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RESEARCH ARTICLE

On adaptive identification of systems having multiple nonlinearities

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Abstract

Objectives. The solution to the relevant problem of identifying systems with multiple nonlinearities depends on such factors as feedback, ways of connecting nonlinear links, and signal properties. The specifics of nonlinear systems affect control systems design methods. As a rule, the basis for the development of a mathematical model involves the linearization of a system. Under conditions of uncertainty, the identification problem becomes even more relevant. Therefore, the present work sets out to develop an approach to the identification of nonlinear dynamical systems under conditions of uncertainty. In order to obtain a solution to the problem, an adaptive identification method is developed by decomposing the system into subsystems.

Methods. Methods applied include the adaptive identification method, implicit identified representation, S-synchronization of a nonlinear system, and the Lyapunov vector function method.

Results. A generalization of the excitation constancy condition based on fulfilling the S-synchronizability for a nonlinear system is proposed along with a method for decomposing the system in the output space. Adaptive algorithms are obtained on the basis of the second Lyapunov method. The boundedness of the adaptive system trajectories in parametric and coordinate spaces is demonstrated. Approaches for self-oscillation generation and nonlinear correction of a nonlinear system are considered along with obtained exponential stability conditions for the adaptive system

Conclusions. Simulation results confirm the possibility of applying the proposed approach to solving the problems of adaptive identification while taking the estimation of the structural identifiability (S-synchronization) of the system nonlinear part into account. The influence of the structure and relations of the system on the quality of the obtained parametric estimates is investigated. The proposed methods can be used in developing identification and control systems for complex dynamic systems.

Keywords: adaptive identification, identifiability, stability, excitation constancy, Lyapunov vector function, self-oscillation

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НАУЧНАЯ СТАТЬЯ

Об адаптивной идентификации систем с несколькими нелинейностями

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Резюме

Цели. Задача идентификации систем с несколькими нелинейностями является актуальной. Решение этой задачи зависит от наличия обратных связей, способов соединения нелинейных звеньев, свойств сигналов. Специфика нелинейных систем накладывает отпечаток на методы синтеза систем управления. В условиях полной априорной определенности обычно применяют линеаризацию систем. Если существует априорная неопределенность, то задача синтеза системы идентификации обеспечения усложняется. Целью настоящей работы является разработка подхода к идентификации нелинейных динамических систем с несколькими нелинейностями. Для решения проблемы применяется подход, основанный на декомпозиции системы на ряд подсистем и разработке метода адаптивной идентификации, использующего только доступную информацию о системе и измерениях. Необходимо оценить частотные свойства сигналов, которые должны гарантировать оценку параметров системы и обеспечивать структурную идентифицируемость нелинейностей в системе; оценить работоспособность синтезированной адаптивной системы.

Методы. Применяются метод адаптивной идентификации системы, неявное идентификационное представление, S-синхронизация нелинейной системы, метод векторных функций Ляпунова.

Результаты. Введено условие постоянства возбуждения переменных состояния с учетом S-синхронизируемости нелинейной части системы. Дано обобщение условия постоянства возбуждения. Предложен способ декомпозиции системы в выходном пространстве. Получены адаптивные алгоритмы на основе второго метода Ляпунова. Доказана ограниченность траекторий адаптивной системы в параметрическом и координатном пространствах на основе векторных функций Ляпунова. Получены условия, гарантирующие экспоненциальную устойчивость траекторий системы. Рассмотрены системы генерации автоколебаний и нелинейной коррекции нелинейной системы.

Выводы. Результаты моделирования подтвердили возможность применения предлагаемого подхода для решения задач адаптивной идентификации с учетом оценки структурной идентифицируемости (S-синхронизируемости) нелинейной части системы. Исследовано влияние структуры и связей системы на качество получаемых параметрических оценок. Предлагаемые методы могут использоваться при разработке систем идентификации и управления сложными динамическими системами.

Ключевые слова: адаптивная идентификация, идентифицируемость, устойчивость, постоянство возбуждения, векторная функция Ляпунова, автоколебания

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INTRODUCTION

A number of studies [1–11] are devoted to the identification of systems having several nonlinearities (SSN). In [1], the case when a system comprises several nonlinearities in series is considered and an identification method proposed. The detection of nonlinearity is performed using sinusoidal tests. A similar approach based on the functional description method is applied in [2] for parametric identification of a system having two nonlinear elements in different locations. In [3], an approach to estimating the parameters of the transfer function of the second-order nonlinear system containing two nonlinearities is proposed. In this case, the harmonic linearization of nonlinearities is performed beforehand. In [4], in noting difficulties of SSN identification, a proposed approach to the parameter estimation is based on function approximation. Various methods based on the nonlinearity approximation are considered in [5–7]. In [7, 8], approaches to identifying discrete systems with feedback are studied. Identifiability conditions are obtained by applying the least squares method [8].

In [9], the identification of the system having nonlinear mechanical oscillations is considered. The proposed model, which has a "gray box" appearance, is based on the application of nonlinear basis functions using a limited number of measured output variables. Other approaches to identification are considered in [10–14]. These are based on considering physical laws when selecting the model structure [11], applying frequency methods for the feedback system [12], along with learning theory [13].

The review [15] is devoted to the analysis of methods for identifying nonlinear processes in structural dynamics. Here, modifications of the frequency approach are mainly applied. Disadvantages of approaches based on linearization, harmonic balance, and the restoring force surface method are noted. In [16], regression analysis and Hilbert transform are applied. In [17], chaos theory methods are used for identifying bifurcation processes.

Various approaches based on estimating parameters of the transfer function of a feedback system are presented in [18–21]. The identification of the feedback system is often reduced to identifying an open system. In [22, 23], difficulties in identifying the feedback system are noted.

Thus, it follows from the presented review that frequency methods are generally used for identifying a nonlinear system. In some cases, approaches to estimating the nonlinearity structure are proposed. Typically, different procedures of nonlinearity linearization from the given class are applied. However, the identification of systems having multiple nonlinearities has received less attention: this is possibly due to the inherent complexity of such systems. In these cases, different approaches and identification methods based on the localization of nonlinearities are used.

The problem of SSN identification is complex and insufficiently studied. It requires solving a number of problems whose consideration is given below.

PROBLEM STATEMENT

We shall consider system $\mathbf{S_F}$

$$\dot{\mathbf{X}}(t) = \mathbf{AX}(t) + \mathbf{DF}_1(\mathbf{X},t) + \mathbf{BU}(t), \tag{1}$$

$$\mathbf{LY}(t) = \mathbf{CX}(t) + \mathbf{F}_2(\mathbf{X},t), \tag{2}$$

where $\mathbf{X} \in \mathbb{R}^m$ is the state vector; $\mathbf{A} \in \mathbb{R}^{m\times m}$ is the state matrix; $D \in \mathbb{R}^{m\times q}$; $F_1(X,t): \mathbb{R}^m \to \mathbb{R}^q$ is the nonlinear vector function; $U \in \mathbb{R}^k$ is the input (control) vector; $B \in \mathbb{R}^{m\times k}$; $Y \in \mathbb{R}^n$ is the output vector; $C \in \mathbb{R}^{n\times m}$, $F_2(X,t): \mathbb{R}^m \to \mathbb{R}^n$ is the disturbance (measurement errors) vector; $\mathcal{L}$ is the operator defining the way of forming vector $\mathbf{Y}$; t is time. In some cases, $\mathcal{L}$ may be the differential operator indicating dynamic properties of the measurement system.

We shall consider the following data set:

$$\mathbb{I}_o = \{Y(t), U(t), t \in [t_0, t_N]\},\ t_N < \infty. \tag{3}$$

Assumption 1. Elements $\varphi_{1,i}(x_j) \in \mathbf{F}_1$, $\varphi_{2,i}(x_j) \in \mathbf{F}_2$ (non-linear functions belonging to $\mathbf{F}$) are smooth single-valued functions.

In some cases, condition $\varphi_i^1(x_j) = \varphi_i^1(\varphi_k^1(x_j))$, $i \neq k$ may be satisfied. For estimating the parameters of matrices **A**, **D**, **B**, and **C**, the following model is applied:

$$\begin{cases} \dot{\hat{\mathbf{X}}}(t) = \hat{\mathbf{A}}(t)\hat{\mathbf{X}}(t) + \hat{\mathbf{D}}(t)\mathbf{F}_1(\mathbf{X},t) + \hat{\mathbf{B}}(t)\mathbf{U}(t), \\ \mathcal{L}\hat{\mathbf{Y}}(t) = \mathbf{CX}(t) + \mathbf{F}_2(\mathbf{X},t), \end{cases} \tag{4}$$

where $\hat{\mathbf{A}}(t)$, $\hat{\mathbf{D}}(t)$, $\hat{\mathbf{B}}(t)$ are matrices with adjustable parameters.

