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RESEARCH ARTICLE

# New energy effect in non-cylindrical domains with a thermally insulated moving boundary

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**Abstract**
**Objectives.** To develop mathematical model representations of the energy effect in non-cylindrical domains having a thermally insulated moving boundary; to introduce a new boundary condition for thermal insulation of a moving boundary both for locally equilibrium heat transfer processes in the framework of classical Fourier phenomenology, as well as for more complex locally non-equilibrium processes in the framework of Maxwell–Cattaneo–Lykov–Vernott phenomenology, taking into account the finite rate of heat propagation into analytical thermophysics and applied thermomechanics; to consider an applied problem of analytical thermophysics according to the theory of thermal shock for a domain with a moving thermally insulated boundary free from external and internal influences; to obtain an exact analytical solution of the formulated mathematical models for hyperbolic type equations; to investigate the solutions obtained using a computational experiment at various values of the parameters included in it; to describe the wave nature of the kinetics of the processes under consideration.
**Methods.** Methods and theorems of operational calculus, Riemann–Mellin contour integrals are used in calculating the originals of complex images with two branch points. A new mathematical apparatus for the equivalence of functional constructions for the originals of the obtained operational solutions, which considers the computational difficulties in finding analytical solutions to boundary value problems for equations of hyperbolic type in the domain with a moving boundary, is developed.
**Results.** Developed mathematical models of locally nonequilibrium heat transfer and the theory of thermal shock for equations of hyperbolic type in a domain with a moving thermally insulated boundary are presented. It is shown that, despite the absence of external and internal sources of heat, the presence of a thermally insulated moving boundary leads to the appearance of a temperature gradient in the domain and, consequently, to the appearance of a temperature field and corresponding thermoelastic stresses in the domain, which have a wave character. A stochastic analysis of this energy effect forms the basis for a proposed transition of the kinetic energy of a moving thermally insulated boundary into the thermal energy of the domain. The presented model representations of the indicated effect confirmed the stated assumption.
**Conclusions.** Mathematical models for locally nonequilibrium heat transfer processes and the theory of thermal stresses are developed and investigated on the basis of constitutive relations of the theory of thermal shock for equations of hyperbolic type in a domain with a thermally isolated moving boundary. A numerical experiment is presented to demonstrate the possibility of transiting from one form of analytical solution of a thermophysical problem to another equivalent form of a new type. The described energy effect manifests itself both for parabolic type equations based on the classical Fourier phenomenology, as well as for hyperbolic type equations based on the generalized Maxwell–Cattaneo–Lykov–Vernott phenomenology.

**Keywords:** moving thermally insulated boundary, temperature field, temperature stresses, equations of hyperbolic type

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НАУЧНАЯ СТАТЬЯ

# Новый энергетический эффект в областях нецилиндрического типа с термоизолированной движущейся границей

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**Резюме**

**Цели.** Разработка математически модельных представлений энергетического эффекта в областях нецилиндрического типа с термоизолированной движущейся границей. Введение в аналитическую теплофизику и прикладную термомеханику нового граничного условия теплоизоляции движущейся границы как для локально равновесных процессов теплопереноса в рамках классической феноменологии Фурье, так и для более сложных локально-неравновесных процессов в рамках феноменологии Максвелла – Каттанео – Лыкова – Вернотта, учитывающих конечную скорость распространения теплоты. Рассмотрение прикладной задачи аналитической теплофизики и теории теплового удара для области с движущейся термоизолированной границей, свободной от внешних и внутренних воздействий. Получение точного аналитического решения сформулированных математических моделей для уравнений гиперболического типа. Исследование полученных решений с помощью вычислительного эксперимента при различных значениях, входящих в него параметров. Описание волнового характера кинетики рассматриваемых процессов.

**Методы.** Использованы методы и теоремы операционного исчисления, контурные интегралы Римана – Меллина при вычислении оригиналов сложных изображений с двумя точками ветвления. С учетом вычислительных трудностей при нахождении аналитических решений краевых задач для уравнений гиперболического типа в области с движущейся границей, развит новый математический аппарат эквивалентности функциональных конструкций для оригиналов полученных операционных решений.

**Результаты.** Представлено развитие новых математических моделей локально-неравновесного теплопереноса и теории теплового удара для уравнений гиперболического типа в области с движущейся термоизолированной границей. Показано, что, несмотря на отсутствие внешних и внутренних источников теплоты, наличие термоизолированной движущейся границы приводит к появлению в области градиента температуры и, следовательно, к появлению в области температурного поля и соответствующих ему термоупругих напряжений, имеющих волновой характер. Стохастический анализ указанного энергетического эффекта позволил высказать предположение о переходе кинетической энергии движущейся термоизолированной границы в тепловую энергию области. Приведенные модельные представления указанного эффекта подтвердили высказанное предположение.

**Выводы.** Развиты и исследованы математические модели для локально-неравновесных процессов теплопереноса и теории термических напряжений на основе определяющих соотношений теории теплового удара для уравнений гиперболического типа в области с термоизолированной движущейся границей. Проведен численный эксперимент и показана возможность перехода от одной формы аналитического решения теплофизической задачи к другой эквивалентной форме нового типа. Описанный энергетический эффект проявляется как для уравнений параболического типа на основе классической феноменологии Фурье, так и для уравнений гиперболического типа на основе обобщенной феноменологии Максвелла – Каттанео – Лыкова – Вернотта.

**Ключевые слова:** движущаяся теплоизолированная граница, температурное поле, температурные напряжения, уравнения гиперболического типа

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## INTRODUCTION

The effect of the concentration gradient emergence in the domain with moving impermeable boundary was encountered by the author for the first time when studying the phenomenon of adsorption reduction of strength and durability of brittle polymers in surface-active media [1]. A review of the literature confirms that, while the above-described phenomenon affects many fields of science and technology, it has yet to be practically described in scientific publications. Considering thermal processes, it will be shown that a temperature gradient exists in a domain having a moving thermally insulated boundary despite the absence of internal or external sources of heat; this is due to the conversion of the kinetic energy of the boundary motion into the thermal energy of the domain. A stochastic analysis of this energy effect for the temperature average based on an analysis of the corresponding dispersion shows the similarity of the dispersion behavior to that arising in the domain of the average value of temperature stresses, creating the risks of cracks and the possible beginning of material destruction [2].

## PROBLEM STATEMENT

We shall briefly consider thermophysical problems in domains with moving boundaries (non-cylindrical domains).

