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RESEARCH ARTICLE

Modeling of spatial spread of COVID-19 pandemic waves in Russia using a kinetic-advection model

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Abstract
Objectives. COVID-19 has a number of specific characteristics that distinguish it from past pandemics. In addition to the high infection rate, the high spread rate is due to the increased mobility of contemporary populations. The aim of the present work is to construct a mathematical model for the spread of the pandemic and identify patterns under the assumption that Moscow comprises the main source of viral infection in Russia. For this purpose, a two-parameter kinetic model describing the spatial spread of the epidemic is developed. The parameters are determined using theoretical constructions alongside statistical vehicle movement and population density data from various countries, additionally taking into account the development of the first wave on the examples of Russia, Italy and Chile with verification of values obtained from subsequent epidemic waves. This paper studies the development of epidemic events in Russia, starting from the third and including the most recent fifth and sixth waves. Our two-parameter model is based on a kinetic equation. The investigated possibility of predicting the spatial spread of the virus according to the time lag of reaching the peak of infections in Russia as a whole as compared to Moscow is connected with geographical features: in Russia, as in some other countries, the main source of infection can be identified. Moscow represents such a source in Russia due to serving as the largest transport hub in the country.
Methods. Mathematical modeling and data analysis methods are used.
Results. A predicted time lag between peaks of daily infections in Russia and Moscow is confirmed. Identified invariant parameters for COVID-19 epidemic waves can be used to predict the spread of the disease. The checks were carried out for the wave sequence for which predictions were made about the development of infection for Russia and when the recession following peak would occur. These forecasts for all waves were confirmed from the third to the last sixth waves to confirm the found pattern, which can be important for predicting future events.
Conclusions. The confirmed forecasts for the timing and rate of the recession can be used to make good predictions about the fifth and sixth waves of infection of the Omicron variant of the COVID-19 virus. Earlier predictions were confirmed by the statistical data.
Keywords: kinetic equation, COVID-19, wave propagation

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НАУЧНАЯ СТАТЬЯ

Моделирование пространственного распространения волн пандемии COVID-19 в России на основе кинетико-переносного описания

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Резюме

Цели. Пандемия COVID-19 обладает рядом важных особенностей по сравнению с прошлыми эпидемиями. Помимо высокой степени заражения, она имеет высокую скорость распространения за счет мобильности населения, связанной, в частности, с возросшей скоростью средств передвижения. Целью данной работы является построение математической модели распространения пандемии и выявление закономерностей в предположении, что основным источником вирусной инфекции в России является г. Москва. Для этого строится двухпараметрическая кинетическая модель, описывающая пространственное распространение эпидемии. Параметры находятся с помощью теоретических построений, оценок известных данных о статистике передвижения транспортных средств и плотности населения в различных странах, а также с учетом развития первой волны на примере России, Италии и Чили с проверкой значений для последующих эпидемических волн. Исследуется возможность предсказывать скорость пространственного распространения вируса по временно́му интервалу запаздывания достижения пика заражений в России по сравнению с Москвой. Это связано с географическими особенностями: в России, как и в некоторых других странах, можно выделить основной источник распространения инфекции. Таким источником в России выступает г. Москва – крупнейший в стране транспортный узел. Для реализации цели в настоящей работе изучается развитие эпидемических событий в России, начиная с 3-й, и вплоть до последних 5-й и 6-й волн.

Методы. Использованы методы математического моделирования и методы обработки статистических данных.

Результаты. Подтверждено, что величина запаздывания достижения пика заражений составляет в среднем 2.5 недели. Выявлена сохраняемость параметров для различных волн, поэтому модель обладает предсказательными возможностями. Проверки проводились для последовательности волн, для которых делались соответствующие предсказания о развитии заражения для России в целом и о том, когда произойдет спад. Данные прогнозы подтвердились для всех волн, начиная с 3-й, и вплоть до последней 6-й волны, что подтверждает найденную закономерность, важную для прогнозирования будущих событий.

Выводы. Прогнозы о начале и скорости выздоровления подтвердились, что дало возможность уверенно прогнозировать, в частности, протекание 5-й и 6-й волн пандемии, связанной с новым вирусным штаммом «омикрон». Предсказания, которые делались заранее, были проверены и получили подтверждение.

Ключевые слова: кинетическое уравнение, COVID-19, распространение волн

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INTRODUCTION

As well as representing a social and medical challenge, the complexity of the COVID-19 pandemic phenomenon entailed the development of novel scientific approaches for its study. This led to an intensification of research activity involving numerous laboratories around the world. The contemporary development of information technology allows large volumes of data to be processed quickly. In addition to more complex mathematical models using powerful computer resources, simple models can also play a significant role in describing the spread of the virus. The development of the COVID-19 pandemic differs from previously known epidemics in a number of ways, some of which can be attributed to the increased rapidity of modern transportation patterns. The aim of this work is to simulate the processes of spatial spread of the pandemic according to a study of rapid virus transport scenarios rather than the diffusion-type equations used in traditional methods on the basis of slow contact processes (see [1–3]).

There are a number of works in which statistical methods are among those used to study the development of epidemics [4–14]. For the most part, susceptible–infected–recovered (SIR) and susceptible–exposed–infected–recovered (SEIR) models are used for studying spatial–local processes occurring in time. However, the conjunction with spatial development in such works is infrequent. Therefore, in the present work, we investigate only the spatial propagation of the epidemic.

For this purpose, a kinetic model is constructed based on model equations similar to those used to study various physical processes [15, 16].

