Russian Technological Journal. 2024;12(3):78–92 ISSN 2500-316X (Online)

Mathematical modeling

Математическое моделирование

UDC 519.63
https://doi.org/10.32362/2500-316X-2024-12-3-78-92
EDN WBOETG

RESEARCH ARTICLE

Analyzing and forecasting the dynamics of Internet resource user sentiments based on the Fokker–Planck equation

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Abstract

Objectives. The study aims to theoretically derive the power law observed in practice for the distribution of characteristics of sociodynamic processes from the stationary Fokker–Planck equation and apply the non-stationary Fokker–Planck equation to describe the dynamics of processes in social systems.

Methods. During the research, stochastic modeling methods were used along with methods and models derived from graph theory, as well as tools and technologies of object-oriented programming for the development of systems for collecting data from mass media sources, and simulation modeling approaches.

Results. The current state of the comment network graph can be described using a vector whose elements are the average value of the mediation coefficient, the average value of the clustering coefficient, and the proportion of users in a corresponding state. The critical state of the network can be specified by the base vector. The time dependence of the distance between the base vector and the current state vector forms a time series whose values can be considered as the "wandering point" whose movement dynamics is described by the non-stationary Fokker–Planck equation. The current state of the comment graph can be determined using text analysis methods.

Conclusions. The power law observed in practice for the dependence of the stationary probability density of news distribution by the number of comments can be obtained from solving the stationary Fokker–Planck equation, while the non-stationary equation can be used to describe processes in complex network structures. The vector representation can be used to describe the comment network states of news media users. Achieving or implementing desired or not desired states of the whole social network can be specified on the basis of base vectors. By solving the non-stationary Fokker–Planck equation, an equation is obtained for the probability density of transitions between system states per unit time, which agree well with the observed data. Analysis of the resulting model using the characteristics of the real time series to change the graph of comments of users of the RIA Novosti portal and the structural parameters of the graph demonstrates its adequacy.

Keywords: social networks, modeling of social processes, network graph, network graph characteristics, Fokker–Planck equation, monitoring, management, nonlinear dynamics, power law of distribution

• **Submitted:** 12.01.2023 • **Revised:** 17.11.2023 • **Accepted:** 08.04.2024

For citation: Perova J.P., Lesko S.A., Ivanov A.A. Analyzing and forecasting the dynamics of Internet resource user sentiments based on the Fokker–Planck equation. *Russ. Technol. J.* 2024;12(3):78–92. https://doi.org/10.32362/2500-316X-2024-12-3-78-92

Financial disclosure: The authors have no a financial or property interest in any material or method mentioned.

The authors declare no conflicts of interest.

НАУЧНАЯ СТАТЬЯ

Анализ и прогнозирование динамики настроений пользователей интернет-ресурсов на основе уравнения Фоккера – Планка

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Резюме
Цели. Цель работы – вывод наблюдаемого на практике степенного закона распределения характеристик социодинамических процессов из стационарного уравнения Фоккера – Планка и проверка возможности применения нестационарного уравнения Фоккера – Планка для описания динамики процессов в социальных системах.
Методы. При проведении исследований были использованы методы моделирования стохастических процессов, методы и модели теории графов, инструменты и технологии объектно-ориентированного программирования для разработки систем сбора данных из массмедиа-источников, методы имитационного моделирования.
Результаты. Наблюдаемое текущее состояние графа сети комментариев может быть описано с помощью вектора, элементами которого являются среднее значение коэффициента посредничества, среднее значение коэффициента кластеризации, доля пользователей в конкретном состоянии. Критическое состояние сети может быть задано базовым вектором. Зависимость от времени расстояния между базовым вектором и текущим вектором состояния образует временной ряд, значения которого можно рассматривать как «блуждающую точку», динамика перемещений которой описывается нестационарным уравнением Фоккера – Планка. Текущее состояние графа комментариев можно определить с помощью методов текстовой аналитики.
Выводы. Наблюдаемый на практике степенной закон зависимости стационарной плотности вероятности распределения новостей по числу комментариев может быть получен из решения стационарного уравнения Фоккера – Планка, а нестационарное уравнение может быть использовано для описания процессов в сложных сетевых структурах. Для описания состояний сети комментариев пользователей новостных массмедиа можно использовать векторное представление. Достижение или реализация желаемых, или нежелательных состояний всей социальной сети могут быть заданы на основе базовых векторов. Решение нестационарного уравнения Фоккера – Планка позволяет получить уравнение для плотности вероятности переходов между состояниями системы в единицу времени, которые хорошо согласуются с наблюдаемыми данными. Анализ полученной модели с использованием характеристик реального временного ряда для изменения графа комментариев читателей официальной страницы в социальной сети «ВКонтакте» информационного агентства «РИА Новости» и структурных параметров графа показывает ее адекватность.

Ключевые слова: социальные сети, моделирование социальных процессов, сетевой граф, характеристики сетевого графа, уравнение Фоккера – Планка, мониторинг, управление, нелинейная динамика, степенной закон распределения

• **Поступила:** 12.01.2023 • **Доработана:** 17.11.2023 • **Принята к опубликованию:** 08.04.2024

Для цитирования: Перова Ю.П., Лесько С.А., Иванов А.А. Анализ и прогнозирование динамики настроений пользователей интернет-ресурсов на основе уравнения Фоккера – Планка. *Russ. Technol. J.* 2024;12(3):78–92. https://doi.org/10.32362/2500-316X-2024-12-3-78-92

Прозрачность финансовой деятельности: Авторы не имеют финансовой заинтересованности в представленных материалах или методах.

Авторы заявляют об отсутствии конфликта интересов.

INTRODUCTION

One of the most important trends in mathematical sociology consists in methods for describing the behavior of users of social networks and information resources. In practical terms, the creation of models describing the manifestation dynamics of user opinions and preferences can be used to develop systems for the automated monitoring of social sentiment along with trends of its change. The advantage of these systems over traditional methods for studying public opinion is their technological feasibility.

Significantly, the dynamics of changes in the opinions and attitudes of Internet users can be attributed to stochastic processes. While on the one hand, the presence of the human factor (many people having different opinions, preferences, etc.) creates randomness of changes due to the great variety of user behavioral patterns, this also introduces elements of purposefulness into the dynamics of changes on the other hand.

In [1], a model describing the spatial and temporal diffusion of information in social networks based on a stochastic partial differential equation is considered. The creation and study of a non-autonomous diffusion logistic model with Dirichlet boundary conditions demonstrates that information diffusion in social networks is strongly influenced by the diffusion coefficient and the internal growth rate (information dissemination or rumors can be considered as resembling viruses that do not possess a physical form).

Among the most promising for modelling dynamics of changes in social sentiment are models created on the basis of stochastic differential equations, for example, the Fokker–Planck equation, which accounts for both ordered ("drift") and random changes ("diffusion"). The Fokker–Planck equation is widely used to analyze and model the behavior of time series when describing processes in complex systems [2–5].

