Russian Technological Journal. 2024;12(1):101–110 ISSN 2500-316X (Online)

Mathematical modeling

Математическое моделирование

UDC 330.4
https://doi.org/10.32362/2500-316X-2024-12-1-101-110

RESEARCH ARTICLE

Implementation of bagging in time series forecasting

Ian V. Gramovich,
Danila Yu. Musatov,
Denis A. Petrusevich @

MIREA – Russian Technological University, Moscow, 119454 Russia
@ Corresponding author, e-mail: petrusevich@mirea.ru, petrdenis@mail.ru

Abstract
Objectives. The purpose of the article is to build different models of bagging, to compare the accuracy of their forecasts for the test period against standard models, and to draw conclusions about the possibility of further use of the bagging technique in time series modeling.
Methods. This study examines the application of bagging to the random component of a time series formed after removing the trend and seasonal part. A bootstrapped series combining into a new random component is constructed. Based on the component thus obtained, a new model of the series is built. According to many authors, this approach allows the accuracy of the time series model to be improved by better estimating the distribution.
Results. The theoretical part summarizes the characteristics of the different bagging models. The difference between them comes down to the bias estimate obtained, since the measurements making up the bootstraps are not random. We present a computational experiment in which time series models are constructed using the index of monetary income of the population, the macroeconomic statistics of the Russian Federation, and the stock price of Sberbank. Forecasts for the test period obtained by standard, neural network and bagging-based models for some time series are compared in the computational experiment. In the simplest implementation, bagging showed results comparable to ARIMA and ETS standard models, while and slightly inferior to neural network models for seasonal series. In the case of non-seasonal series, the ARIMA and ETS standard models gave the best results, while bagging models gave close results. Both groups of models significantly surpassed the result of neural network models.
Conclusions. When using bagging, the best results are obtained when modeling seasonal time series. The quality of forecasts of seigniorage models is somewhat inferior to the quality of forecasts of neural network models, but is at the same level as that of standard ARIMA and ETS models. Bagging-based models should be used for time series modeling. Different functions over the values of the series when constructing bootstraps should be studied in future work.

Keywords: dynamic series, macroeconomic statistics, ARIMA, nonoverlapping block bootstrap (NBB), moving block bootstrap (MBB), stationary bagging (SB)

• **Submitted:** 21.06.2023 • **Revised:** 19.07.2023 • **Accepted:** 14.12.2023

For citation: Gramovich Ia.V., Musatov D.Yu., Petrusevich D.A. Implementation of bagging in time series forecasting. *Russ. Technol. J.* 2024;12(1):101–110. https://doi.org/10.32362/2500-316X-2024-12-1-101-110

Financial disclosure: The authors have no a financial or property interest in any material or method mentioned.

The authors declare no conflicts of interest.

НАУЧНАЯ СТАТЬЯ

Применение беггинга в прогнозировании временных рядов

Я.В. Грамович,
Д.Ю. Мусатов,
Д.А. Петрусевич @

МИРЭА – Российский технологический университет, Москва, 119454 Россия
@ *Автор для переписки, e-mail: petrusevich@mirea.ru, petrdenis@mail.ru*

Резюме
Цели. Цель работы состоит в построении различных моделей беггинга, сопоставлении точности их прогнозов на тестовый период со стандартными моделями и получении выводов о возможности дальнейшего использования техники беггинга при моделировании временных рядов.
Методы. Исследуется применение беггинга к случайной составляющей временного ряда, формируемой после удаления тренда и сезонной части. Строится серия псевдовыборок, совмещающихся в новую случайную составляющую. На основе полученной компоненты строится новая модель ряда. По мнению многих авторов такой подход позволяет повысить точность модели временного ряда, лучшим образом оценив распределение.
Результаты. В теоретической части приведены характеристики различных моделей беггинга. Разница между ними сводится к оценке смещения, получаемой из-за того, что измерения, которые составляют псевдовыборки, не являются случайными. Представлен вычислительный эксперимент, в котором модели временных рядов строятся по индексу денежных доходов населения макроэкономической статистики Российской Федерации и по курсу акций Сбербанка. Прогнозы на тестовый период, полученные стандартными, нейросетевыми моделями и моделями на основе беггинга для некоторых временных рядов, сравниваются в вычислительном эксперименте. В самой простой реализации беггинг показал результаты, сравнимые со стандартными моделями ARIMA и ETS и несколько уступающие нейросетевым моделям для сезонных рядов; для несезонных рядов лучшие результаты дали стандартные модели ARIMA и ETS, модели беггинга дали близкие результаты. Обе группы моделей существенно превзошли результат нейросетевых моделей.
Выводы. При использовании беггинга лучшие результаты получены при моделировании сезонных временных рядов. Качество прогнозов моделей беггинга несколько уступает качеству прогнозов нейросетевых моделей, но оказывается на том же уровне, что у стандартных моделей ARIMA и ETS. Модели на основе беггинга следует использовать для моделирования временных рядов, различные функции над значениями ряда при построении псевдовыборок должны быть исследованы в дальнейшей работе.

Ключевые слова: динамические ряды, макроэкономическая статистика, ARIMA, псевдовыборка неперекрывающихся блоков, псевдовыборка перекрывающихся блоков, стационарный беггинг

• **Поступила:** 21.06.2023 • **Доработана:** 19.07.2023 • **Принята к опубликованию:** 14.12.2023

Для цитирования: Грамович Я.В., Мусатов Д.Ю., Петрусевич Д.А. Применение беггинга в прогнозировании временных рядов. *Russ. Technol. J.* 2024;12(1):101–110. https://doi.org/10.32362/2500-316X-2024-12-1-101-110

Прозрачность финансовой деятельности: Авторы не имеют финансовой заинтересованности в представленных материалах или методах.

