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Математическое моделирование

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RESEARCH ARTICLE

Investigation of influence of objective function valley ratio on the determination error of its minimum coordinates

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Abstract

Objectives. A valley is a region of an objective function landscape in which the function varies along one direction more slowly than along other directions. In order to determine the error of the objective function minimum location in such regions, it is necessary to analyze relations of valley parameters.

Methods. A special test function was used in numerical experiments to model valleys with variables across wide ranges of parameters. The position and other valley parameters were defined randomly. Valley dimensionality and ratio were estimated from eigenvalues of the approximated Hessian of objective function in the termination point of minimum search. The error was defined as the Euclidian distance between the known minimum position and the minimum search termination point. Linear regression analysis and approximation with an artificial neural network model were used for statistical processing of experimental data.

Results. A linear relation of logarithm of valley ratio to logarithm of minimum position error was obtained. Here, the determination coefficient R^2 was ~0.88. By additionally taking into account the Euclidian norm of the objective function gradient in the termination point, R^2 can be augmented to ~0.95. However, by using the artificial neural network model, an approximation R^2 ~ 0.97 was achieved.

Conclusions. The obtained relations may be used for estimating the expected error of extremum coordinates in optimization problems. The described method can be extended to functions having a valley dimensionality of more than one and to other types of hard-to-optimize algorithms regions of objective function landscapes.

Keywords: objective function landscape, valley landscape, valley ratio, valley dimensionality, Hessian eigenvalues, linear regression, approximation, artificial neural network

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НАУЧНАЯ СТАТЬЯ

Исследование влияния степени овражности целевой функции на погрешность определения координат ее минимума

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Резюме
Цели. Целью работы было исследование зависимостей, связывающих характеристики оврагов, т.е. участков рельефа минимизируемой функции, на которых ее изменение по одному из направлений значительно медленнее, чем по другим направлениям, с погрешностью определения координат ее минимума.
Методы. В экспериментах использовалась специально разработанная тестовая функция с изменяемыми в широких пределах параметрами овражности. В сериях опытов случайно задавались положение и параметры оврага и координаты стартовой точки поиска. Размерность и степень овражности оценивались по собственным числам аппроксимированного гессиана функции в точке окончания поиска минимума. Погрешность определялась как эвклидово расстояние между заданным положением минимума функции и конечной точкой поиска. Для статистической обработки результатов применены линейный регрессионный анализ и аппроксимация с помощью модели искусственной нейронной сети (ИНС).
Результаты. Установлено наличие линейной зависимости между логарифмами степени овражности и погрешности определения координат минимума функции. Коэффициент детерминации R^2 ~ 0.88. Дополнительный учет эвклидовой нормы градиента функции в точке окончания поиска позволил повысить коэффициент детерминации до R^2 ~ 0.95, а при использовании модели ИНС – до R^2 ~ 0.97.
Выводы. Найденные зависимости можно использовать для оценки ожидаемой погрешности определения координат экстремумов оптимизируемых функций. В дальнейшем необходимо расширить методику на функции с размерностью оврагов более единицы и на другие типы сложных для алгоритмов оптимизации участков рельефа.

Ключевые слова: рельеф целевой функции, овражность рельефа, степень овражности, размерность овражности, собственные значения гессиана, линейная регрессия, аппроксимация, искусственная нейронная сеть

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INTRODUCTION

The problem of searching for an optimal solution of $\mathbf{x}_{\text{opt}}$ is formulated as follows:

$$\mathbf{x}_{\text{opt}} = \arg\min_{\mathbf{x}\in X} f(\mathbf{x}), \qquad (1)$$

where X is the search area, while $f(\mathbf{x})$ is the objective function (OF). Numerous methods for solving problem (1) are known, both those having a sufficiently rigorous mathematical justification and which are applicable in cases where OF satisfies certain conditions (convexity, smoothness, etc.) [1, 2], as well as heuristic methods that do not impose strict requirements on the OF properties, but also do not guarantee finding the optimal solution [3, 4].

The possibility and accuracy of solving problem (1) are determined by the properties of both the OF and the search algorithm. In this connection, considerable research attention is attracted to an analysis of the OF landscape where the landscape is understood as a set of pairs $\{\mathbf{x} \in X, f(\mathbf{x})\}$. In this case, since the analytical expression of the function $f(\mathbf{x})$ is absent, its values have to be found by modeling the optimized system (black-box optimization problems). This research direction is referred to as exploratory landscape analysis (ELA).