Problem: for system (1) satisfying assumption 1, construct model (4) on the basis of the analysis of $\mathbb{I}_o$ and find such regularities of adjusting parameters of matrices $\hat{\mathbf{A}}(t)$, $\hat{\mathbf{D}}(t)$, and $\hat{\mathbf{B}}(t)$, that

$$\lim_{t\to\infty} \left\| \hat{\mathbf{Y}}(t) - \mathbf{Y}(t) \right\| \le \delta_y,$$

where $\|\cdot\|$ is the Euclidean norm, while $\delta_y \geq 0$ specifies the model (4) accuracy.

EXCITATION CONSTANCY CONDITION

The excitation constancy (EC) condition plays an important role in parametric estimation problems. If the system is nonlinear, then the fulfillment of this condition may not be sufficient. As shown in [24, 25], the system should have the property of S-synchronizability in order to take into account the nonlinear properties of the system.

Let there exist: (1) bounded vector $P \in \mathbb{R}^m$ and its corresponding set of frequencies $\Omega_\mathrm{P}(\omega)$; (2) a set of allowable input frequencies $\Omega_\mathrm{S}(\omega)$ providing S-synchronizability of the system. Then the EC condition for matrix $\mathbf{B_P}(t) = \mathbf{P}(t)\mathbf{P}^\mathrm{T}(t)$ has the following form:

$$\mathcal{PE}^{\mathrm{S}}_{\alpha,\bar{\alpha}}:\ \left(\alpha \mathbf{I}_l \leq \mathbf{B}_P(t) \leq \bar{\alpha}\mathbf{I}_l\right) \& \left(\Omega_\mathbf{P}(\omega) \subseteq \Omega_\mathrm{S}(\omega)\right) \qquad (5)$$

for $\alpha > 0$ and $\forall t \geq t_0$ on some interval $T > 0$, where $\bar{\alpha} > 0$ is some number, while $I_l \in \mathbb{R}^{m\times m}$ is the identity matrix. Usually, $\mathbf{P}(t)$ is the vector of measurements and state variables.

THE STRUCTURAL-PARAMETRIC APPROACH TO IDENTIFICATION

The following sets out the procedure for identifying system $\mathbf{S_F}$ based on the structural-parametric approach (SPA) [26]. Depending on the available a priori information, several stages implementing SPA can be applied. The system (1), (2) has a complex form; moreover, the synthesis of adaptive algorithms requires a priori information about its structure. The composition of the subsystems included in system $\mathbf{S_F}$ is assumed to be known. Hence, based on the dimensionality of the system output vector, matrix $\mathbf{A}$ can be divided into n blocks (subsystems $\mathbf{S}_{\mathbf{F},\, n} \subseteq \mathbf{S_F}$, $\{\mathbf{s}_{\mathbf{F},\, i}\} \in \mathbf{S}_{\mathbf{F},\, n}$, $j < n$). Analyzing the subsystems (blocks), we shall distinguish those containing nonlinearities, i.e., $\mathbf{S}_{\mathbf{F},\,\mathbf{nonlin}} \subseteq \mathbf{S}_{\mathbf{F},\, n}$. Then we shall apply SPA to each $\mathbf{S}_{\mathbf{F},\,\mathbf{nonlin},\,k} \in \mathbf{S}_{\mathbf{F},\,\mathbf{nonlin}}$ element. If subsystem $\{\mathbf{s}_{\mathbf{F},\, i}\} \in \mathbf{S}_{\mathbf{F},\, n} \setminus \mathbf{S}_{\mathbf{F},\,\mathbf{nonlin}}$ does not contain nonlinearities, then the adaptive identification procedure is applied to it.

Remark 1. The structural-parametric approach is based on the S-synchronizability of the system and the fulfillment of condition (5).

Under uncertainty, SPA can be divided into two procedures: (1) structural S_F analysis and (2) parametric estimation (adaptive identification). These stages are described in detail in [26].

Remark 2. The structural identifiability (S-synchronizability) of a system is greatly influenced by the means of connecting subsystems and mutual influence of variables. In this case, estimating the identifiability of the system nonlinear elements requires the construction of a diagram of the mutual influence in the system [27]. By analyzing the interrelations, the effect of influencing variables can be excluded to determine the structural identifiability of the system (nonlinearity). Constructing the mutual influence diagram is possible only if condition $\mathcal{PE}^{\mathrm{S}}_{\alpha,\bar{\alpha}}$ is satisfied.

Remark 3. Estimating the S-synchronizability of the system is based on the analysis of a special class of dynamic structures $\mathbf{S}_{ey}$ indicating the nonlinearity structure of the corresponding subsystem $\mathbf{S}_{\mathbf{F},\,\mathbf{nonlin},\,k}$. The method of their construction is described in [24, 25].

Remark 4. The obtained estimates of the nonlinearity structure in (1), (2) comprise the basis for implementing the adaptive parametric identification of system $\mathbf{S_F}$.

Remark 5. If a priori information about the nonlinear properties is known, then the structural analysis stage of system $\mathbf{S_F}$ may be skipped.

ADAPTIVE IDENTIFICATION OF THE $\mathbf{S_F}$ SYSTEM

We shall consider subsystem $\mathbf{s}_{\mathbf{F},\, i} \in \mathbf{S_F}$, $\dim \mathbf{s}_{\mathbf{F},\, i} = n_i$, $\mathcal{L}$ being the linear operator in (2). Let set $\mathbb{I}_{\mathrm{o},i} \subset \mathbb{I}_\mathrm{o}$ be known for $\mathbf{s}_{\mathbf{F},\, i}$. Subsystem $\mathbf{s}_{\mathbf{F},\, i}$ is described by the following equations:

$$\begin{cases} \dot{\mathbf{X}}_{\mathbf{s}_{\mathbf{F},i}} = \mathbf{A}_{\mathbf{s}_{\mathbf{F},i}}\mathbf{X}_{\mathbf{s}_{\mathbf{F},i}} + \mathbf{D}_{1,\mathbf{s}_{\mathbf{F},i}}\mathbf{F}_{1,\mathbf{s}_{\mathbf{F},i}}(\mathbf{X}) + \mathbf{B}_{\mathbf{s}_{\mathbf{F},i}}\mathbf{U}_{\mathbf{s}_{\mathbf{F},i}}, \\ y_{\mathbf{s}_{\mathbf{F},i}} = x^1_{\mathbf{s}_{\mathbf{F},i}}. \end{cases} \qquad (6)$$

We shall represent (6) in the form of the n_ith order differential equation, as follows:

$$W_{\mathbf{s}_{\mathbf{F},i}}(\upsilon)y_{\mathbf{s}_{\mathbf{F},i}} = \sum_{k=1}^{n_i}\sum_{j=1}^{n_i}\left(d^{k,h}_{1,\mathbf{s}_{\mathbf{F},i}} f^{h,j}_{\mathbf{s}_{\mathbf{F},i}} + b^{k,j}_{\mathbf{s}_{\mathbf{F},i}} u^j_{\mathbf{s}_{\mathbf{F},i}}\right), \qquad (7)$$

where $\mathbf{X}_{\mathbf{s}_{\mathbf{F},i}} \in \mathbb{R}^{n_i}$ is the state vector of subsystem $\mathbf{s}_{\mathbf{F},\, i}$, $x^1_{\mathbf{s}_{\mathbf{F},i}}$ being the first $\mathbf{X}$ element; $\mathbf{D}_{1,\mathbf{s}_{\mathbf{F},i}}$, $\mathbf{F}_{1,\mathbf{s}_{\mathbf{F},i}}$ and $\mathbf{B}_{\mathbf{s}_{\mathbf{F},i}}$ are matrices of appropriate dimensions; $d^{k,h}_{1,\mathbf{s}_{\mathbf{F},i}} \in \mathbf{D}_{1,\mathbf{s}_{\mathbf{F},i}}$, $f^j_{\mathbf{s}_{\mathbf{F},i}} \in \mathbf{F}_{1,\mathbf{s}_{\mathbf{F},i}}$, $b^{k,j}_{\mathbf{s}_{\mathbf{F},i}} \in \mathbf{B}_{\mathbf{s}_{\mathbf{F},i}}$, $u^j_{\mathbf{s}_{\mathbf{F},i}} \in \mathbf{U}_{\mathbf{s}_{\mathbf{F},i}}$; $\upsilon = d/dt$, $W_{\mathbf{s}_{\mathbf{F},i}}(\upsilon)$ is the polynomial of n_i degree. Matrix $\mathbf{A}_{\mathbf{s}_{\mathbf{F},i}} \in \mathbb{R}^{n_i \times n_i}$ is the Hurwitz matrix. We shall divide left and right parts of (7) by polynomials of $n_i - 1$ degree