Авторы заявляют об отсутствии конфликта интересов.

INTRODUCTION

This work considers the application of bagging [1–5] in time series modeling. The use of bagging in time series modeling can be considered as an expression of the general idea of building a more accurate model based on several available models. The approach of making a weighted combination of forecasts of several time series models and averaging several forecasts is discussed in [6, 7]. The main difference between bagging and combining forecasts of time series models is that it combines only noise components. The main objective in both approaches is to improve the quality of forecasts on the basis of building a combination of forecasts of several time series models.

The approach under consideration is relevant due to the expediency of improving the accuracy of time series forecasting based on the best estimate of the distribution of the random component. The article contains new research results expressed in the experimental realization of models built on the basis of bagging of time series and comparison of forecasting results against results obtained using alternative ARIMA[1] and neural network models. The aim of the work is to build different bagging models, to compare the accuracy of their forecasts for the test period with standard models and to draw conclusions about the possibility of further use of the bagging method in time series modeling.

The time series is represented as a combination of three parts: seasonal component S_t, trend T_t, and noise R_t in additive or multiplicative form (index t stands for time):

$$y_t = S_t + T_t + R_t, \quad (1)$$

$$y_t = S_t \times T_t \times R_t. \quad (2)$$

Bagging is applied to the noise component R_t. This strategy was originally successfully applied in the classification task, where it involves building an ensemble model by training independent classifiers on different samples [8]. The predictions obtained by each model are then averaged, in order to obtain the final result (the weighted averaging can be applied depending on how accurate the predictions of each model participating in the ensemble are on the test sample). In this way, the forecasting accuracy is improved.

In addition to the idea of combining models, bagging is based on bootstrap. This approach consists in replacing the unknown distribution of data (characterizing the time process under consideration) with an empirical distribution constructed by the researcher. When using bootstrap in classification tasks, the data have no temporal dimension, so they can be mixed randomly. Things get more complicated when such ideas are applied to time series. In this case, the different sample values must follow each other according to the time dimension, even if chosen randomly. Here, the idea is transformed into constructing a set of bootstraps based on the original time series data. Several times in fact, (the number of patterns is specified by the user), based on a certain principle, values are selected from the series data to represent a new time sequence. Since there are usually many values of the time series, it is possible to build a set of new time series based on the original one, randomly selecting new values for each bootstrap. It is assumed that the characteristics of the time series under study will be close to the parameters of the resulting bootstraps.

CONSIDERED BAGGING METHODS

The approaches to obtaining bootstraps from the time series values are as follows:

1. Construction of bootstraps from nonoverlapping blocks (nonoverlapping block bootstrap, block bootstrap, circular bootstrap, NBB) [9, 10]. The time series data is divided into a given number of nonoverlapping blocks. The block length is a customizable parameter. When constructing bootstraps, each block can fall into any of them with some probability. For example, let us build blocks with the length of 3 elements from a row with 12 values: $X = \{X_1, ..., X_{12}\}$:

   $(X_1, X_2, X_3), (X_4, X_5, X_6), (X_7, X_8, X_9), (X_{10}, X_{11}, X_{12})$.

   When compiling a bootstrap, any blocks can be selected from them with return. If the length of the bootstrap is 12, you can take 4 blocks, e.g.:

   $(X_4, X_5, X_6), (X_1, X_2, X_3), (X_{10}, X_{11}, X_{12}), (X_4, X_5, X_6)$.

   Note that blocks can be repeated. The measurements in the bootstrap do not have to follow the same temporal order as the original data, so the stationarity of the original time series does not have to be preserved.
2. Constructing a bootstrap from overlapping blocks (moving block bootstrap, MBB) [11–13]. The blocks into which the time series data are divided can overlap. The block length is a customizable parameter. When constructing bootstraps, each block can fall into any block with some probability. In general, this case differs from the first, in that the blocks can overlap. The example from the previous paragraph can be transformed as follows:

[1] ARIMA is an autoregressive integrated moving average model or Box–Jenkins model.

$$(X_1, X_2, X_3), (X_3, X_4, X_5), (X_5, X_6, X_7), (X_7, X_8, X_9),$$
$$(X_9, X_{10}, X_{11}), (X_{10}, X_{11}, X_{12}).$$

Note that the beginning of each block (except for the first one) overlaps with the end of the previous block. The number of overlapping elements is, of course, adjustable. In general, further construction of the bootstrap follows point 1, so stationarity of the initial series, if any, does not guarantee stationarity of the bootstraps.