In addition to the Fokker–Planck equation, approaches for modeling on the basis of differential equations include the Liouville equations [5, 6], diffusion equations [4, 7], and some others.

In addition to describing dynamic processes from the Fokker–Planck equation, stationary solutions can be obtained, which can describe the condition of any system in a stationary state, for example, when its evolution has already ended resulting in no changes having occurred.

For modeling social processes, not only models based on partial differential equations are used but also such as those based on game-theory approaches, as well as management decisions carried out on their basis [8].

Since the dynamics of processes in network structures is inextricably linked to their topology, their structural characteristics should be taken into account. In [9], for example, a methodology for analyzing topics in a social network is presented, which includes collecting, processing, and classifying information, as well as measuring the time between publications. This data is then used to create a timeline.

On this basis, a graph is constructed for tracking changes in the popularity of specific topics discussed on social media. This graph can also be used to visualize related events based on social sentiment, as well as to identify periods of active discussions on specific topics. In [10], the Kronecker graph model is used to study community detection on graphs, overlapping community detection, as well as community detection in incomplete networks with missing links and in complete networks.

The use of nature-like algorithms is also gaining popularity for investigating the link structure of social networks. In particular, in [11], swarm-like algorithms are used in social network analysis to optimize the process of solving link prediction and community detection problems. With increasing network sizes, finding similarities between their nodes becomes a rather resource-intensive process.

Although the study of processes occurring in complex systems involving the human factor shows that the power law of distribution $\rho(x) \sim x^{-\gamma}$ (where γ is the characteristic degree) is fulfilled very often for the observed parameter characteristics of these processes [12–17], its theoretical foundation requires further study. In order to carry out a deeper study of the behavior and analysis of complex social systems, it will be necessary to identify the nature of processes that give rise to the degree law.

In this regard, studying the possibility of applying the Fokker–Planck equation to model the dynamics of social processes looks very promising.

1. DATA COLLECTION AND PROCESSING

Several news portals and one of the online communities of the VKontakte social network dedicated to discussing the news of the RIA Novosti information resource[1] were selected for study. This resource was chosen based on its recognizability and popularity in Russian society; it ranks first among media resources (for March 2022) according to Brand Analytics[2], is among the top 3 most quoted news agencies in mass media and social media (for March 2022), and ranks first[3] by these indicators in the Russian Internet segment.

First, a special software application (parser) was used to download the desired range of news from January 1, 2019 to April 2022 from the official RIA Novosti community in the VKontakte social network using the developed parser and the network application programming interface[4]. Within the social network, each post has its own unique address, where {owner_id} is the community unique identifier (for RIA_Novosti, it is "-15755094"), while {post_id} is the unique identifier of the post (news). Each post (news) has a number of basic parameters such as: unique identifier of the post in the social network; post text; date and time of publication; and number of views and user comments. Comments have the following parameters: unique identifier in the social network community; unique user identifier; comment text; date and time of appearance; comment hierarchy level; and connection by comment level with parent comment (which user comments on which other user when discussing the news).

Since comments could be left by chatbots, spammers, and unscrupulous users who write comments on a professional basis, it is necessary to introduce data-scrubbing rules. Users writing more than 7,365 comments during a year (more than 20 comments per day on average) or who write with a frequency of more than one comment per 5 min are considered as unscrupulous.

When analyzing the obtained data, it is necessary to determine to what distribution law the observed distribution density obeys. For this, the following three most frequently observed distribution laws are examined: Gaussian $\rho(x) = e^{-\frac{x^2}{2\sigma^2}} \Big/ \sigma\sqrt{2\pi}$; exponential $\rho(x) = a e^{-\alpha x}$, and power series $\rho(x) \sim \beta x^{-\gamma}$. When processing the collected data using linearization in appropriate coordinates, the closest linearization is observed for the power law (Fig. 1). For other laws, worse linearization is observed.

The straight line drawn in Fig. 1 shows that the trend line is well described by the linear approximation $y = -0.76 - 1.48z$, where $y = \ln\{\rho(x)\}$, $z = \ln\{x\}$, $\ln\{\beta\} = -0.76$, and the correlation coefficient is 0.95.

In order to confirm the conclusion about linear approximation, the behavior of the residuals can be studied in order to test the hypothesis that they are normally distributed with mean equal to zero and have homogeneous variance. The equation is derived from residuals calculated based on the actually observed values for the natural logarithm of the fraction of commenters who have written a certain number of comments. The calculated value of the mathematical expectation for the distribution of residuals is equal to 0.25, while the variance is 0.13. The test of the slope hypothesis (two-sample F-test for variances) shows that the variance of residuals (calculated relative to the trend line) equal to 2.11 (0.13 << 2.11) is significantly smaller than that of the deviation of linear regression points from the mean value of the observed data ($\Sigma y_i/n = \Sigma \ln\{\rho(x_i)\}/n$). Thus, it can be concluded from the obtained data that the distribution of residuals is very close to normal, while the obtained regression is significant. This supports the conclusion that the natural logarithm of the fraction of commenters who have written these comments depends linearly on the natural logarithm of the number of comments, which confirms the fulfillment of the power law.

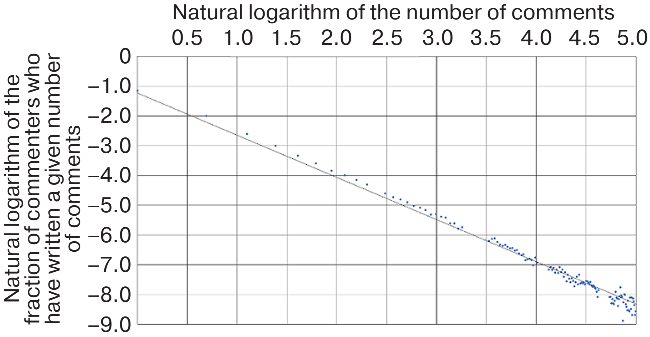

Fig. 1. Linearization of the observed data for the power series distribution of the fraction of commenters to the number of comments made

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[1] https://vk.com/ria (in Russ.). Accessed September 02, 2023.

[2] https://br-analytics.ru/mediatrends/media/?period=202203 (in Russ.). Accessed September 02, 2023.

[3] https://www.mlg.ru/ratings/media/federal/11110/#internet (in Russ.). Accessed September 02, 2023.

[4] https://dev.vk.com/guide (in Russ.). Accessed September 02, 2023.