$$H_{\mathbf{s}_{\mathbf{F},i}}(\upsilon) = \prod_{k=1}^{n_i-1}\left(\upsilon + \mu_k\right), \qquad (8)$$

and obtain

$$\dot{y}_{\mathbf{s}_{\mathbf{F},i}} = -\eta_{\mathbf{s}_{\mathbf{F},i}} y_{\mathbf{s}_{\mathbf{F},i}} + \sum_{k=1}^{n_i-1}\sum_{j=1}^{n_i-1}\left(\tilde{d}^{k,h}_{\mathbf{s}_{\mathbf{F},i}} p_{f^{h,j}_{\mathbf{s}_{\mathbf{F},i}}} + \tilde{b}^{k,j}_{\mathbf{s}_{\mathbf{F},i}} p_{u^{j}_{\mathbf{s}_{\mathbf{F},i}}}\right), \quad (9)$$

where υ, μ_k are positive numbers, $\eta_{\mathbf{s}_{\mathbf{F},i}} > 0$, $\tilde{d}^{k,h}_{\mathbf{s}_{\mathbf{F},i}}$ and $\tilde{b}^{k,j}_{\mathbf{s}_{\mathbf{F},i}}$ depend on parameters of subsystem $\mathbf{s}_{\mathbf{F},i}$ and μ_k, and variables $p_{z^{j}_{\mathbf{s}_{\mathbf{F},i}}}$ ($z = f$, u) satisfy the following equation:

$$\dot{p}_{z^{j}_{\mathbf{s}_{\mathbf{F},i}}} = -\mu_j p_{z^{j}_{\mathbf{s}_{\mathbf{F},i}}} + z^{j}_{\mathbf{s}_{\mathbf{F},i}}. \quad (10)$$

Remark 6. The structure of the right part of (9) is determined by the type of matrix $\mathbf{A}_{\mathbf{s}_{\mathbf{F},i}}$ and polynomial $H_{\mathbf{s}_{\mathbf{F},i}}(\upsilon)$.

The adaptive model for estimating parameters (9), (10) is the following:

$$\dot{\hat{y}}_{\mathbf{s}_{\mathbf{F},i}} = -k_{\mathbf{s}_{\mathbf{F},i}}\left(\hat{y}_{\mathbf{s}_{\mathbf{F},i}} - y_{\mathbf{s}_{\mathbf{F},i}}\right) + \hat{\kappa}_{\mathbf{s}_{\mathbf{F},i}} y_{\mathbf{s}_{\mathbf{F},i}} + \sum_{k=1}^{n_i-1}\sum_{j=1}^{n_i-1}\left(\hat{\tilde{d}}^{k,h}_{\mathbf{s}_{\mathbf{F},i}} p_{f^{h,j}_{\mathbf{s}_{\mathbf{F},i}}} + \hat{\tilde{b}}^{k,j}_{\mathbf{s}_{\mathbf{F},i}} p_{u^{j}_{\mathbf{s}_{\mathbf{F},i}}}\right), \quad (11)$$

where $k_{\mathbf{s}_{\mathbf{F},i}} > 0$, $\hat{\kappa}_{\mathbf{s}_{\mathbf{F},i}}$, $\hat{\tilde{d}}^{k,h}_{\mathbf{s}_{\mathbf{F},i}}$, and $\hat{\tilde{b}}^{k,j}_{\mathbf{s}_{\mathbf{F},i}}$ are adjustable parameters.

The equation for the identification error is the following:

$$\dot{e}_{\mathbf{s}_{\mathbf{F},i}} = -k_{\mathbf{s}_{\mathbf{F},i}} e_{\mathbf{s}_{\mathbf{F},i}} + \Delta\kappa_{\mathbf{s}_{\mathbf{F},i}} y_{\mathbf{s}_{\mathbf{F},i}} + \sum_{k=1}^{n_i-1}\sum_{j=1}^{n_i-1}\left(\Delta\tilde{d}^{k,h}_{\mathbf{s}_{\mathbf{F},i}} p_{f^{h,j}_{\mathbf{s}_{\mathbf{F},i}}} + \Delta\tilde{b}^{k,j}_{\mathbf{s}_{\mathbf{F},i}} p_{u^{j}_{\mathbf{s}_{\mathbf{F},i}}}\right), \quad (12)$$

where $\Delta\tilde{d}^{k,h}_{\mathbf{s}_{\mathbf{F},i}} = \hat{\tilde{d}}^{k,h}_{\mathbf{s}_{\mathbf{F},i}} - \tilde{d}^{k,h}_{\mathbf{s}_{\mathbf{F},i}}$, $\Delta\tilde{b}^{k,j}_{\mathbf{s}_{\mathbf{F},i}} = \hat{\tilde{b}}^{k,j}_{\mathbf{s}_{\mathbf{F},i}} - \tilde{b}^{k,j}_{\mathbf{s}_{\mathbf{F},i}}$, $e_{\mathbf{s}_{\mathbf{F},i}} = \hat{y}_{\mathbf{s}_{\mathbf{F},i}} - y_{\mathbf{s}_{\mathbf{F},i}}$, $\Delta\kappa_{\mathbf{s}_{\mathbf{F},i}} = \hat{\kappa}_{\mathbf{s}_{\mathbf{F},i}} - \eta_{\mathbf{s}_{\mathbf{F},i}}$.

We shall consider the Lyapunov function $V_{e,i}\left(e_{\mathbf{s}_{\mathbf{F},i}}\right) = 0.5e^2_{\mathbf{s}_{\mathbf{F},i}}$. Then the following is obtained for $\dot{V}_{e,i}\left(e_{\mathbf{s}_{\mathbf{F},i}}\right)$:

$$\dot{V}_{e,i}\left(e_{\mathbf{s}_{\mathbf{F},i}}\right) = e_{\mathbf{s}_{\mathbf{F},i}}\left[-k_{\mathbf{s}_{\mathbf{F},i}} e_{\mathbf{s}_{\mathbf{F},i}} + \Delta\kappa_{\mathbf{s}_{\mathbf{F},i}} y_{\mathbf{s}_{\mathbf{F},i}} + \sum_{k=1}^{n_i-1}\sum_{j=1}^{n_i-1}\left(\Delta\tilde{d}^{k,h}_{\mathbf{s}_{\mathbf{F},i}} p_{f^{h,j}_{\mathbf{s}_{\mathbf{F},i}}} + \Delta\tilde{b}^{k,j}_{\mathbf{s}_{\mathbf{F},i}} p_{u^{j}_{\mathbf{s}_{\mathbf{F},i}}}\right)\right].$$

If variables $p_{f^{h,j}_{\mathbf{s}_{\mathbf{F},i}}}$, $p_{u^{j}_{\mathbf{s}_{\mathbf{F},i}}}$ have property $\mathcal{PE}^{\mathrm{S}}_{\alpha,\overline{\alpha}}$, then the following is obtained from condition $\dot{V}_i\left(e_{\mathbf{s}_{\mathbf{F},i}}\right) < 0$:

$$\begin{aligned} \Delta\dot{\kappa}_{\mathbf{s}_{\mathbf{F},i}} &= -\gamma_{\kappa,\mathbf{s}_{\mathbf{F},i}} e_{\mathbf{s}_{\mathbf{F},i}} y_{\mathbf{s}_{\mathbf{F},i}}, \\ \Delta\dot{\tilde{d}}^{k,h}_{1,\mathbf{s}_{\mathbf{F},i}} &= -\gamma_{k,h,\mathbf{s}_{\mathbf{F},i}} e_{\mathbf{s}_{\mathbf{F},i}} p_{f^{h,j}_{\mathbf{s}_{\mathbf{F},i}}}, \\ \Delta\dot{\tilde{b}}^{k,j}_{\mathbf{s}_{\mathbf{F},i}} p_{u^{j}_{\mathbf{s}_{\mathbf{F},i}}} &= -\gamma_{k,j,\mathbf{s}_{\mathbf{F},i}} e_{\mathbf{s}_{\mathbf{F},i}} p_{u^{j}_{\mathbf{s}_{\mathbf{F},i}}}, \end{aligned} \quad (13)$$

where $\gamma_{k,j,\mathbf{s}_{\mathbf{F},i}} > 0$ is the amplification factor.