3. Constructing a stationary bootstrap [14]. This differs from the first two cases in that the researchers set the idea of preserving the stationarity property for the extracted bootstraps, provided that the original time series X is stationary. The length of the blocks is not fixed. Instead, a certain block termination probability p is given. The first element of the block X_i is selected randomly. Then each subsequent element either falls into the block with probability $1 - p$, or the block is terminated and a new one begins. The block lengths L_1, L_2, ... are subject to geometric distribution, so the probability of obtaining a block of length l:

$$p(L_j = l) = (1 - p)^{l-1} p.$$

The length L_j and initial position X_i of a block are set. We thus obtain the set of blocks $B_j(i, L_j) = \{X_1^*, X_2^*, ..., X_{L_j}^*\}$. Here the asterisk denotes that the values selected from the series do not have to form a continuous interval, but that the elements are selected following the initial element X_i of the bootstrap: $X_1^* = X_i$. Figure 1 schematically represents the process of selecting elements of the time series into the bootstrap when applying stationary bagging: X_i is the sequence of values of the time series, X_i^* is the bootstrap selected by bagging). Each subsequent element must be later than the previously selected element (X_{i+1}^* is always later than the moment corresponding to the element of the row X_i^*). That said, there may be gaps between them.

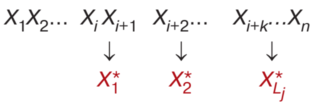


Fig. 1. Example of selecting time series elements X into the bootstrap X^* when applying the stationary bagging (element X_{i+1}^* always comes later than the previously selected X_i^*)

The work [15] studies the selection of the optimal block length and concludes that the length should be proportional to the cube root of the length of the time series.

The present work also considers the fourth method which in many respects repeats stationary bagging. The main difference is the prohibition to use blocks (values in the next block could refer to an earlier time interval than the previous one). Instead, a single block is actually used, where each previous value refers to an earlier measurement than the next. Interpolation is used when it is necessary to align the length of the bootstrap with the length of the row.

In [16] the author compares methods by the bias of the expectation (which appears due to the fact that independent quantities cannot be extracted from the time process), while in [15] a simpler bias estimation for the mathematical expectation E and the dispersion V is suggested:

$$B(\hat{E}(b)) = \frac{A_1}{b} + \overline{o}\left(\frac{1}{b}\right),$$
$$B(\hat{V}(b)) = \frac{A_2}{b} + \overline{o}\left(\frac{1}{b}\right). \quad (3)$$

Here b is the block length in the bagging scheme. A_1, A_2 are constants, the calculation details of which are given in [16]. Thus, when considering first-order estimates, the different approaches to bagging remain theoretically identical.

The MBB (overlapping blocks) method has smaller second order moments compared to NBB (non-overlapping blocks) and stationary bagging [15, 16]. The estimates for each method are given in formulas (4)–(6):

$$V_{\text{NBB}}(\hat{E}(b)) = \frac{4\pi^2 g_1(0)}{3n^3} b + \overline{o}\left(\frac{b}{n^3}\right),$$
$$V_{\text{NBB}}(\hat{V}(b)) = \frac{4\pi^2 g_2(0)}{3n^3} b + \overline{o}\left(\frac{b}{n^3}\right), \quad (4)$$

$$V_{\text{MBB}}(\hat{E}(b)) = \frac{2\pi^2 g_1(0)}{n^3} b + \overline{o}\left(\frac{b}{n^3}\right),$$
$$V_{\text{MBB}}(\hat{V}(b)) = \frac{2\pi^2 g_2(0)}{n^3} b + \overline{o}\left(\frac{b}{n^3}\right), \quad (5)$$

$$V_{\text{SB}}(\hat{E}(b)) = \frac{4\pi^2 g_1(0) + 2\pi G_1}{n^3} b + \overline{o}\left(\frac{b}{n^3}\right),$$
$$V_{\text{SB}}(\hat{V}(b)) = \frac{4\pi^2 g_2(0) + 2\pi G_2}{n^3} b + \overline{o}\left(\frac{b}{n^3}\right), \quad (6)$$

where g_1, g_2, G_1, G_2 are functions, the type and properties of which are described in [15, 16]. Here n is the number of time series elements. The method based on overlapping MBB blocks has lower second order moments than NBB.

Table 1. Comparison of the groups of models involved in the calculation experiment

Group of models	Learning algorithm (principle of model fitting to series values)	Additional model comparison indicators
Standard (ARIMA, ETS)	Principle of maximum plausibility	Akaike, Bayes (Schwartz) information criteria
Neural networks (LSTM, GRU, RNN, fully-connected neural networks) [19]	Error back propagation algorithm (with added batch normalization, dropout)/ gradient descent	Absent
Models based on bagging	After dividing by the trend-seasonality residual using STL processing the residual and rebuilding the model with the new residual	Absent

The bias estimation for stationary bagging differs significantly in the type of expression from the other two cases, so the comparison is difficult. The variance for stationary bagging is believed to be higher. At the same time, it has certain advantages. In [14] the properties of stationary bagging are studied. Here it is shown that the bootstrap is a Markovian chain, the order of which depends on how many matching blocks fall into the bootstrap.

Various statistical packages mainly implement the MBB algorithm as theoretically superior to other basic bagging strategies. Modifications of bagging for time series are widely used for modeling and forecasting of time processes [2–5, 17].

The algorithm for processing of time series values to apply one of the bootstrap strategies is presented in [17, 18]. Its block diagram is shown in Fig. 2.

In this way, the standard models ARIMA and exponential time smoothing (ETS), neural network models (long-short term memory (LSTM), gated recurrent unit (GRU), recurrent neural network (RNN), fully-connected multilayer perseptron) are presented in the computational experiment. Their comparison is presented in Table 1.

The purpose of the work is to compare the forecast accuracy of models built using different bagging approaches: with each other; and with other models often used for time series modeling and forecasting.

