Mathematical modeling

Математическое моделирование

UDC 621.391:53.08
https://doi.org/10.32362/2500-316X-2023-11-4-94-104

RESEARCH ARTICLE

Restoration of a blurred photographic image of a moving object obtained at the resolution limit

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Abstract

Objectives. When processing images of the Earth's surface obtained from satellites, the problem of restoring a blurry image of a moving object is of great practical importance. The aim of this work is to study the possibility of improving the quality of restoration of blurry images obtained at the limit of the resolution of the camera.

Methods. Digital signal processing methods informed by the theory of incorrect and ill-conditioned problems were used.

Results. The proposed method for restoring a blurred photographic image of a moving object differs from traditional approaches in that the discrete convolution equation, to which the problem of restoring a blurred image is reduced, is obtained by approximating the corresponding integral equation based on the Kotelnikov interpolation series rather than on the traditional basis of the quadrature formula. In the work, formulas are obtained for calculating the kernel of the convolution obtained using the Kotelnikov interpolation series. The discrete convolution inversion problem, which belongs to the class of ill-posed problems, requires regularization. Results of traditional approaches to restoring blurred images using the quadrature formula with Tikhonov regularization and the proposed method based on the Kotelnikov interpolation series are compared. Although the quality of the blurred image restoration is almost the same in both cases, in the quadrature formula the blur value is expressed as an integer number of pixels, while, when using the Kotelnikov series, this value can also be specified in fractions of a pixel.

Conclusions. The expediency of discretizing the convolution describing the image distortion of the blur type on the basis of the Kotelnikov interpolation series when processing a blurred image obtained at the limit of the resolution of the camera is demonstrated. In this case, the amount of blur can be expressed in fractions of a pixel. This situation typically arises when processing satellite photography of the Earth's surface.

Keywords: blurred image of a moving object, resolution, image restoration, Tikhonov regularization, inverse problem, Kotelnikov interpolation series, edge effect

• **Submitted:** 05.12.2022 • **Revised:** 09.03.2023 • **Accepted:** 06.06.2023

For citation: Fedorov V.B., Kharlamov S.G., Starikovskiy A.I. Restoration of a blurred photographic image of a moving object obtained at the resolution limit. *Russ. Technol. J.* 2023;11(4):94–104. https://doi.org/10.32362/2500-316X-2023-11-4-94-104

Financial disclosure: The authors have no a financial or property interest in any material or method mentioned.

The authors declare no conflicts of interest.

НАУЧНАЯ СТАТЬЯ

Восстановление смазанного фотографического изображения движущегося объекта, получаемого на пределе разрешающей способности

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Резюме
Цели. Задача восстановления смазанного изображения движущегося объекта имеет большое практическое значение, в частности, при обработке изображений поверхности Земли, получаемых со спутников. Целью работы является исследование возможности повышения качества восстановления смазанных изображений, получаемых на пределе разрешающей способности фотоаппарата.
Методы. Использованы методы цифровой обработки сигналов, методы теории некорректных и плохо обусловленных задач.
Результаты. Предложен метод восстановления «смазанного» фотографического изображения движущегося объекта, отличающийся от традиционных подходов тем, что уравнение дискретной свертки, к решению которого сводится задача восстановления смазанного изображения, получается путем аппроксимации соответствующего интегрального уравнения на основе интерполяционного ряда Котельникова, а не на основе квадратурной формулы, как это делается традиционно. В работе получены формулы для вычисления ядра свертки, получаемой с применением интерполяционного ряда Котельникова. Как известно, задача обращения дискретной свертки относится к классу некорректных задач и требует регуляризации. Дано сравнение результатов восстановления смазанных изображений (с использованием регуляризации по Тихонову), осуществляемого как традиционным путем, т.е. с применением квадратурной формулы, так и предлагаемым способом, основывающимся на интерполяционном ряде Котельникова. Показано, что качество восстановления смазанного изображения в обоих случаях получается практически одинаковым. Однако использование квадратурной формулы предполагает, что величина «смаза» выражена целым числом пикселей, в то время как в случае использования ряда Котельникова эта величина может задаваться и долями пикселя.
Выводы. Показано, что дискретизацию свертки, описывающей искажение изображения типа «смаз», целесообразно осуществлять на основе интерполяционного ряда Котельникова в случае, когда осуществляется обработка смазанного изображения, получаемого на пределе разрешающей способности фотоаппарата. Это обусловлено тем, что в этом случае величина «смаза» может составлять доли пикселя. Такая ситуация характерна, например, для спутниковой фотосъемки поверхности Земли.