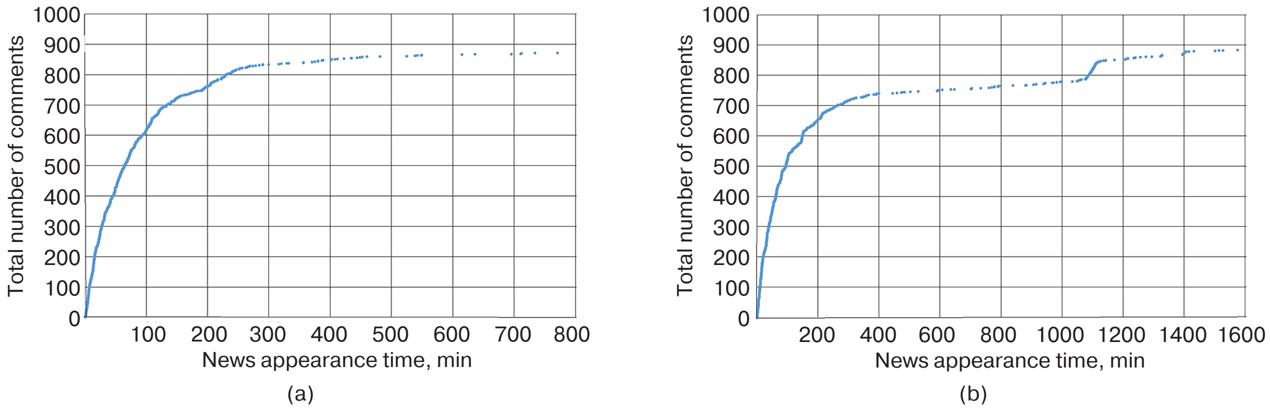

Fig. 2. Observed dynamics of change in the number of comments to news items

It is also of interest in research to analyze the dynamics of change in the number of comments on news of great public interest over time (such news gathers hundreds of comments during the viewing period).

Observing this dynamic of change shows that it can have both S-shaped and two-stage character. For demonstrating such dynamics of commenting on RIA Novosti news by VKontakte users, the following several news items are selected:

1. "Zelensky had left Ukraine and moved to Poland, Volodin said" (see Fig. 2a, where the dynamics of comments has an S-shaped character)[5, 6], origin date and time: 2022-03-04 16:13:27 UTC +03:00). The total number of comments is 894. The number of first-level comments (comments on the news itself) is 433, while the number of second-level comments (first-level comments on the comments) is 461. The total number of views is 118764. The average appearance time of the first-level comments is 73 min while that of the second-level comments is 74 min.
2. "Soon the city will be liberated, he stated" (see Fig. 2b, where the dynamics of comments has a two-stage character)[7, 8], origin date and time: 2022-04-10 17:14:40 UTC +03:00). The total number of comments is 901. The number of first-level comments is 173 and the number of second-level comments is 728. The total number of views is 173607. The average appearance time of the first-level comments is 75 min and that of the second-level comments is 82 min.

In our view, this can be related to the difference in the average appearance time of the second-level comments (the time interval between the appearance of the first-level comments and the commentary on these comments), as well as to the ratio between the number of comments of the first and second levels. For the first news item, the average appearance time intervals of the first and second-level comments practically coincide; for the second news item, there is a slight increase in the average appearance time interval of the second-level comments (time lag occurs). In addition, their number significantly exceeds the number of comments of the first-level for the second news item.

The following task of further theoretical research can be formulated: what is the nature of commenting news and blogs, and what features of these complex social systems result in describing the probability density of comment distribution on the number of comments using the power law, whose dynamics have a complex two-step character over many cases.

2. SOLVING THE STATIONARY FOKKER–PLANCK EQUATION

In general, the Fokker–Planck equation has the following form:

$$\frac{\partial \rho(x,t)}{\partial t} = -\frac{\partial}{\partial x}[\mu(x)\rho(x,t)] + \frac{1}{2}\cdot\frac{\partial^2}{\partial x^2}[D(x)\rho(x,t)], \qquad (1)$$

where $\rho(x, t)$ is the time t dependent probability density of distribution over states x (in our case, state x is the number of comments observed at time t); $D(x)$ is the state-dependent coefficient determining the random change of state x (diffusion); $\mu(x)$ is the state x dependent coefficient determining the purposeful change of state x (drift).

As applied to our model, $D(x)$ can be interpreted as the user activities caused by a spontaneous impulse arising from reading news item or comments of other users in case the event described in the news item or blog is not significantly important. The user is ready to

[5] https://vk.com/ria?w=wall-15755094_34243579 (in Russ.). Accessed September 02, 2023.

[6] https://ria.ru/20220304/zelenskiy-1776545154.html (in Russ.). Accessed September 02, 2023.

[7] https://vk.com/ria?w=wall-15755094_35202266 (in Russ.). Accessed September 02, 2023.

[8] https://ria.ru/20220410/ukraina-1782778315.html (in Russ.). Accessed September 02, 2023.

spend time on commenting or responding to another commenter (the user has a spontaneous desire to respond to this news item). The coefficient $\mu(x)$ can be interpreted as purposeful behavior caused by the desire to respond to a news item or blog event that is essential for the user as well as to comment on another user's comment in case it addresses an important topic from this user perspective (the user is constantly interested in this topic).

For building the model, assumptions about the dependence of $D(x)$ and $\mu(x)$ on state x should be made and two conditions should be considered. First, the dimensionality of the terms included in Eq. (1) should be taken into account; second, it can be assumed that as the state x grows (the number of possible comments, the importance of the news or blog), values $D(x)$ and $\mu(x)$ should also increase. Physical considerations suggest that all terms of Eq. (1) should have the same dimensionality, which $\rho(x, t)$ has. Both the first and the second condition would be satisfied if dependencies of $D(x)$ and $\mu(x)$ on state x have the following form: $\mu(x) = \mu_0 x$ and $D(x) = D_0 x^2$. The coefficients μ_0 and D_0 have dimensionality $1/t$.

The solution of the stationary Fokker–Planck equation is written as follows:

$$-\frac{d}{\partial x}[\mu(x)\rho(x)] + \frac{1}{2}\cdot\frac{d^2}{dx^2}[D(x)\rho(x)] = 0$$

under the following assumptions: $\mu(x) = \mu_0 x$ and $D(x) = D_0 x^2$ has the form $\rho(x) = [\gamma - 1]x^{-\gamma}$, which corresponds to the power law observed in practice.

According to the results obtained from analyzing the observed data, $\gamma = 1.48$, and $\gamma - 1 = 0.48$, the natural logarithm $(\gamma - 1)$ is -0.73, which is equal to $\ln\{\beta\} = -0.76$ with sufficiently high precision (see the resulting linearization equation: $y = -0.76 - 1.48z$, where $y = \ln\{\rho(x)\}$, $z = \ln\{x\}$, $\ln\{\beta\} = \ln\{\gamma - 1\} - 0.76$, and the correlation coefficient is 0.95). This generally indicates the adequacy of the developed model. Note that the terms of the stationary equation for $\rho(x)$ should also have the same dimensionality. The solution of the stationary equation includes expression $2\left[1 - \frac{\mu_0}{D_0}\right] = \gamma$ which does not depend on t.

Deriving the power law can be described as follows. We substitute expressions for $\mu(x)$ and $D(x)$ into the stationary Fokker–Planck equation.