CALCULATION EXPERIMENT

The computational experiment considers several time series models: real personal income (HHI)[2]; and real agricultural production (AGR)[3] according to macroeconomic statistics of the Russian Federation; as

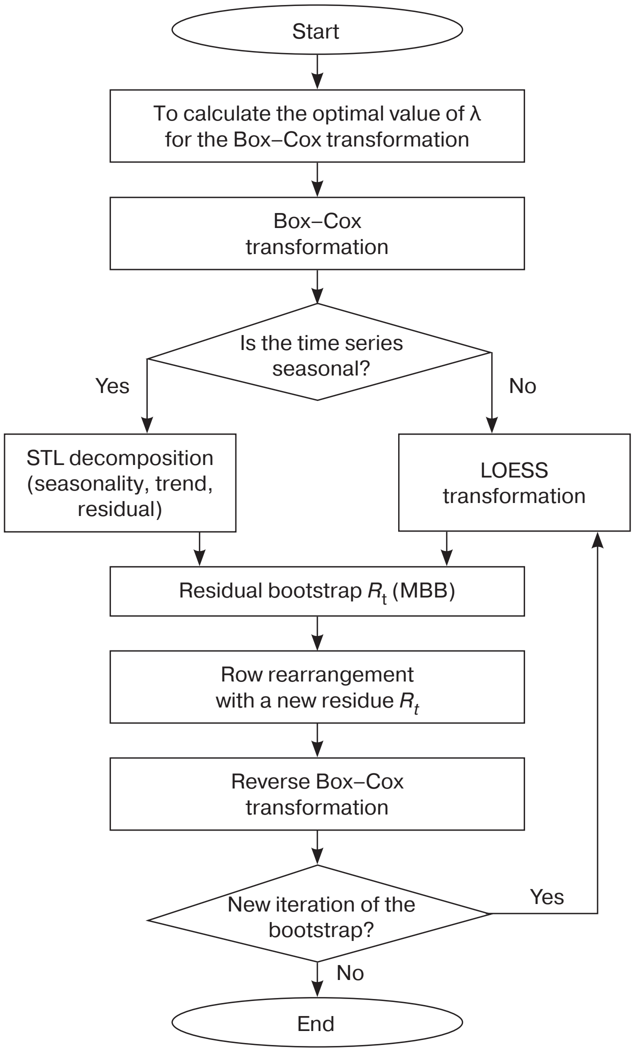


Fig. 2. Illustration of an example of selecting time series elements X into the bootstrap X^* when using stationary bagging (element X^*_{i+1} always comes later than the previously selected X^*_i).
λ is the parameter for the Box–Cox transformation;
LOESS—locally estimated scatterplot smoothing;
STL (seasonal and trend decomposition using LOESS)—method of time series decomposition into trend, seasonality, and residuals

[2] Unified archive of economic and sociological data. Dynamic series of macroeconomic statistics of the Russian Federation. Index of money incomes of the population. http://sophist.hse.ru/hse/1/tables/HHI_M_I.htm (in Russ.). Accessed September 01, 2023.

[3] Unified archive of economic and sociological data. Dynamic series of macroeconomic statistics of the Russian Federation. Index of real agricultural production. http://sophist.hse.ru/hse/1/tables/AGR_M_I.htm (in Russ.). Accessed September 01, 2023.

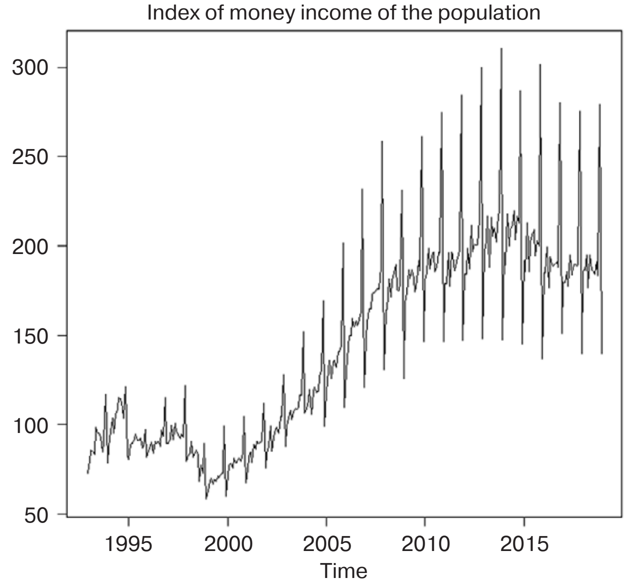


Fig. 3. Time series of the index of money income of the population (in %) according to macroeconomic statistics of the Russian Federation for 1993–2019