Ключевые слова: «смаз» изображения движущегося объекта, разрешающая способность, восстановление изображения, тихоновская регуляризация, обратная задача, интерполяционный ряд Котельникова, краевой эффект

• **Поступила:** 05.12.2022 • **Доработана:** 09.03.2023 • **Принята к опубликованию:** 06.06.2023

Для цитирования: Федоров В.Б., Харламов С.Г., Стариковский А.И. Восстановление смазанного фотографического изображения движущегося объекта, получаемого на пределе разрешающей способности. *Russ. Technol. J.* 2023;11(4):94–104. https://doi.org/10.32362/2500-316X-2023-11-4-94-104

Прозрачность финансовой деятельности: Авторы не имеют финансовой заинтересованности в представленных материалах или методах.

Авторы заявляют об отсутствии конфликта интересов.

INTRODUCTION

The problem of restoring a blurred image of a moving object has been quite well studied[1] [1–14]. Thus, in the case when the velocity of the moving object is known a priori, its solution is reduced to an inversion of the discrete convolution. This is obtained by approximating the corresponding integral convolution by replacing the integral with a quadrature formula (typically the trapezium quadrature formula). However, another possibility, proposed in this work, is based on the replacement of the integrand by its Kotelnikov interpolation series. In this case, the integrand function expresses the dependence of the brightness of the image point on its coordinates.

If we consider an image whose smallest details are significantly larger than the pixel size, then, when restoring it after blurring, the value of blurring can be set to the accuracy of the pixel size, i.e., expressed by an integer number of pixels. In this case, as we will show, the result of restoration is almost independent of the way in which the convolution was discretized. However, this is not the case when it is necessary to restore a blurred image obtained at the resolution limit of the camera, i.e., when the smallest details of the image are close to the pixel size. In this case, it may be necessary to express the amount of blur in fractions of a pixel in order to maximize resolution. In practice, this may imply the need to interpolate the blurred image prior to its restoration, i.e., to virtually represent it as if by smaller pixels. By doing this, it will be possible to express the exact value of blur by a whole number of these smaller pixels and then apply a traditional restoration method.

However, such approaches are associated with additional consumption of computer memory. For example, if the pixel size has to be reduced by a factor of 10, then the size of the corresponding image array will increase by a factor of 100.

The proposed method of integral convolution approximation based on the Kotelnikov interpolation series allows blur to be specified in fractions of a pixel: consequently, no interpolation of the image implying additional consumption of computer memory is required.

METHOD

Basic integral equation

A basic integral equation linking the brightness of points of the blurred image $Q(x, y)$ with the brightness of points of the restored image $P(x, y)$ has the form:

$$Q(x,y) = w^2 \int_0^T P(x - v_x t; y - v_y t)dt,$$

where (x, y) are Cartesian coordinates of the current point; (v_x, v_y) are Cartesian coordinates of the image velocity on the CCD matrix surface (by assumption all image points move with the same velocity); w is the pixel size; T is the exposure time [1–3].

If the vertical component of velocity is absent, then we have a Fredholm integral equation of the first kind of convolution type:

$$Q(x) = \frac{w^2}{v_x} \int_0^{v_x T} P(x - \xi)d\xi. \qquad (1)$$

Discretization of the convolution based on the quadrature formula

By replacing the integral in Eq. (1) with the trapezium quadrature formula, we obtain

$$Q(x) \approx \frac{w^2}{v_x} \sum_{k=0}^{n} c_k P(x - kw),$$

where $c_0 = c_n = 0.5$, $c_1 = \ldots = c_{n-1} = 1$ are the coefficients of the quadrature formula; $n = v_x/w$ is the amount of image shift during exposure time, expressed as an integer number of pixels; $k \in \{0, \ldots, n\}$.