The present work comprises a continuation, development, and generalization of our earlier research [17], in which common features of the method were determined by considering and studying the first and second pandemic waves. The developed model is applied to study all subsequent waves of the COVID-19 pandemic in Russia. Using the proposed kinetic approach, the nature of the contemporary pandemic spread it is investigated for the countries to which the one-dimensional model is applicable. For the first wave of the pandemic, processes were studied in Italy, Chile and Russia, while the study of the second wave was mainly limited to Russia. These countries are assumed to comprise major centers from which the spread of infection originated, which in turn determines the delay in disease development in individual regions and across the entire country. This assumption is used to make predictions for the unfolding of subsequent pandemic waves, as demonstrated by the spread of such waves in Russia.

It is possible to distinguish between two mechanisms of infection: transportable (studied in the present work) and contact, realized in the regions. The superposition of these two factors gives the sum of the number of infections. The first phase, corresponding to transmissible infection, lays the foundations for the next phase, consisting in the development of contact-related infection. We study the spread of carriers of the virus, which is associated with the use of various types of vehicles for transportation. Average travel speeds take into account differences between airplane, train, bus, and car transportation modes. Another model parameter consists in resistance to the advancement of virus carriers, which has the dimension of the frequency of disembarkation of passengers from vehicles during movement to their places of residence. This parameter mainly depends on population density. The same character of the course of the disease is subsequently assumed in different places. The third phase is associated with the spread of recovery, which is determined by reaching a maximum of new infections per day for each region.

The geographical features of Russia allow a simple one-dimensional model to be applied as for Italy and Chile (the pandemic spread was considered for these countries for the first wave). At the same time, taking into account the nature of infection for Russia, the main source in all waves is identified with Moscow (the regions to the east of the capital were studied). This forms a basis for judgements about the shift in the time of the beginning of recovery.

The third wave was associated with the emergence of the delta virus strain, which mostly entered Russian from India through Moscow airports. While the fourth wave was similar to the second in being mainly caused by people returning from summer vacations, in this case Moscow was also the main source of the new wave development. However, since other cities also contributed, the development of the processes was somewhat blurred in comparison with the development of the processes in the first and third waves. The fifth wave has its own characteristics, since the explosive nature of the infection with the Omicron strain can lead to some corrections in the spatial spread, accelerating this process to some extent.

The general patterns determined in the first wave and confirmed in the second wave remain unchanged due to the accepted accounting of infection associated with the movement of transport of infected passengers leading to a repetition of the pandemic wave spread pattern. Here, the average rate of spread depends not on the intensity of infection by new viruses, but the average speed of vehicles. Therefore, the model is also applied to describe subsequent pandemic waves. This also applies to the most recent sixth wave.

This study uses a one-dimensional kinetic model to predict and then verify the propagation patterns of the third, fourth, fifth, and sixth waves of the pandemic. A lag of the maximum infection in Russia as a whole compared to the maximum infection in Moscow by about 2.5 weeks was detected (taking into account the inevitable statistical error, the maximum is within two to three weeks). Thus, the timing of the pandemic wave in Russia can be judged from the development of the disease in Moscow. While infection values per day vary from wave to wave, some invariants of the waveform can be identified. Therefore, an attempt is made to find functions describing recurring forms of infection waves, which take into account different infection intensities for different strains—and, accordingly, different amplitude of fluctuations in the number of infected per day.

We considered whether the duration of the incubation period of infection affects the presented results. For the first wave, the period was seven to ten days, while for subsequent waves it was the same or less. However, this value is the same for Moscow and Russian regions, so there is practically no effect of this parameter on the lag time of infection.

DEVELOPED TRANSFER-KINETIC MODEL AND ITS ADJUSTMENT

Here is a brief description of the one-dimensional two-parameter advection model, taking into account the kinetic term:

$$\frac{\partial n}{\partial t} + U\frac{\partial n}{\partial x} = -\sigma n(t,x), \qquad (1)$$

where t is time; x is distance; $n(t, x)$ is the density of moving virus carriers in the vehicle; U is the average speed of the vehicle; σ is the resistance factor (having the dimension of frequency) to the movement of infected elements mainly due to dropping off passengers from vehicles at places of residence.

The initial condition for the Cauchy problem is taken as:

$$n_0(x) = H(-x),$$

where $H(x)$ is the Heaviside function. This formulation of the problem means that in fact the virus carriers enter the studied area $x > 0$ through the boundary $x = 0$. The linear equation (1) is solved by standard methods; the analytical solution has the form:

$$n(t,x) = n_0(x-Ut)\mathrm{e}^{-\sigma\frac{x}{U}}. \qquad (2)$$

We denote the density of virus carriers disembarking from the vehicle at a given point as $n_M(t, x)$. This density grows in the same way $n(t, x)$ decreases, hence, we can write as follows:

$$\frac{dn}{dt} + \frac{dn_M}{dt} = 0,$$
$$\frac{dn}{dt} = \frac{\partial n}{\partial t} + U\frac{\partial n}{\partial x},$$
$$\frac{dn_M}{dt} = \frac{\partial n_M}{\partial t}.$$

Note that the total and partial derivatives of n differ from each other due to the progression of elements (virus carriers) with the speed U. For n_M, these derivatives are coincident, since the elements with this density are stationary.

Given (1), the equation for $n_M(t, x)$ takes the following form:

$$\frac{\partial n_M}{\partial t} = \sigma n. \qquad (3)$$

The solution to this equation is written as follows

$$n_M(t,x) = n_{M_0}(x) + \int_0^t \sigma n(\tau,x)\,d\tau.$$

By substituting expression (2) and the initial conditions and $n_{M_0}(x) = 0$ and $n_0(x) = H(-x)$, we obtain an expression for $n_M(t, x)$

$$n_M(t,x) = \frac{\sigma}{U}(Ut - x) \cdot H(Ut - x) e^{-\sigma \frac{x}{U}}. \quad (4)$$

In a large series of calculations, it was sometimes more convenient to use a numerical approach in place of an analytical expression; here, a simple Courant–Isaacson–Rees scheme is applied.