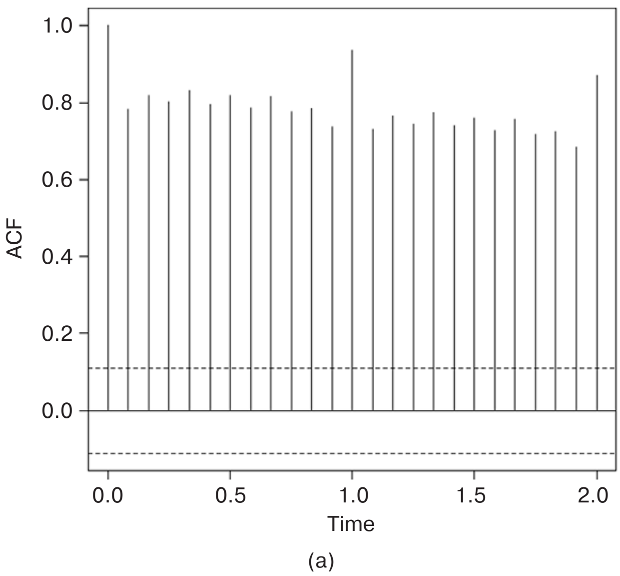
well as Sberbank shares on the Moscow stock exchange[4]. This article does not address economic issues. The data is used for modeling and forecasting. All data except the last year is used for training purposes. The test period for which the forecast is made is the last year of the time series. It should be emphasized that the beginning of the global economic crisis in 2008 and the beginning of the crisis relating to the shift in power in Ukraine in 2014 are excluded from consideration. This is because the behavior of indicators at this time undergoes significant change (changes in the mathematical expectation, variance of the series, heteroscedasticity appears). The data of the previous and the next year are glued together with respect to the crisis year. The graph for the series of real monetary income of the population (the ratio of the average per capita money income in the current month to the same indicator for the corresponding month of the last year) and its autocorrelation function (ACF) and partial autocorrelation function (PACF) [1] are presented in Figs. 3 and 4. The graphs for the series of real agricultural production are presented in Figs. 5 and 6.

Mean absolute error (MAE) and root mean square error (RMSE) estimates are measured as similarity metrics [1]. The results of processing the index of money income of the population are presented in Table 2 (the best models according to various criteria are marked in bold, accuracy is 0.01). In addition to models based on bagging and standard ARIMA and ETS models [20], models based on neural networks GRU, LSTM, RNN [21–24] are also presented in the experiment.

[4] Sberbank (SBER) stock price. https://www.moex.com/ru/issue.aspx?board=TQBR&code=SBER (in Russ.). Accessed September 01, 2023.

Table 2. Monetary income index models according to macroeconomic statistics of the Russian Federation and their forecasts for the test period

Time series model	MAE	RMSE
NBB	4.67	5.53
MBB	4.78	5.57
Stationary bagging	**4.10**	**4.91**
LOESS method	**3.49**	**4.57**
ARIMA	5.86	7.01
ETS	6.57	8.47
RNN	**3.88**	**4.45**
LSTM model	5.91	6.63
GRU model	**3.94**	**4.36**



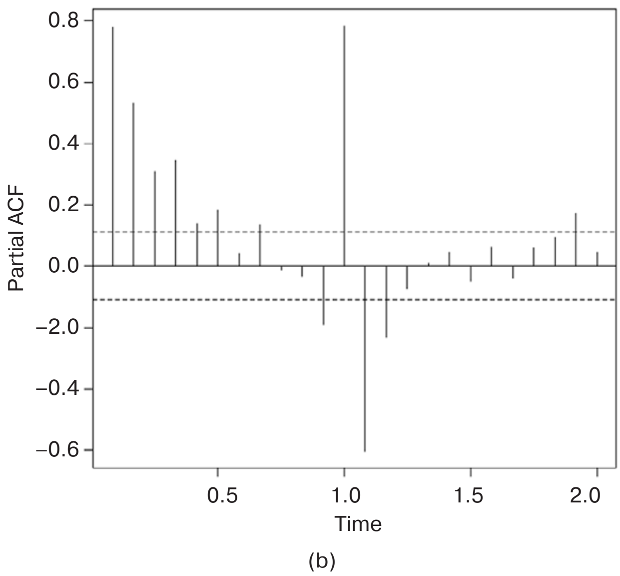


Fig. 4. Diagrams of ACF (a) and PACF (b) functions for the time series of money incomes of the population according to macroeconomic statistics of the Russian Federation

Bagging-based time series models show better results than the ARIMA and ETS standard series models. Among them, the best forecast was given by the model based on stationary bagging. At the same time, the forecast quality of the model based on stationary bagging is inferior to certain neural network models (RNN and GRU) and LOESS method (STL series decomposition).

Experiment 2 considers the index of real agricultural production in Russia for the period 2000–2020. Figures 5 and 6 show the plots of series and functions of ACF and PACF. All the models considered were adjusted for the training period 2000–2020 (the crisis years 2008 and 2014 were removed from it, the data were glued together). The results of their forecasts for the test period (2021) are compared in Table 3.

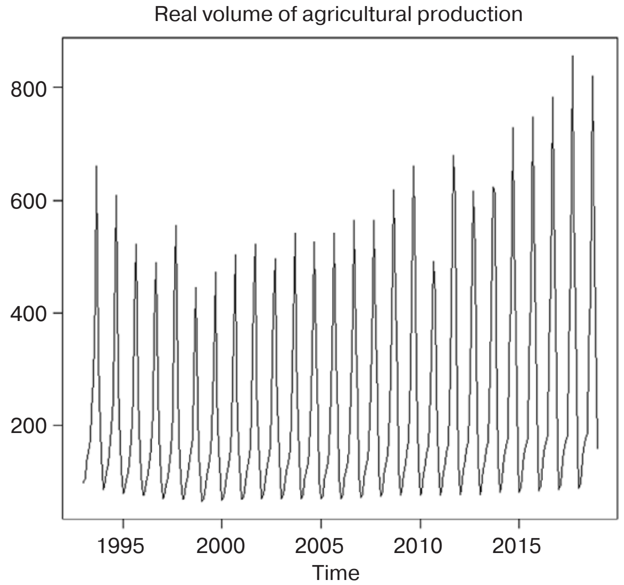


Fig. 5. Time series of real volume of agricultural production (in %) according to macroeconomic statistics of the Russian Federation