Or, since within a single pixel the brightness must be considered constant, we have

$$Q(m) \approx \sum_{k=0}^{\min\{n,m\}} \frac{w^2}{v_x} c_k P(m - k), \qquad (2)$$

where m is the pixel number; $m \in \{0, \ldots, M - 1\}$; M is the number of pixels in one row, $k \in \{0, \ldots, m\}$; $Q[m] = Q(mw)$; $P[m] = P(mw)$.

Then the equality in (2) will be considered as exact.

Convolution (2) can also be written in matrix form

$$\mathbf{Q} = \mathbf{A} \cdot \mathbf{P}, \qquad (3)$$

where, for example, if $M = 8$ and $n = 3$, the Toeplitz matrix $\mathbf{A}$ would have the form:

$$\mathbf{A} = \frac{w^2}{v_x} \begin{bmatrix} 0.5 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 1.0 & 0.5 & 0.0 & 0.0 & 0.0 & 0.0 \\ 1.0 & 1.0 & 0.5 & 0.0 & 0.0 & 0.0 \\ 0.5 & 1.0 & 1.0 & 0.5 & 0.0 & 0.0 \\ 0.0 & 0.5 & 1.0 & 1.0 & 0.5 & 0.0 \\ 0.0 & 0.0 & 0.5 & 1.0 & 1.0 & 0.5 \end{bmatrix}.$$

[1] Gruzman I.S., Kirichuk V.S., Kosykh V.P., Peretyagin G.I., Spector A.A. *Digital Image Processing in Information Systems*: Tutorial. Novosibirsk: NSTU; 2002. 352 p. (in Russ.).

Convolution discretization based on the Kotelnikov interpolation series

Assuming that the maximum spatial frequency in the Fourier transform of the image $P(x)$ does not exceed $1/2w$, the subintegral function in (1) can be represented by the Kotelnikov interpolation series:

$$P(x) = \sum_{k=-\infty}^{+\infty} P[k] sinc\left(\frac{x}{w} - k\right).$$

Inserting this expression into (1), we obtain

$$Q(x) = \frac{w^2}{v_x} \sum_{k=-\infty}^{+\infty} P[k] \int_0^{v_x T} sinc\left(\frac{x}{w} - \frac{\xi}{w} - k\right) d\xi$$

or

$$Q[m] = \sum_{k=-\infty}^{+\infty} P[k] a[m-k] = \sum_{k=-\infty}^{+\infty} a[k] P[m-k].$$

However, since the horizontal dimensions of the image are limited to M pixels, we arrive at an approximate equality (which we will further assume to be exact):

$$Q[m] \approx \sum_{k=0}^{M-1} P[k] a[m-k], \qquad (4)$$

where the kernel of this discrete convolution is defined by the formula

$$a[k] = \frac{w^2}{v_x \pi}\left(Si\left(\pi\left(\frac{v_x T}{w} - k\right)\right) + Si(\pi k)\right). \qquad (5)$$

Corresponding diagrams are shown in Fig. 1.

As we can see from the above diagrams, the convolution kernel (4) approaches the convolution kernel (2) as the amount of image displacement for the exposure time increases. In the case of convolution (4), the shift value can be expressed not only by an integer number of pixels, but also by fractions of a pixel. In the case of convolution (2), however, this value is always expressed as an integer number of pixels.

Convolution (4) can also be written in matrix form:

$$\mathbf{Q} = \mathbf{A} \cdot \mathbf{P}. \qquad (6)$$

However, here, unlike case (3), the Toeplitz matrix **A** will no longer be a lower triangular matrix (it will not have zero diagonals).