The question of specifying the parameters U and σ is important. The estimation of the first parameter takes the value of the second to be equal to that obtained in our previous works. Using the values of the first of these parameters obtained in the study of the first and second waves, predictions are made about the speeds of the pandemic spread and convalescence waves, respectively. The theoretically estimated average generalized speed of movement vehicle is compared it with the real data on the spread of the disease; here, it is deemed acceptable to make some corrections to the resulting speed parameter for subsequent waves. We consider four main modes of transportation: car, bus, train, and airplane (while bicycles, motorcycles, scooters, and walking also contribute, for moving a large numbers of passengers over appreciable distances, these comprise the main modes of transport). It is necessary to introduce some weight averaging, taking into account the relative proportions of passengers using one or another vehicle type.

More accurate quantitative estimates were also obtained for the average weighted speed of trains. According to the Russian Railways press release[1], $9.614 \cdot 10^8$ passengers were transported by short-distance trains in Russia in 2021, who travelled a total of $2.9 \cdot 10^{10}$ km. Long-distance trains carried $9.2 \cdot 10^7$ passengers, who traveled a total of $7.44 \cdot 10^{10}$ km. Thus, on average, each passenger travelled 30.16 km on short distance trains and 808.7 km on long distance trains. According to the Ministry of Transport of the Russian Federation[2] the average speed of trains in Russia in 2021 is in the range from 57 to 65 km/h, i.e., passengers of both short-distance and long-distance trains on average reached their destination in less than a day. Since the average distances obtained earlier can be assumed to be covered in a day, the average speed of short-distance trains is taken to be 30.16 km/day, and long-distance trains—808.7 km/day. As a result, the average speed of trains in Russia in 2021 is

$$U = \frac{9.614}{9.614 + 0.92} \cdot 30.16 + \frac{0.92}{9.614 + 0.92} \cdot 808.7 = 98.15 \text{ km/day}.$$

One would expect the real data to be close to this value, since this was the case for all of the studied pandemic waves. Thus, the approximation of the weighted average speed of the vehicle and the speed of the epidemic spread in the first few days is true for all waves. Differences of 10–15% lie within statistical error, confirming the assumed possibility of using the U parameter in the model.

Note that some semi-empirical calculations based on an analysis of annual passenger turnover values for other vehicles confirm the above estimates, but we do not cite them because they require more careful consideration, which is not the purpose of this article.

SPREAD OF THE THIRD WAVE OF THE COVID-19 PANDEMIC IN RUSSIA

The results obtained in [17] are used to analyze the development of subsequent waves. First of all, let us consider the character of the third-wave propagation. Firstly, in [17] the data of pandemic development for Moscow are given. Once the maximum of infections for a day is reached, the beginning of recovery is judged by the calendar number corresponding to the maximum. A prediction is made as to the possible day of reaching such a maximum and the beginning of recovery for Russia. This hypothesis is then tested.

According to the data processing in [17], the value of the parameter of the average speed of vehicles was obtained. This speed was estimated as U = 75–90 km/day, which is close to the above estimates of average speed with an accuracy of 10% for the value of 90–100 km/day.

The time of movement of the wave from Moscow to the center of masses of the Russian population can be calculated. According to the density distribution graph given in [11], this point approximately corresponds to a distance of 1000–1200 km from Moscow; therefore, a significant outbreak in Moscow can be expected to affect the whole of Russia in about two weeks. However, since this is a relatively crude estimate, such a lag can in practice be expected to extend for up to three weeks. Then, for example, the maximum infection rate per day in Moscow appears about two weeks earlier than the infection peak in Russia as a whole. Since this characteristic point is interpreted as the beginning of recovery, the recovery wave also has a corresponding lag.

[1] More than 1 bn passengers carried on the Russian Railways network in 2021 | Press releases | Company (rzd.ru). https://company.rzd.ru/ru/9397/page/104069?id=269758. Accessed December 21, 2022 (in Russ.).

[2] Average speed of passenger trains in Russia may grow to 65 km/h by 2031. Ministry of Transport of the Russian Federation. https://mintrans.gov.ru/press-center/branch-news/595. Accessed December 21, 2022 (in Russ.).

