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RESEARCH ARTICLE

Models of waveguides combining gradient and nonlinear optical layers

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Abstract

Objectives. Theoretical studies of the waveguide properties of interfaces between nonlinear optical and graded-index media are important for application in optoelectronics. Waveguides combining layers with different optical properties seem to be the most promising, since they can be matched to optimal characteristics using a wide range of control parameters. The paper aims to develop a theory of composite optically nonlinear graded-index waveguides with an arbitrary profile, within which it is possible to obtain exact analytical expressions for surface waves and waveguide modes in an explicit form. The main feature of the theory proposed in this paper is its applicability for describing surface waves and waveguide modes, in which the field is concentrated inside the gradient layer and does not exceed its boundary, avoiding contact with the nonlinear layer.

Methods. Analytical methods of the theory of optical waveguides and nonlinear optics are used.

Results. A theoretical description of the waveguide properties of the interface between two media having significantly different optical characteristics is carried out. The formulated model of a plane waveguide is applicable to media having an arbitrary spatial permittivity profile. An analytical expression describing a surface wave propagating along the interface between a medium having stepwise nonlinearity and a gradient layer with an arbitrary permittivity profile is obtained. Additionally, analytical expressions for surface waves propagating along the interface between a medium with Kerr nonlinearity (both self-focusing and defocusing), as well as graded-index media characterized by exponential and linear permittivity profiles, are obtained.

Conclusions. The proposed theory supports a visual description in an explicit analytical form of a narrowly localized light beam within such waveguides. It is shown that by combining different semiconductor crystals in a composite waveguide, it is possible to obtain a nonlinear optical layer on one side of the waveguide interface and a layer with a graded-index dielectric permittivity profile on the other.

Keywords: nonlinear optics, nonlinear waves, optical nonlinearity, Kerr nonlinearity, optical waveguide, graded-index waveguide

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НАУЧНАЯ СТАТЬЯ

Модели волноводов, сочетающих градиентные и нелинейно-оптические слои

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Резюме
Цели. Теоретические исследования волноводных свойств границ раздела нелинейно-оптических и градиентных сред являются важными для использования в оптоэлектронике. Комбинированные волноводы, сочетающие слои с различными оптическими свойствами, представляются наиболее перспективными, поскольку для них можно подобрать оптимальные характеристики с помощью широкого ряда управляющих параметров. Цель работы – разработка теории композитных оптически-нелинейных градиентных волноводов с произвольным профилем, в рамках которой возможно получение точных аналитических выражений для поверхностных волн и волноводных мод в явном виде. Основной особенностью предлагаемой в данной работе теории является то, что она применима для описания поверхностных волн и волноводных мод, поле в которых сосредоточено внутри градиентного слоя и не выходит за его границу, не контактирующую с нелинейным слоем.
Методы. Использованы аналитические методы теории оптических волноводов, нелинейной оптики.
Результаты. Проведено теоретическое описание волноводных свойств границы раздела двух сред с принципиально различными оптическими характеристиками. Сформулированная модель плоского волновода применима для сред с произвольным распределением пространственного профиля диэлектрической проницаемости. Получено аналитическое выражение, описывающее поверхностную волну, распространяющуюся вдоль границы раздела среды со ступенчатой нелинейностью и градиентного слоя с произвольным профилем диэлектрической проницаемости. Также получены аналитические выражения для поверхностных волн, распространяющихся вдоль границы раздела среды с керровской нелинейностью (как самофокусирующей, так и дефокусирующей) с градиентными средами, характеризующимися экспоненциальным и линейным профилями диэлектрической проницаемости.
Выводы. Предложенная теория позволяет наглядно описать в явном аналитическом виде узко локализованные световые потоки в таких волноводах. Показано, что сочетание различных полупроводниковых кристаллов в композитном волноводе позволяет получить с одной стороны от волноведущего интерфейса нелинейно-оптический слой, а с другой – слой с градиентным профилем диэлектрической проницаемости.