Simulation of a blurred image (direct problem solving)

For numerical experiments on blurred image restoration, it is necessary to have the images blurred in the right way. It is possible to obtain such images using relations (2) or (4), considering function $P[m]$ as given and function $Q[m]$ as unknown. The mentioned relations can be reduced to a cyclic convolution, which is effectively computed (and reversed) on the basis of the fast Fourier transform (FFT). For example, Figs. 2 and 3 show the original image and the horizontally blurred 20 pixels image obtained in this way, respectively.

When simulating a blurred image, the essential point is the presence or absence of transient edges in the blurred image. In the horizontally blurred image shown in Fig. 3, such transient edges (left and right) are present. Each of these transient edges comprises a strip of width

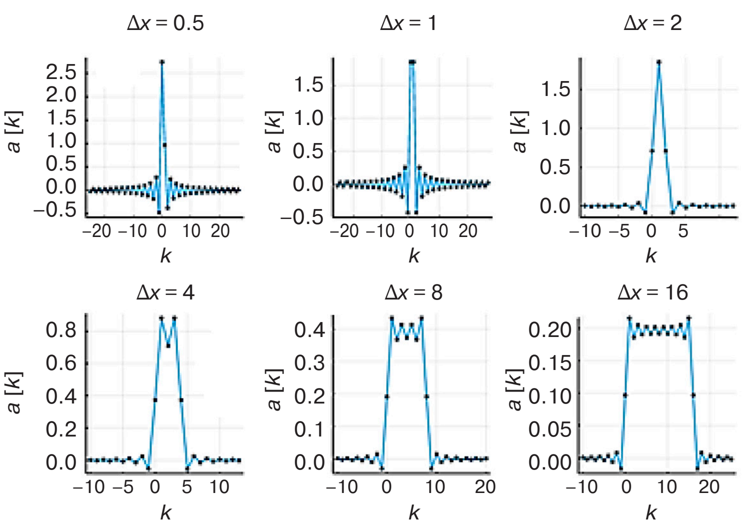

Fig. 1. Diagram of the kernel of the discrete convolution (4) obtained using the Kotelnikov interpolation theorem and corresponding to different values of the image blur $\Delta x = v_x T/w$

Fig. 2. Source image

Fig. 3. Simulated blurred image with uncropped edges (horizontal blur size—20 pixels)

equal to the amount of image displacement, whose brightness gradually decreases to zero.

However, since the prototype of the obtained image is like an infinite ribbon with no edges at both ends, the real blurred photographic image will have no such edges. Therefore, in order to conform the obtained blurred image to reality, its edges should be cropped.

RESULTS

Results of blurred image restoration without the application of regularization

Reversal of the simulated blurred image, as well as obtaining the solution of the direct problem, is feasible on the basis of FFT. Thus, a discrete convolution of the form (2) or (4) can be represented as a cyclic convolution by augmenting each of the finite sequences included in the convolution with the required number of zeros

$$\mathbf{Q}[m] = \sum_{k=0}^{M-1} \mathbf{P}[k]\mathbf{a}[m-k], \tag{7}$$

where **Q**, **P**, and **a** are arrays of length equal to length(**P**) + + length(**a**) − 1 obtained from the corresponding arrays Q, P, a (included in (2) and (4)) by adding the required number of zeros.

Then, to reverse this convolution a so-called inverse filter can be used, whose effect in the frequency domain is as follows:

$$\mathcal{F}[\mathbf{P}] = \frac{1}{\mathcal{F}[\mathbf{a}]}\mathcal{F}[\mathbf{Q}],$$

where $\mathcal{F}[\cdot]$ denotes the discrete Fourier transform; $1/\mathcal{F}[\mathbf{a}]$ is the so-called transfer function of the inverse filter.

If in this case the transient edges of the simulated blurred image are not cut off, the restoration result will be almost perfect even without the use of regularization. An example of an image restored in this way is shown in Fig. 4. Experiments show that, in order to obtain this result, the length of tails of the Kotelnikov kernel should be not less than 3 in cases where the edges of a blurred image $\Delta x = \frac{v_x T}{w} = 20$ remain uncropped. The length of tails refers to the number of counts of the kernel with negative indices, which is also equal to the number of counts with indices greater than Δx.