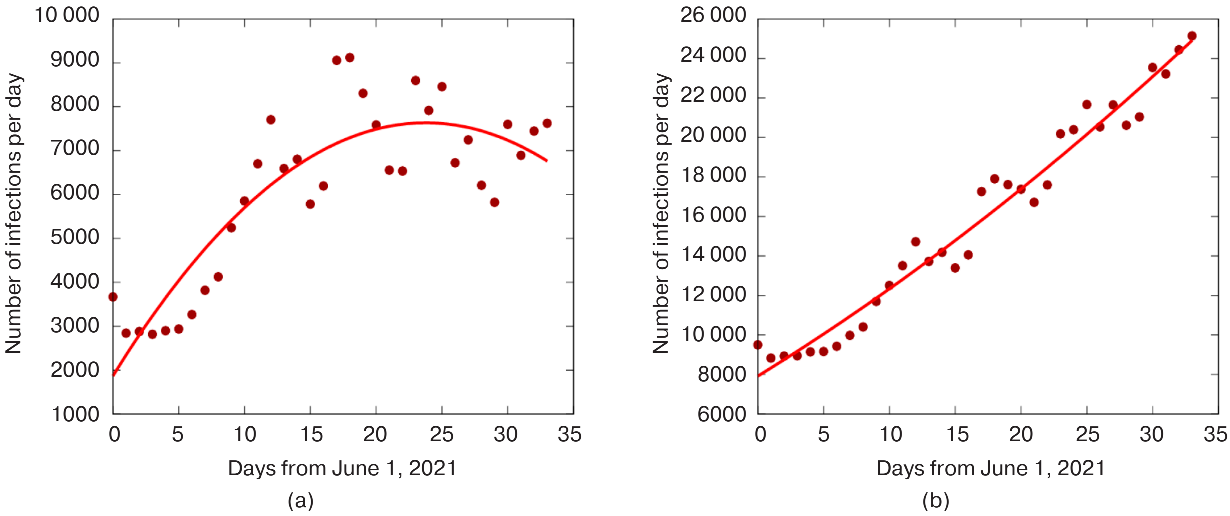


Fig. 1. Number of infections per day for Moscow (a) and Russia (b) in June 2021

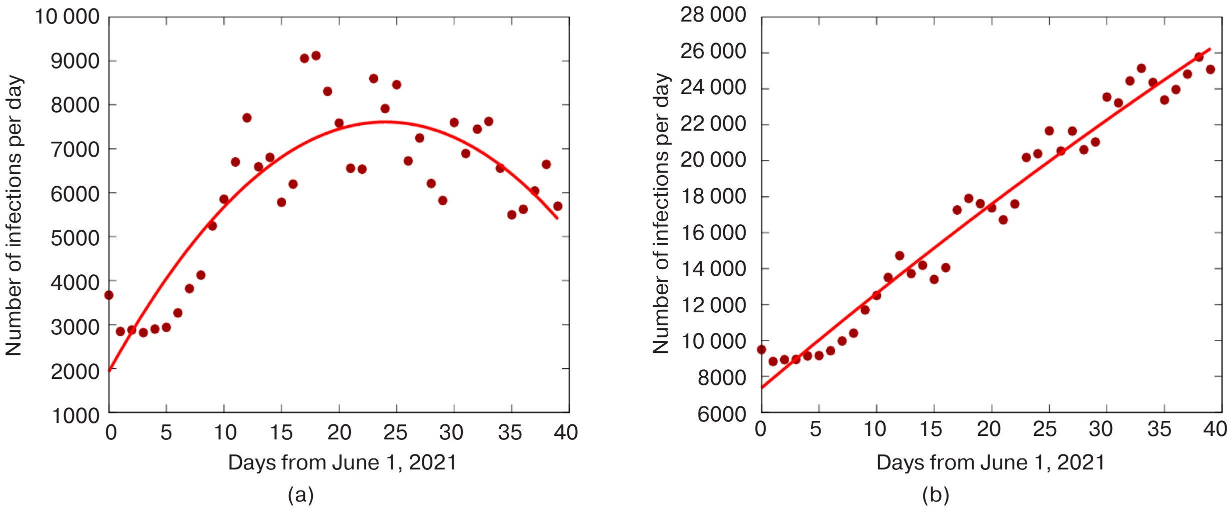


Fig. 2. Number of infections per day for Moscow (a) and Russia (b) in June–July 2021

The diagrams (Fig. 1) concerning the month of June 2021 (the data until June 29th) are constructed on the basis of the data on the number of infections in Moscow and Russia as a whole. It can be seen that the maximum of infection for Moscow passed at the end of the month (according to the applied mean square approximation), while for Russia there was only an increase in the number of infections per day. Therefore, we could expect that the maximum infection in Russia would be reached by the middle of July.

Figure 2 shows infection data for Moscow and Russia as of July 11, which is 12 days later than the previous graphs in Fig. 1. We can see a clearly formed "hump" of infections in Moscow, while in Russia as a whole, the maximum is only beginning to emerge.

Figure 3 shows the expected development parabola of events (number of infections in Russia) based on the received data up to July 15, 2021. So, it seems that the parabola line would have to be clarified, since only an extrapolation is given.

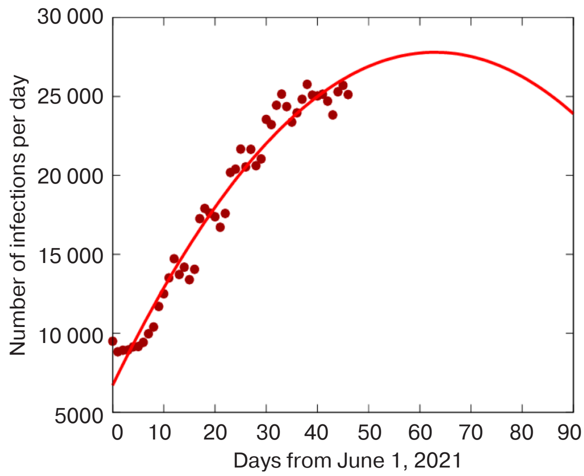


Fig. 3. Expected number of infections for the third wave for Russia based on the received data until mid-July 2021

We can conclude that the predictions turned out to be justified. The maximum for Moscow was reached by June 25. Therefore, we assumed that the maximum