It is not difficult to obtain algorithms for the parameters of the model (11) from (13).

Thus, the adaptive identification system of the subsystem $\mathbf{S}_{\mathbf{F},i}$ is described by Eqs. (12) and (13). We shall denote it as $\mathbf{AS}_{\mathbf{F},i}$.

We shall consider the Lyapunov function

$$V_{\Delta,i} = 0.5\gamma^{-1}_{\kappa,\mathbf{s}_{\mathbf{F},i}}\left(\Delta\kappa_{\mathbf{s}_{\mathbf{F},i}}\right)^2 + 0.5\mathrm{Sp}\left(\Delta\mathbf{D}_{1,\mathbf{s}_{\mathbf{F},i}} \mathbf{\Gamma}^{-1}_{k,h,\mathbf{s}_{\mathbf{F},i}} \Delta\mathbf{D}^{\mathrm{T}}_{1,\mathbf{s}_{\mathbf{F},i}}\right) + 0.5\mathrm{Sp}\left(\Delta\mathbf{B}_{\mathbf{s}_{\mathbf{F},i}} \mathbf{\Gamma}^{-1}_{k,j,\mathbf{s}_{\mathbf{F},i}} \Delta\mathbf{B}^{\mathrm{T}}_{\mathbf{s}_{\mathbf{F},i}}\right), \quad (14)$$

where $\mathbf{\Gamma}_{k,h,\mathbf{s}_{\mathbf{F},i}} = \mathrm{diag}(\gamma_{k,h,\mathbf{s}_{\mathbf{F},i}})$, $\mathrm{Sp}(\cdot)$ is the spur of matrix, $\mathbf{\Gamma}^{-1}_{k,j,\mathbf{s}_{\mathbf{F},i}} = \mathrm{diag}(\gamma_{k,j,\mathbf{s}_{\mathbf{F},i}})$, $\Delta\mathbf{D}_{1,\mathbf{s}_{\mathbf{F},i}}$ and $\Delta\mathbf{B}_{\mathbf{s}_{\mathbf{F},i}}$ contain elements $\Delta\tilde{d}^{k,h}_{\mathbf{s}_{\mathbf{F},i}}$ and $\Delta\tilde{b}^{k,j}_{\mathbf{s}_{\mathbf{F},i}}$, respectively. Let $\Delta\mathbf{K}_{\mathbf{s}_{\mathbf{F},i}} \triangleq \left[\Delta\mathbf{D}_{1,\mathbf{s}_{\mathbf{F},i}}, \Delta\mathbf{B}_{\mathbf{s}_{\mathbf{F},i}}\right]$ and $V_{\mathbf{s}_{\mathbf{F},i}}(t) = V_e(t) + V_{\Delta,i}(t)$.

Theorem 1. Let: 1) functions $V_{e,i}(t)$, $V_{\Delta,i}(t)$ are positively definite and admit the infinitesimal higher limit at $|e| \to \infty$ and $\left\|\Delta\mathbf{K}_{\mathbf{s}_{\mathbf{F},i}}\right\| \to \infty$, $\left|\Delta\kappa_{\mathbf{s}_{\mathbf{F},i}}\right| \to \infty$; 2) matrix $\mathbf{A}_{\mathbf{s}_{\mathbf{F},i}} \in \mathbb{R}^{n_i \times n_i}$ is the Hurwitz matrix; 3) $p_{f^{h,j}_{\mathbf{s}_{\mathbf{F},i}}}$, $p_{u^{j}_{\mathbf{s}_{\mathbf{F},i}}}$ have property $\mathcal{PE}^{\mathrm{S}}_{\alpha,\overline{\alpha}}$. Then all trajectories of system $\mathbf{AS}_{\mathbf{F},i}$ are bounded, lie in the following domain:

$$G_t = \left\{ \left(e_{\mathbf{s}_{\mathbf{F},i}}, \Delta\kappa_{\mathbf{s}_{\mathbf{F},i}}, \Delta\mathbf{K}_{\mathbf{s}_{\mathbf{F},i}} \right) : V_{\mathbf{s}_{\mathbf{F},i}}(t) \le V_{\mathbf{s}_{\mathbf{F},i}}(t_0) \right\}$$

and the following estimation is valid:

$$2k_{\mathbf{s}_{\mathbf{F},i}} \int_{t_0}^{t} V_{e,i}(\tau)\, d\tau \le V_{\mathbf{s}_{\mathbf{F},i}}(t_0) - V_{\mathbf{s}_{\mathbf{F},i}}(t).$$

Let estimation called A1 be true for $V_{\Delta,i}(t)$:

$$0.5\,\underline{\vartheta} \left\{ \mathrm{Sp}\left(\Delta\mathbf{K}_{\mathbf{s}_{\mathbf{F},i}}(t) \Delta\mathbf{K}^{\mathrm{T}}_{\mathbf{s}_{\mathbf{F},i}}(t) \right) + \left(\Delta\kappa_{\mathbf{s}_{\mathbf{F},i}} \right)^2 \right\} \le V_{\Delta,i}(t) \le$$

$$\le 0.5\,\overline{\vartheta} \left\{ \mathrm{Sp}\left(\Delta\mathbf{K}_{\mathbf{s}_{\mathbf{F},i}}(t) \Delta\mathbf{K}^{\mathrm{T}}_{\mathbf{s}_{\mathbf{F},i}}(t) \right) + \left(\Delta\kappa_{\mathbf{s}_{\mathbf{F},i}} \right)^2 \right\},$$

where $\mathbf{\Gamma}_{\mathbf{s}_{\mathbf{F},i}} = \mathbf{\Gamma}_{k,h,\mathbf{s}_{\mathbf{F},i}} \dotplus \mathbf{\Gamma}_{k,j,\mathbf{s}_{\mathbf{F},i}}$, $\dotplus$ is the sign of the direct sum of matrices, $\beta_1\left(\mathbf{\Gamma}_{\mathbf{s}_{\mathbf{F},i}}\right)$, $\beta_l\left(\mathbf{\Gamma}_{\mathbf{s}_{\mathbf{F},i}}\right)$ are the minimal and maximal eigenvalues of matrix $\mathbf{\Gamma}_{\mathbf{s}_{\mathbf{F},i}}$, $\underline{\vartheta} = \min\left(\beta_l^{-1}\left(\mathbf{\Gamma}_{\mathbf{s}_{\mathbf{F},i}}\right), \gamma^{-1}_{\kappa,\mathbf{s}_{\mathbf{F},i}} \right)$, and $\overline{\vartheta} = \max\left(\beta_1^{-1}\left(\mathbf{\Gamma}_{\mathbf{s}_{\mathbf{F},i}}\right), \gamma^{-1}_{\kappa,\mathbf{s}_{\mathbf{F},i}} \right)$.

Theorem 2. Let the following conditions be satisfied: 1) positive definite Lyapunov functions $V_{e,i}(t) = 0.5 e_{\mathbf{s}_{\mathbf{F},i}}(t)$,

$$V_{\Delta,i}(t) = 0.5\gamma^{-1}_{\kappa,\mathbf{s}_{\mathbf{F},i}} \left(\Delta\kappa_{\mathbf{s}_{\mathbf{F},i}} \right)^2 +$$

$$+ 0.5\mathrm{Sp}\left(\Delta\mathbf{D}_{1,\mathbf{s}_{\mathbf{F},i}} \mathbf{\Gamma}^{-1}_{k,h,\mathbf{s}_{\mathbf{F},i}} \Delta\mathbf{D}^{\mathrm{T}}_{1,\mathbf{s}_{\mathbf{F},i}} \right) +$$

$$+ 0.5\mathrm{Sp}\left(\Delta\mathbf{B}_{\mathbf{s}_{\mathbf{F},i}} \mathbf{\Gamma}^{-1}_{k,j,\mathbf{s}_{\mathbf{F},i}} \Delta\mathbf{B}^{\mathrm{T}}_{\mathbf{s}_{\mathbf{F},i}} \right)$$