In [5] and other works by the same group of authors, the classification of high-level OF landscape properties, determined qualitatively by the method of expert evaluation, and low-level properties, evaluated quantitatively by processing the results of the OF calculations at sampling points and the results of searching for OF extrema from the starting points, is given. High-level properties include multimodality, i.e., the presence of many local extrema, regularity and uniformity of the OF landscape properties in the search area, the presence of plateaus, and others. Low-level properties include statistics of the OF values, curvature and convexity indices, correlation indices of differences between OF values and distances between sampling points, and many others. In [6], more than 300 low-level properties are considered; a list of publications on this subject is also provided. Machine learning technologies [5, 7] are used to search for statistical dependencies between low- and high-level properties, as well as between landscape properties and the performance of various optimization algorithms on this landscape.

However, the above mentioned and other works known to us almost do not consider such OF landscape objects as valleys, i.e., areas in which OF along one or more directions changes significantly slower than along other directions [8], and walls representing OF sharp changes along any direction [9]. Meanwhile, in the presence of such objects, the search may end not at the extremum, but at some other point at the bottom of the valley or at the foot of the wall. In such case, optimization algorithms would find incorrect solutions. The problems related to the detection of these objects in the OF landscape and estimation of their quantitative characteristics have been little investigated.

Theoretical aspects of the occurrence of valleys in the landscape and methods of solution search in their presence are considered in [8]. Of the several definitions of valley proposed in this study, we use the most convenient for use in applications, which we present below with some simplification (by omitting additional conditions).

Let D be some region of the n-dimensional space R^n; let $J(\mathbf{x}) \in C^2(D)$ be a functional with continuous second derivatives in D; let $\mathbf{H}(\mathbf{x})$ be the matrix of second derivatives (Hessian) of functional $J(\mathbf{x})$ at point $\mathbf{x}$; and let $\lambda_i[\mathbf{H}(\mathbf{x})]$, $i = \overline{1,n}$ be eigenvalues of Hessian $\mathbf{H}(\mathbf{x})$ at point $\mathbf{x}$ ordered by descending.

The functional is called valley, i.e., it contains a valley, if there is such number $\sigma \gg 1$ and set $Q \subset D$, that

$$\begin{gathered} \forall \mathbf{x} \in Q \;\; \lambda_1\left[\mathbf{H}(\mathbf{x})\right] \geq \ldots \geq \lambda_{n-r}\left[\mathbf{H}(\mathbf{x})\right] \geq \\ \geq \sigma\lambda_{n-r+1}\left[\mathbf{H}(\mathbf{x})\right] \geq \ldots \geq \sigma\lambda_n\left[\mathbf{H}(\mathbf{x})\right]. \end{gathered} \tag{2}$$

This means that the largest $(n - r)$ eigenvalues of the Hessian are significantly larger than the other r eigenvalues at all points $\mathbf{x}$ belonging to the valley region Q. The number r is called the valley dimensionality; the number σ is called the valley ratio. The valley ratio shows to what extent in a given valley the rate of change of the OF along its bottom is smaller than along the directions orthogonal to the bottom. These indicators can be generalized as characteristics or indicators of the landscape's valley.

The result of the presence of valleys in the OF landscape, as noted above, consists in the error in determining the coordinates of the OF extrema; therefore, this error can serve as an objective characteristic of the valley. In practice, it is impossible to estimate the error directly when searching for the OF minimum since the true position of the minimum is unknown. At the same time, the valley ratio can be estimated on the basis of definition (2). In this connection, it is of interest to investigate the dependence linking the above error with the valley ratio. This task is not considered in [8] or other relevant works known to us; a practical means for estimating the valley ratio when searching for the OF minimum is also absent.

We also note [10], which introduces the definition of a valley as a one-dimensional set using the notion of topological homeomorphism. It also presents a method for determining the position and direction of a valley based on selecting a subset of points with the lowest OF values from a set of sampling points and applying the principal component analysis method to this subset. Quantitative characteristics of the valley are not considered in this study. In [11], the OF landscape properties of the well-known combinatorial traveling salesman problem are studied; this landscape is shown to contain groups of closely spaced depressions, also called valleys; however, these results are not applicable to optimization problems for functions of continuous variables.

The present work aims to investigate the dependence of the error in determining the coordinates of the sought OF minimum on the valley characteristics in the neighborhood of the search end points.

Achieving this goal requires performing a series of experiments on searching from different starting points for the OF minimum with varied valley parameters including its position, orientation relative to the coordinate axes, slope steepness and curvature, etc.