Ключевые слова: нелинейная оптика, нелинейные волны, оптическая нелинейность, керровская нелинейность, оптический волновод, градиентный волновод

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INTRODUCTION

Waveguide structures are typically based on differences in the optical properties of their layers [1, 2]. In optoelectronics, waveguides whose layers have constant refractive index values [3], those having a variable spatial distribution profile (graded-index) [4], as well as those providing a nonlinear optical response when the refractive index depends on the light flux intensity [5], are widely used. Combined waveguides combining such layers appear to be the most promising for optoelectronics, since optimal characteristics can be selected for them using a wide range of control parameters [6, 7].

In recent years, theoretical studies on the waveguide properties of the interfaces between nonlinear-optical and graded-index media have intensified [8, 9]. In particular, waveguide modes having a layered structure consisting of a gradient layer with linear [10, 11] and exponential [12, 13] profiles in contact with the nonlinear Kerr medium have been obtained. Our recent works present the results of theoretical studies on the waveguide properties of structures combining pairs of different gradient and nonlinear media. In particular, linear [14–19], parabolic [20, 21], and exponential profiles [22, 23] of the refractive index/dielectric permittivity of media in contact with media of different nonlinearities, such as stepwise [14, 22], Kerr [20] and its generalizations [16–18, 20, 21], as well as those of a photorefractive diffusive type [21, 23], are considered. Symmetric three-layer structures consisting of a linear-gradient layer in a medium with Kerr nonlinearity [24] and in a medium with photorefractive nonlinearity [25] are also described.

The paper proposes the theoretical description of surface waves propagating along a flat media interface, one of which has a nonlinear response, while the other is characterized by a spatial gradient of dielectric permittivity. The novelty of the work lies in the proposed theory of such composite optically nonlinear graded-index waveguides with an arbitrary profile, within the framework of which analytical expressions for surface waves can be obtained. The main feature of this theory is its applicability to the description of surface waves and waveguide modes, in which the field is concentrated inside the gradient layer and does not go beyond its boundary not contacting with a nonlinear layer. The proposed theory permits a clear description in an explicit analytical form of the narrowly localized light fluxes in such waveguides. Examples of semiconductor materials that could be used for designing waveguides of this type are also considered.

1. THEORY AND RESEARCH METHODS

A flat interface between two nonmagnetic media of fundamentally different optical types is considered as a simple waveguide structure. In particular, a medium with a smooth change in the spatial profile of the refractive index/dielectric permittivity (i.e., $n = n(x)$ or $\varepsilon = \varepsilon(x)$, respectively; x being the spatial coordinate in the direction perpendicular to the contact plane $x = 0$) is on one side of the boundary, and a non-linear optical medium wherein the refractive index/permittivity depends on light intensity I (i.e., $n = n(I)$ or $\varepsilon = \varepsilon(\mathrm{I})$, $I = |E|^2$; E being the amplitude of electric field strength) is on the other side. It is assumed that the considered media have no dielectric losses; consequently, all their optical characteristics are valid values.

The origin is located in the yz plane such that the x-axis is perpendicular to the interface located in the $x = 0$ plane. We shall consider a transverse wave propagating along the interface with a non-zero component of the electric field strength: $E_y = \psi(x)\exp(i\beta z - i\omega t)$, where $\beta = kn$ is the propagation constant; $n = ck/\omega$ is the effective refractive index; c is the speed of light in vacuum; ω is a frequency; $k = 2\pi/\lambda$ is a wave number; λ is a wavelength; $\psi(x)$ is the spatial distribution of electric field strength in the direction transverse to the interface. As is well known [1, 2], function $\psi(x)$ obeys the stationary equation (magnetic permeability is assumed to be equal to one):

$$\psi''(x) + \{\varepsilon(x, I) - n^2\}k^2\psi(x) = 0, \tag{1}$$

where the dielectric permittivity of the waveguide system is written in the following form:

$$\varepsilon(x, I) = \begin{cases} \varepsilon_G(x), & x < 0, \\ \varepsilon_N(I), & x > 0. \end{cases} \tag{2}$$

For definiteness, the graded-index medium is located in the left half-space and characterized by dielectric permittivity $\varepsilon_G(x)$, while the optically nonlinear medium is located in the right half-space and characterized by dielectric permittivity $\varepsilon_N(I)$.