admit the infinitesimal higher limit at $\left| e_{\mathbf{s}_{\mathbf{F},i}}(t) \right| \to 0$, $\left| \Delta\kappa_{\mathbf{s}_{\mathbf{F},i}} \right| \to 0$, $\left\| \Delta\mathbf{K}_{\mathbf{s}_{\mathbf{F},i}} \right\| \to 0$; 2) matrix $\mathbf{W}_{\mathbf{s}_{\mathbf{F},i}}(t) = \mathbf{P}^{\mathrm{T}}_{f^{h,j}_{\mathbf{s}_{\mathbf{F},i}}, u^{j}_{\mathbf{s}_{\mathbf{F},i}}}(t) \mathbf{P}_{f^{h,j}_{\mathbf{s}_{\mathbf{F},i}}, u^{j}_{\mathbf{s}_{\mathbf{F},i}}}(t)$ and $y_{\mathbf{s}_{\mathbf{F},i}}$ are piecewise continuous bounded and $\mathbf{W}_{\mathbf{s}_{\mathbf{F},i}}(t) \in \mathcal{PE}^{\mathrm{S}}_{\alpha,\overline{\alpha}}$, $y_{\mathbf{s}_{\mathbf{F},i}}(t) \in \mathcal{PE}^{\mathrm{S}}_{\alpha_{y_i},\overline{\alpha}_{y_i}}$; 3) the following equality is valid:

$$e_{\mathbf{s}_{\mathbf{F},i}} \sum_{k=1}^{n_i-1} \sum_{j=1}^{n_i-1} \left(\Delta\tilde{d}^{k,h}_{\mathbf{s}_{\mathbf{F},i}} p_{f^{h,j}_{\mathbf{s}_{\mathbf{F},i}}} + \Delta\tilde{b}^{k,j}_{\mathbf{s}_{\mathbf{F},i}} p_{u^{j}_{\mathbf{s}_{\mathbf{F},i}}} \right) =$$

$$= \pi \left\{ \mathrm{Sp}\left(\Delta\mathbf{K}_{\mathbf{s}_{\mathbf{F},i}}(t) P^{\mathrm{T}}_{f^{h,j}_{\mathbf{s}_{\mathbf{F},i}}, u^{j}_{\mathbf{s}_{\mathbf{F},i}}} P_{f^{h,j}_{\mathbf{s}_{\mathbf{F},i}}, u^{j}_{\mathbf{s}_{\mathbf{F},i}}} \Delta\mathbf{K}^{\mathrm{T}}_{\mathbf{s}_{\mathbf{F},i}}(t) \right) + \left(\Delta\hat{e}_{\mathbf{s}_{\mathbf{F},i}} y_{\mathbf{s}_{\mathbf{F},i}} \right)^2 + e^2_{\mathbf{s}_{\mathbf{F},i}} \right\}$$

in domain $O_{\mathrm{v}}(O)$, where $\pi > 0$, $O = \{0,\ 0^{n_i \times n_i}\} \subset \mathbb{R} \times \mathbb{R}^{n_i \times n_i} \times \mathbb{J}_{0,\infty}$, $0^{n_i \times n_i} \in \mathfrak{R}^{n_i \times n_i}$ is a zero matrix, O_{v} is some neighborhood of point O, $t \in [0,\infty] = \mathbb{J}_{0,\infty}$ is time interval; 4) estimation A1 is valid for function $V_{\Delta,i}(t)$; 5) the following system of inequalities is fulfilled:

$$\begin{bmatrix} \dot{V}_{e,i} \\ \dot{V}_{\Delta,i} \end{bmatrix} \le \underbrace{\begin{bmatrix} -k_{\mathbf{s}_{\mathbf{F},i}} & \dfrac{\overline{\alpha}\overline{\vartheta}}{2k_{\mathbf{s}_{\mathbf{F},i}}} \\ \dfrac{8}{3}\pi & -\dfrac{3}{8}\pi\alpha_i \end{bmatrix}}_{\mathbf{A}_{\mathbf{V}}} \begin{bmatrix} V_{e,i} \\ V_{\Delta,i} \end{bmatrix}$$

for $\dot{V}_{e,i}, \dot{V}_{\Delta,i}$; 6) the upper solution for $\mathbf{V}_{e_i,\Delta_i}(t) = \left[V_{e,i}(t)\ V_{\Delta,i}(t) \right]^{\mathrm{T}}$ satisfies equation $\dot{\mathbf{S}} = \mathbf{A}_{\mathbf{V}}\mathbf{S}$, if the following inequality is valid:

$$V_\rho(t) \le s_\rho(t)\ \forall (t \ge t_0)\ \&\ (V_\rho(t_0) \le s_\rho(t_0)),$$

$\rho = e, i; \Delta, i$ for elements $V_{e,i}(t)$, $V_{\Delta,i}(t)$. Then adaptive system $\mathbf{AS}_{\mathbf{F},i}$ is exponentially stable with the following estimate:

$$\mathbf{V}_{e_i,\Delta_i}(t) \le e^{\mathbf{A}_{\mathbf{V}}(t-t_0)} \mathbf{S}(t_0),$$

if

$$k_{\mathbf{s}_{\mathbf{F},i}} > 0, k_{\mathbf{s}_{\mathbf{F},i}} \ge \frac{4}{3}\sqrt{\frac{2\overline{\alpha}\overline{\vartheta}}{\alpha_i}}. \qquad (15)$$

The proof of theorems is based on the approach described in [26].

Theorem 2 shows: if the information matrix

$$\mathbf{W}_{\mathbf{s}_{\mathbf{F},i}}(t) = \mathbf{P}^{\mathrm{T}}_{f^{h,j}_{\mathbf{s}_{\mathbf{F},i}}, u^{j}_{\mathbf{s}_{\mathbf{F},i}}}(t) \mathbf{P}_{f^{h,j}_{\mathbf{s}_{\mathbf{F},i}}, u^{j}_{\mathbf{s}_{\mathbf{F},i}}}(t)$$

is continuously excitable, then the adaptive system $\mathbf{AS}_{\mathbf{F},i}$ allows obtaining true estimates of the system $\mathbf{S}_{\mathbf{F},i}$ parameters. In this case, the system parameters should satisfy conditions (15).

EXAMPLES

We shall consider the system with nonlinear correction of the nonlinear system. It contains the amplifier with electric motor and the relay control described by function $f_1(u)$. The nonlinear feedback (fb in an index notation) on speed with parabolic characteristic $f_2(x_2)$ is used as a correction device, as follows:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3, \\ \dot{x}_3 = -a_1 x_2 - a_2 x_3 + b f_1(u), \\ y = x_1, \end{cases} \quad (16)$$

$$f_1(u) = \begin{cases} c, \text{ if } u \geq d, \\ 0, \text{ if } -d \leq u \leq d, \\ -c, \text{ if } u < -d, \end{cases}$$

where $u = g - x_1 - f_2(x_2)$ is control; $f_2(x_2) = k_{\text{fb}} x_2^2 \text{sign}(x_2)$; g is the system input; $c > 0$, $d > 0$; $k_{\text{fb}} > 0$; x_i is a state variable; and a_i, b are system parameters.

Equation (13) and polynomials in (7), (8) have the following form:

$$\left(\upsilon + \eta - \frac{v_1}{\upsilon + \mu_1} - \frac{v_2}{\upsilon + \mu_2}\right) y = \left(\frac{b_1}{\upsilon + \mu_1} + \frac{b_2}{\upsilon + \mu_2}\right) f_1(u). \quad (17)$$

Equation (17) is obtained as follows. First, system (16) is written in the space ($y(t)$, $f_1(u)$). So, Eq. (7) is obtained. Since the system has the third order, to find the equation for $\dot{y}(t)$, according to the results of the ADAPTIVE IDENTIFICATION OF THE $\mathbf{S_F}$ SYSTEM section, both parts of the obtained Eq. (7) are divided by polynomial $H(\upsilon) = (\upsilon + \mu_1)(\upsilon + \mu_2)$. Then the left and right parts of (7) are decomposed into prime fractions, and Eq. (17) is derived.

We assume $\upsilon = d/dt$. The analogue of Eqs. (8) and (9) for system (16) is obtained from (17), as follows:

$$\begin{aligned} \dot{y} = {} & -\eta y + v_1 p_{y,\mu_1} + v_2 p_{y,\mu_2} + \\ & + b_1 p_{f_1,\mu_1} + b_2 p_{f_1,\mu_2}, \end{aligned} \quad (18)$$

$$\begin{aligned} \dot{p}_{y,\mu_1} &= -\mu_1 p_{y,\mu_1} + y, \\ \dot{p}_{y,\mu_2} &= -\mu_2 p_{y,\mu_2} + y, \\ \dot{p}_{f_1,\mu_1} &= -\mu_1 p_{f_1,\mu_1} + f_1, \\ \dot{p}_{f_1,\mu_2} &= -\mu_2 p_{f_1,\mu_2} + f_1, \end{aligned} \quad (19)$$

where $b_1 = 1.4$, $b_2 = -0.4$, $\eta = 0.35$, $v_1 = -1$, $v_2 = 1.35$, $\mu_1 = 2.05$, $\mu_2 = 2.25$. The parameters of nonlinearity f_1: $c_1 = 2$, $d = 0.5$. Input $g(t) = \sin(0.2t)$.