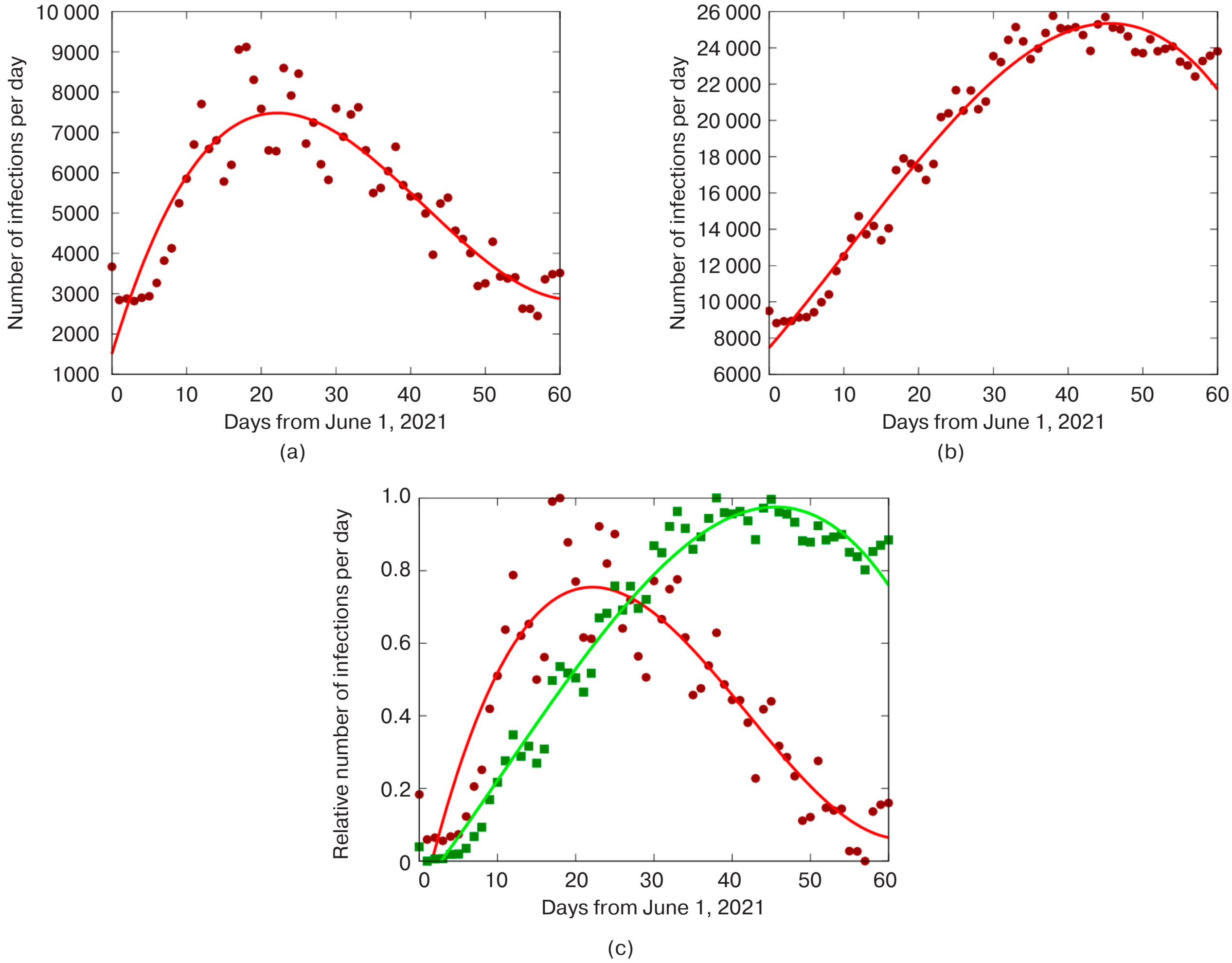


Fig. 4. Real data and averaging lines by LSM: total number of infected for Moscow (a) and for Russia as a whole (b); relative number of infections for Moscow and for Russia as a whole (c)

for Russia should be reached (taking into account the parameters obtained from the study of previous pandemic waves) in about two to three weeks, i.e., by mid-July. The curves constructed using the least squares method (LSM) corresponded to the forecast. But it is interesting that "peaks," i.e., absolute maximums of infections in Moscow and Russia are displaced by this very value: they occur around June 25 and July 15 respectively (generally speaking, such values are not sufficiently representative due to not very representative statistics and random outliers). As time passes and new data become available each day, the LSM parabola shifts somewhat to the right, a trend that has been noted before. But the magnitude of the lag remains the same: the maxima are now July 5 and July 25, respectively.

For the third wave, the shift in the time of onset of recovery-maximum infection for Moscow and Russia as a whole is three weeks. The maximum for Moscow in Fig. 4 corresponds approximately to the beginning to the middle of the third decade of June, while the maximum for Russia as a whole corresponds approximately to the middle of July. The graphs in Fig. 4c present relative values of infections, which were calculated according to the formula $A_{rel} = (A - A_{min})/(A_{max} - A_{min})$, where A_{max} and A_{min} mean the maximum and minimum of this value, respectively (Figs. 4a and 4b).

DEVELOPMENT OF THE FOURTH WAVE OF THE PANDEMIC

The regularities identified for the previous waves were used to predict the behavior of the fourth wave. In general, the predictions were confirmed as applying both to the nature of the infection curves and to the shifts of Russia relative to Moscow.

For the fourth wave, the dates of maximum infections for Moscow and Russia as a whole were late October and mid-November, separated by two to three weeks (Fig. 5). Moreover, this shift for the fourth wave is somewhat less than for the third wave, which can be explained by the return of patients from vacations not only through Moscow, but also through other cities.