If the transverse field distribution is represented as:

$$\psi(x) = \begin{cases} \psi_G(x), & x < 0, \\ \psi_N(x), & x > 0, \end{cases} \tag{3}$$

then, considering (2), Eq. (1) splits into the following two:

$$\psi''_G(x) + \{\varepsilon_G(x) - n^2\}k^2\psi_G(x) = 0, \; x < 0, \tag{4}$$

$$\psi''_N(x) + \{\varepsilon_N(I) - n^2\}k^2\psi_N(x) = 0, \; x > 0. \tag{5}$$

Equations (4) and (5) should be supplemented with boundary conditions describing the continuity requirement for the field components at the interface:

$$\psi_N(+0)=\psi_G(-0),\quad \psi'_N(+0)=\psi'_G(-0), \tag{6}$$

as well as boundedness at infinity: $|\psi(x)| \to 0$, $|x| \to \infty$.

Since the localization of the light beam energy along the waveguide layer represents an important task, the focus is on such waves for whom the depth of field penetration is less than the thickness of the gradient layer. It has been shown in a number of our works [14–22] that, in certain intervals of the waveguide system parameters, the field amplitude at distances of the order of the gradient layer thickness may be much smaller than the amplitude at the interface and can therefore be neglected. Consequently, there is no need to use the boundary conditions at the distance of the gradient layer thickness in this case; the boundary conditions at the interface are sufficient for use when describing narrowly localized surface waves.

The solution of Eq. (4) may be written in the following form:

$$\psi_G(x)=\psi_0\frac{F(g(x))}{F(g(0))}, \tag{7}$$

where ψ_0—field amplitude at the interface; $F(g)$—special function with an auxiliary argument $g(x)$ solving Eq. (4) analytically for a given permittivity profile $\varepsilon_G(x)$ and satisfying the following equation:

$$F''g' + F'g'' + \{\varepsilon_G(x)-n^2\}k^2F=0. \tag{8}$$

The explicit form of dependence $g(x)$ is related to the substitution of variables when reducing Eq. (8) to the form for which it is known that the exact solution is expressed through some special function. For example, in the case of linear replacement $g(x) = ax + b$ in (8), it would be $g' = a$ and $g'' = 0$; then, it would be simplified as $aF''+\{\varepsilon_G(x)-n^2\}k^2F=0$.

When choosing to write the solution in the form (7), it is necessary to consider the requirement at infinity: $|F(g(x))| \to 0$, $|x| \to \infty$.

We shall briefly summarize the basic provisions of the theory of the composite waveguides under consideration. The waveguide comprises an ultrathin boundary separating the medium with a dielectric permittivity graded-index and a nonlinear optical medium. The transverse wave is localized so that the field does not extend beyond the boundary of the gradient layer. The electric field distribution in the direction transverse to the interface is determined from the stationary one-dimensional nonlinear equation to satisfy the conjunction condition at the interface and the disappearance at infinity condition.

The explicit form of the solution of Eq. (5) depends on selecting the nonlinearity model of the medium. We shall further consider the waves arising in such waveguides with profiles narrowly localized along the interface of the media using the examples of Kerr and stepwise nonlinearities and the graded-index exponential profile.