The adaptive model and algorithms are the following:

$$\begin{aligned} \dot{\hat{y}} = {} & -k_y e - \hat{\eta} y + \hat{v}_1 p_{y,\mu_1} + \\ & + \hat{v}_2 p_{y,\mu_2} + \hat{b}_1 p_{f_1,\mu_1} + \hat{b}_2 p_{f_1,\mu_2}, \end{aligned} \quad (20)$$

$$\begin{aligned} & \dot{\hat{\eta}} = -\gamma_\eta e y, \dot{\hat{v}}_1 = -\gamma_{v_1} e p_{y,\mu_1}, \\ & \dot{\hat{v}}_2 = -\gamma_{v_2} e p_{y,\mu_2}, \dot{\hat{b}}_1 = -\gamma_{b_1} p_{f_1,\mu_1}, \\ & \dot{\hat{b}}_2 = -\gamma_{b_2} p_{f_1,\mu_2}, \end{aligned} \quad (21)$$

where $e = \hat{y} - y$.

The equation for error $e(t)$ is the following:

$$\begin{aligned} \dot{e} = {} & -ke - \Delta\eta y + \Delta v_1 p_{y,\mu_1} + \\ & + \Delta v_2 p_{y,\mu_2} + \Delta b_1 p_{f_1,\mu_1} + \Delta b_2 p_{f_1,\mu_2}, \end{aligned} \quad (22)$$

where $k > 0$, $\Delta\sigma = \hat{\sigma} - \sigma$, $\sigma = \eta$, v_1, v_2, b_2, b_2. Coefficients γ_i in (21) vary in the range (0.002; 0.009).

Let $\Delta\mathbf{K} \triangleq \left[\Delta\eta, \Delta v_1, \Delta v_2, \Delta b_1, \Delta b_2\right]^{\text{T}}$. The adjustment law for $\Delta\mathbf{K}$ follows from (21):

$$\Delta\dot{\mathbf{K}} = -\mathbf{\Gamma}_{\text{K}} e \mathbf{P}_y, \quad (23)$$

where $\mathbf{\Gamma}_{\text{K}} = \text{diag}\left(\gamma_\eta, \gamma_{v_1}, \gamma_{v_2}, \gamma_{b_1}, \gamma_{b_2}\right)$,

$$\mathbf{P}_y = \left[y, p_{y,\mu_1}, p_{y,\mu_2}, p_{f_1,\mu_1}, p_{f_1,\mu_2}\right]^{\text{T}}.$$

The boundedness of the system (22), (23) trajectories follows from Theorem 1. The adaptive system results are shown in Figs. 1–4. The adjustment of the model (20) parameters is shown in Figs. 1 and 2. The change in estimation error $e(t)$ is shown in Fig. 3. This change in the error is related to the change in the system output.

It is noted in [28] that by function $f_2(x_2)$, the system is unidentifiable on the set of measurements. In this case, the indirect information about the dependence of $u_1 = \omega - x_1$ on x_2 may be used. This is true since there is a relationship between u_1 and u.

2. We shall consider the self-oscillation generation system consisting of an object (variables y_1, y_2), nonlinear (variable y_3) and linear (variable y_4) converters, and the linear amplifier-converter with nonlinear actuator (variable y_5). Function $f_i(x)$ ($i = 1, 3$) is the saturation function with a dead zone

$$f_i(x) = \begin{cases} c, & \text{if } \; x \geq d_{2,i}, \\ 2(x - d_{1,i}), & \text{if } \; d_{1,i} < x < d_{2,i}, \\ 0, & \text{if } \; -d_{1,i} \leq x \leq d_{1,i}, \\ 2(x + d_{1,i}), & \text{if } \; -d_{1,i} < x, \\ -c, & \text{if } \; x < -d_{2,i}, \end{cases}$$

where $c > 0$, $d_{1,i} > 0$, and $d_{2,i} > 0$ are some numbers,

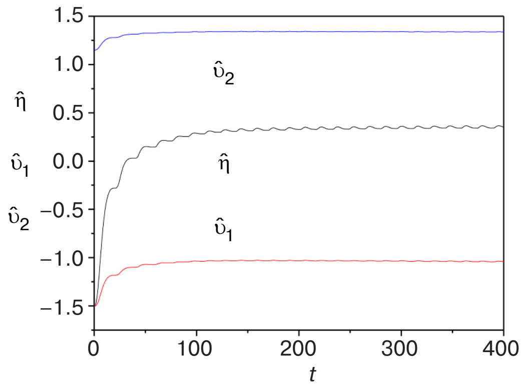

Fig. 1. Adjusting parameters of model (20)

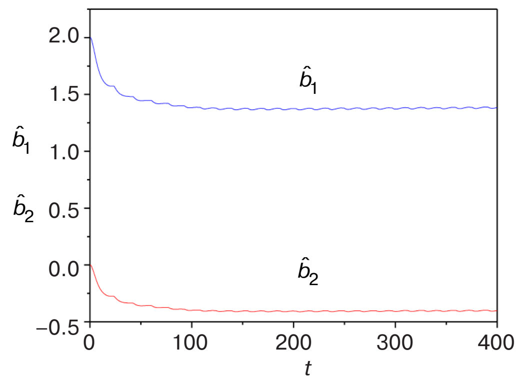

Fig. 2. Adjusting parameters $\hat{b}_i$ of model (20)

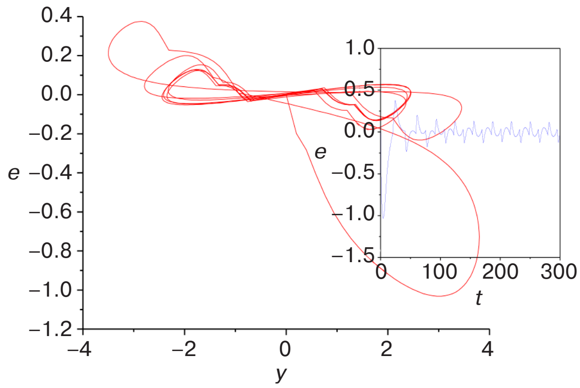

Fig. 3. Change in the estimation error

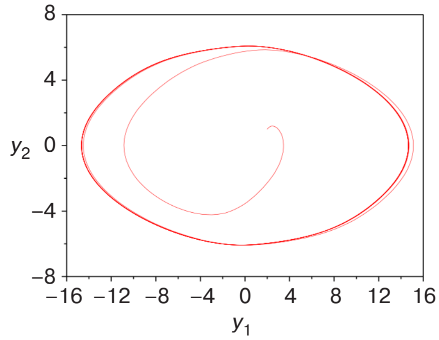

Fig. 4. Phase portrait of object (24)

$$\dot{\mathbf{Y}} = \mathbf{AY} + \mathbf{DF}(\mathbf{Y})$$

$$\mathbf{Y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix}, \mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & -g & 0 & 0 & k_0 \\ 0 & 0 & -\frac{1}{T_1} & 0 & 0 \\ 0 & \frac{k_2}{T_2} & 0 & -\frac{1}{T_2} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{T_3} \end{bmatrix}, \tag{24}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{T_1} & 0 \\ 0 & 0 \\ 0 & -\frac{1}{T_3} \end{bmatrix}, \mathbf{F}(\mathbf{Y}) = \begin{bmatrix} f_1(y_1) \\ f_3(y_3 + y_4) \end{bmatrix},$$

where $T_i > 0$ is a time constant, $g \geq 0$. Variable y_5 is used as the input.