A very wide range of issues arise when considering boundary value problems of nonstationary heat conduction in non-cylindrical domains of the type $[0, y(t)]$, $t > 0$ or $[y(t), \infty)$, $t > 0$, where $y(t)$ is continuous function. Similar problems arise in the theoretical study of energy transfer processes related to changes in the aggregate state of matter, as well as in strength theory, dam theory, soil mechanics, oil-reservoir thermic and electrodynamic problems, filtration problems, the theory of zone cleaning of materials, kinetic theory of crystal growth, thermomechanics in the study of thermal shock, etc. [3].

In mathematical terms, boundary transfer problems in the domain with moving boundaries are fundamentally different from classical ones. Due to the dependence of the domain boundary on time, the classical methods of equations of mathematical physics are inapplicable to this type of problems: it is impossible to match the solution of the heat conduction equation with the motion of the domain boundary while remaining within the framework of such methods. This explains why only the simplest cases with a uniformly moving boundary, or partially with a root dependence, have been considered in analytical thermophysics to date.

Let $\bar{\Omega}_t$ be a non-cylindrical domain, whose cross-section by characteristic plane $t = \text{const} \geq t_0 > 0$ is convex domain $D_t$ of change $M(x, y, z)$ with boundary $S_t$ depending on time $t \geq 0$, $\bar{\mathbf{n}}$ is the external normal to $S_t$, representing a vector continuous at points $S_t$, so that $\bar{\Omega}_t = \left\{M \in \bar{D}_t = D_t + S_t, t \geq 0\right\}$.

Let $T(M, t)$ be the temperature function satisfying the conditions of the problem, $a$ be thermal conductivity, and $f$ be the source function. $\Phi_0$ is initial temperature, $\beta_1$, $\beta_2$ are coefficients, while $C^0$, $C^1$, $C^2$ are function classes.

$$\frac{\partial T}{\partial t} = a\Delta T(M,t) + f(M,t),\ M \in D_t,\ t > 0, \qquad (1)$$

$$T(M,t)\big|_{t=0} = \Phi_0(M),\ M \in \bar{D}_{t=0}, \qquad (2)$$

$$\beta_1 \frac{\partial T(M,t)}{\partial n} + \beta_2 T(M,t) = \varphi(M,t),\ M \in S_t,\ t \geq 0. \qquad (3)$$

Here,

$$f(M,t) \in C^0(\overline{\Omega}_t);\ \Phi_0(M) \in C^1(\overline{\Omega}_t);$$
$$\varphi(M,t) \in C^0(S_t \times t \geq 0);\ \beta_1^2 + \beta_2^2 > 0. \quad (4)$$

The desired solution: $T(M,t) \in C^2(\Omega_t) \cap C^0(\overline{\Omega}_t)$, $\mathrm{grad}_M T(M,t) \in C^0(\overline{\Omega}_t)$.

The boundary condition (3) includes the cases of temperature heating, thermal heating, and heating by the medium (or cooling in all three cases). If $\overline{D}$ is a canonical (cylindrical) domain with a fixed boundary *S* (elastic half-space, infinite plate, cylinder, ball, etc.), the thermal insulation condition of boundary *S* of domain *D* is written in the following form:

$$\frac{\partial T(M,t)}{\partial n}\Big|_{M\in S} = 0,\ t > 0 \quad (5)$$

and is a classical boundary condition in analytical thermophysics when formulating appropriate problems for parabolic type equations. However, the presence of a moving boundary fundamentally changes the form of the boundary condition for its thermal insulation; moreover, it is this circumstance that is not generally considered in the literature describing various kinds of applications related to the thermal insulation of the moving boundary.

For deriving the above condition, we shall consider domain $\Omega_t = (0 < z < y(t),\ t > 0)$. Here, $y(t)$ is a continuous-differentiable function; $v(t) = dy(t)/dt$ is the boundary movement rate; $T(z, t)$ is temperature field in $\Omega_t$; $F(z, t)$ is nonstationary heat source ($F(z, t)/c\rho = f(z, t)$) continuously distributed in $\Omega_t$, where $c$ is heat capacity and $\rho$ is density. We have for $\Omega_t$:

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial z^2} + f(z,t),\ \ (z,t) \in \Omega_t. \quad (6)$$

We shall write the heat balance equation at time $(t + \Delta t)$ considering boundary $z = y(t)$ insulated as follows:

$$-\lambda\frac{\partial T(0,t)}{\partial z}\Delta t + c\rho\Delta t\int_0^{y(t)+\Delta y} f(z,t)dz =$$
$$= c\rho\int_0^{y(t)}\left[T(z,t+\Delta t) - T(z,t)\right]dz + c\rho\int_{y(t)}^{y(t)+\Delta y} T(z,t+\Delta t)dz,$$

where $\lambda$ is heat conduction.

For the second of the integrals on the right, we apply the average theorem, as follows:

$$-a\frac{\partial T(0,t)}{\partial z}\Delta t + \Delta t\int_0^{y(t)+\Delta y} f(z,t)dz =$$
$$= \int_0^{y(t)}\left[T(z,t+\Delta t) - T(z,t)\right]dz + T(z,t+\Delta t)\Big|_{z=y+\theta\Delta y}\Delta y,$$

where $0 < \theta < 1$. Dividing both parts of the equation by $\Delta t$ and going to the limit at $\Delta t \to 0$, we obtain the following:

$$-a\frac{\partial T(0,t)}{\partial z} + \int_0^{y(t)} f(z,t)dz = \int_0^{y(t)}\frac{\partial T}{\partial t}dz + v(t)T(z,t)\Big|_{z=y(t)}.$$

We shall substitute under the integral sign the right-hand side of heat conduction Eq. (6) for $\partial T/\partial t$, integrate it, and add similar terms. The final result is the following condition:

$$\frac{\partial T(z,t)}{\partial z}\Big|_{z=y(t)} + \frac{v(t)}{a}T(z,t)\Big|_{z=y(t)} = 0,\ \ t > 0, \quad (7)$$

which represents the thermal insulation condition for the moving boundary. If the boundary movement rate $v(t) = 0$, then we arrive at the condition $(\partial T/\partial n)|_S = 0$ implying thermal insulation of the stationary boundary surface.