2. RESULTS AND DISCUSSION

2.1. Kerr nonlinearity

The most common form of nonlinear response of an optical system is the Kerr response, for which the dielectric permittivity depends linearly on the light intensity:

$$\varepsilon_N(I)=\varepsilon_{0N}+\alpha I, \tag{9}$$

where ε_{0N}—unperturbed dielectric constant (positive); α—Kerr nonlinearity coefficient whose positive and negative value corresponds to the self-focusing medium and defocusing one, respectively.

Then, considering dielectric permittivity (9), Eq. (5) may be written in the following form:

$$\psi''_N(x)-q_N^2\psi_N(x)+\alpha k^2\psi_N^3(x)=0, \tag{10}$$

where $q_N^2=k^2(n^2-\varepsilon_{0N})$.

The solution of nonlinear Eq. (10) at $n^2 > \varepsilon_{0N}$, satisfying the condition $|\psi(x)| \to 0$, $|x| \to \infty$, may be written as follows:

$$\psi_N(x)=\begin{cases}\sqrt{\dfrac{2}{\alpha}}\cdot\dfrac{q_N}{k\,\mathrm{ch}\,q_N(x-x_N)}, & \alpha>0,\\[2ex] \sqrt{\dfrac{2}{|\alpha|}}\cdot\dfrac{q_N}{k\,\mathrm{sh}\,q_N(x-x_N)}, & \alpha<0,\end{cases} \tag{12}$$

where the x_N value characterizes the position of the "soliton" center and is determined from the boundary conditions (6). Substituting (7) and (12) into (6), the following may be written:

$$x_N=\begin{cases}\dfrac{1}{k\sqrt{n^2-\varepsilon_{0N}}}\cdot\mathrm{arctg}\left(\dfrac{\varepsilon_{G\,\mathrm{eff}}}{n^2-\varepsilon_{0N}}\right)^{1/2}, & \alpha>0,\\[2ex] \dfrac{1}{k\sqrt{n^2-\varepsilon_{0N}}}\cdot\mathrm{arcctg}\left(\dfrac{\varepsilon_{G\,\mathrm{eff}}}{n^2-\varepsilon_{0N}}\right)^{1/2}, & \alpha<0,\end{cases} \tag{13}$$

where the effective permittivity of the gradient layer is introduced, as follows:

$$\varepsilon_{G\,\mathrm{eff}}=\frac{F'(g(0))}{F(g(0))}\cdot\frac{g'(0)}{k}. \tag{14}$$

Similarly, after substituting (7) and (12) into (6), the field intensity at the interface is obtained as follows:

$$I_0=\psi_0^2=\begin{cases}2(n^2-\varepsilon_{0\mathrm{N}}-\varepsilon_{\mathrm{G\,eff}})/\alpha, & \alpha>0,\\ 2(\varepsilon_{\mathrm{G\,eff}}+\varepsilon_{0\mathrm{N}}-n^2)/|\alpha|, & \alpha<0.\end{cases} \quad (15)$$

As an example, the exponential dielectric permittivity profile is considered:

$$\varepsilon_{\mathrm{G}}(x)=\varepsilon_{\mathrm{e}}+(\varepsilon_0-\varepsilon_{\mathrm{e}})\mathrm{e}^{2x/h}, \quad (16)$$

where ε_0 and ε_{e} are dielectric constants (positive) at the interface and at the end of the gradient layer of characteristic thickness h, respectively.

Substituting the profile (16) into Eq. (4), the following may be written:

$$\psi''_{\mathrm{G}}(x)+(V/a)^2\exp(2x/a)-q^2\}\psi_{\mathrm{G}}(x)=0, \quad (17)$$

where $V^2=a^2k^2(\varepsilon_0-\varepsilon_{\mathrm{e}})$, $q^2=k^2(n^2-\varepsilon_{\mathrm{e}})$.