The phase portrait of the object is shown in Fig. 4. It shows that self-oscillations arise in the system. For identifying parameters of system (24), we shall use the ideas of the adaptive observer for the object described above. It is necessary to transform only the first two equations in (24). For this, we represent them in the form (38) and divide the resulting equation by $\upsilon + \mu$. Then variables y_1, y_5 are transformed, as follows:

$$\begin{aligned} \dot{p}_{y_1,\mu} &= -\mu p_{y_1,\mu} + y_1, \\ \dot{p}_{y_5,\mu} &= -\mu p_{y_5,\mu} + y_5, \end{aligned} \tag{25}$$

and the following identification representation is obtained:

$$\begin{aligned} \dot{y}_1 &= a_{11} y_1 + a_{12} p_{y_1} + a_{15} p_{y_5}, \\ \dot{y}_3 &= -a_3 y_3 + a_{31} f_1(y_1), \\ \dot{y}_4 &= -a_4 y_4 + a_{42} y_2, \\ \dot{y}_5 &= -a_5 y_5 + a_{53} f_3(y_3 + y_4). \end{aligned} \tag{26}$$

The adaptive system for estimating parameters of system (26) has the following form:

$$\begin{aligned} \dot{\hat{y}}_1 &= -k_1 e_1 + \hat{a}_{11} y_1 + \hat{a}_{12} p_{y_1} + \hat{a}_{15} p_{y_5}, \\ \dot{\hat{y}}_2 &= +\hat{a}_{11} y_1 + \hat{a}_{12} p_{y_1} + \hat{a}_{15} p_{y_5}, \\ \dot{\hat{y}}_3 &= -k_3 e_3 + \hat{a}_{31} \left(f_1(y_1) - y_3 \right), \\ \dot{\hat{y}}_4 &= -k_4 e_4 + \hat{a}_{41} y_4 + \hat{a}_{42} \hat{y}_2, \\ \dot{\hat{y}}_5 &= -k_5 e_5 + \hat{a}_{51} \left(y_5 + f_3(y_3 + y_4) \right), \end{aligned} \tag{27}$$

where $e_i = \hat{y}_i - y_i$, $i = 1, 3, 4, 5$.

If Lyapunov functions $V_i(e_i)=0.5e_i^2$ are introduced, then adaptive algorithms for adjusting parameters of system (27) are obtained from condition $\dot{V}_i \le 0$, as follows:

$$\begin{aligned} &\dot{\hat{a}}_{11}=-\gamma_{11}e_1y_1, \quad \dot{\hat{a}}_{12}=-\gamma_{12}e_1p_{y_1}, \\ &\dot{\hat{a}}_{15}=-\gamma_{15}e_1p_{y_5}, \quad \dot{\hat{a}}_{31}=-\gamma_{31}e_3\left(f_1(y_1)-y_3\right), \\ &\dot{\hat{a}}_{41}=-\gamma_{41}e_4y_4, \quad \dot{\hat{a}}_{42}=-\gamma_{42}e_4\hat{y}_2, \\ &\dot{\hat{a}}_{51}=-\gamma_{51}e_5\left(y_5+f_3(y_3+y_4)\right), \end{aligned} \tag{28}$$

where $\gamma_{ij}>0$.

Systems (26) and (27) are simulated with the following parameters: $a_{11}=0.55$, $a_{12}=-0.6$, $a_{15}=-1.15$, $k_1=2$, $\mu=0.5$, $a_{31}=2.21$, $k_3=0.8$, $k_4=1.5$, $a_{41}=1.15$, $a_{42}=0.56$, $k_5=1.25$, $a_{51}=1.1$. Parameters of functions f_1, f_3: $d_{11}=0.5$, $d_{21}=1.5$, $c=2$, $d_{13}=0.25$, $d_{23}=1.25$. Coefficients γ_{ij} vary in the range (0.001; 0.05).

The adaptive system has the following form:

$$\begin{aligned} &\dot{e}_1=-k_1e_1+\Delta a_{11}y_1+\Delta a_{12}p_{y_1}+\Delta a_{15}p_{y_5}, \\ &e_2=+\Delta a_{11}y_1+\Delta a_{12}p_{y_1}+\Delta a_{15}p_{y_5}, \\ &\dot{e}_3=-k_3e_3+\Delta a_{31}\left(f_1(y_1)-y_3\right), \\ &\dot{e}_4=-k_4e_4-\Delta a_{41}y_4+\Delta a_{42}\hat{y}_2, \\ &\dot{e}_5=-k_5e_5-\Delta a_{51}\left(y_5+f_3(y_3+y_4)\right), \end{aligned} \tag{29}$$

$$\begin{aligned} &\Delta\dot{a}_{11}=-\gamma_{11}e_1y_1, \quad \Delta\dot{a}_{12}=-\gamma_{12}e_1p_{y_1}, \\ &\Delta\dot{a}_{15}=-\gamma_{15}e_1p_{y_5}, \quad \Delta\dot{a}_{31}=-\gamma_{31}e_3\left(f_1(y_1)-y_3\right), \\ &\Delta\dot{a}_{41}=\gamma_{41}e_4y_4, \quad \Delta\dot{a}_{42}=-\gamma_{42}e_4\hat{y}_2, \\ &\Delta\dot{a}_{51}=\gamma_{51}e_5\left(y_5+f_3(y_3+y_4)\right), \end{aligned} \tag{30}$$

where $\Delta a_{ij}(t)=\hat{a}_{ij}(t)-a_{ij}$.

The results of system (29), (30) are shown in Figs. 5–7. The process of adjusting parameters of the system (27) is shown in Figs. 5, and 6, while the change in the output discrepancies of system (27) is shown in Fig. 7. The results confirm the boundedness of the adaptive system trajectories.

Despite the fulfillment of condition $y_5 \in \mathcal{PE}^{S}_{\alpha,\bar{\alpha}}$ for the input of system (24) and the S-synchronizability of the system, it is not possible to ensure the condition of asymptotic stability. This is due to the presence of nonlinearities in the system.

We shall consider the Lyapunov functions

$$V_{e,1-5}(t)=0.5\sum_{i=1,\, i\ne 2}^{5} e_i^2(t),\ \mathbf{V}_{e,\Delta}=$$

$$=\left[V_{e,1},V_{\Delta,1},V_{e,3},V_{\Delta,3},V_{e,4},V_{\Delta,4},V_{e,5},V_{\Delta,5}\right]^{\mathrm{T}},$$

$$\begin{aligned} V_{\Delta,1-5}(t)=\ &\underbrace{0.5\left(\gamma_{11}^{-1}\Delta a_{11}^2(t)+\gamma_{12}^{-1}\Delta a_{12}^2(t)+\gamma_{15}^{-1}\Delta a_{15}^2(t)\right)}_{V_{\Delta,1}}+ \\ &+\underbrace{0.5\gamma_{31}^{-1}\Delta a_{31}^2(t)}_{V_{\Delta,3}}+\underbrace{0.5\left(\gamma_{41}^{-1}\Delta a_{41}^2(t)+\gamma_{42}^{-1}\Delta a_{42}^2(t)\right)}_{V_{\Delta,4}}+ \\ &+\underbrace{0.5\gamma_{51}^{-1}\Delta a_{51}^2(t)}_{V_{\Delta,5}}. \end{aligned} \tag{31}$$

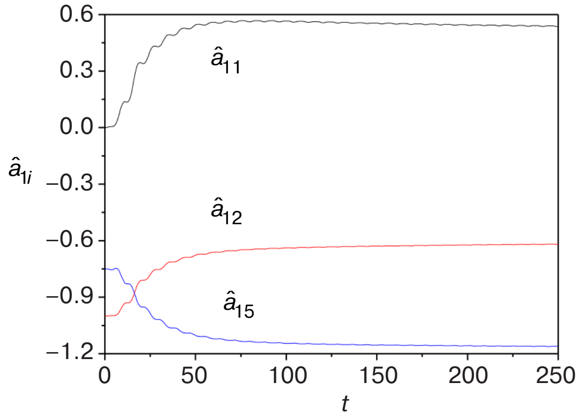

Fig. 5. Adjusting parameters of the models for estimating y_1

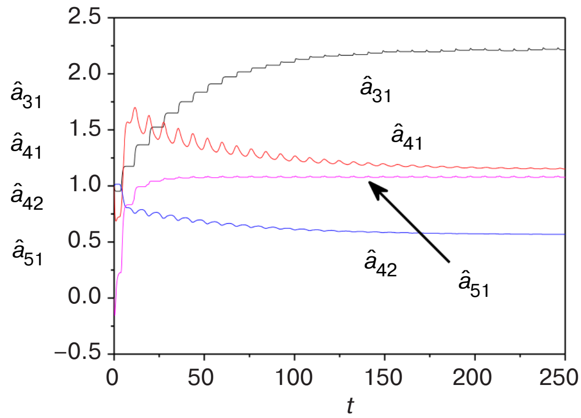

Fig. 6. Adjusting parameters of the models for estimating y_3, y_4, and y_5

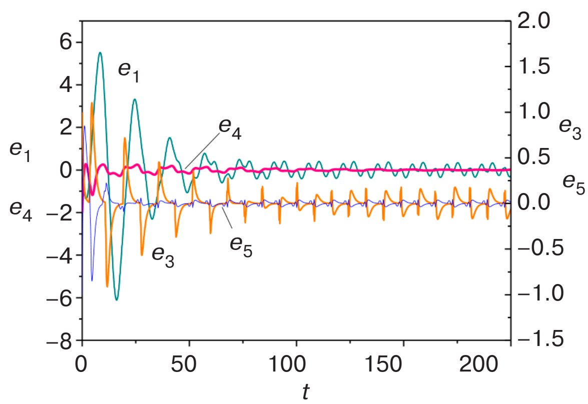

Fig. 7. Change in the discrepancy of outputs of models (27) (green line is e_1, red line is e_4, orange line is e_3, and blue line is e_5)