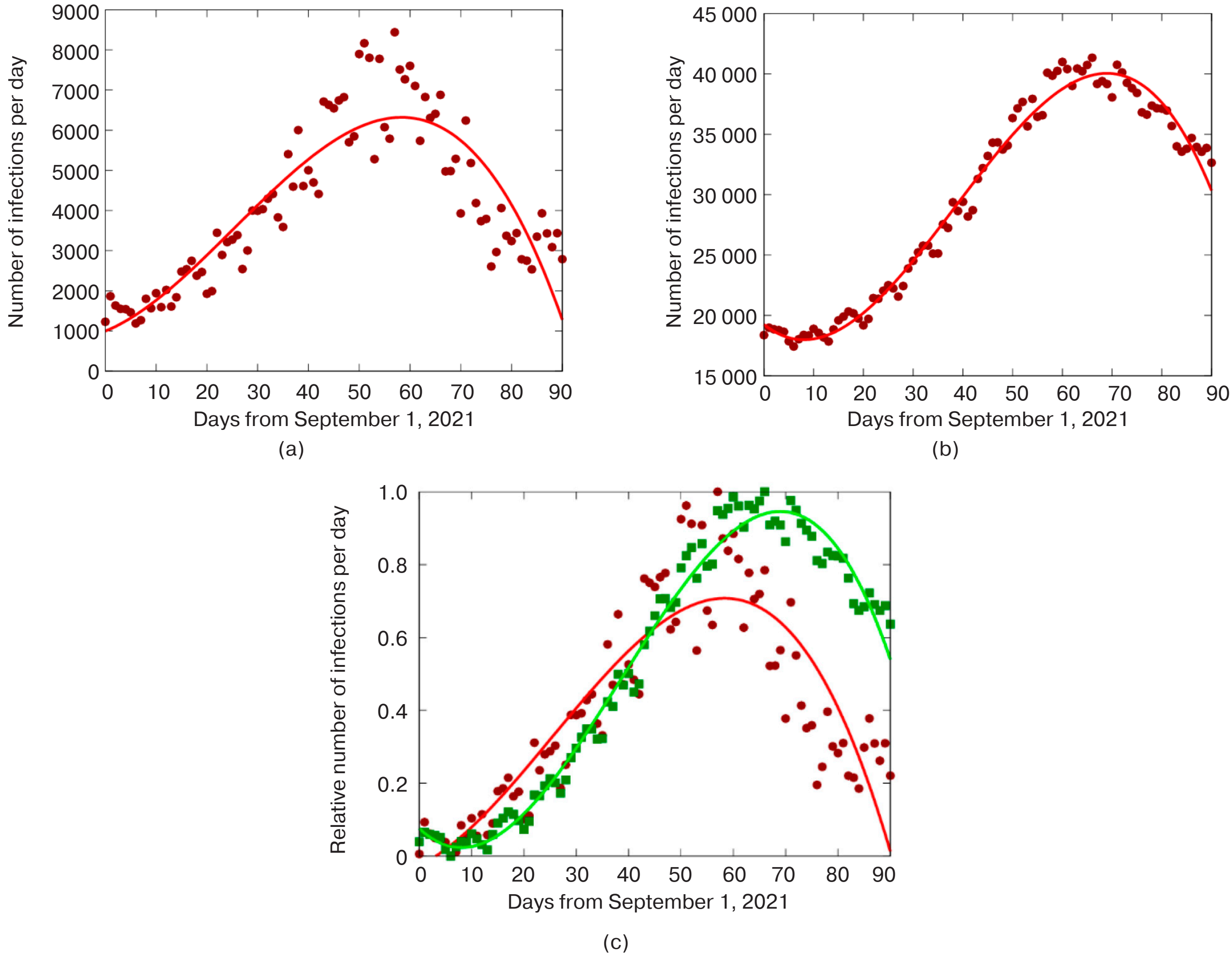


Fig. 5. Real data and averaging lines by LSM, total number of infected for Moscow (a) and for Russia (b), relative number of infections for Moscow and for Russia (c)

MODEL VALIDATION FOR THE FIFTH WAVE OF THE PANDEMIC

Similar processes for the fifth wave are considered. Note that on February 10, 2022, the method of counting sick people changed, which increased the readings, so a correction factor was added to the calculations. Due to the new Omicron strain, the infection was more intense, but the spatial spread was not expected to be much affected. It was assumed that since the maximum for Moscow corresponded to approximately February 1, the maximum for Russia would be around February 14.

The development of this wave is characterized by its own peculiarities. Here possible sharp outbreaks of local infection, particularly in Moscow, correspond to the greater infectability of this strain. Thus, Fig. 6, which shows the number of infections in the first three weeks of the new wave, we can see that the number of cases per day sharply increased starting from January 18. Thus, the expected manifestation of the disease for Russia as a whole involves a two- to three-week lag, which was indeed from around February 3.

The graph in Fig. 6 gives the number of cases per day for Russia without taking into account the Moscow figures.

Based on the results of January, the predicted decline in infection (the beginning of recovery) for Russia occurred by mid-February (Fig. 7). However, the question as to whether the speed of spread of the spatial wave is affected by the nature of the virus remains open. Figure 7 shows graphs of absolute increments of infections per day, as well as more indicative relative values of infections.

The build-up of relative values can help to identify universal properties of the model.

Figure 8 shows the results of the number of infections per day based on actual data as of mid-February. As expected, the maximum of infections across Russia as a whole is formed by mid-February. We note a certain outlier in the data associated with the official recalculation of statistics. The forecast of a shift of 12–14 days was confirmed.

The distributions for Moscow and Russia at the end of February were also built-up (Fig. 9). The data indicate