In this case, $F(g(x))=J_\nu(g(x))$, where $J_\nu(g)$ is the Bessel function of the first kind, $g(x)=V\mathrm{e}^{x/a}$; then Eq. (7) defining the solution of Eq. (17) at $n^2>\varepsilon_{\mathrm{e}}$ and satisfying the condition $|\psi(x)|\to 0$, $x\to-\infty$ may be written in the following form:

$$\psi_{\mathrm{G}}(x)=\psi_0 J_{aq}(V\mathrm{e}^{x/a})/J_{aq}(V). \quad (18)$$

The effective permittivity of the exponential gradient layer is written as follows:

$$\varepsilon_{\mathrm{G\,eff}}=\frac{J'_{aq}(V)}{J_{aq}(V)}\cdot\frac{V}{ak}. \quad (19)$$

Thus, using Eqs. (3), (12), and (18), the wave propagating along the interface of the self-focusing Kerr nonlinear medium and the exponential gradient layer may be written in the following form:

$$\psi(x)=\begin{cases}\sqrt{\dfrac{2(n^2-\varepsilon_{0\mathrm{N}}-\varepsilon_{\mathrm{G\,eff}})}{\alpha}}\cdot\dfrac{J_{aq}(V\mathrm{e}^{x/a})}{J_{aq}(V)}, & x<0,\\ \sqrt{\dfrac{2}{\alpha}}\cdot\dfrac{q_{\mathrm{N}}}{k\,\mathrm{ch}\,q_{\mathrm{N}}(x-x_{\mathrm{N}})}, & x>0,\end{cases} \quad (20)$$

where the dielectric permittivity of the exponential gradient layer is defined by Eq. (19).

As a second example, the linear dielectric permittivity profile is considered:

$$\varepsilon_{\mathrm{G}}(x)=\varepsilon_0+(\varepsilon_0-\varepsilon_{\mathrm{e}})(x/h). \quad (21)$$

where h is the layer thickness in the case of a linear function.

Substituting profile (21) into Eq. (4), the following may be written:

$$\psi''_{\mathrm{G}}(x)+\{\varepsilon_0-n^2+(\varepsilon_0-\varepsilon_{\mathrm{e}})(x/h)\}k^2\psi_{\mathrm{G}}(x)=0. \quad (22)$$

In this case, $F(g(x))=\mathrm{Ai}(g(x))$, where $\mathrm{Ai}(g)$ is the Airy function of the first kind, $g(x)=-x/x_{\mathrm{G}}+\delta$ and $x_{\mathrm{G}}=\{a/k_0^2(\varepsilon_0-\varepsilon_{\mathrm{e}})\}^{1/3}$, $\delta=-(\varepsilon_0-n^2)h/x_{\mathrm{G}}(\varepsilon_0-\varepsilon_{\mathrm{e}})$. Then Eq. (7) defining the solution of Eq. (22) at $\varepsilon_{\mathrm{e}}<n^2<\varepsilon_0$ and satisfying the condition $|\psi(x)|\to 0$, $x\to-\infty$ may be written in the following form:

$$\psi_{\mathrm{G}}(x)=\psi_0\mathrm{Ai}(-x/x_{\mathrm{L}}+\delta)/\mathrm{Ai}(\delta). \quad (23)$$

The effective permittivity of the linear gradient layer is written as follows:

$$\varepsilon_{\mathrm{G\,eff}}=-\frac{1}{kx_{\mathrm{L}}}\cdot\frac{\mathrm{Ai}'(\delta)}{\mathrm{Ai}(\delta)}. \quad (24)$$

Thus, using Eqs. (3), (12), and (24), the wave localized along the interface of the self-focusing Kerr nonlinear medium and the linear gradient layer may be written in the following form:

$$\psi(x)=\begin{cases}\sqrt{\dfrac{2(n^2-\varepsilon_{0\mathrm{N}}-\varepsilon_{\mathrm{G\,eff}})}{\alpha}}\cdot\dfrac{\mathrm{Ai}(-x/x_{\mathrm{L}}+\delta)}{\mathrm{Ai}(\delta)}, & x<0,\\ \sqrt{\dfrac{2}{\alpha}}\cdot\dfrac{q_N}{k\,\mathrm{ch}\,q_{\mathrm{N}}(x-x_{\mathrm{N}})}, & x>0,\end{cases} \quad (25)$$

where the dielectric permittivity of the exponential gradient layer is determined by Eq. (24).