Substituting $D(x)$ and $\mu(x)$ into Eq. (1), the following is obtained:

$$-\mu_0 x\frac{d\rho(x)}{dx} - \mu_0\rho(x) + \frac{1}{2}D_0 x^2\frac{d^2\rho(x)}{dx^2} + 2D_0 x\frac{d\rho(x)}{dx} + D_0 x\rho(x) = 0.$$

We denote $2\left[1 - \frac{\mu_0}{D_0}\right] = \gamma$, then

$$x^2\frac{d^2\rho(x)}{dx^2} + [2 + \gamma]x\frac{d\rho(x)}{dx} + \gamma\rho(x) = 0.$$

The solution of the resulting equation can be sought in the form $\rho(x) = \sum_k C_k x^q$, where C_k are constant coefficients at corresponding roots of the characteristic equation having the form $q(q-1) + [2 + \gamma]q + \gamma = 0$. This equation has two roots: $q_1 = -1$ and $q_2 = -\gamma$. Thus, for $\rho(x)$, we obtain: $\rho(x) = C_1 x^{-1} + C_2 x^{-\gamma}$. The constant coefficients C_1 and C_2 are found using the normalization condition for function $\rho(x)$, as follows:

$$\int_1^\infty \rho(x)dx = C_1 \ln x\Big|_1^\infty + C_2\frac{x^{1-\gamma}}{1-\gamma}\Big|_1^\infty \equiv 1.$$

The integral is calculated from 1 to ∞, since there may be users who have written a very large number of comments on news items, but there cannot be commenters who have written less than one comment. Given that at $x \to \infty$, $\ln x\big|_\infty = \infty, C_1 = 0$ and $C_2 = \gamma - 1$, respectively, we finally obtain: $\rho(x) = [\gamma - 1]x^{-\gamma}$.

3. SOLVING THE NON-STATIONARY FOKKER–PLANCK EQUATION AND ANALYZING THE MODEL

We briefly derive the solution for the non-stationary Fokker–Planck equation. Using the Laplace transform method for the Fokker–Planck equation, the following can be written:

$$s\overline{G(s,x)} - \rho(x,0) = = -\frac{d}{dx}\left[\mu(x)\overline{G(s,x)}\right] + \frac{1}{2}\cdot\frac{d^2}{dx^2}\left[D(x)\overline{G(s,x)}\right]. \quad (2)$$

Substituting the corresponding derivatives and dependencies of $\mu(x)$ and $D(x)$ into Eq. (2), we obtain the following:

$$x^2\frac{d^2\overline{G(s,x)}}{dx^2} + 2\left[2 - \frac{\mu_0}{D_0}\right]x\frac{d\overline{G(s,x)}}{dx} + 2\left[1 - \frac{\mu_0 + s}{D_0}\right]\overline{G(s,x)} = -\frac{\delta(x - x_0)}{D_0}. \quad (3)$$

The solution of Eq. (3) will be found in the form: $\overline{G(s,x)} = \sum_k C_k x^q$, where C_k are the coefficients for the roots of the characteristic equation which has the following form:

$$q(q-1)+2\left[2-\frac{\mu_0}{D_0}\right]q+2\left[1-\frac{\mu_0+s}{D_0}\right]=0.$$

The solution to the non-stationary Fokker–Planck Eq. (3) under the $\mu(x)$ and $D(x)$ assumptions takes the following form:

$$\rho(x,t)=\int\frac{\left[\frac{[\ln x]^2}{D_0 t}+\left[\frac{1}{2}-\frac{\mu_0}{D_0}\right]\ln x-1\right]}{\sqrt{2\pi D_0 t^3}}\times \\ \times\, e^{-\left[\frac{[\ln x]^2}{2D_0 t}+\left[\frac{3}{2}-\frac{\mu_0}{D_0}\right]\ln x+\left[\frac{1}{2}-\frac{\mu_0}{D_0}\right]^2\frac{D_0 t}{2}\right]}dt. \quad (4)$$

The probability that the number of comments by time t reaches a certain number L can be found by the formula: $P(L,t)=1-\int_0^L\rho(x,t)dx$. The dependence of the number of comments on time t is described by equation: $N(t) = P(L, t)L$.

To analyze the obtained solution, simulation modeling is carried out. As an example, we select $L = 100$ and three sets of μ_0 and D_0 values: $\mu_0 = 0.45$ and $D_0 = 0.50$ conventional units ($\mu_0 < D_0$ is curve 1 in Fig. 3a), $\mu_0 = 0.50$ and $D_0 = 0.50$ conventional units ($\mu_0 = D_0$ is curve 2 in Fig. 3), and $\mu_0 = 0.55$ and $D_0 = 0.50$ conventional units ($\mu_0 > D_0$ is curve 3 in Fig. 3a). The calculations show that, as μ_0 increases relative to D_0, the growth rate of curves for the number of comments $N(t)$ at the selected model parameters μ_0, D_0, and L increases (Fig. 3a).

A two-step curve can be obtained by using the distribution density function accounting for delay time τ, as follows:

$$\rho(x,t-\tau)=\int\frac{\left[\frac{[\ln x]^2}{D_0(t-\tau)}+\left[\frac{1}{2}-\frac{\mu_0}{D_0}\right]\ln x-1\right]}{\sqrt{2\pi D_0[t-\tau]^3}}\times \\ \times\, e^{-\left[\frac{[\ln x]^2}{2D_0[t-\tau]}+\left[\frac{3}{2}-\frac{\mu_0}{D_0}\right]\ln x+\left[\frac{1}{2}-\frac{\mu_0}{D_0}\right]^2\frac{D_0(t-\tau)}{2}\right]}dt. \quad (5)$$

Confirmation of the compliance between the theoretical model and the observed data (Figs. 2 and 3b) can be obtained on the assumption that two processes with different coefficients μ_0 and D_0 can occur simultaneously. Moreover, the sum of partial fractions of processes should be equal to 1, i.e., $P_{com}(L, t) = \alpha_1 P_1(L, t) + \alpha_2 P_2(L, t)$. As a purely illustrative modeling example, the following model parameters are selected for the process of commenting on the news item or blog itself: $\mu_{0,1} = 0.55$, $D_{0,1} = 0.50$. For commenting on comments: $\mu_{0,2} = 0.50$, $D_{0,2} = 0.50$, $\tau = 50$ conventional units, $\alpha_1 = 0.75$, $\alpha_2 = 0.25$, and $L = 100$ ($\mu_{0,1} > \mu_{0,2}$ is selected based on the assumption that commenting on a news item is a more primary process for users than commenting on comments), where $\alpha_1 + \alpha_2 = 1$.