However, as already noted, the real blurred image is distinguished by the fact that it has no transient edges. If, in accordance with reality, the resulting transient edges are cut off, the result of restoration (without regularization) will be extremely poor. A corresponding example of the result of restoration is shown in Fig. 5.

It is possible to try to correct the situation by performing a preliminary restoration of the cropped edges. Such restoration can simply entail the addition of two vertical bars equal to the width of the blur to the left and right of the cropped image, whose pixel brightness of the horizontal rows will increase/decrease (by

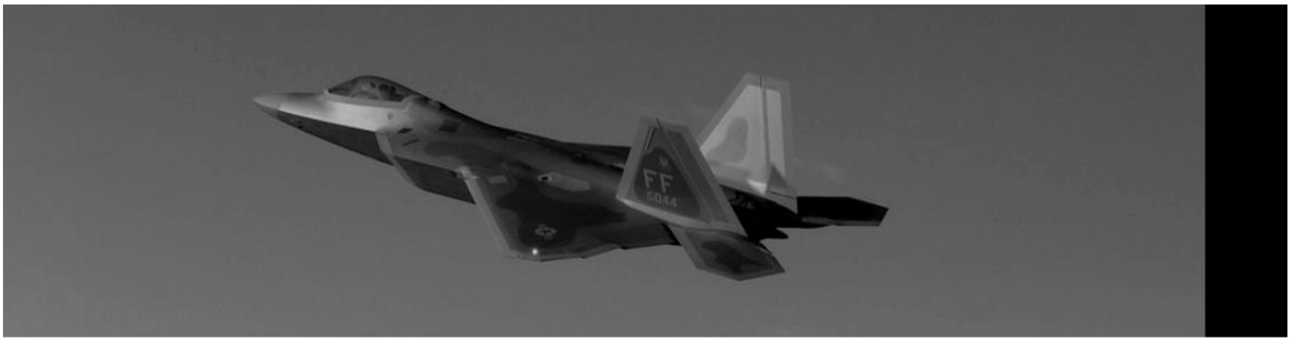

Fig. 4. Blurred image with uncropped edges restored with an inverse filter using the Kotelnikov kernel and without regularization

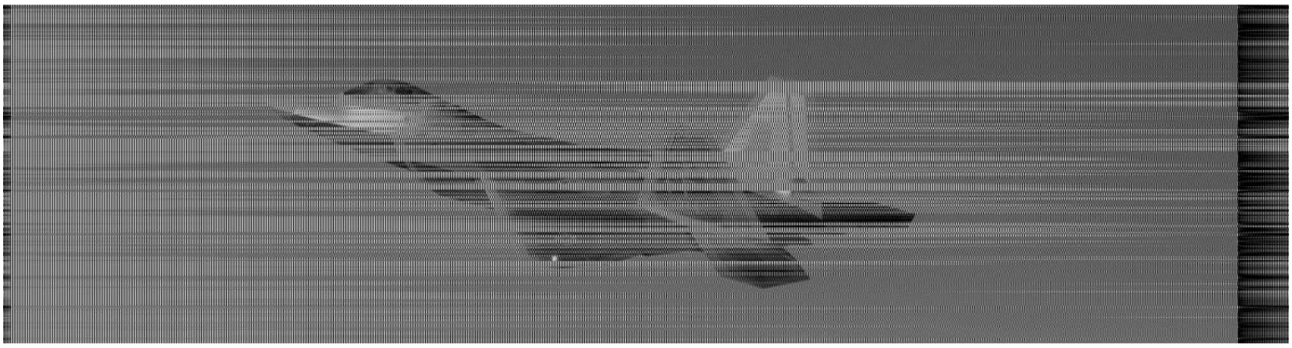

Fig. 5. Blurred image with uncropped edges restored with an inverse filter using the trapezium quadrature formula and no regularization

continuity) according to a linear law. Figure 6 shows an example of a simulated blurred image with cropped transient edges, while Figs. 7 and 8 depict the results of its restoration after the restoration of the cropped edges.