2.2. Stepwise nonlinearity

Let now a nonlinear medium whose dielectric permittivity is described by the step function (the model of the simplest nonlinear medium [26] or the "sharp step" model [27]) contact the gradient layer:

$$\varepsilon_{\mathrm{N}}(|E|)=\begin{cases}\varepsilon_1, |E|<E_{\mathrm{s}},\\ \varepsilon_2, |E|>E_{\mathrm{s}},\end{cases} \quad (26)$$

where E_{s} is the field value; when reached, the abrupt (instantaneous) switching from one value of the dielectric constant ε_1 to another ε_2 ($\varepsilon_2>\varepsilon_1$) occurs.

Thus, near the contact in the nonlinear medium where $|E|>E_{\mathrm{s}}$, there is a region (near-surface domain) of width x_{s} in which the dielectric constant has value ε_2, while outside of it where $|E|<E_{\mathrm{s}}$, the dielectric constant has value ε_1. The position of the boundary of the near-surface domain x_{s} is determined by the following conditions:

$$\begin{gathered}\psi_{\mathrm{N}}(x_{\mathrm{s}}+0)=\psi_{\mathrm{N}}(x_{\mathrm{s}}-0)=E_{\mathrm{s}},\\ \psi'_N(x_{\mathrm{s}}+0)=\psi'_{\mathrm{N}}(x_{\mathrm{s}}-0).\end{gathered} \quad (27)$$

As shown in [26], in the stepwise nonlinearity model, Eq. (5) with permittivity (26) decomposes into two:

$$\psi''_{\mathrm{N}}(x) - (n^2 - \varepsilon_1)k^2\psi_{\mathrm{N}}(x) = 0,\ |E| < E_{\mathrm{s}}, \quad (28)$$

$$\psi''_{\mathrm{N}}(x) + (\varepsilon_2 - n^2)k^2\psi_{\mathrm{N}}(x) = 0,\ |E| > E_{\mathrm{s}}. \quad (29)$$

The solution of Eq. (28) at $n^2 > \varepsilon_1$ is written as follows:

$$\psi_{\mathrm{N}}(x) = E_{\mathrm{s}} \mathrm{e}^{-q_1(x - x_{\mathrm{s}})}, \quad (30)$$

where $q_1^2 = (n^2 - \varepsilon_1)k^2$, while the solution of Eq. (28) at $n^2 < \varepsilon_2$ is written in the following form:

$$\psi_{\mathrm{N}}(x) = \psi_0 \cos(p_2(x - x_m))/\cos(p_2 x_m), \quad (31)$$

where $p_2^2 = (\varepsilon_2 - n^2)k^2$, while values x_{s}, x_{m} are determined from the boundary conditions.

Substituting solutions (7), (30), and (31) into boundary conditions (6) and (27), the following may be written:

$$x_{\mathrm{m}} = \frac{1}{k\sqrt{\varepsilon_2 - n^2}} \cdot \mathrm{arctg}\left(\frac{\varepsilon_{\mathrm{G\,eff}}}{\varepsilon_2 - n^2}\right)^{1/2}, \quad (32)$$

$$x_{\mathrm{s}} = x_{\mathrm{m}} + q_1 / p_2^2, \quad (33)$$

$$I_0 = \psi_0^2 = E_{\mathrm{s}}^2 \cdot \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_{\mathrm{G\,eff}} + \varepsilon_2 - n^2}, \quad (34)$$

where the dielectric permittivity of the exponential gradient layer is determined by Eq. (19).