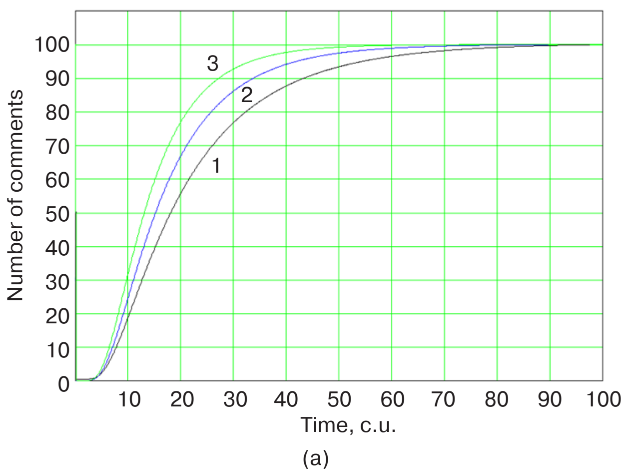

Fig. 3. Dynamics of change over time in the number of comments to the news item in the simulation model based on the Fokker–Planck equation:
(a) 1 process,
(b) 2 parallel processes

As indicated earlier, the plateau in curve 1 in Fig. 3b may be due to a significant difference in the average appearance time of the second-level comments (the time interval between the appearance of the first-level comment and the comment on a given comment), which may result in implementation of two-stage dynamics in the appearance of comments.

The results of modeling the dynamics of the change in the number of comments $N(t)$ over time with allowance for two processes running in parallel are shown in Fig. 3b. The results show a good compliance between the real data and theoretical calculations.

4. MODELING THE DYNAMICS OF THE INTERNET MASS MEDIA USER SENTIMENT BASED ON THE FOKKER–PLANK EQUATION AND CHANGES IN THE PARAMETERS OF THEIR COMMENT NETWORK GRAPHS

It is proposed to use vector representation to describe the states of the comment network. The elements of the vectors serve as allowed values of the network parameters (density, mediation coefficient average, value clustering coefficient average, elasticity, and others), as well as characteristics like fraction of users who can be attributed, based on the analysis of comment texts, to one of the following groups: 1) loyalist (unconditionally supports the actions of the government and authorities); 2) oppositionist; 3) "troll" (a user who uses the resource to blow the whistle); and 4) undecided or neutral. Achieving or implementing the desirable or undesirable states of the entire social network as a whole can be based on base vectors.

The time change in the value of the distance between the base vector and the vector of the current state can be considered as a "wandering point" on segment $[L_{min}, L_{max}]$ or as a random (or almost random) time series. Some given value of this distance (the state upon reaching which management decisions should be made) can be regarded as a trap or a point of the acceptable implementation threshold, where a "wandering point" can fall over time. This allows building probabilistic sociodynamic models for forecasting the dynamics of social sentiment.

The diffusion model is commonly used in the conventional description of the "wandering point" behavior. However, it cannot be considered reliable in this case. As a rule, time series describing processes in complex systems (for example, financial indicators of stock and commodity exchanges) are not stationary due to various reasons including the presence of human factor. Their sample distribution functions have the time dependent mathematical expectation, which contradicts the simple diffusion model and shows the non-stationarity of time series.

In this regard, it is intended to consider more complex models of the "wandering point" behavior, for example, based on the Fokker–Planck equations.

The graph structure obtained from processing news comments is shown in Fig. 4. When visualizing the obtained data in color, the graph nodes, depending on their state (attribution to one of 4 types), can be marked in different colors (oppositionists in green-sand color, "trolls" in red, loyalists in blue, and undecided in green). The links in the figure show mutual commenting of users by each other. Thus, one can estimate their state by the color of the nodes and the interaction by the graph edges.

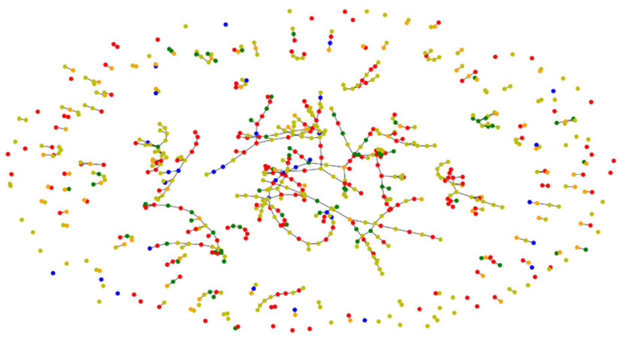

Fig. 4. Graph structure for comments to the news item under consideration

It is evident from Fig. 4 that this structure has many unconnected single vertices. In a connected component of the graph presented separately in Fig. 5, the closed oval lines (representing the core of the graph in Fig. 4) show the users commenting on themselves.

We consider the elements of the network state vector to be used in our model:

1. The fraction of nodes having a certain state (e.g., users who are negative about some social life event).
2. The clustering coefficient is a measure for the link density of a given node with its neighbors. The ratio of the real number of links connecting the nearest neighbors of a given node *i* to the maximum possible one (such that all the nearest neighbors of a given node would be connected to each other directly) is called the node clustering coefficient; its value lies on interval [0, 1]. The higher its value is, the more significant this node is in the process of information exchange.
3. The mediation degree shows the ratio of the number of shortest paths between all pairs of network nodes passing through a given node to the total number of all shortest paths in the network. Its value lies on the interval [0, 1]. The larger its value, the more significant is the role of a given node in information exchange.

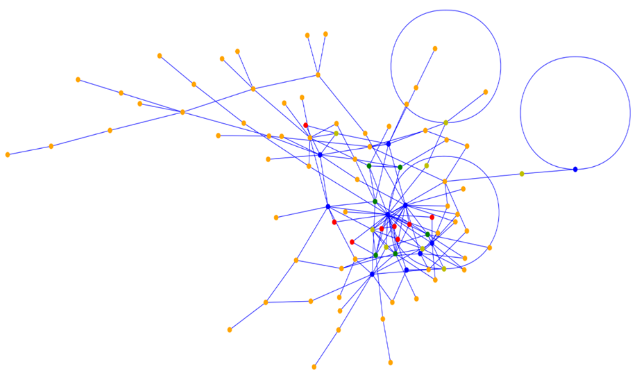

Fig. 5. Graph of user interconnections by comments

We define the values for the elements of the network state base vector (denoted by θ). They specify the threshold values, whose passing through is undesirable in terms of state management. Given that any community usually contains from 0.10 to 0.15 fraction of participants always disagreeing on any issue, we take the fraction of those who are negative about the considered event to be 0.12. The desired average value (over all nodes) of the clustering coefficient for such network is also assumed to be small, for example, equal to 0.05; the average degree of node mediation in such network is also equal to 0.05. Thus, the basis vector would have the following form: θ = (0.12; 0.05; 0.05).

Note that the number of parameters used to describe the network state may be larger. Only those considered the most important are selected. In addition, the selected parameters are normalized (lie in interval [0, 1]) so that they have the same effect on calculating the distance metric.

In the proposed approach, different graphs of news commenting on certain topics on a selected info resource during a day can be combined into a single structure through links between nodes that refer to users. In this way, a large graph describing the activity of users of a given network info resource during a day can be identified. Next, it is possible to define the elements of the current state vector that describes its characteristics.

Changes in this vector components for each day during a certain time would form some time series.