Table 3. Models of the index of real volume of agricultural production according to macroeconomic statistics of the Russian Federation and their forecasts for the test period

Time series model	MAE	RMSE
NBB	15.01	22.47
MBB	16.63	25.80
Stationary bagging	17.11	25.59
ARIMA	13.24	18.51
ETS	17.22	25,40
LSTM model	**8.78**	**15.41**
GRU model	**10.11**	**16.34**
RNN	**10.51**	**16.17**

In this experiment, the NBB approach (based on non-intersecting blocks) showed the best result among the bagging-based models. It showed approximately equal characteristics in terms of forecast quality for the test period with the ARIMA and ETS standard models. At the same time, the neural network models LSTM, GRU and RNN outperformed the standard and bagging-based models in terms of forecasting (the former—significantly, the latter two—insignificantly).

Let us separately consider a series of exchange rate of exchange-traded shares: those of Sberbank of the Russian Federation. This series has heteroscedasticity. Since the stock rate is non-seasonal, only two approaches are possible for each neural network system: to make a forecast for the entire test period at once (integral); or to make step-by-step forecasts, declaring each new step as a part of the training sample to move to the next point in time. The plots of the ACF and PACF functions are shown in Fig. 7.

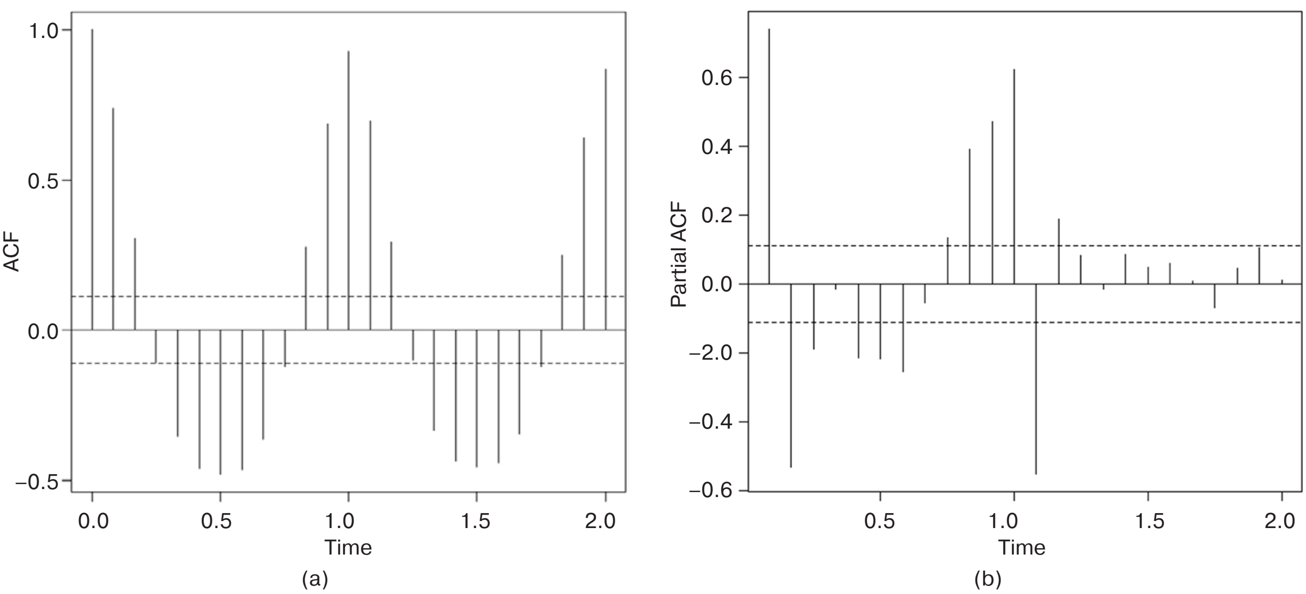


Fig. 6. Diagrams of ACF (a) and PACF (b) functions for the time series of real volume of agricultural production according to macroeconomic statistics of the Russian Federation