Thus, the surface wave propagating along the interface between the medium with stepwise nonlinearity and a gradient layer with an arbitrary profile may be written in the following form:

$$\psi(x) = E_{\mathrm{s}} \cdot \begin{cases} \left(\dfrac{\varepsilon_2 - \varepsilon_1}{\varepsilon_{\mathrm{G\,eff}} + \varepsilon_2 - n^2}\right)^{1/2} \dfrac{F(g(x))}{F(g(0))},\ x < 0, \\ \left(\dfrac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 - n^2}\right)^{1/2} \cos(p_2(x - x_{\mathrm{m}})),\ 0 < x < x_{\mathrm{s}}, \\ \mathrm{e}^{-q_1(x - x_s)},\ x > x_{\mathrm{s}}, \end{cases} \quad (35)$$

where the position of the near-surface domain boundary is determined by Eq. (33).

In [22], the case of contact between a medium with abrupt nonlinearity and a medium with an exponential dielectric permittivity profile is considered. In this work, the impact of optical parameters of the corresponding waveguide structures on the wave profiles and their controlled localization is analyzed (and illustrated) in detail.

2.3. Discussion

The resulting Eq. (20) for surface waves differs from the one given in [14]. Wave (20) can propagate at the effective refractive index (propagation constant) varying arbitrarily in the admissible range of values. In [14], the authors present and analyze the dispersion equation linking the effective refractive index and the optical characteristics of the waveguide, i.e., its value is fixed at given values of the waveguide characteristics. In particular, they analyze the dependence of the effective refractive index n on the nonlinearity parameter αI_0 being the product of the Kerr nonlinearity coefficient on the field intensity at the media interface.

As is known, the propagation constant is related to the angle of incidence of the beam exciting the surface wave, which can vary under experimental conditions. Therefore, such a parameter should be considered as controlling, i.e., varying in experiments on the selected waveguide structure. In the work, Eq. (15) determining the dependence of the nonlinearity parameter αI_0 on the effective refractive index n is obtained. In our opinion, this dependence is more practical from the experimental point of view.

The difference between the obtained Eq. (25) describing the wave narrowly localized along the interface of a self-focusing Kerr nonlinear medium and the linear gradient layer should also be emphasized. The surface wave profile obtained in [10, 12] transverse to the interface of such media can go beyond the gradient layer. The developed theory is suitable for describing waves whose spatial intensity distribution is completely concentrated in the gradient layer and does not go beyond its limits.

The following are examples of materials on which composite waveguides combining nonlinear optical layers and gradient layers with a spatial distribution of optical characteristics can be based.

In nonlinear optical crystals, localization of the light beam occurs due to the nonlinear response of the medium. Under certain conditions, field localization occurs not only in a medium with self-focusing nonlinearity, but also with defocusing nonlinearity [28].

The dielectric permittivity—or square of the refractive index—depends on the square of the electric field strength, i.e., linearly on the light intensity (Kerr nonlinearity), which is characteristic of KDP and $LiNbO_3$ crystals at certain wavelengths. This form of nonlinear response is observed in multilayer Co/TiO_2 nanocomposite films in a wavelength range of 400–1000 nm at temperatures of 10–50°C [29]. In [30], the Kerr effect strengthening in cobalt-based thin films in the same wavelength range is noted.

It is indicated in [31, 32] that a high-intensity light beam can change the optical characteristics of crystals in narrow regions along the direction of its propagation. As a result, the formation of a near-surface layer with optical characteristics different from the rest of the crystal is observed. Such changes are caused by the nonlinear response of the crystal depending on the intensity of the light beam propagating along its surface. The theoretical description of such phenomena is based on models such as "sharp" stepwise nonlinearity [26, 33], "smooth" stepwise nonlinearity [27], and saturable nonlinearity in various formulations [34, 35].