For describing the change in the value of the distance between the value of the current state vector and a given basis vector over time, the solution of the non-stationary Fokker–Planck equation is considered, which may allow the construction of probabilistic sociodynamic models for predicting the dynamics of social sentiment.

We formulate the boundary value problem, whose solution describes the process of changing the value of the distance between the value of the current state vector in the comment network graph and a given basis vector over time.

First boundary condition. When selecting the first boundary condition, we assume the following: state $x = L_{\min}$ (the left boundary of the segment of possible states) determines the state whose passing through should be avoided (the area to the left of this state on the segment is undesirable). The probability of detecting such a system state may be non-zero, while the probability density determining the flow in state $x = L_{\min}$ should be taken as equal to 0, since the states should not exceed this boundary (here the reflection condition is implemented). Thus:

$$\rho(x,t)\big|_{x=L_{\min}} = 0. \tag{6a}$$

Second boundary condition. We limit the area of possible states to the right by some value $x = L_{\max}$ (the metric used in calculations cannot be larger than the value of the vector whose elements have maximum values in the space of selected coordinates). The probability of detecting such state over time is different from zero. However, the probability density determining the flow in state $x = L_{\max}$ should be set equal to zero (the distance between the current and the base state vector is limited by the maximum values of possible coordinates in the applied vector space (the condition of reflection from the boundary is implemented)):

$$\rho(x,t)\big|_{x=L_{\max}} = 0. \tag{6b}$$

When formulating the boundary value problem, the initial condition should be specified. Since the state of the system (the distance between the basis vector and the current state vector) can be equal to some value x_0 at time $t = 0$, the initial condition can be specified in the following form:

$$\rho(x,t=0) = \begin{cases} \int \delta(x-x_0)dx = 1, & x = x_0, \\ 0, & x \neq x_0. \end{cases} \tag{6c}$$

Briefly, the solution to this boundary value problem has the following form. Using the Laplace transform method for the Fokker–Planck equation, the following can be written:

$$s\overline{G(s,x)} - \rho(x,0) = \\ = -\frac{d}{dx}\left[\mu(x)s\overline{G(s,x)}\right] + \frac{1}{2}\cdot\frac{d^2}{dx^2}\left[D(x)s\overline{G(s,x)}\right]. \tag{7}$$

Substituting the corresponding derivatives and dependencies of $\mu(x)$ and $D(x)$ into Eq. (7), we obtain the following:

$$x^2\frac{d^2\overline{G(s,x)}}{dx^2} + 2\left[2 - \frac{\mu_0}{D_0}\right]x\frac{d\overline{G(s,x)}}{dx} + \\ + 2\left[1 - \frac{\mu_0 + s}{D_0}\right]\overline{G(s,x)} = -\frac{\delta(x-x_0)}{D_0}. \tag{8}$$

The solution to this equation is found in the form: $\overline{G(s,x)} = \sum_k C_k x^q$, where C_k are the coefficients for the roots of the following characteristic equation: $q(q-1) + 2\left[2 - \frac{\mu_0}{D_0}\right]q + 2\left[1 - \frac{\mu_0 + s}{D_0}\right] = 0.$

We shall seek the solution in the form of two functions: $\overline{G_1(s,x)}$ on interval $[L, x_0]$ and $\overline{G_2(s,x)}$ on interval $[x_0, 1]$ using the cross-linking condition of functions $\overline{G_1(s,x)}$ and $\overline{G_2(s,x)}$ at point $x = x_0$. The presence of the δ-function in the equation results in the solution remaining continuous at point $x = x_0$ but having a discontinuity of the derivative there.

At $L_{\min} \le x \le x_0$

$$\overline{G_1(s,x)} = C_1 x^{-[1+\alpha]+\beta\sqrt{k+s}} + C_2 x^{-[1+\alpha]-\beta\sqrt{k+s}}.$$

At $x_0 \le x \le L_{\max}$

$$\overline{G_2(s,x)} = C_3 x^{-[1+\alpha]+\beta\sqrt{k+s}} + C_4 x^{-[1+\alpha]-\beta\sqrt{k+s}}.$$

After appropriate transformations, we obtain the following:

- at $L_{\min} \le x \le x_0$,

$$\overline{G_1(s,x)} = \frac{x_0^{\alpha} x^{-[1+\alpha]} \operatorname{sh}\left\{\left[\beta \ln\left(\frac{L_{\max}}{x_0}\right)\right]\sqrt{k+s}\right\}}{\beta D_0 \sqrt{k+s}} \times \frac{\operatorname{sh}\left\{\left[\beta \ln\left(\frac{x}{L_{\min}}\right)\right]\sqrt{k+s}\right\}}{\operatorname{sh}\left\{\left[\beta \ln\left(\frac{L_{\max}}{L_{\min}}\right)\right]\sqrt{k+s}\right\}};$$

- at $x_0 \le x \le L_{\max}$,

$$\overline{G_2(s,x)} = \frac{x_0^{\alpha} x^{-[1+\alpha]} \operatorname{sh}\left\{\left[\beta \ln\left(\frac{x_0}{L_{\min}}\right)\right]\sqrt{k+s}\right\}}{\beta D_0 \sqrt{k+s}} \times \frac{\operatorname{sh}\left\{\left[\beta \ln\left(\frac{x}{L_{\max}}\right)\right]\sqrt{k+s}\right\}}{\operatorname{sh}\left\{\left[\beta \ln\left(\frac{L_{\max}}{L_{\min}}\right)\right]\sqrt{k+s}\right\}}.$$

We perform the following inverse Laplace transformations:

$$\overline{G_1(s,x)} = \frac{A(s)}{B(s)} = \sum_{n=1}^{M} \frac{A(s_n)}{B(s_n)} \cdot e^{s_n t},$$

where s_n are simple non-zero roots of $B(s)$,

$$B(s) = \sqrt{k+s} \cdot \operatorname{sh}\left\{[\beta \ln L]\sqrt{k+s}\right\},$$

$$A(s) = sh\left\{[\beta \ln x_0]\sqrt{k+s}\right\} \cdot \operatorname{sh}\left\{\left[\beta \ln \frac{x}{L}\right]\sqrt{k+s}\right\}.$$

After appropriate transformations, we obtain the following at $L_{\min} \le x \le x_0$:

$$\rho_1(x,t) = 2 \cdot \frac{x_0^{\alpha} x^{-[1+\alpha]} e^{-kt}}{D_0 \beta^2 \ln\left(\frac{L_{\max}}{L_{\min}}\right)} \cdot \sum_{n=1}^{M} \frac{\sin\left\{\pi n \frac{\ln\left(\frac{L_{\max}}{x_0}\right)}{\ln\left(\frac{L_{\max}}{L_{\min}}\right)}\right\} \cdot \sin\left\{\pi n \frac{\ln\left(\frac{x}{L_{\min}}\right)}{\ln\left(\frac{L_{\max}}{L_{\min}}\right)}\right\}}{\cos \pi n} \cdot e^{-\frac{\pi^2 n^2 t}{\left[\beta \ln\left(\frac{L_{\max}}{L_{\min}}\right)\right]^2}}.$$