At the end of each search, it is necessary to determine the OF Hessian eigenvalues, and according to them, the valley dimensionality and the valley ratio in accordance with definition (2). In addition, the error equal to the distance of the search end point to the true position of the OF minimum must be calculated in order to analyze the statistical relationships linking the valley characteristics and the error in determining the OF minimum coordinates.

MATERIALS AND METHODS

First of all, it is necessary to choose a method for finding the OF minimum. The error of the minimum coordinates in the presence of valleys may significantly differ for different optimization algorithms. After selecting an algorithm and performing experimental studies, it would be possible to construct the valley estimation scale against which the results of other algorithms could be compared.

In the paper, the quasi-Newton (QN) local search algorithm implemented in *MATLAB* software environment by the *fmincon*(..) function is used. This type of algorithm is recommended in [6] as an exemplary one for searching local extrema of test functions when evaluating OF landscape properties. In addition, in QN methods, the Hessian approximation is an integral part of the algorithm at each iteration and, therefore, is automatically obtained at the final search point [1].

The list of output variables of the *fmincon*(..) function includes the vector of coordinates of the search end point $\mathbf{x}_{\text{fin}}$, the OF value f_{fin} at point $\mathbf{x}_{\text{fin}}$, the reason indicator for the search end *ExitFlag*, the OF gradient vector, and the Hessian approximation at point $\mathbf{x}_{\text{fin}}$ in the form of the real numbers matrix. In the input variables, we set the boundaries of the search area

$$-5 \le x_i \le 5, \quad i = \overline{1, ND}, \tag{3}$$

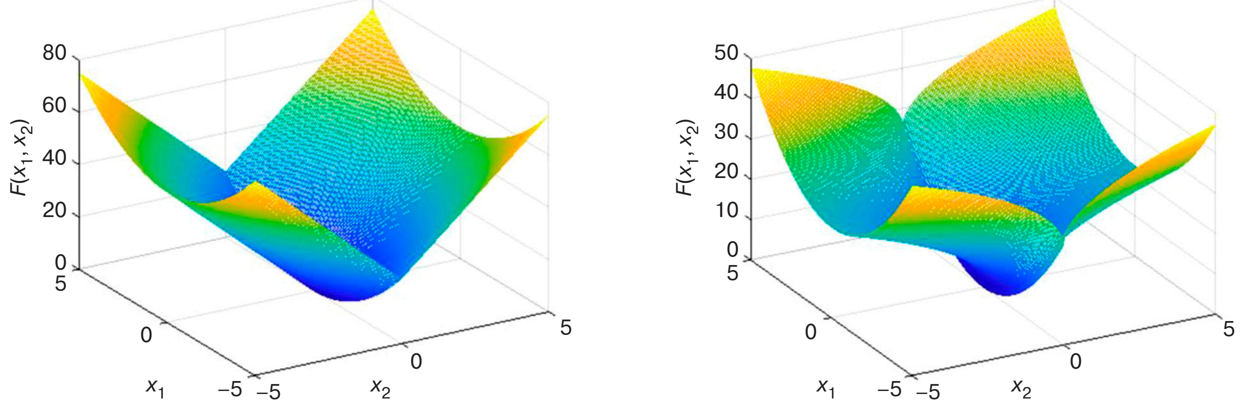
variant of the sequential quadratic programming (SQP) search algorithm, maximum number of iterations in each search is 1000, and other settings are default.

Next, we consider the OF used in the experiments. Sets of test functions [12, 13] are used for testing and comparing search algorithms for extrema. Although some of them possess the valley property, there is no function in which the valley parameters could be changed within wide limits. For this reason, the *TestValley*(..) function, whose text in *MATLAB* language is shown in Fig. 1, has been developed.

Here, $\mathbf{x}$ stands for coordinates of the point where the function value is calculated, $\mathbf{x}_{\text{opt}}$ and f_{opt} are specified coordinates of the minimum point and the function value in it, and $\mathbf{R}$ is the orthonormalized matrix specifying the rotation of coordinate axes. These parameters allow different positions and orientations of the valley to be obtained in the search space. Parameter N defines the valley dimensionality. Parameter W defines the curvature of the valley slopes. At W = 1, OF is quadratic, i.e., convex and smooth. At $1 < W < 0.5$, OF is convex, but not smooth. At $W = 0.5$, OF increases linearly. Finally, at $W < 0.5$, OF is concave. Parameter K sets a uniform scale

```
%%TestValley - Valley modeling with variable parameters
function f = TestValley(x,fopt,xopt,R,P,W,K,N)
%%P - Type of scaling on different coordinates
%%P=0 - Same acceleration rate for all x(i), i>N
%%P>0 - At j > i, the accretion rate along x(j) is faster than along x(i)
%%W - Type of dependence on the distance to the bottom of the valley
%%W=0.5 - Linear function
%%W<0.5 - Concave function; W>0.5 - convex function
%%K - Total scale factor in directions from the valley axis
%%N - Valley dimesionality
    n=length(x);
    z0=(x-xopt)*R;
    L=eye(n);
    for n1=1:n
        L(n1,n1)=10^(P*(n1-1)/2/(n-1));
    end
    z1=z0*L;
    z3=z1.^2;
    f =sum(z3(1:N),2)+K*(sum(z3(N+1:n),2))^W+fopt;
end
```