Since the late 1960s, systematic publications on hyperbolic transfer models that take into account the finite rate of heat propagation have appeared[1, 2] [4–16]. Nowadays, it is increasingly common practice to distinguish a large class of models based on the following equation:

$$\frac{\partial T(M,t)}{\partial t} = a\Delta T(M,t) - \tau_r\frac{\partial^2 T(M,t)}{\partial t^2},\ (M,t) \in \Omega_t, \quad (8)$$

where $\tau_r$ is the thermal flux relaxation time related to the heat propagation rate by relation $v_T = \sqrt{a/\tau_r}$.

The boundary problems for Eq. (8) describe high-intensity heat exchange in pulse and laser devices, laser metal processing, plasma spraying processes, processes occurring in energy channels of nuclear reactors, as well as in a fluidized bed and disperse systems, granular materials, and layered semiconductor structures. The problems also arise in descriptions of electronic heat conduction

[1] Eremin A.V. *Modeling methodology of heat and mass transfer, elastic vibrations and electromagnetic waves with allowance for spatial and temporal nonlocality*. Abstract. Cand. Sci. Thesis (Eng.). Samara; 2021. 30 p. (in Russ.).

[2] Zhukov V.V. *Investigation of internal mechanisms of heat, mass, and momentum transfer with allowance for relaxation phenomena*. Abstract. Cand. Sci. Thesis (Eng.). Samara; 2021. 18 p. (in Russ.).

and high-temperature plasma, in mathematical modeling of thermal decomposition front processes, in catalyst crystals, and the growth of homoepitaxial germanium films during exothermic chemical reactions, etc. In [3], the issues of correct formulation of boundary value problems for Eq. (8) are studied: it is shown that the writing of boundary conditions of the second and third kind significantly differs from (3) for equations of parabolic type. However, the question about the heat isolation of the moving boundary for Eq. (8) remains open. For this, we shall consider the phenomenological Maxwell–Cattaneo–Lykov–Vernott relation [4, 6–7]:

$$\bar{\mathbf{q}}(M,t) = -\lambda\ \mathrm{grad}T(M,t) - \tau_\mathrm{r}\frac{\partial\bar{\mathbf{q}}(M,t)}{\partial t},$$

here, $\bar{\mathbf{q}}$ is the vector of heat flux density, which forms the basis for the analytical theory of local nonequilibrium processes of heat transfer in a non-cylindrical domain.

We shall write this equation in the following form:

$$\left(1+\tau_\mathrm{r}\frac{\partial}{\partial t}\right)\bar{\mathbf{q}}(M,t) = -\lambda\ \mathrm{grad}T(M,t),\ M\in D_t,\ t>0, \quad (9)$$

or

$$\bar{\mathbf{q}}(M,t+\tau_\mathrm{r}) = -\lambda\ \mathrm{grad}T(M,t),\ M\in D_t,\ t>0, \quad (10)$$

using the Maclaurin series Eq. (9) of function $\bar{\mathbf{q}}(M,t+\tau_\mathrm{r})$ in the vicinity of point $\tau_\mathrm{r}=0$.

Equation (10) may be rewritten in the following form:

$$\bar{\mathbf{q}}(M,t) = -\lambda\ \mathrm{grad}T(M,t-\tau_\mathrm{r}),\ M\in D_{t-\tau_\mathrm{r}},\ t>\tau_\mathrm{r}. \quad (11)$$

Using energy equation $c\rho\partial T(M,t)/\partial t = -\mathrm{div}\left[\bar{\mathbf{q}}(M,t)\right]$ and relation (11), Eq. (8) may be written in the following form:

$$\frac{\partial T(M,t)}{\partial t} = a\Delta T(M,t-\tau_\mathrm{r}),\ M\in D_{t-\tau_\mathrm{r}},\ t>\tau_\mathrm{r}. \quad (12)$$

Consider now the domain of interest $z > y(t)$, $t > 0$, wherein (12) is the following:

$$\frac{\partial T(z,t)}{\partial t} = a\frac{\partial^2 T(z,t-\tau_\mathrm{r})}{\partial z^2},\ z>y(t-\tau_\mathrm{r}),\ t>\tau_\mathrm{r}. \quad (13)$$

Under constant initial conditions, as well as in the absence of internal heat sources and external heating conditions at the thermal insulation of the moving boundary, the following condition is true:

$$c\rho\int_{y(t-\tau_\mathrm{r})}^{\infty} T(z,t)dz = \mathrm{const},\ t>\tau_\mathrm{r}. \quad (14)$$

Differentiating both parts of (14) by $t$ and using Eq. (13), the following relation is obtained:

$$\left[\frac{\partial T(z,t-\tau_\mathrm{r})}{\partial z} + \frac{1}{a}\cdot\frac{dy(t-\tau_\mathrm{r})}{\partial t}T(z,t)\right]_{z=y(t-\tau_\mathrm{r})} = \quad (15)$$
$$= 0,\quad t>\tau_\mathrm{r},$$

which may be rewritten as:

$$\left[\frac{\partial T(z,t)}{\partial z} + \frac{v(t)}{a}T(z,t+\tau_\mathrm{r})\right]_{z=y(t)} = 0,\quad t>0, \quad (16)$$

where $v(t) = dy/dt$.

Equation (16) is the moving boundary heat isolation condition for locally nonequilibrium heat transfer processes described by hyperbolic type equations. In particular cases (local-equilibrium processes, $\tau_\mathrm{r} = 0$) or cylindrical type domain ($v(t) = 0$), the thermal insulation conditions discussed above are obtained.

## THE TEMPERATURE GRADIENT EFFECT IN THE DOMAIN WITH A MOVING THERMALLY INSULATED BOUNDARY

In the corresponding model representations of nonstationary heat conduction, boundary conditions (7), (16) create the temperature gradient effect in the domain and consequent appearance of corresponding thermoelastic stresses, which occur despite the absence of external and internal thermal impacts. While formally, there is an idea concerning the impossibility of manifestation of the described effect, analytical solutions of model problems demonstrate the opposite. In [2], it is suggested that the kinetic energy of the moving insulated boundary is converted into the thermal energy of the domain, thus causing heat and thermal effects. In this connection, in formulating the thermal problem for the hyperbolic type equation, we shall consider elastic half-space $z > l + vt$, $t > 0$ with a uniformly moving insulated boundary in the absence of external and internal thermal loads:

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial z^2} - \tau_\mathrm{r}\frac{\partial^2 T}{\partial t^2},\quad z>l+vt,\quad t>0, \quad (17)$$

$$T(z,t)\big|_{t=0} = T_0,\ \left.\frac{\partial T(z,t)}{\partial t}\right|_{t=0} = 0,\quad z\ge l, \quad (18)$$

$$\left[\frac{\partial T(z,t)}{\partial z} + \frac{v}{a}T(z,t+\tau_\mathrm{r})\right]\Bigg|_{z=l+vt} = 0,\quad t>0, \quad (19)$$

$$|T(z,t)| < \infty,\ z\ge l+vt,\ t\ge 0. \quad (20)$$

The boundary condition (19) may be written in the following form:

$$\left[\frac{\partial T(z,t)}{\partial z}+\frac{v}{a}T(z,t)+\frac{v\tau_r}{a}\cdot\frac{\partial T(z,t)}{\partial t}\right]\Bigg|_{z=l+vt}= \quad (21)$$
$$=0,\quad t>0.$$

We shall introduce dimensionless variables

$$z'=(z-l)/l;\quad \tau=at/l^2;\ v_0=vl/a;\ \tau_0=a\tau_r/l^2;$$
$$T^*(z',\tau)=\left[T(z,t)-T_0\right]/T_0$$

and then the moving coordinate system $\xi=z'-v_0\tau,\ \tau>0$, assuming $T^*(z',\tau)=W(\xi,\tau)$.

Relations (17)–(21) have the following form:

$$\frac{\partial W}{\partial\tau}=(1-\tau_0v_0^2)\frac{\partial^2W}{\partial\xi^2}+v_0\frac{\partial W}{\partial\xi}+ \quad (22)$$
$$+\,2v_0\tau_0\frac{\partial^2W}{\partial\xi\partial\tau}-\tau_0\frac{\partial^2W}{\partial\tau^2}=0,\ \xi>0,\ \tau>0,$$

$$W(\xi,\tau)\big|_{\tau=0}=0,\ \frac{\partial W(\xi,\tau)}{\partial\tau}\bigg|_{\tau=0}= \quad (23)$$
$$=v_0\frac{\partial W(\xi,\tau)}{\partial\xi}\bigg|_{\tau=0}=0,\ \xi\ge0,$$

$$(1-\tau_0v_0^2)\frac{\partial W(\xi,\tau)}{\partial\xi}\bigg|_{\xi=0}= \quad (24)$$
$$=-v_0\left[W(\xi,\tau)+\tau_0\frac{\partial W(\xi,\tau)}{\partial\tau}+1\right]\Bigg|_{\xi=0},\tau>0,$$

$$|W(\xi,\tau)|<\infty,\quad \xi\ge0,\quad \tau\ge0. \quad (25)$$

In the Laplace image space:

$$\overline{W}(\xi,p)=\int_0^\infty W(\xi,\tau)\exp(-p\tau)d\tau$$

the operational solution of the transformed problem (22)–(25)

$$(1-\tau_0v_0^2)\frac{d^2\overline{W}}{d\xi^2}+v_0(1+2\tau_0p)\frac{d\overline{W}}{d\xi}-$$
$$-\ p(1+\tau_0p)\overline{W}=0,\ \xi>0,$$

$$(1-\tau_0v_0^2)\frac{d\overline{W}}{d\xi}\bigg|_{\xi=0}=-v_0\left[(1+\tau_0p)\overline{W}+\frac{1}{p}\right]_{\xi=0},$$

$$\left|\overline{W}(\xi,p)\right|<\infty,\ \xi\ge0$$

may be written in the following form:

$$\overline{W}(\xi,p)=\overline{\Psi}_1(\xi,p)\overline{\Psi}_2(\xi,p), \quad (26)$$

where

$$\overline{\Psi}_1(\xi,p)=\frac{v_0}{\left[(-v_0/2)+\sqrt{\tau_0}\sqrt{(p+2\alpha)(p+2\beta)}\right]}\times \quad (27)$$
$$\times\exp\left[-\frac{(v_0/2)\xi}{1-\tau_0v_0^2}\right],$$

$$\overline{\Psi}_2(\xi,p)=\frac{1}{p}\times$$
$$\times\exp\left\{-\left[\frac{\xi\sqrt{\tau_0}}{1-\tau_0v_0^2}\sqrt{(p+2\alpha)(p+2\beta)}+\frac{\tau_0v_0\xi}{1-\tau_0v_0^2}p\right]\right\}, \quad (28)$$

$$\alpha=\frac{1+\sqrt{1-\tau_0v_0^2}}{4\tau_0};\ \beta=\frac{1-\sqrt{1-\tau_0v_0^2}}{4\tau_0}.$$

For finding the originals of images (27)–(28), we shall first consider new transformations of operational calculus that are of interest for hyperbolic transfer models. In [3], the original image is

$$\frac{1}{p}\exp\left[-\xi\sqrt{(p+2\alpha)(p+2\beta)}\right]\leftarrow$$
$$\leftarrow\left[\exp(-\rho\xi)+\sigma\xi\int_\xi^t\exp(-\rho\tau)\frac{I_1\left(\sigma\sqrt{\tau^2-\xi^2}\right)}{\sqrt{\tau^2-\xi^2}}d\tau\right]\times \quad (29)$$
$$\times\,\eta(t-\xi)=W_1(\xi,t)\eta(t-\xi).$$

Here, $\sigma=\alpha-\beta$, $\rho=\alpha+\beta$, $I_1(z)$ is the modified Bessel function and $\eta(z)$ is the Heaviside function. On the other hand, calculating the original image on the left in (29) using the Riemann–Mellin contour integral with two branching points by the method developed in [3], we find:

$$\frac{1}{p}\exp\left[-\xi\sqrt{(p+2\alpha)(p+2\beta)}\right]\leftarrow$$
$$\leftarrow\left\{\exp(-2\xi\sqrt{\alpha\beta})-\frac{1}{\pi}\int_0^{2\sigma}\frac{\sin\xi\sqrt{y(2\sigma-y)}}{y+2\beta}\times\right. \quad (30)$$
$$\left.\times\exp\left[-(y+2\beta)t\right]dy\right\}\eta(t-\xi)=W_2(\xi,t)\eta(t-\xi).$$

We shall show that $W_1(\xi,t)=W_2(\xi,t)$.