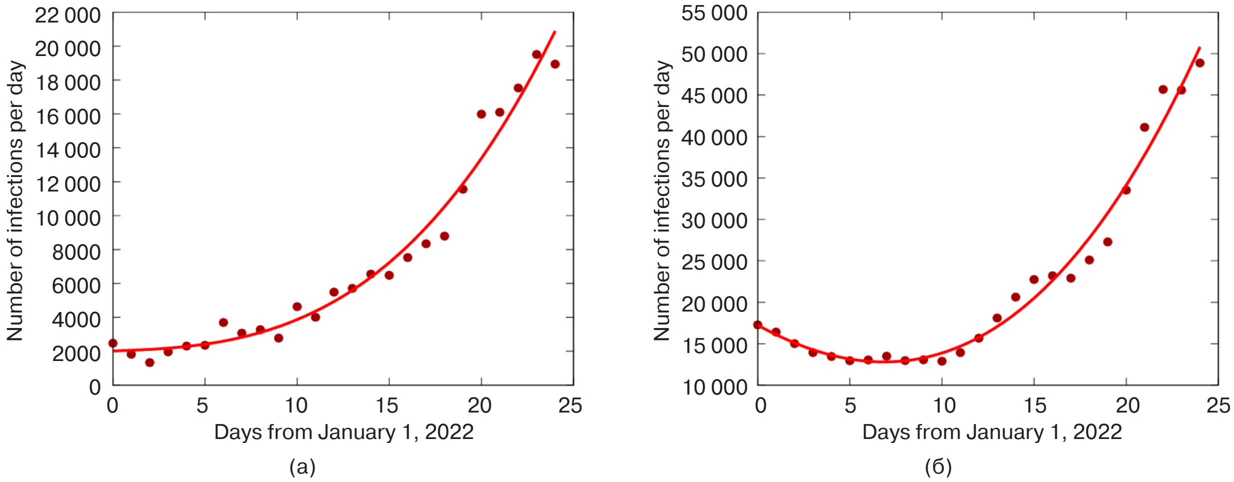


Fig. 6. Number of infections (absolute value) per day in Moscow (a) and in Russia (b)

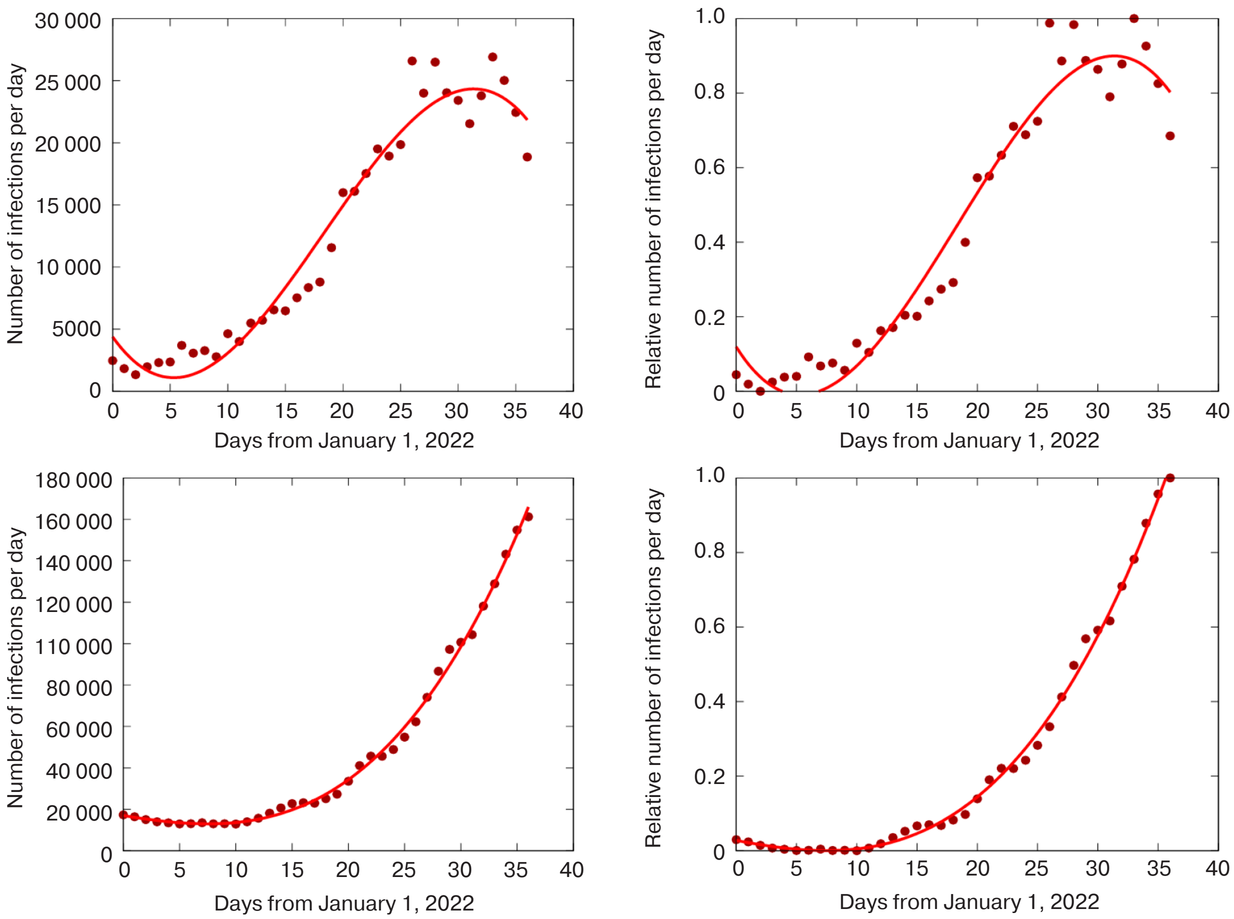


Fig. 7. Number of infections per day from January 1 to February 7, 2022, upper figures refer to Moscow, lower figures refer to Russia excluding Moscow figures

that the curves of the fifth wave have already been formed. The lines resemble those presented in Figs. 7 and 8.

Thus, the predictions for the fifth wave made in late January are confirmed by real data.