Fig. 1. Text of the program implementing the *TestValley*(..) function



(a) (b)

Fig. 2. Graphs of the *TestValley*(..) function: (a) $W = 0.5$; (b) $W = 0.25$

of the OF growth rate in all directions, while parameter P affects the OF anisotropy. At $P = 0$, OF grows at the same rate in all directions while at $P > 0$, the growth rate in different directions is different; these differences are greater the greater P is.

Examples of function graphs at the search space dimensionality $ND = 2$, parameters $P = 0$, $K = 10$, $N = 1$, and different values of parameter W are shown in Fig. 2.

Coordinates of starting points in the number of *NPnt* within the boundaries of the search area (3) are set using the Latin hypercube sampling algorithm implemented in *MATLAB* by the *lhsdesign*(..) function. The value of function f_{opt} in the minimum is set equal to 0. In the paper, only one-dimensional valleys at $N = 1$ are investigated. The values of other parameters are set by random numbers with uniform distributions in the following ranges:

$$\begin{aligned} &-3 \le x_{\text{opt}i} \le 3, \quad i = \overline{1, ND}; \\ &0 \le P \le 1; \quad 0.25 \le W \le 1.25; \\ &0 \le \lg K \le 4. \end{aligned} \tag{4}$$

The rotation matrix **R** is formed as a square matrix $ND \times ND$ of random numbers uniformly distributed in the interval (0, 1) with subsequent orthogonalization using the *MATLAB orth*(..) function. Setting the above parameters is possible both separately for each start, as well as once for the whole series of *NPnt* starts.

Calling the minimum search function and processing the results returned by it are explained by the program fragment in Fig. 3. In variables $\mathbf{X}_1$ and GF_1, the coordinates of the search end point $\mathbf{x}_{\text{fin}}$ and the OF value f_{fin} in it are returned, respectively. The arrays **grad** and **hess** contain the gradient vector and the approximated Hessian matrix, respectively. The @FEval pointer contains a reference to call the *TestValley*(..) function, which sets its input parameters as described above.

The program finds the eigenvalue vector of the **Ehess** Hessian and orders them by ascending absolute value in the **HessEV** array. Then the relations of adjacent values stored in the **S0hess** array are calculated. Finally, the estimation of the valley ratio *SValley* as the maximum of these relation values and the valley dimensionality *NValley* as the number of the maximum value in the array is determined. This definition of the valley ratio and dimensionality corresponds to the above definition (2), with the unprincipled difference that the ordering of the Hessian eigenvalues is performed in ascending rather than descending order. The error in determining the coordinates of the minimum DX_{opt} is calculated as the Euclidean distance between points $\mathbf{x}_{\text{opt}}$ and $\mathbf{x}_{\text{fin}}$.

The dependencies between variables are analyzed using two methods. The first one is linear regression analysis [14]. The *MATLAB fitlm*(..) function which approximates the linear model using the original data is used for implementing it. The second method is training the artificial neural network (ANN) model that approximates the desired dependence [15]. For this, the *fitnet*(..) function creating the ANN model with a given structure and the *train*(..) function performing the model training and testing are used.

```
[X1,GF1,ExitFlag,~,~,grad,hess]=fmincon(@FEval,Xin,[],[],[],[],Lb,Ub,[],MICoptions);
Ehess=eig(hess);
HessEV=sort(abs(Ehess));
S0hess=HessEV(2:end)./HessEV(1:end-1);
[SValley,NValley]=max(S0hess); % valley ratio and dimensionality
```

Fig. 3. Search function call and estimation of valley parameters at its end point

RESEARCH RESULTS

First, we consider the results of preliminary experiments presented in Fig. 4 giving insight into the influence of the valley ratio on the error of finding the minimum point of the *TestValley*(..) function. The dimensionality of problem $ND = 4$, the number of starts $NPnt = 40$, the position of the minimum, and the valley rotation are set once for the whole series. The input parameters of the function are shown above the diagrams showing the movement from the start point (red markers) to the end point $\mathbf{x}_{fin}$ (blue markers). One diagram shows the changes in all four coordinates, with circles marking coordinates x_1 and x_2 and triangles marking coordinates x_3 and x_4.