As can be seen, the result of restoration by reversing the discrete convolution with the Kotelnikov kernel turned out to be much better. However, as shown by numerical experiments, the quality of the restoration significantly depends on how many counts in the tails of the Kotelnikov kernel, defined by formula (5), are taken into account. In this case, only counts with indices from 0 to 20 (with a blur value of 20 pixels) were taken into account, i.e., the tails of the kernel were completely discarded. In general, we cannot claim that the treatment of the discrete convolution with the Kotelnikov kernel without the application of regularization in all cases guarantees a satisfactory result.

As can be seen from the examples, although edge restoration slightly improves the result of image restoration, its quality is still generally unsatisfactory. The reason is that the inversion of discrete convolutions (2) or (4) can be considered as a solution of a system of linear algebraic equations (SLAE) of the form (3) or (6) respectively.

Errors of restoration (and moreover the complete absence of such restoration) should be considered as

Fig. 6. Simulated blurred image with cropped transient edges at a horizontal blur value of 20 pixels

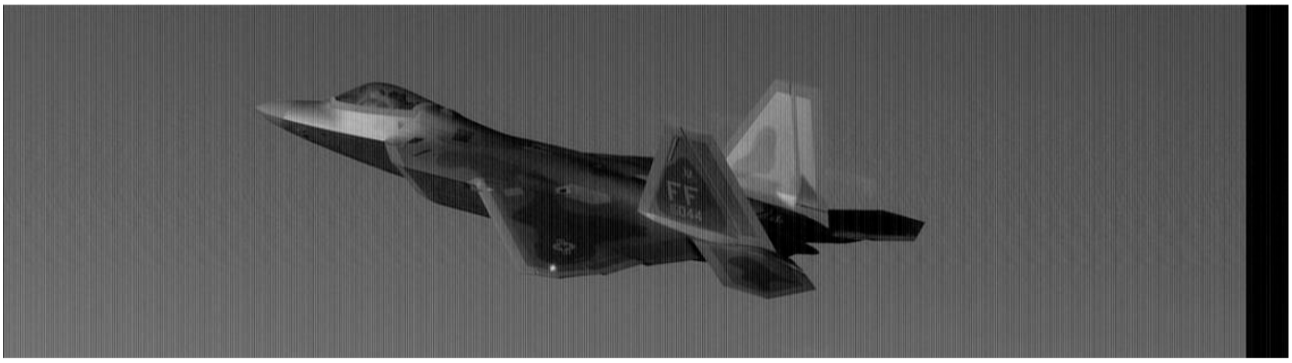

Fig. 7. Blurred image with cropped edges restored with an inverse filter using the Kotelnikov kernel and no regularization

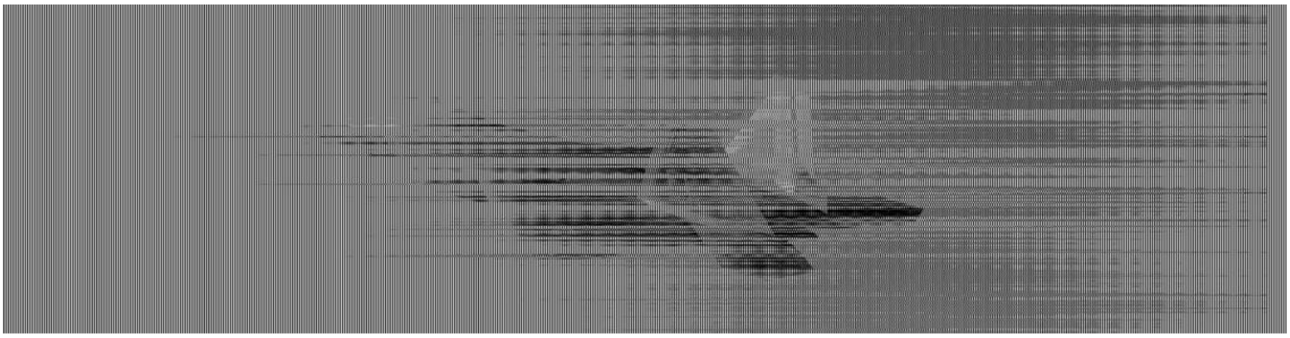

Fig. 8. Blurred image with cropped edges restored with an inverse filter using the trapezium quadrature formula and no regularization