We have:

$$W_1(\xi,t)=\frac{\partial}{\partial\xi}\left[-\int_{\xi}^{t}\exp(-\rho\tau)I_0(\sigma\sqrt{\tau^2-\xi^2})d\tau\right]= =\frac{\partial}{\partial\xi}\left[-\int_{\xi}^{t}\exp(-\rho\tau)J_0(\sigma\sqrt{\xi^2-\tau^2})d\tau\right]. \quad (31)$$

We shall differentiate both parts of (31) by $t$:

$$\left[W_1(\xi,t)\right]_t'=\frac{\partial}{\partial\xi}\left[-\exp(-\rho t)J_0(\sigma\sqrt{\xi^2-t^2})\right]= =\frac{\partial}{\partial\xi}\left[-\exp(-2\beta t)\exp(-\sigma t)J_0(\sigma\sqrt{\xi^2-t^2})\right].$$

We shall use further a rather rare integral [3]:

$$\int_0^a\frac{\exp(-px)}{\sqrt{ax-x^2}}\cos c\sqrt{ax-x^2}\,dx= =\pi\exp(-ap/2)J_0\left(\frac{a}{2}\sqrt{c^2-p^2}\right).$$

Hence:

$$\left[W_1(\xi,t)\right]_t'=\frac{1}{\pi}\int_0^{2\sigma}\sin\xi\sqrt{y(2\sigma-y)}\exp\left[-(y+2\sigma)t\right]dy. \quad (32)$$

Integrating both parts of (32) by $t$ and using the finite value theorem $\lim_{t\to\infty}f(t)=\lim_{p\to 0}p\bar{f}(p)$ giving $C=\exp(-2\xi\sqrt{\alpha\beta})$ in (30) to find the integration constant, the following is finally obtained:

$$W_1(\xi,t)=\exp(-2\xi\sqrt{\alpha\beta})- -\frac{1}{\pi}\int_0^{2\sigma}\frac{\sin\xi\sqrt{y(2\sigma-y)}}{y+2\beta}\exp\left[-(y+2\beta)t\right]dy=W_2(\xi,t).$$

Thus, the original is found:

$$\frac{1}{p}\exp\left[-\frac{\xi\sqrt{\tau_0}}{1-\tau_0 v_0^2}\sqrt{(p+2\alpha)(p+2\beta)}\right]\leftarrow \leftarrow\left\{\exp\left[-\frac{(v_0/2)\xi}{1-\tau_0 v_0^2}\right]-\frac{1}{\pi}\int_0^{2\sigma}\frac{1}{y+2\beta}\sin\frac{\xi\sqrt{\tau_0}\sqrt{y(2\sigma-y)}}{1-\tau_0 v_0^2}\times \times\exp\left[-(y+2\beta)\tau\right]dy\right\}\times\eta\left(\tau-\frac{\xi\sqrt{\tau_0}}{1-\tau_0 v_0^2}\right). \quad (33)$$

Now, using (33), we find the original image $\overline{\Psi}_2(\xi,p)$ (28):

$$\Psi_2(\xi,\tau)= =\left\{\exp\left[-\frac{(v_0/2)\xi}{1-\tau_0 v_0^2}\right]-\frac{1}{\pi}\int_0^{2\sigma}\frac{1}{y+2\beta}\sin\frac{\xi\sqrt{\tau_0}\sqrt{y(2\sigma-y)}}{1-\tau_0 v_0^2}\times \times\exp\left[-(y+2\beta)\left(\tau-\frac{\tau_0 v_0\xi}{1-\tau_0 v_0^2}\right)\right]dy\right\}\times\eta\left(\tau-\frac{\xi\sqrt{\tau_0}}{1-v_0\sqrt{\tau_0}}\right). \quad (34)$$

The original image $\overline{\Psi}_1(\xi,p)$ (27) has the following form:

$$\Psi_1(\xi,\tau)=\exp\left(-\frac{(v_0/2)\xi}{1-\tau_0 v_0^2}\right)\times \times\left[\frac{v_0\sqrt{\tau_0}}{\sqrt{\pi}}\int_{2\beta}^{2\alpha}\frac{\sqrt{(2\alpha-y)(y-2\beta)}\exp(-y\tau)}{(v_0^2/4)+\tau_0(2\alpha-y)(y-2\beta)}dy\right]. \quad (35)$$

The desired original image $\overline{W}(\xi,p)$ (26) may be written now in the following form:

$$W(\xi,\tau)=\left[\int_{\frac{\xi\sqrt{\tau_0}}{1-v_0\sqrt{\tau_0}}}^{\tau}\Psi_1(\xi,\tau-\tau')\Psi_2(\xi,\tau')d\tau'\right]\times \times\eta\left(\tau-\frac{\xi\sqrt{\tau_0}}{1-v_0\sqrt{\tau_0}}\right). \quad (36)$$

Figure 1 shows curves of the temperature function (36) versus $\tau$ in the cross section $\xi=1$ for different $v_0$ at $\tau=0.25$. The curves in Fig. 1 clearly show the peculiarities of the thermal response of the domain for locally nonequilibrium processes (the analytical solution (36) contains the Heaviside function explaining the delay in the onset of heat propagation in the fixed cross section).

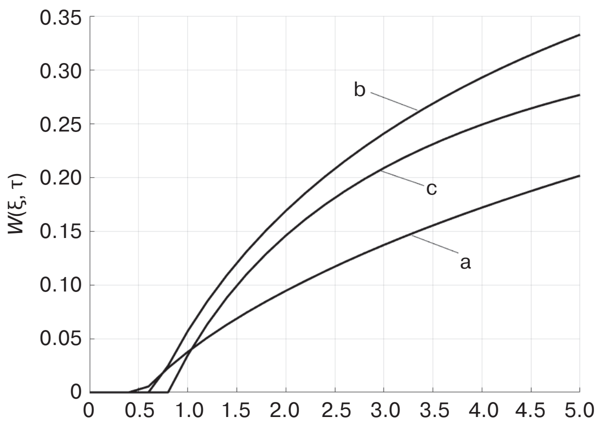


**Fig. 1.** Dependence of the temperature function $W(\xi,\tau)$ (36) on $\tau$ in cross section $\xi=1$ for different $v_0$: $v_0=0.16$ (a); $v_0=0.5$ (b); $v_0=0.75$ (c) at $\tau_0=0.25$