Theorem 3. Let the following conditions be satisfied:

1) positive definite Lyapunov functions $V_{\Delta,i}(t) = 0.5\gamma_{ij}^{-1}\left(\Delta a_{ij}\right)^2$, $V_{e,i}(t) = 0.5e_i(t)$, $V_{\Delta,1}(t) = 0.5\left(\gamma_{11}^{-1}\Delta a_{11}^2(t) + \gamma_{12}^{-1}\Delta a_{12}^2(t) + \gamma_{15}^{-1}\Delta a_{15}^2(t)\right)$ admit the infinitesimal higher limit at $|e_i(t)| \to 0$, $|\Delta a_{ij}| \to 0$;

2) y_i are piecewise continuous bounded while $\alpha = \max\left(\bar{\alpha}_{y_1}, \bar{\alpha}_{p_{y_1}}, \bar{\alpha}_{p_{y_5}}\right)$, $y_i(t) \in \mathcal{PE}^{\mathrm{S}}_{\alpha_{y_i}, \bar{\alpha}_{y_i}}$, $p_{y_1} \in \mathcal{PE}^{\mathrm{S}}_{\alpha_{p_{y_1}}, \bar{\alpha}_{p_{y_1}}}$, $p_{y_5} \in \mathcal{PE}^{\mathrm{S}}_{\alpha_{p_{y_5}}, \bar{\alpha}_{p_{y_5}}}$, and $i = 1, 3, 4, 5$;

3) for $V_{\Delta,1}(t) = 0.5\Delta\mathbf{G}_1^{\mathrm{T}}(t)\mathbf{\Gamma}_1^{-1}\Delta\mathbf{G}_1(t)$, the following inequation is valid:

$$0.5\underline{\vartheta}_1\Delta\mathbf{G}_1^{\mathrm{T}}(t)\Delta\mathbf{G}_1(t) \le V_{\Delta,1}(t) \le 0.5\overline{\vartheta}_1\Delta\mathbf{G}_1^{\mathrm{T}}(t)\Delta\mathbf{G}_1(t),$$

where $\mathbf{\Gamma}_1 = \mathrm{diag}\left(\gamma_{11}, \gamma_{11}, \gamma_{15}\right)$, $\underline{\vartheta}_1 = \beta_l^{-1}\left(\mathbf{\Gamma}_1\right)$, $\Delta\mathbf{G}_1(t) = \left[\Delta a_{11}(t), \Delta a_{12}(t), \Delta a_{15}(t)\right]^{\mathrm{T}}$, $\overline{\vartheta}_1 = \beta_1^{-1}\left(\mathbf{\Gamma}_1\right)$, $\beta_1(\mathbf{\Gamma}_1)$, $\beta_l(\mathbf{\Gamma}_1)$ are minimal and maximal eigenvalues of matrix $\mathbf{\Gamma}_1$;

4) $\alpha_{f_1,y_3} - \underline{\upsilon_3} \le \left(f_1\left(y_1\right) - y_3\right)^2 \le \bar{\alpha}_{f_1,y_3} + \upsilon_3$, $\underline{\upsilon_3} \ge 0$, $\upsilon_3 \ge 0$;

5) $\gamma_4 = \max(\gamma_{41}, \gamma_{42})$;

6) equality

$$e_i\Delta a_{ij}\omega_i = \pi_i\left(e_i^2 + \left(\Delta a_{ij}\right)^2\omega_i^2\right)$$

is satisfied in domain $O_{\mathrm{v}}(O)$, where $\pi_i > 0$, $O = \{0^4, 0^n\} \subset \mathbb{R}^4 \times \mathbb{R}^n \times \mathbb{J}_{0,\infty}$, 0^4, 0^n are zero vectors, n is the number of adjustable parameters, O_{v} is some neighborhood of point O, $t \in [0, \infty] = \mathbb{J}_{0,\infty}$;

7) for $V_{e,\Delta}(t)$, matrix system of inequalities $\dot{\mathbf{V}}_{e,\Delta} \le \mathbf{A}_{e,\Delta}\mathbf{V}_{e,\Delta} + \mathbf{B}_{e,\Delta}$ is valid, where $\mathbf{A}_{e,\Delta}$ is the following block diagonal matrix:

$$\mathbf{A}_{e,\Delta} = \mathrm{diag}\left(\mathbf{A}_{e,\Delta,1}, \mathbf{A}_{e,\Delta,3}, \mathbf{A}_{e,\Delta,4}, \mathbf{A}_{e,\Delta,5}\right),$$

$$\mathbf{B}_{e,\Delta} = \begin{bmatrix} 0 & 0 & \tilde{\upsilon}_3\vartheta_3 & \pi_3\chi & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}},$$

submatrices $\mathbf{A}_{e,\Delta,i}$ have the form similar to $\mathbf{A}_{\mathbf{V}}$ from Theorem 2;

8) the upper solution for $V_{e,\Delta}(t)$ satisfies equation $\dot{\mathbf{S}}_{e,\Delta} = \mathbf{A}_{e,\Delta}\mathbf{S}_{e,\Delta} + \mathbf{B}_{e,\Delta}$, if there exist such functions $s_i(t) \ge 0$, that $V_{e,\Delta,i}(t) \le s_i(t)$ $\forall(t \ge t_0)$ & $V_{e,\Delta,i}(t_0) \le s_i(t_0)$, $i = 1, 3, 4, 5$, where $S_{e,\Delta} \in \mathbb{R}^6$, s_i are elements of vector $\mathbf{S}_{e,\Delta}$. Then adaptive system (29), (30) is exponentially dissipative with the following estimation:

$$\mathbf{V}_{e,\Delta}(t) \le e^{\mathbf{A}_{e,\Delta}(t-t_0)}\mathbf{S}(t_0) + \int_{t_0}^{t} e^{\mathbf{A}_{e,\Delta}(t-\tau)}\mathbf{B}_{e,\Delta}d\tau,$$

if $k_1 \ge \frac{2}{3}\sqrt{\frac{\alpha\beta_l\left(\tilde{\mathbf{A}}_1\right)}{\alpha_{P_1}k_1\beta_1\left(\tilde{\mathbf{A}}_1\right)}}$, $k_3 \ge \frac{2}{3}\sqrt{\frac{\bar{\alpha}_{f_1,y_3}}{\left(\alpha_{f_1,y_3} - \underline{\upsilon_3}\right)}}$, $k_4 \ge \frac{2}{3}\sqrt{2\frac{\alpha_{y_1,\hat{y}_2}}{\tilde{\alpha}_{y_1,\hat{y}_2}}}$, $k_5 \ge \frac{2}{3}\sqrt{2\frac{\bar{\alpha}_{y_5} + c^2}{\alpha_{y_5 \cdot f_3}}}$.

The proof of Theorem 3 is similar to the proof of Theorem 2.

It follows from Theorem 3 that bound properties of system (29), (30) depend on nonlinear properties, feedback, and compliance with the excitation constancy condition. In particular, this applies to block 3.

CONCLUSIONS

In the paper, the approach to adaptive identification of systems with several nonlinearities is proposed. It is based on the transformation of a system in order to exclude unmeasurable state variables. The synthesis of an adaptive identification system is presented. In order to simplify the adaptation process, an approach involving the decomposition of a system into a number of subsystems is proposed. The boundedness of trajectories in the adaptive system is proven. The problem of the S-synchronizability of a system is considered taking into account the modification of the excitation constancy condition of the system information, which is set with allowance for the specifics of structural identifiability of the nonlinear part of the system. The method of Lyapunov vector functions is applied for proving the exponential stability of the identification system.

An approach to the nonlinear correction of a nonlinear system is considered. Adaptive algorithms for estimating system parameters are obtained. The boundedness of system trajectories is shown. Considering a nonlinear self-oscillation generation system with nonlinear feedback, an adaptive system of parametric identification is proposed. The influence of feedback and nonlinearities on the boundedness of trajectories is investigated. The simulation results confirm theoretical conclusions. The proposed methods could be used in developing identification and control systems for complex dynamic systems.

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