In the case of weak valley (Fig. 4a), the search from all starting points comes to the neighborhood of the minimum point of the test function. The average value of error DX_{opt} is 0.019 in this case. In the case of a strong valley (Fig. 4b), the searches starting from different points end in different points scattered along the valley bottom. In this case, the average DX_{opt} value reaches 9.2.

Then a series of experiments are performed to reveal the dependence between the estimation of the valley ratio *SValley* and the error of finding the minimum point of the test function DX_{opt}. The dimensionality of the *ND* space varies within the range from 2 to 12. Each experiment includes $12 \cdot 10^3$ starts, in each of which the random position of the search starting point, the position of the minimum $\mathbf{x}_{opt}$, and the rotation $\mathbf{R}$ of the *TestValley*(..) function, as well as the valley parameters *P*, *W*, and *K* in the ranges defined by inequalities (4), are set.

The experimental results for all *ND* values are similar. As an example, Fig. 5 shows histograms of the values of the experimental results for $ND = 4$. Due to the wide ranges of *SValley* and DX_{opt} values, their logarithms are analyzed and plotted. It can be concluded from the scatter plot of these variables shown in Fig. 6 that there is a stochastic dependence between them.

Notably, the value $ExitFlag = 1$ corresponds to the search end when the gradient modulus at the reached point does not exceed the specified *OptimalityTolerance* value (10^{-6} by default), while the value $ExitFlag = 2$ corresponds to the search end when the last movement during the search does not exceed the specified *StepTolerance* value (also 10^{-6} by default). In the second case, the modulus of the OF gradient at the search end point can be much larger than *OptimalityTolerance* since the OF smoothness conditions are violated on the valley axis (Fig. 2).

There is not a single case when the search ends due to exceeding the specified number of iterations ($MaxIteration = 1000$) or due to the algorithm being unable to find an acceptable point for further movement. Thus, all search starts end at points that the algorithm determines to be a local minimum. Similar results are recorded for all dimensionality values of the *ND* space.

Next, in most starts, the valley dimensionality is correctly determined at the search end point $NValley = 1$. At $ND = 4$, the valley dimensionality is determined incorrectly at 1092 points. These points (colored in black in Fig. 6) are all located in the region where the search error DX_{opt} is negligible and the valley ratio $SValley < 1000$. At $ND > 4$, estimations $NValley > 3$ are encountered, but also only in the region $\lg(DX_{opt}) < -4$. At $ND = 2$, values $NValley > 1$ are obviously impossible.

We proceed to the statistical processing of the data collected in the experiments. The influence of parameters