errors in the setting of initial data (i.e. errors in several extreme values $Q[0], \ldots, Q[M-1]$). The estimate of the relative error of the obtained solution is known to be proportional to the relative error of the right part of SLAE with the proportionality coefficient equal to the conditionality number of SLAE matrix. This number, whose value depends both on the matrix size and on the value of blur, is typically quite large. Thus, even a small relative error of initial data can lead to very significant relative errors of the obtained solution. To mitigate this phenomenon, the Tikhonov regularization method is usually used in the convolution reversal [1–3].

Results of blurred image restoration by inverse filtering with Tikhonov regularization

The image restoration problem is known to be very sensitive to errors of initial data (the solution of the Fredholm 1-grade integral equation (1) belongs to the class of incorrectly posed problems). Therefore, in the presence of any significant errors in the initial data, the convolution reversal requires the application of regularization.

The transfer function of the Tikhonov regularized discrete inverse filter has the form:

$$\frac{A[m]}{|A[m]|^2 + \alpha W^{2p}[m]},$$

where $A = \mathcal{F}[\mathbf{a}]$, W^{2p} is the stabilizing function of the order $p = 0, 1, 2, \ldots$; α is the regularization parameter ($\alpha \geq 0$, with $\alpha = 0$ there is no regularization; this parameter can be chosen, for example, experimentally); array a is defined in Eq. (7) [1–3].

Stabilizing function of zero order is $W^0 = \mathcal{F}([1, 0, \ldots, 0])$, i.e., identically equal to 1. The first-order regularizing function is defined as $W^2 = \mathcal{F}([1, -2, 1, 0, \ldots, 0])$, i.e., the Fourier transform of the 2nd-order finite-difference filter. Regularizing functions of higher orders are the degrees of W^2. The results of restoration with edge restoration and application of regularization of horizontally blurred image by 20 pixels are shown in Figs. 9 and 10.

As we can see from the obtained results, the quality of the blurred image restoration is equally good when using the quadrature formula as in the case of using the Kotelnikov series.

Results of numerical experiments on image restoration at blur values expressed in fractions of a pixel

Let us now consider the situation when we need to restore a blurred image obtained at the resolution limit of the camera, i.e., when the instantaneous images of the smallest details of the moving image are close to the size of a pixel. In this case, we will consider a test image

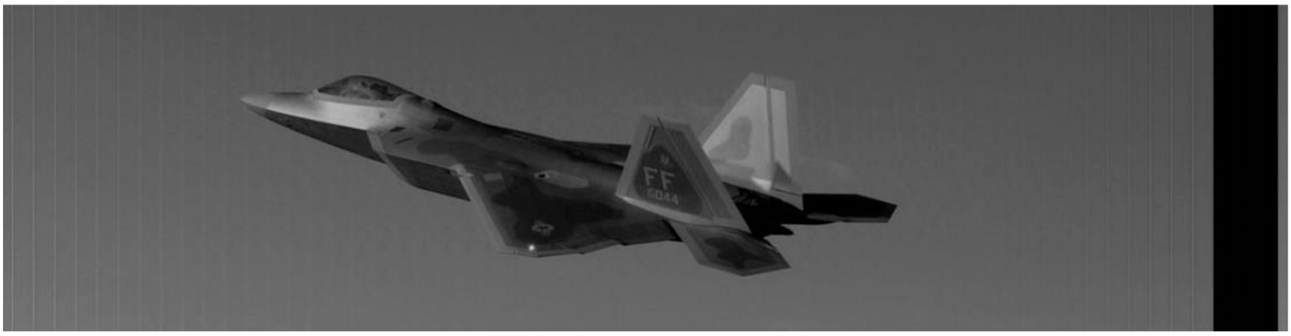

Fig. 9. Blurred image after cropped edge restoration, restored via inverse filter using the Kotelnikov kernel ($\alpha = 10^{-3}$, $p = 1$)