Similarly, at $x_0 \le x \le L_{\max}$,

$$\rho_2(x,t) = -4 \cdot \frac{x_0^{\alpha} x^{-[1+\alpha]} e^{-kt}}{D_0 \beta^2 \ln L} \cdot \sum_{n=1}^{M} \frac{\sin\left\{\pi n \frac{\ln\left(\frac{x_0}{L}\right)}{\ln L}\right\} \cdot \sin\left\{\pi n \frac{\ln x}{\ln L}\right\}}{\cos \pi n} \cdot e^{-\frac{\pi^2 n^2 t}{[\beta \ln L]^2}}.$$

Accounting for $\alpha = \frac{1 - 2 \cdot \frac{\mu_0}{D_0}}{2} = \frac{1}{2} - \frac{\mu_0}{D_0}$, $\beta^2 = \frac{2}{D_0}$ and $k = \frac{\alpha^2}{\beta^2}$, we obtain the following:

- at $L_{\min} \le x \le x_0$,

$$\rho_1(x,t) = -2\cdot\frac{x_0^{\alpha}x^{-[1+\alpha]}\cdot e^{-\frac{D_0\alpha^2}{2}t}}{\ln\left(\frac{L_{\max}}{L_{\min}}\right)}\cdot\sum_{n=1}^{M}\frac{\sin\left\{\pi n\frac{\ln\left(\frac{L_{\max}}{x_0}\right)}{\ln\left(\frac{L_{\max}}{L_{\min}}\right)}\right\}\cdot\sin\left\{\pi n\frac{\ln\left(\frac{x}{L_{\min}}\right)}{\ln\left(\frac{L_{\max}}{L_{\min}}\right)}\right\}}{\cos\pi n}\cdot e^{-\frac{\pi^2n^2D_0t}{2\left[\ln\left(\frac{L_{\max}}{L_{\min}}\right)\right]^2}}; \tag{9a}$$

- at $x_0 \le x \le L_{\max}$,

$$\rho_2(x,t) = 2\cdot\frac{x_0^{\alpha}x^{-[1+\alpha]}\cdot e^{-\frac{D_0\alpha^2}{2}t}}{\ln\left(\frac{L_{\max}}{L_{\min}}\right)}\cdot\sum_{n=1}^{M}\frac{\sin\left\{\pi n\frac{\ln\left(\frac{x_0}{L_{\min}}\right)}{\ln\left(\frac{L_{\max}}{L_{\min}}\right)}\right\}\cdot\sin\left\{\pi n\frac{\ln\left(\frac{x}{L_{\max}}\right)}{\ln\left(\frac{L_{\max}}{L_{\min}}\right)}\right\}}{\cos\pi n}\cdot e^{-\frac{\pi^2n^2D_0t}{2\left[\ln\left(\frac{L_{\max}}{L_{\min}}\right)\right]^2}},\ \alpha = \frac{1}{2}-\frac{\mu_0}{D_0}. \tag{9b}$$

The probability that the system state will be in the interval from $L_{\min}$ to $L_{\max}$ by time t (i.e., the threshold state (θ) will not be reached) can be calculated as follows:

$$P(\theta,t) = \int_{L_{\min}}^{x_0}\rho_2(x,t)dx + \int_{x_0}^{L_{\max}}\rho_1(x,t)dx. \tag{10}$$

The probability $Q(\theta, t)$ that the threshold state θ will be reached or surpassed by time t is calculated using the following formula:

$$Q(\theta, t) = 1 - P(\theta, t). \tag{11}$$

Defining the boundaries of the interval of possible states from $L_{\min}$ to $L_{\max}$ will be discussed later on.

Using sentiment analysis tools, all user comments during a day can be categorized into positive and negative to obtain a time series. The time series of daily activity of users (with negative attitudes) of the RIA Novosti official community on the VKontakte social network on commenting on news from January 2013 to December 2020 is presented in Fig. 6. The activity is defined as the ratio of the total number of unique comments left by users on all news items during the day to the total number of unique views of all published news items by users during the day.

It should be noted that visitors in general are very reluctant to leave comments. The total number of those who make them is unlikely to exceed 1–2% of all portal viewers.

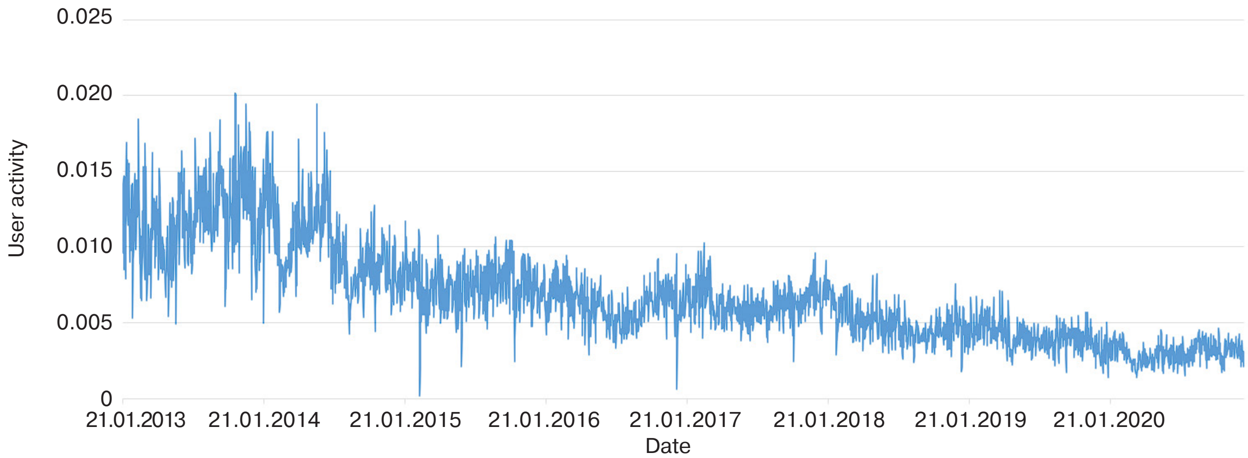

Fig. 6. Time series of RIA Novosti portal user activity in commenting on news from January 2013 to December 2020

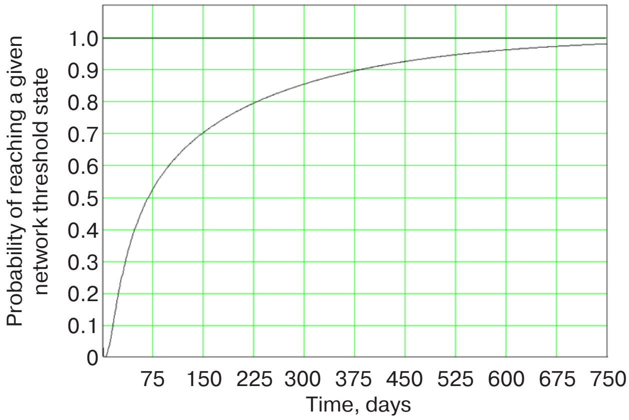

Fig. 7. Time dependence of the probability of reaching a given network threshold state for the considered example at time point May 15, 2019

It is possible to construct an aggregated comment graph for a day, as well as to define its characteristics and state vector (changing from day to day).