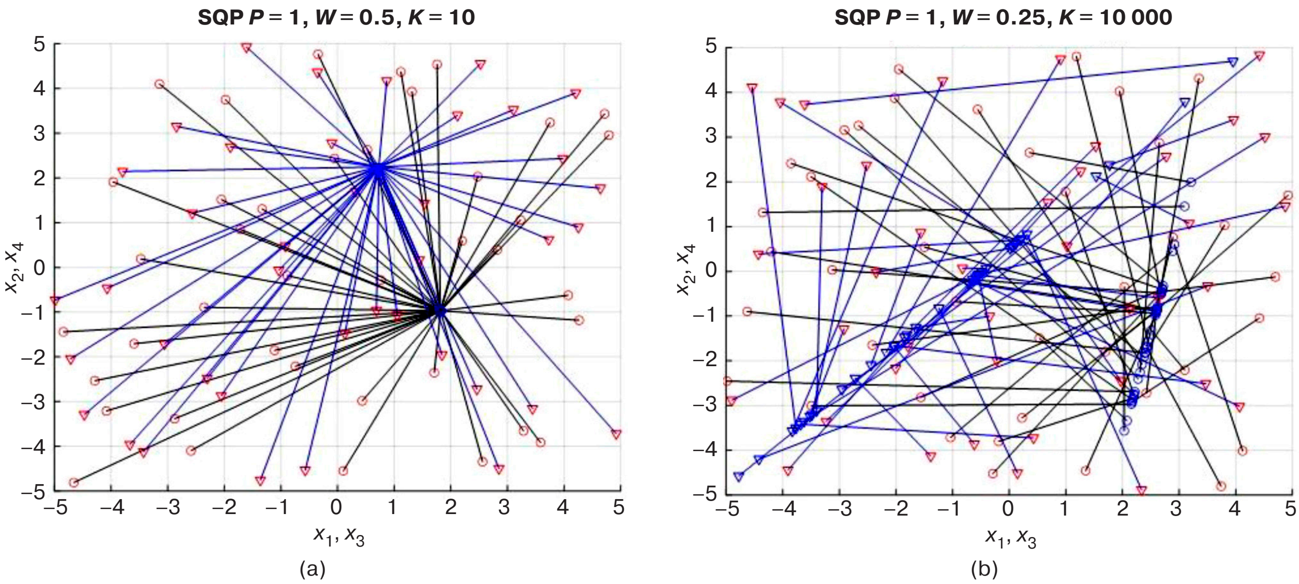


Fig. 4. Results of *TestValley*(..) minimum search for weak (a) and strong (b) valley

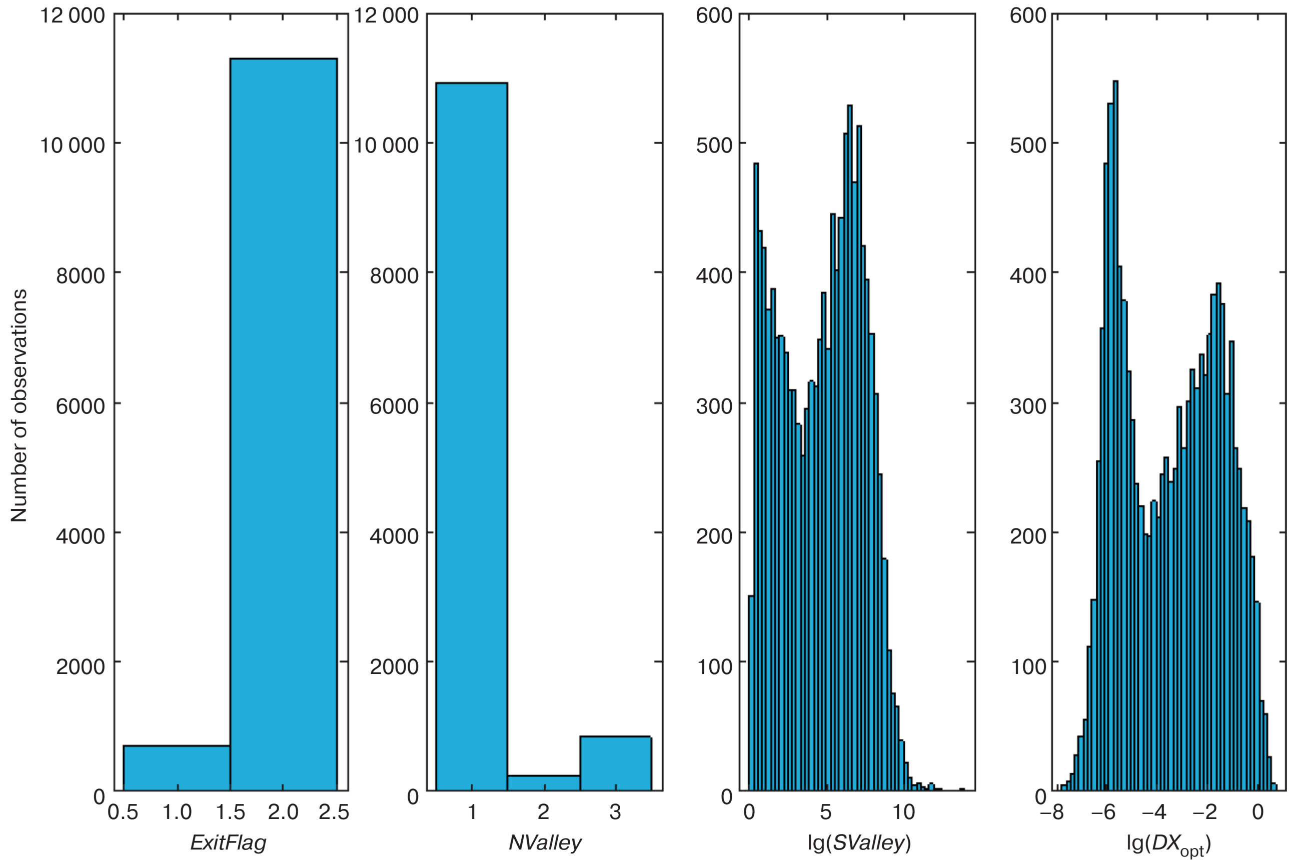


Fig. 5. Histograms of the main experiment results. *ExitFlag* is the reason indicator for search end; *NValley* is the estimation of valley dimensionality; *SValley* is the estimation of valley ratio; DX_{opt} is the error in determining the position of the OF minimum