## THERMAL RESPONSE OF THE DOMAIN TO THE TEMPERATURE FIELD $W(\xi, \tau)$

The next step is investigating the thermal response of domain $\overline{\Omega}_t = (z \geq l + vt,\, t \geq 0)$ with a moving thermally insulated boundary in the framework of the model problem (17)–(20). We shall consider, as above, an elastic half-space, which is of practical interest for many areas of science and technology described in [3]. We shall write down the defining relations of dynamic thermoelasticity for domain $\overline{\Omega}_t = \{M(x,y,z) \in \overline{D}_t = D_t + S_t,\, t \geq 0\}$ with temperature function $T(M, t)$. Let $T_0$ be the initial temperature at which the domain is in the undeformed and unstressed state; $\sigma_{ij}(M, t)$, $\varepsilon_{ij}(M, t)$, $U_i(M, t)$ $(i = x, y, z)$ are the components of stress tensors, strain, and displacement vectors, respectively, satisfying the basic equations of (unbounded) thermoelasticity (in index notations) [19, 20]:

$$\sigma_{ij,j}(M,t) = \rho \ddot{U}_i(M,t), \tag{37}$$

$$\varepsilon_{ij}(M,t) = (1/2)\left[U_{i,j}(M,t) + U_{j,i}(M,t)\right], \tag{38}$$

$$\sigma_{ij}(M,t) = 2\mu\varepsilon_{ij}(M,t) + \left[\lambda\varepsilon_{ii}(M,t) - (3\lambda + 2\mu)\alpha_T(T(M,t) - T_0)\right]\delta_{ij}, \tag{39}$$

where $\rho$ is density; $\mu = G$, $G$ is shear modulus; $\lambda = 2Gv/(1 - 2v)$ are isothermal Lame coefficients; $v$ is Poisson ratio with $2G(1 + v) = E$, $E$ is Young modulus; $\alpha_T$ is linear thermal expansion coefficient; $\delta_{ij}$ is the Kronecker symbol; and $\varepsilon_{ii}(M,t) = \overline{e}(M,t) = U_{i,i}(M,t)$ is volume strain related to the sum of normal stresses $\overline{\sigma}(M,t) = \sigma_{nn}(M,t),\, n = x, y, z$ by the following relation:

$$\overline{e}(M,t) = \frac{1 - 2v}{E}\overline{\sigma}(M,t) + 3\alpha_T\left[T(M,t) - T_0\right]. \tag{40}$$

For the case of one-dimensional motion $M = M(z, t)$, $z > l + vt$, $t > 0$, we have the following from (37)–(40):

$$U_x = U_y = 0,\ U_z = U_z(z, t),\ \varepsilon_{xx} = \varepsilon_{yy} = 0,$$
$$\varepsilon_{zz}(z, t) = \partial U_z(z, t)/\partial z = [1/(1 - v)] \times$$
$$\times \{[(1 - 2v)/(2G)]\sigma_{zz}(z, t) + (1 + v)\alpha_T T(z, t) - T_0]\},$$

$$\frac{\partial \sigma_{zz}(z,t)}{\partial z} = \rho \frac{\partial^2 U_z(z,t)}{\partial t^2}.$$

Differentiating this relation by $z$ and substituting the value $\partial U_z(z, t)/\partial z$, the equation of the following form is obtained:

$$\frac{\partial^2 \sigma_{zz}}{\partial z^2} - \frac{1}{v_p^2}\cdot\frac{\partial^2 \sigma_{zz}}{\partial t^2} = \frac{(1+\nu)}{(1-\nu)}\alpha_T\rho\frac{\partial^2\left[T(z,t) - T_0\right]}{\partial t^2},\ z > l + vt,\ t > 0 \tag{41}$$

with the following boundary conditions:

$$\sigma_{zz}(z,t)\big|_{t=0} = 0,\ \left.\frac{\partial \sigma_{zz}(z,t)}{\partial t}\right|_{t=0} = 0,\ z \geq l, \tag{42}$$

$$\sigma_{zz}(z,t)\big|_{z=l+vt} = 0,\ t > 0,\quad \left|\sigma_{zz}(z,t)\right| < \infty,\ z \geq l + vt,\ t \geq 0. \tag{43}$$

In (41), $v_\text{p} = \sqrt{2G(1-\nu)/\left[\rho(1-2\nu)\right]} = \sqrt{(\lambda + 2\mu)/\rho}$ is the expansion wave propagation velocity in the elastic medium close to the speed of sound. The remaining nonzero components of the stress tensor, according to (37)–(39), have the following form:

$$\sigma_{xx}(z,t) = \sigma_{yy}(z,t) = \frac{\nu}{1-\nu}\sigma_{zz}(z,t) - \frac{E\alpha_T\left[T(z,t) - T_0\right]}{1-\nu}.$$

In addition,

$$\varepsilon_{zz}(z,t) = \frac{1-2\nu}{2G(1-\nu)}\sigma_{zz}(z,t) + \frac{(1+\nu)}{(1-\nu)}\alpha_T\left[T(z,t) - T_0\right].$$

The function $T(z, t)$ in (41)–(43) satisfies conditions (17)–(20). For solving the problem (41)–(43), we move to the coordinate system $(z', \tau)$ by the above relations, assuming that

$$\alpha_0 = v_\text{p} l/a,\ S_\text{T} = \alpha_\text{T}(3\lambda + 2\mu) = \frac{\alpha_\text{T} E}{(1-2\nu)},$$
$$\sigma_{z'z'}(z',\tau) = \frac{\sigma_{zz}(z,t)}{S_\text{T} T_0}.$$

Omitting intermediate calculations of the transition, we shall further introduce the moving coordinate system: $\xi = z' - v_0\tau$, assuming $\sigma_{\xi\xi}(\xi,\tau) = \sigma_{z'z'}(z',\tau)$, $T^*(z',\tau) = W(\xi,\tau)$. Relations (41)–(43) may be written now in the following form:

$$(\alpha_0^2 - v_0^2)\frac{\partial^2 \sigma_{\xi\xi}}{\partial \xi^2} + 2v_0\frac{\partial^2 \sigma_{\xi\xi}}{\partial \xi \partial \tau} - \frac{\partial^2 \sigma_{\xi\xi}}{\partial \tau^2} = \frac{\partial^2 W}{\partial \tau^2} - 2v_0\frac{\partial^2 W}{\partial \xi \partial \tau} + v_0^2\frac{\partial^2 W}{\partial \xi^2},\ \xi > 0,\ \tau > 0, \tag{44}$$

$$\left.\begin{array}{l}\sigma_{\xi\xi}(\xi,\tau)\big|_{\tau=0}=\dfrac{\partial\sigma_{\xi\xi}(\xi,\tau)}{\partial\tau}\Bigg|_{\tau=0}=0,\ \xi\ge 0,\\ \sigma_{\xi\xi}(\xi,\tau)\big|_{\xi=0}=0,\ \tau>0,\ \left|\sigma_{\xi\xi}(\xi,\tau)\right|<\infty,\ \xi\ge 0,\ \tau\ge 0\end{array}\right\}.\quad(45)$$