We shall discuss selecting the boundaries for the interval of possible states from L_{min} to L_{max}. For the comment graph of the RIA Novosti portal users, the elements of the current state vector at the time moment of May 15, 2019 (which is taken as $t = 0$) are equal to $X(t) = (0.0035; 0.07; 0.12)$: the fraction of negative comments is 0.0035, while the clustering coefficient average is 0.07, and the average degree of node mediation is 0.12. The baseline vector of the desired state is defined as $\theta = (0.0025; 0.05; 0.05)$: the fraction of negative comments is 0.0025, the clustering coefficient average is 0.05, and the average degree of node mediation is 0.05. The value for the fraction of negative comments equal to 0.0025 is selected for model validation, since the time of observing this value actually in the time series graph is known. The model's adequacy can be evaluated by comparing the forecast modeling results with the real value. The distance between the given base vector of the desired state $\theta = (0.0025; 0.05; 0.05)$ and the current state vector $X(t)$ at time $t = 0$ is $x_0 = 0.073$. Analyzing the dynamics of the time series of changes in the state of the network for several previous days and using the model equations, the inverse problem can be solved to determine values for model parameters μ_0 and D_0. In this case, $\mu_0 = 0.0002$ and $D_0 = 0.0009$.

The right boundary of the interval of possible states L_{max} can be given as the distance between the basis vector (θ) and the vector of maximum possible values of network parameters $X(t) = (1; 1; 1)$. In the considered case, $L_{max} = 1.70$. For reliability of consideration, the left boundary can be defined, for example, as half the length of a given basis vector (in this case, $|\theta| = 0.071$), so that L_{min} is equal to 0.035.

The results show that under the prevailing conditions with no impact on the user network, the desired state can be achieved with a probability of 0.8 on the 225th day, and with a probability of 0.9 on the 375th day (Fig. 7), which is roughly what actually happened, according to the observed data.

5. ALGORITHM FOR PREDICTING THE ACHIEVEMENT OF A GIVEN STATE OF THE COMMENT NETWORK GRAPH OF NEWS MEDIA USERS

On the basis of the studies and the developed model for the dynamics of change in the sentiment of Internet media users based on the Fokker–Planck equation, as well as changes in the parameters of the graphs of their comment networks, an algorithm is created for their prediction consisting of the following steps:

a) Date and time stamped collection of text comments and metadata of users on a certain topic from online news media resources.
b) Data processing using text analytics tools and sentiment analysis, obtaining the user comment graph on a certain topic, and calculating its characteristics (network density, mediation coefficient average, clustering coefficient average, elasticity, and fraction of users with a particular sentiment).
c) Specifying the value for elements of the base vector determining the achievement of the desired or undesired state (θ), as well as forming a time series of changes in the graph of user comments on a certain topic over time based on the processed data and the given vector.
d) Specifying step duration τ (hour, day, week, etc.). Based on the values of the time series at several steps for a given τ, determining model parameters μ_0 and D_0 by numerical calculations using the observed data and Eqs. (5) and (6).
e) Accepting the last average value of the distance metric between the base vector and the current network state vector as initial state x_0 and using the obtained values for μ_0 and D_0 along with Eqs. (5) and (6). Performing calculations and obtaining the time dependence of the probability of achieving the desired or undesired state. Then the probability value can be specified (e.g., 0.95), as well as the time to reach the specified probability level can be estimated (time prediction).

CONCLUSIONS

In the work, the following results have been obtained:

1. The stationary distribution of news in terms of the number of comments to them observed in practice corresponds to the power law: $\rho(x) = [\gamma - 1]x^{-\gamma}$, where $\rho(x)$ is the fraction of news in their total

number having x comments while γ is the index of power.

2. The dynamics of change over time in the number of comments to a news item or blog can have both S-shaped form and two-stage form due to a significant difference in the average appearance time of second-level comments (the time interval between the appearance of the first-level comment and the comment to this comment), i.e., the value of the average delay.
3. On the basis of certain assumptions, the power law for dependence of the stationary probability density of news distribution on the number of comments (states x) observed in practice can be obtained from the solution to the stationary Fokker–Planck equation. In particular, assuming that coefficient $\mu(x)$ responsible in the Fokker–Planck equation for the purposeful change of the system state x (x is the current number of comments on the news item) depends linearly on state x, while coefficient $D(x)$ responsible for the random change depends on x quadratically. All this suggests that the Fokker–Planck equation can be used for describing processes occurring in complex network structures.
4. Solving the non-stationary Fokker–Planck equation under the assumptions of a linear dependence of $\mu(x)$ on state x and a quadratic dependence of $D(x)$ on state x yields an equation for the probability density of transitions between states of the system per unit time, which corresponds well with the observed data with regard for the effect of the delay time between the appearance of a first-level comment and the comment on that comment.
5. The good correspondence of the models developed on the basis of the Fokker–Planck equation with the observed data provides a basis for the creation of algorithms for monitoring and predicting the public opinion evolution for users of news info resources.
6. The parameters of comment network graphs can be obtained using both off-the-shelf tools and libraries of the Python language used to analyze complex networks, as well as the additionally developed software.
7. The analysis of the resulting model using the characteristics of the real time series to change the comment graph for users of the RIA Novosti portal and structural parameters of the graph shows its adequacy and consistency. The results show that given no impact on the user network, the desired state in terms of the number of negative commenters can be reached with probability 0.8 on the 225th day and with probability 0.9 on the 375th day, which is approximately what actually happened, according to the observed data.
8. The model based on the Fokker–Planck equation justifies the development of a practically significant algorithm for predicting the achievement of a given state of the comment network graph for news media users.
9. The complex dynamics of processes in complex social systems can be described by models other than those based on the Fokker–Planck equation. For example, the models developed by the present authors for describing the stochastic dynamics of state changes in complex social systems with provision for processes of self-organization and the presence of memory are discussed in [18–21]. For the development of the present model, the authors considered graphical schemes of transition probabilities between possible states of the described systems taking previous states into account, which accounts for memory and describes not only Markovian but also non-Markovian processes.

ACKNOWLEDGMENTS

The study is financially supported by the Russian Science Foundation, grant No. 22-21-00109 “Development of models for forecasting the dynamics of social sentiments based on analyzing time series of text content of social networks using the Fokker–Planck equations and nonlinear diffusion.”

Authors’ contribution. All authors equally contributed to the research work.

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Translated from Russian into English by K. Nazarov
Edited for English language and spelling by Thomas A. Beavitt