P, *W*, and *K* of the *TestValley*(..) function on the valley ratio *SValley* is analyzed beforehand. The following linear model is studied:

$$\lg(SValley) = k_1P + k_2W + k_3\lg(K) + b. \quad (5)$$

The regression analysis of this model shows that the valley ratio is most strongly influenced by parameter *W* determining the curvature and convexity or concavity of the valley slopes. Parameter *K* is the next to contribute to the result, while the influence of parameter *P* is the least significant, although it cannot be neglected. The values of the coefficient of determination R^2, used for determining the adequacy of model [14], are within the range of 0.88–0.90 for different *ND*s.

Next, the linear model linking the error of finding the OF minimum point with the estimation of the valley ratio is considered:

$$\lg(DX_{opt}) = k\lg(SValley) + b. \quad (6)$$

The regression analysis results are shown in Table 1, where the first group of columns corresponds to the accounting of all points while the second group excludes

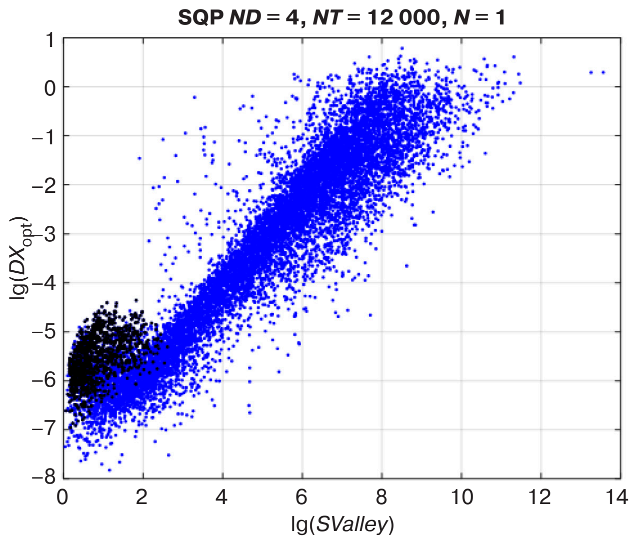


Fig. 6. Scatter plot of the logarithm values of the valley ratio *SValley* and the error in determining coordinates of the minimum DX_{opt}

points with *NValley* > 1. This data selection results in some improvement in the model accuracy expressed in increasing coefficient of determination R^2 and decreasing root-mean-square (RMS) error (residual) of regression *StdErr*.

Next, the possibility of improving the model accuracy by taking into account the Euclidean norm (length) of the OF gradient vector ||**grad**|| at the search end point is investigated. The following linear model is considered:

$$\lg\left(DX_{\text{opt}}\right) = k_1 \lg\left(SValley\right) + k_2 \lg\left(\|\mathbf{grad}\|\right) + b. \quad (7)$$

The regression analysis results of this model are given in Table 2. As for the previous model, cases including all points and excluding points with *NValley* > 1 are distinguished. Notably, the correlation coefficient of values lg(||**grad**||) and lg(*SValley*) at different dimensions of *ND* space varies from 0.91 to 0.95, i.e., the correlation is significant. Nevertheless, taking the gradient norm into account provides additional

Table 1. Results of the regression analysis of model (6)

ND	All points				Points with *NValley* > 1 are excluded			
	b	*k*	R^2	*StdErr*	*b*	*k*	R^2	*StdErr*
2	−6.97	0.564	0.846	0.825	–	–	–	–
3	−6.81	0.675	0.855	0.755	−6.98	0.702	0.856	0.747
4	−6.80	0.714	0.881	0.686	−7.08	0.758	0.885	0.666
5	−6.82	0.744	0.882	0.685	−7.15	0.797	0.884	0.667
6	−6.79	0.756	0.884	0.674	−7.09	0.806	0.885	0.660
7	−6.72	0.757	0.883	0.673	−7.03	0.808	0.886	0.656
8	−6.69	0.766	0.876	0.700	−7.00	0.817	0.878	0.687
9	−6.63	0.764	0.874	0.700	−6.94	0.816	0.877	0.684
10	−6.60	0.772	0.868	0.712	−6.93	0.828	0.869	0.699
11	−6.59	0.777	0.865	0.716	−6.90	0.830	0.868	0.702
12	−6.54	0.779	0.857	0.740	−6.86	0.834	0.859	0.729