As noted in Section 2.2, the model of "sharp" stepwise nonlinearity describes the change in dielectric permittivity abruptly from one constant value to another when the intensity of the light beam reaches a certain threshold value. Within the framework of such a model, many authors have been able to obtain in explicit analytical form the results applied to the theoretical description of the features of surface wave propagation [26], self-reflection [33], self-localized optical pulses [27], and optical bistabilities [36]. This dependence can be considered as a limiting case of the model of "smooth" stepwise nonlinearity in the case of a sharp increase in dielectric permittivity at a small increase in the light beam intensity.

It is shown in [37] that the change in the dielectric permittivity of a semiconductor with exciton-exciton interaction in a certain spectral range may occur quite abruptly. The authors [38, 39] noted that such behavior can be observed in semiconductor crystals such as CdS and CdSe crystals with low biexciton binding energy within the range of approximately 0.5–3.0 MeV. The authors explain this phenomenon by the fact that coherent photons passing through the semiconductor film excite coherent excitons with the same wave vector- and phase values as the photons. This optical interaction results in biexcitons determining both the polarization of the crystal and the concentration of quasiparticles responsible for its optical properties. Optical nonlinear effects can be observed at relatively low intensities of the incident light beam.

It should be noted that some media under intensive illumination undergo rapid changes in their optical properties: in particular, in semiconductor-doped CdSSe and Schott OG 550 glasses [40, 41], ion-doped $GdAlO_3{:}Cr^{3+}$ crystals [42], and thin films formed from the photochromic protein bacteriorhodopsin [43]. A practically discontinuous change of the refractive index from one value to another depending on the intensity of the light beam was observed in them in the limit at small relaxation times.

On the other hand, many semiconductor heterostructures used in modern optoelectronics exhibit a dependence of the refractive index or dielectric function on the spatial distance [3]. In particular, the refractive indices of GaAs/GaAlAs [44, 45], InGaAs/InAlAs [46], and InGaAsP/InP [47] semiconductor photonic crystals are described by spatially graded-index profiles. Therefore, these materials can be referred to as optically graded-index media. It should be noted that the use of gallium arsenide, as well as heterostructures based on it such as Ga1–*x*Al*x*As, Ga1–*x*In*x*As and Ga1–*x*Al*x*N, Ga1–*x*In*x*N, seems very promising due to their enhanced radiation resistance [48] in comparison with other crystals used in semiconductor optoelectronics.

Spatial distributions of refractive indices are often created by implanting ions into glasses, which induce ion-exchange processes creating stress induction near the surface due to the large difference between the ionic radii of the exchanged ions. For example, graded refractive index profiles are formed in BK7 glasses by introducing K^+-Na^+ ions and in lime-sodium glass by introducing Ag^+-Na^+ ions [49]. It should be noted that the profiles obtained by thermal diffusion of metal ions into the glass substrate are close to the exponential profile [50].

Thus, by combining different semiconductor crystals in a composite waveguide, it becomes possible to obtain a nonlinear optical layer on one side of the waveguide interface and a layer with a graded dielectric permittivity profile on the other.

CONCLUSIONS

In the present work, a theoretical description of waveguide properties of the interface between two media with fundamentally different optical characteristics is presented. The formulated flat waveguide model is applicable to media having an arbitrary distribution of the spatial dielectric permittivity profile.

An analytical expression describing the surface wave propagating along the interface between the medium with a stepwise nonlinearity and the gradient layer with an arbitrary dielectric permittivity profile is obtained. Analytical expressions for surface waves propagating along the interface of the medium with Kerr nonlinearity (both self-focusing and defocusing) with graded-index media characterized by exponential and linear dielectric permittivity profiles are also presented.

The above analysis of materials shows that it is possible to select semiconductor crystals on which composite waveguides combining nonlinear optical layers and gradient layers with spatial distribution of optical characteristics may be based.

The results obtained in this work may be of value in designing elements of optical devices based on the possibility to control the localization of light beams along the waveguiding surfaces of the interface of contacting media.

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