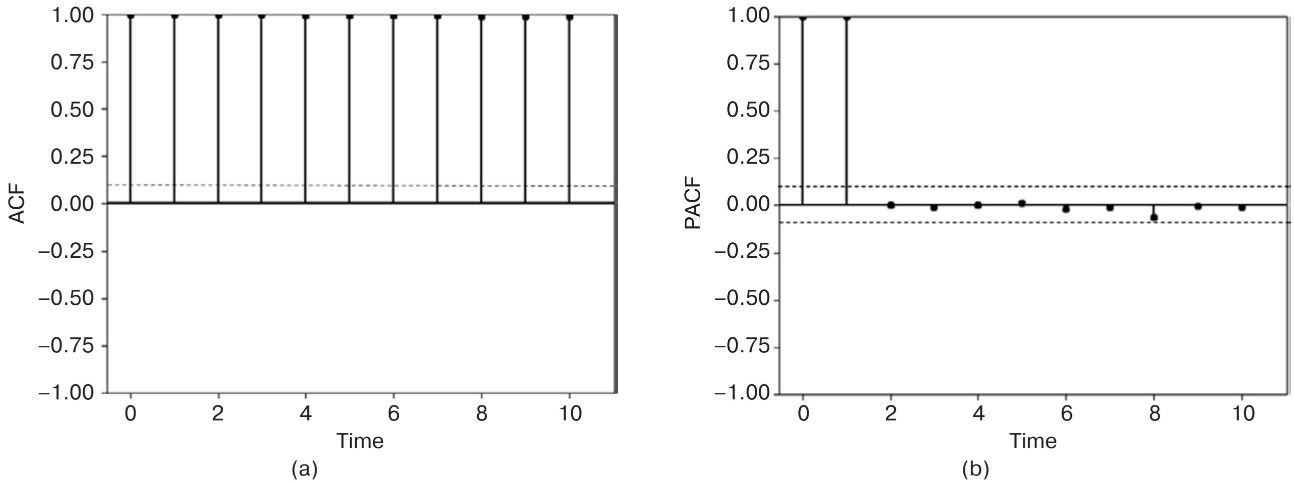


Fig. 7. Diagrams of the ACF (a) and PACF (b) functions for the time series of Sberbank of the Russian Federation stock price

Table 4. Stock price time series models for Sberbank of the Russian Federation

Time series model	MAE	RMSE
NBB	23.78	25.53
MBB	24.60	26.39
Stationary bagging	20.94	22,61
ARIMA	11.23	42.11
ETS	**4.95**	**20.68**
RNN network	80.53	86.39
LSTM model	76.95	81.40
GRU model	24.66	85.05

The best results are shown by classical methods of series modeling: ETS and ARIMA models. Stationary bagging shows slightly worse results, although significantly outperforming all neural network models. It should be noted that the standard ARIMA and ETS models describe the time series statistically better in the absence of seasonality. The main idea of bagging is to determine the properties of the noise component of the series. Obviously, it makes sense to do this for series with seasonal or cyclical patterns. Modeling noise for non-seasonal series does not lead to better forecasting (standard models gave better forecasts than models based on bagging application).

CONCLUSIONS

The work presents an analysis of different approaches to time series bagging and examples of their application to non-seasonal and seasonal time series. In computational experiments, the results of models applying bagging are compared with the forecasts of standard models (ARIMA and ETS), and models based on neural networks (RNN, LSTM, GRU).

When processing a non-seasonal time series, modeling of the noise component did not improve the modeling of the whole series and its forecast. In this experiment, the best results among all three groups of models were obtained by ARIMA and ETS standard models. It should be noted that neural network models, often used in modeling processes of a different nature, gave forecasts of worse quality compared to ARIMA and ETS models (Table 4).

When modeling seasonal time series, the best results were shown by neural network models, actively used in time series modeling, and the LOESS method. Bagging-based models outperformed the standard ARIMA and ETS models. Bagging was better able to model the residual of the series (which is obtained by removing the trend and seasonal component of the series). Thus, work on various bootstrap schemes should be continued and their accuracy improved. In addition, it may be possible to improve the accuracy of modeling and forecasting by working separately on the trend, seasonality, and residual. At the same time, it is not possible to determine which bootstrap type will best model the residual of a given series. Each type is best suited for a different set of seasonal time series. In this work, the different bootstrap approaches are implemented in the simplest form. Based on the experimental results, the work should be continued by editing the differing bootstrap features and combining the various approaches to model trend, noise and residual.

Authors' contribution. All authors equally contributed to the research work.