In the Laplace image space $\bar{\sigma}_{\xi\xi}(\xi,p)=\int_0^\infty\sigma_{\xi\xi}(\xi,\tau)\exp(-p\tau)d\tau$, relations (44)–(45) are written in the following form:

$$(\alpha_0^2-v_0^2)\frac{d^2\bar{\sigma}_{\xi\xi}}{d\xi^2}+2v_0p\frac{d\bar{\sigma}_{\xi\xi}}{d\xi}-p^2\bar{\sigma}_{\xi\xi}=\frac{p(p+v_0^2)}{1-\tau_0v_0^2}\overline{W}-\frac{v_0(v_0^2+2p)}{1-\tau_0v_0^2}\frac{d\overline{W}}{d\xi},\ \xi>0,\quad(46)$$

$$\bar{\sigma}_{\xi\xi}(\xi,p)\big|_{\xi=0}=0,\ \left|\bar{\sigma}_{\xi\xi}(\xi,p)\right|<\infty,\ \xi\ge 0.\quad(47)$$

Here, the following relation is used:

$$\frac{d^2\overline{W}}{d\xi^2}=\frac{p(1+\tau_0p)}{1-\tau_0v_0^2}\overline{W}-\frac{v_0(1+2\tau_0p)}{1-\tau_0v_0^2}\frac{d\overline{W}}{d\xi},$$

derived from the operational form of Eq. (22). For reducing the awkwardness in solving problem (46)–(47), we shall take into account the fact that inertial effects in (41) operate at times of microsecond duration. Then expression $\sqrt{\tau_0p^2+p+v_0^2/4}$ included in general solution (26) may be written in the following form:

$$\sqrt{\tau_0p^2+p+v_0^2/4}\simeq p\sqrt{\tau_0}\left(1+1/(2\tau_0p)\right)$$

and solution (26) takes the following form:

$$\overline{W}(\xi,p)=\frac{v_0/\sqrt{\tau_0}}{p\left[p+(1-v_0\sqrt{\tau_0})/(2\tau_0)\right]}\times\exp\left[-\frac{(2\tau_0p+1)\xi}{2\sqrt{\tau_0}(1-v_0\sqrt{\tau_0})}\right].$$

The desired voltage in the image space may be written now as follows:

$$\bar{\sigma}_{\xi\xi}(\xi,p)=\overline{F}(p)\left\{\exp\left[-\bar{\gamma}(p)\xi\right]-\exp\left(-\frac{\xi}{\alpha_0-v_0}p\right)\right\},\quad(48)$$

$$\overline{F}(p)=\left[\frac{A_{11}}{(p+\gamma_{11})(p+\gamma_{12})}+\frac{A_{12}}{p(p+\gamma_{11})(p+\gamma_{12})}+\frac{A_{13}}{p^2(p+\gamma_{11})(p+\gamma_{12})}\right]-\left[\frac{A_{21}}{(p+\gamma_{11})(p+\gamma_{13})}+\frac{A_{31}}{p(p+\gamma_{11})(p+\gamma_{13})}+\frac{A_{32}}{p^2(p+\gamma_{11})(p+\gamma_{13})}\right];$$

$$A_{11}=\frac{v_0(\alpha_0-v_0)(1+2\tau_0-v_0\sqrt{\tau_0})}{2\alpha_0\sqrt{\tau_0}(\alpha_0\sqrt{\tau_0}-1)(1-\tau_0v_0^2)};$$

$$A_{12}=A_{11}\left[v_0^2+(1+\tau_0v_0^2)/(2\tau_0)\right],$$

$$A_{13}=\frac{v_0^3(\alpha_0-v_0)}{4\alpha_0\sqrt{\tau_0}(1-\tau_0v_0^2)(\alpha_0\sqrt{\tau_0}-1)},$$

$$A_{21}=\frac{v_0(\alpha_0+v_0)(1+2\tau_0-v_0\sqrt{\tau_0})}{2\alpha_0\sqrt{\tau_0}(\alpha_0\sqrt{\tau_0}+1)(1-\tau_0v_0^2)},$$

$$A_{31}=A_{21}\left[v_0^2+(1+\tau_0v_0^2)/(2\tau_0)\right],$$

$$A_{32}=\frac{v_0^3(\alpha_0+v_0)}{4\alpha_0\sqrt{\tau_0}(1-\tau_0v_0^2)(\alpha_0\sqrt{\tau_0}+1)},$$

$$\gamma_{11}=\frac{1-v_0\sqrt{\tau_0}}{2\tau_0},\quad\gamma_{12}=\frac{\alpha_0-v_0}{2\sqrt{\tau_0}(\alpha_0\sqrt{\tau_0}-1)},$$

$$\gamma_{13}=\frac{\alpha_0+v_0}{2\sqrt{\tau_0}(\alpha_0\sqrt{\tau_0}+1)},$$

$$\bar{\gamma}(p)=\frac{2\tau_0p+1}{2\sqrt{\tau_0}(1-v_0)\sqrt{\tau_0}}.$$

Now from (48), we find for the desired voltage:

- at $\alpha_0\sqrt{\tau_0}=v_\text{p}/v_\text{T}>1$

$$\sigma_{\xi\xi}(\xi,\tau)=\begin{cases}0 & \tau<\dfrac{\xi}{\alpha_0-v_0},\\ \sigma_{\xi\xi}^{(2)}(\xi,\tau) & \dfrac{\xi}{\alpha_0-v_0}<\tau<\dfrac{\xi\sqrt{\tau_0}}{1-v_0\sqrt{\tau_0}},\\ \sigma_{\xi\xi}^{(1)}(\xi,\tau)+\sigma_{\xi\xi}^{(2)}(\xi,\tau) & \tau>\dfrac{\xi\sqrt{\tau_0}}{1-v_0\sqrt{\tau_0}};\end{cases}\quad(49)$$

- at $\alpha_0\sqrt{\tau_0} = v_\mathrm{p}/v_\mathrm{T} < 1$

$$\sigma_{\xi\xi}(\xi,\tau) = \\ = \begin{cases} 0, & \tau < \dfrac{\xi\sqrt{\tau_0}}{1-v_0\sqrt{\tau_0}}, \\ \sigma_{\xi\xi}^{(1)}(\xi,\tau), & \dfrac{\xi\sqrt{\tau_0}}{1-v_0\sqrt{\tau_0}} < \tau < \dfrac{\xi}{\alpha_0 - v_0}, \\ \sigma_{\xi\xi}^{(1)}(\xi,\tau) + \sigma_{\xi\xi}^{(2)}(\xi,\tau), & \tau > \dfrac{\xi}{\alpha_0 - v_0}. \end{cases} \quad (50)$$

