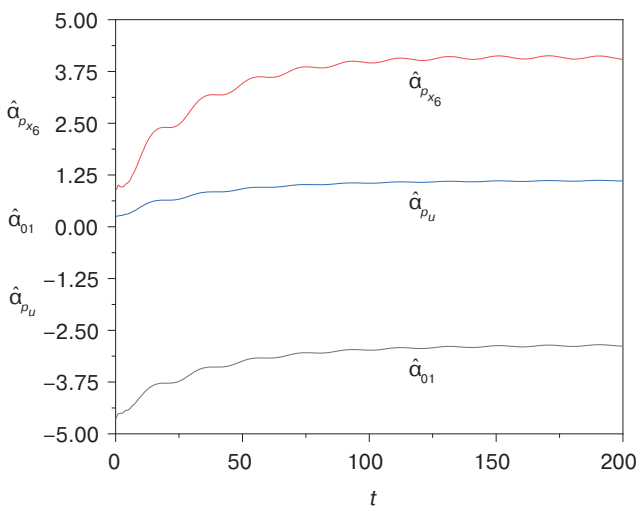


Fig. 11. Adjusting parameters of model (41)

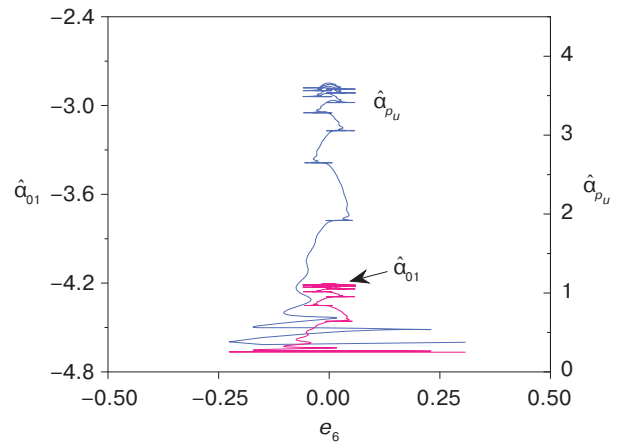


Fig. 12. Dynamics of the adjustment loop for model (41)

The dynamics of processes in adjusting model (40) is more complicated. Here, the input influence is encountered. For TCS, the statement of Theorem 5 is valid.

CONCLUSIONS

The proposed approach to identifying ICS is based on features of adaptive identification of a TCS with cross-connections. Structural aspects of TCS identification are based on obtained conditions of the system identifiability in the state space and output space, as well as the influence of input properties on the possibility of estimating system parameters. Adaptive algorithms for identifying TCS with identical channels are obtained. The identifiability of a TCS in terms of state and output is demonstrated. The results confirm the convergence of estimates for system parameters. The properties of the adaptive identification system depend on the system cross-connection parameters and informative properties of signals in the TCS. When considering the case of ICS, the role of a priori information while accounting for existing interconnections is emphasized. At appropriate ICS splitting, it would be possible to apply the approaches proposed for a TCS. The examples of adaptive identification of real systems are given. The case of TCS with nonlinear control and complex relationships is considered.

Due to the multifaceted nature of the subject area, the paper covers only certain aspects. The proposed approach can be used to estimate the dynamics of processes in an adaptive system taking into account the impact of system elements on the quality of the parameter adjustment process.

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