Fig. 10. Blurred image after cropped edge restoration, restored using an inverse filter using the trapezium quadrature formula ($\alpha = 10^{-2}$, $p = 1$)

consisting of parallel strips of decreasing width down to the size of one pixel (in the image space) as a model of the subject. Figure 11 shows a momentary image of such a test image (taking into account the finiteness of pixel size of the camera CCD matrix). Figure 12 shows a horizontally blurred image by 5.5 pixels of the same test image.

Figure 13 shows the restored blurred image with uncropped edges using the inverse filter using the Kotelnikov kernel and no regularization. To obtain the best restoration result for a given value of blur of the image, the tail length of the Kotelnikov kernel should be chosen experimentally. Thus, the result shown in Fig. 13 is obtained with a tail length equal to 2.

As we can see from comparison of Fig. 14 and 15, the contrast of the restored blurred picture of the test image using the Kotelnikov kernel is higher than when using the quadrature formula. This is not surprising, since the value of blur is equal to 5.5 pixels: when using the quadrature formula, only the shift by an integer number of pixels can be taken into account. Thus, in case of using the quadrature formula, the inverse filter has been adjusted to a blur value equal not to 5.5, but to 6 (because when using the quadrature formula there is no possibility to consider the fractional part of the blur value).

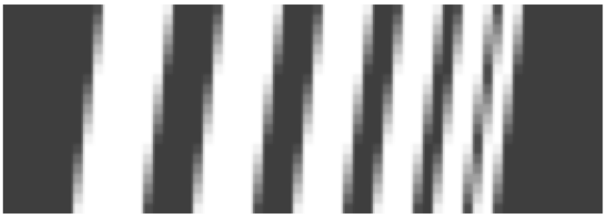

Fig. 11. Instantaneous (unblurred) picture of the test image, obtained at the resolution limit

Fig. 12. Horizontally blurred image of the world by 5.5 pixels (the edges are not cropped)

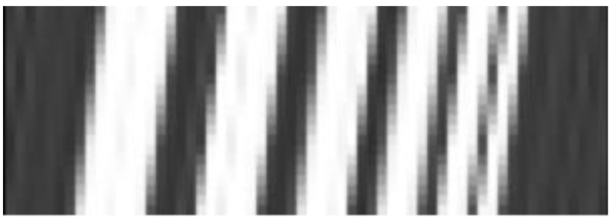

Fig. 13. Blurred picture of a test image with uncropped edges, restored with an inverse filter using the Kotelnikov kernel and no regularization

Fig. 14. Result of the cropped blurred picture restoration of the test image with the inverse filter using the Kotelnikov kernel with preliminary restoration of the cropped edges ($\alpha = 10^{-3}$, $p = 1$)

Fig. 15. Result of the cropped blurred picture restoration of the test image with uncropped edges using the inverse filter using the trapezoid quadrature formula with preliminary restoration of the cropped edges ($\alpha = 10^{-2}$, $p = 1$)

CONCLUSIONS

Results of numerical experiments show that, in cases when the blur value is expressed by a whole number of pixels, the proposed approach based on the Kotelnikov kernel works as well as the traditional technique based on the use of quadrature formulas. However, the use of the Kotelnikov series will have an advantage if the blur value of the image is expressed in fractions of pixels, because in this case, to fully account for a priori information about the blur value, there is no need to interpolate the original image, as would be the case with quadrature formulas.

Thus, if blur to be eliminated is a fraction of a pixel in size, the use of the Kotelnikov kernel is preferable when processing a blurred image obtained at the resolution limit of the camera. Such a situation is typical, for example, when processing satellite photography of the Earth’s surface.

Authors’ contribution. All authors equally contributed to the present work.

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Translated from Russian into English by Lyudmila O. Bychkova
Edited for English language and spelling by Thomas A. Beavitt