Here:

$$\sigma_{\xi\xi}^{(1)}(\xi,\tau) = \\ = F\left(\tau - \frac{\xi\sqrt{\tau_0}}{1-v_0\sqrt{\tau_0}}\right)\exp\left[-\frac{\xi}{2\sqrt{\tau_0}(1-v_0\sqrt{\tau_0})}\right],$$

$$\sigma_{\xi\xi}^{(2)}(\xi,\tau) = -F\left(\tau - \frac{\xi}{\alpha_0 - v_0}\right),$$

$$F(\tau) = \left[B_{11}\exp(-\gamma_{11}\tau) + B_{12}\exp(-\gamma_{12}\tau) + \frac{A_{13}\tau}{\gamma_{11}\gamma_{12}} + B_{13}\right] - \\ -\left[B_{21}\exp(-\gamma_{11}\tau) + B_{31}\exp(-\gamma_{13}\tau) + \frac{A_{32}\tau}{\gamma_{11}\gamma_{13}} + B_{32}\right],$$

$$B_{11} = \frac{\gamma_{11}^2 A_{11} - \gamma_{11}A_{12} + A_{13}}{\gamma_{11}^2(\gamma_{12} - \gamma_{11})},$$

$$B_{12} = \frac{A_{12}\gamma_{12} - A_{11}\gamma_{12}^2 - A_{13}}{\gamma_{12}^2(\gamma_{12} - \gamma_{11})},$$

$$B_{13} = \frac{A_{12}}{\gamma_{11}\gamma_{12}} - \frac{(\gamma_{11}+\gamma_{12})A_{13}}{\gamma_{11}^2\gamma_{12}^2},$$

$$B_{21} = \frac{\gamma_{11}^2 A_{21} - \gamma_{11}A_{13} + A_{32}}{\gamma_{11}^2(\gamma_{13} - \gamma_{11})},$$

$$B_{31} = \frac{\gamma_{13}A_{31} - \gamma_{13}^2 A_{21} - A_{32}}{\gamma_{13}^2(\gamma_{13} - \gamma_{11})},$$

$$B_{32} = \frac{A_{31}}{\gamma_{11}\gamma_{13}} - \frac{(\gamma_{11}+\gamma_{13})A_{32}}{\gamma_{11}^2\gamma_{13}^2}.$$

Figure 2 shows the dependence graph of the dynamic temperature stress (49) on the dimensionless time in cross section $\xi = 1$ at $\tau_0 = 0.25$, $\alpha_0 = 3$ ($\alpha_0\sqrt{\tau_0} = v_p/v_T = 1.5 > 1$; for metals $v_\mathrm{p}/v_\mathrm{T} > 1$, for polymer glasses $v_\mathrm{p}/v_\mathrm{T} < 1$) for values $v_0 = 0.16$ and 0.65. The curves show that for locally nonequilibrium processes, accounting for the finite rate of heat propagation results in a significant change in the stress pattern compared to the corresponding curves in the framework of classical Fourier phenomenology [21]. We shall take an arbitrary point (cross section $\xi$ = const). In the beginning, the stresses in it are zero. At the moment of time $\tau = \xi/(\alpha_0 - v_0)(t = (z-l)/v_\mathrm{p})$, the longitudinal elastic stress wave which front moves at speed $v_\mathrm{p}$ approaches this point. The compression stress changes abruptly and then decreases (increases in absolute value). At the moment of time $\tau = \xi\sqrt{\tau_0}/(1-v_0\sqrt{\tau_0})$, the heat wave which front moves at speed $v_\mathrm{T}$ approaches this point (cross section); the stress changes abruptly and then approaches the value close to the quasi-static value.

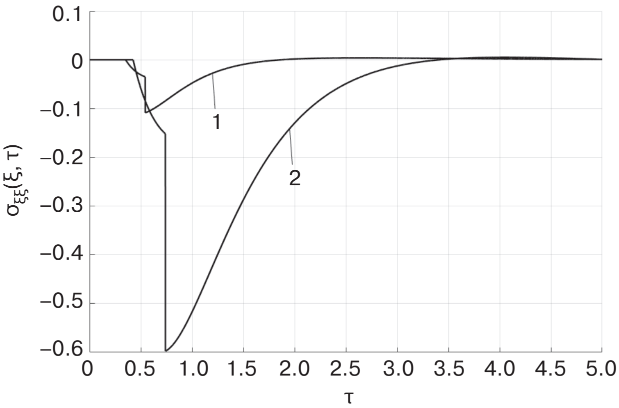


**Fig. 2.** Dependence of stress $\sigma_{\xi\xi}(\xi, \tau)$ (49) on $\tau$ in cross section $\xi = 1$ at $\tau_0 = 0.25$, $v_0 = 0.16$ (1) and $v_0 = 0.65$ (2); $\alpha_0 = 3$

Thus, two waves propagate in a massive solid body (an elastic half-space with a moving thermally insulated boundary), which comprise a thermal wave and an elastic wave; here, the elastic wave front precedes that of the thermal wave. The present author's earlier studies on the effect of heat transfer at the moving boundary of the domain indicate that the dynamic temperature stresses decrease as the heat transfer from the surface of the half-space decreases. If, in the classical case [3], the presence of finite heat transfer from the surface of the half-space boundary results in the disappearance of temperature stress discontinuities, then, in the case of the generalized dynamic thermoelasticity problem [20], the stress character remains the same as at the infinitely large value of the heat transfer coefficient (the first kind boundary condition). It is hoped that this earlier part of the research, being very voluminous in its content, will be published at some point in the future.

## CONCLUSIONS

The above model representations provide a basis for the following statement. A new effect of analytical thermophysics and applied thermomechanics is described. In a domain with a moving thermally insulated boundary, the temperature gradient occurs resulting in the appearance of the temperature field and corresponding temperature stresses despite the absence of external and internal heat sources. This is due to the kinetic energy of the moving boundary being converted into the thermal energy of the domain. The above-described effect is manifested both in the classical Fourier phenomenology (parabolic type equations), as well as in the generalized phenomenology for locally nonequilibrium processes (hyperbolic type equations).

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