REFERENCES

1. Hyndman R.J., Athanasopoulos G. *Forecasting: Principles and Practice.* 2nd ed. Publisher OTexts; 2018. 382 p. ISBN 978-0-9875-0711-2
2. Özen K., Yildirim D. Application of bagging in day-ahead electricity price forecasting and factor augmentation. *Energy Econ*. 2021;103(2):105573. http://doi.org/10.1016/j.eneco.2021.105573
3. Meira E., Oliveira F.L.C., de Menezes L.M. Forecasting natural gas consumption using Bagging and modified regularization techniques. *Energy Econ*. 2022;106(3):105760. http://doi.org/10.1016/j.eneco.2021.105760
4. Kim D., Baek J-G. Bagging ensemble-based novel data generation method for univariate time series forecasting. *Expert Syst. Appl*. 2022;203(1):117366. http://doi.org/10.1016/j.eswa.2022.117366
5. Liu X., Liu A., Chen J.L., Li G. Impact of decomposition on time series bagging forecasting performance. *Tourism Manag.* 2023;97(2):104725. https://doi.org/10.1016/j.tourman.2023.104725
6. Beletskaya N., Petrusevich D. Linear Combinations of Time Series Models with Minimal Forecast Variance. *J. Commun. Technol. Electron.* 2023;67(1):144–158. http://doi.org/10.1134/S1064226922130022
7. Musatov D., Petrusevich D. Modeling of forecasts variance reduction at multiple time series prediction averaging with ARMA(1, q) functions. In: *Proceedings of MIP Computing-V 2022*: *V International Scientific Workshop on Modeling, Information Processing and Computing*. 2022;3091:1–11. https://doi.org/10.47813/dnit-mip5/2022-3091-1-11
8. Golov V.A., Petrusevich D.A. Data analysis methods in astronomic objects classification (Sloan Digital Sky Survey DR14). *Russ. Technol. J*. 2021;9(3):66–77 (in Russ.). https://doi.org/10.32362/2500-316X-2021-9-3-66-77
9. Carlstein E. The use of subseries methods for estimating the variance of a general statistic from a stationary time series. *Ann. Statist*. 1986;14(3):1171–1179. https://doi.org/10.1214/aos/1176350057
10. Carlstein E., Do K-A., Hall P., Hesterberg T., Künsch H.R. *Matched-Block Bootstrap for Dependent Data*. Research Report 74. Seminar für Statistik. ETH Zentrum. Zürich, Switzerland; 1995. 25 p.
11. Künsch H.R. The jackknife and the bootstrap for general stationary observations. *Ann. Statist*. 1989;17(3):1217–1261. https://doi.org/10.1214/aos/1176347265
12. Götze F., Künsch H.R. Second-order correctness of the blockwise bootstrap for stationary observations. *Ann. Statist*. 1996;24(5):1914–1933. http://doi.org/10.1214/aos/1069362303
13. Bühlman P., Künsch H.P. Block length selection in the bootstrap for time series. *Comput. Stat. Data Anal*. 1999;31(3):295–310. https://doi.org/10.1016/S0167-9473(99)00014-6
14. Politis D.N., Romano J.P. The stationary bootstrap. *J. Am. Stat. Assoc*. 1994;89(428):1303–1313. https://doi.org/10.1080/01621459.1994.10476870
15. Politis D.N., White H. Automatic Block-Length Selection for the Dependent Bootstrap. *Econometric Rev*. 2004;23(1):53–70. https://doi.org/10.1081/ETC-120028836
16. Lahiri S.N. Theoretical comparison of block bootstrap methods. *Ann. Statist*. 1999;27(1):386–404. https://doi.org/10.1214/aos/1018031117
17. Bergmeir C., Hyndman R. J., Benitez J.M. Bagging exponential smoothing methods using STL decomposition and Box–Cox transformation. *Int. J. Forecast.* 2016;32(2):303–312. https://doi.org/10.1016/j.ijforecast.2015.07.002
18. Petropoulos F., Hyndman R.J., Bergmeir C. Exploring the sources of uncertainty: Why does bagging for time series forecasting work? *Eur. J. Oper. Res*. 2018;268(2):545–554. http://doi.org/10.1016/j.ejor.2018.01.045
19. Haykin S. *Neural Networks and Learning Machines*. Pearson Education; 2011. 936 p. ISBN 978-0-1330-0255-3
20. Box G., Jenkins G. *Time Series Analysis: Forecast and Management*. John Wiley & Sons; 2015. 712 p. ISBN 978-1-1185-6749-18
21. Rasjid Z.E., Setiawan R., Effendi A. A Comparison: Prediction of Death and Infected COVID-19 Cases in Indonesia Using Time Series Smoothing and LSTM Neural Network. *Procedia Comput. Sci*. 2021;179(5):982–988. http://doi.org/10.1016/j.procs.2021.01.102
22. Liu X., Lin Z., Feng Z. Short-term offshore wind speed forecast by seasonal ARIMA – A comparison against GRU and LSTM. *Energy*. 2021;227:120492. http://doi.org/10.1016/j.energy.2021.120492
23. Shahid F., Zameer A., Muneeb M. Predictions for COVID-19 with deep learning models of LSTM, GRU and Bi-LSTM. *Chaos, Solitons & Fractals*. 2020;140:110212. https://doi.org/10.1016/j.chaos.2020.110212
24. Amalou I., Mouhni N., Abdali A. Multivariate time series prediction by RNN architectures for energy consumption forecasting. *Energy Rep*. 2022;8:1084–1091. https://doi.org/10.1016/j.egyr.2022.07.139

About the authors

Ian V. Gramovich, Student, Institute of Artificial Intelligence, MIREA – Russian Technological University (78, Vernadskogo pr., Moscow, 119454 Russia). https://orcid.org/0009-0000-2908-7611

Danila Yu. Musatov, Student, Institute of Artificial Intelligence, MIREA – Russian Technological University (78, Vernadskogo pr., Moscow, 119454 Russia). Scopus Author ID 57469172700, https://orcid.org/0000-0003-0673-5393

Denis A. Petrusevich, Cand. Sci. (Phys.-Math.), Associate Professor, Higher Mathematics Department, Institute of Artificial Intelligence, MIREA – Russian Technological University (78, Vernadskogo pr., Moscow, 119454 Russia). E-mail: petrusevich@mirea.ru, petrdenis@mail.ru. Scopus Author ID 55900513600, ResearcherID AAA-6661-2020, RSCI SPIN-code 7999-6345, https://orcid.org/0000-0001-5325-6198

Об авторах

Грамович Ян Вадимович, студент, ФГБОУ ВО «МИРЭА – Российский технологический университет» (119454, Россия, Москва, пр-т Вернадского, д. 78). https://orcid.org/0009-0000-2908-7611

Мусатов Данила Юрьевич, студент, ФГБОУ ВО «МИРЭА – Российский технологический университет» (119454, Россия, Москва, пр-т Вернадского, д. 78). Scopus Author ID 57469172700, https://orcid.org/0000-0003-0673-5393

Петрусевич Денис Андреевич, к.ф.-м.н., доцент, кафедра высшей математики Института искусственного интеллекта ФГБОУ ВО «МИРЭА – Российский технологический университет» (119454, Россия, Москва, пр-т Вернадского, д. 78). E-mail: petrusevich@mirea.ru, petrdenis@mail.ru. Scopus Author ID 55900513600, ResearcherID AAA-6661-2020, SPIN-код РИНЦ 7999-6345, https://orcid.org/0000-0001-5325-6198

Translated from Russian into English by Lyudmila O. Bychkova
Edited for English language and spelling by Dr. David Mossop