Table 2. Results of the regression analysis of model (7)

ND	All points					Points with *NValley* > 1 are excluded				
	b	k_1	k_2	R^2	*StdErr*	*b*	k_1	k_2	R^2	*StdErr*
2	−3.53	0.041	0.526	0.936	0.534	–	–	–	–	–
3	−3.84	0.125	0.464	0.929	0.529	−3.97	0.145	0.469	0.937	0.495
4	−3.91	0.163	0.453	0.935	0.507	−4.08	0.184	0.475	0.950	0.439
5	−4.01	0.196	0.438	0.936	0.502	−4.15	0.209	0.479	0.956	0.411
6	−3.98	0.198	0.442	0.937	0.495	−4.07	0.201	0.490	0.956	0.407
7	−3.95	0.199	0.444	0.938	0.490	−4.06	0.204	0.491	0.959	0.395
8	−3.86	0.184	0.460	0.939	0.492	−3.99	0.196	0.501	0.959	0.398
9	−3.89	0.195	0.451	0.937	0.493	−4.02	0.208	0.492	0.959	0.394
10	−3.84	0.190	0.456	0.937	0.492	−4.02	0.211	0.493	0.958	0.397
11	−3.84	0.194	0.458	0.936	0.494	−4.00	0.212	0.496	0.958	0.397
12	−3.84	0.195	0.454	0.934	0.501	−4.03	0.218	0.490	0.957	0.401

Table 3. Approximation results using ANN models

ND	Approximation along lg(*SValley*)				Along lg(*SValley*) and lg(||**grad**||)			
	All points		Without *NValley* > 1		All points		Without *NValley* > 1	
	R^2	*StdErr*	R^2	*StdErr*	R^2	*StdErr*	R^2	*StdErr*
2	0.863	0.777	–	–	0.954	0.451	–	–
3	0.881	0.685	0.877	0.691	0.952	0.437	0.953	0.425
4	0.906	0.609	0.904	0.608	0.959	0.404	0.963	0.378
5	0.904	0.617	0.900	0.618	0.960	0.399	0.966	0.360
6	0.907	0.602	0.904	0.603	0.959	0.402	0.965	0.364
7	0.908	0.598	0.906	0.595	0.961	0.388	0.969	0.341
8	0.903	0.621	0.900	0.622	0.961	0.391	0.970	0.340
9	0.903	0.613	0.901	0.613	0.964	0.376	0.972	0.329
10	0.895	0.634	0.891	0.640	0.960	0.391	0.969	0.339
11	0.891	0.644	0.888	0.648	0.961	0.384	0.970	0.336
12	0.883	0.668	0.879	0.676	0.959	0.398	0.970	0.337

information for estimating the error DX_{opt}. Compared to model (6), the coefficient of determination R^2 becomes closer to one while the RMS error *StdErr* decreases. At the same time, the exclusion of points with erroneously defined valley dimensionality improves the model performance, as in the previous case.

An alternative approach to the approximation of dependencies between data collected in experiments is based on training ANN models. It is known that ANNs with hidden layers and a sufficient number of neurons can be used to approximate any continuous function of several variables [15]. Here, the ANN model with one hidden layer containing 5 neurons is used. The same data used for regression analysis of models (6) and (7) is used as a training sample. All *MATLAB train*(..) function settings are default. The approximation results are presented in Table 3.

The comparison with the results from Tables 1 and 2 shows that ANN models provide a more accurate approximation of the required dependence on the same initial data than linear regression models. Notably, the results are not significantly improved by increasing the number of neurons up to 10.

CONCLUSIONS

An objective stochastic dependence between the valley ratio estimation of the OF landscape in the neighborhood of the minimum search end point and the error in determining the coordinates of the true position of the OF minimum has been demonstrated. When determining the coordinates of the minimum point, this dependence can be identified and recorded in the form of a linear regression equation or in the form of a trained ANN model, and then used to estimate the expected error.

ANN models were found to provide higher accuracy in predicting the magnitude of the error compared to linear regression models. Moreover, the accuracy of both types of models increases taking into account not only the estimate of the valley ratio, but also the Euclidean norm of the OF gradient at the search end point.

In the future, it is planned to expand the methodology to apply to functions with a valley dimensionality greater than one, as well as to other types of landscape areas presenting difficulties for optimization algorithms.

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