It is useful to compare the curves for the relative values in the fourth and fifth pandemic waves (Fig. 10). Let us note some differences in the character of the lines, which is also related to the properties of the LSM used in the averaging. For the fourth wave, the smooth graph for Russia has a noticeably higher maximum than the graph for Moscow, which is associated with a relatively slower decline in the number of infections in Russia compared

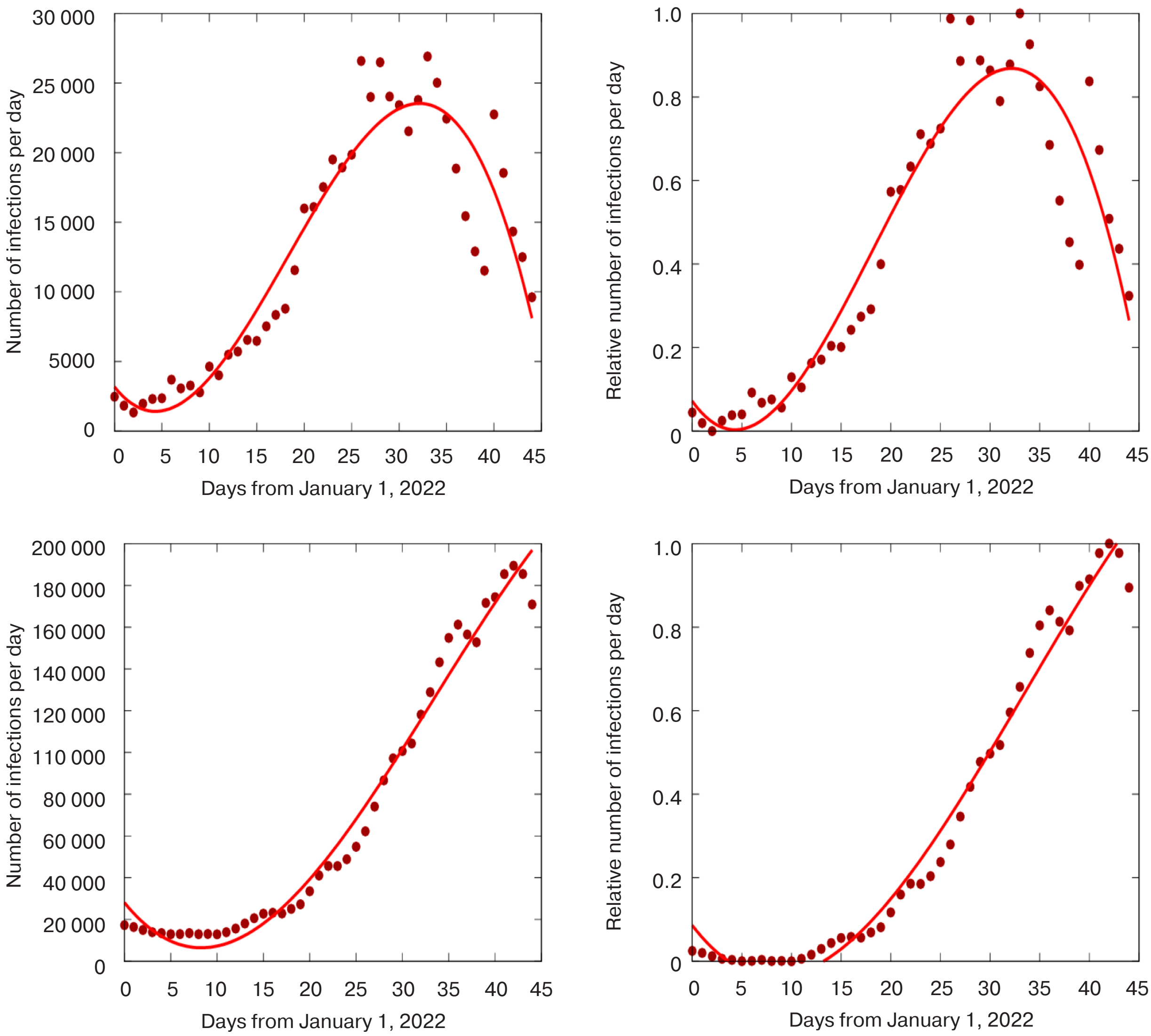


Fig. 8. Pandemic development in Moscow (top) and Russia without Moscow indicators (bottom) from January 1 to February 15, 2022

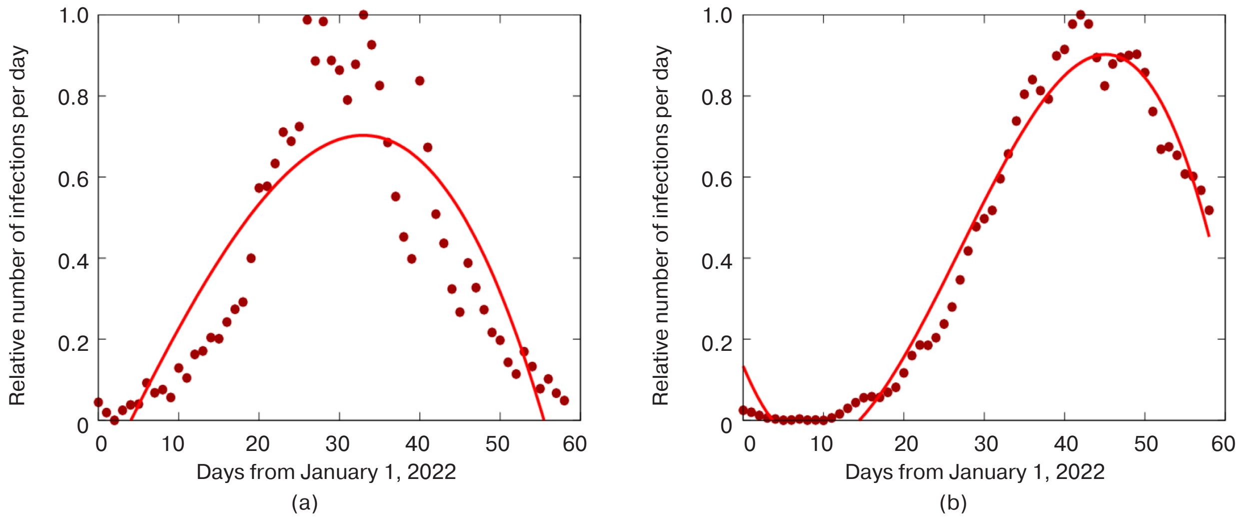
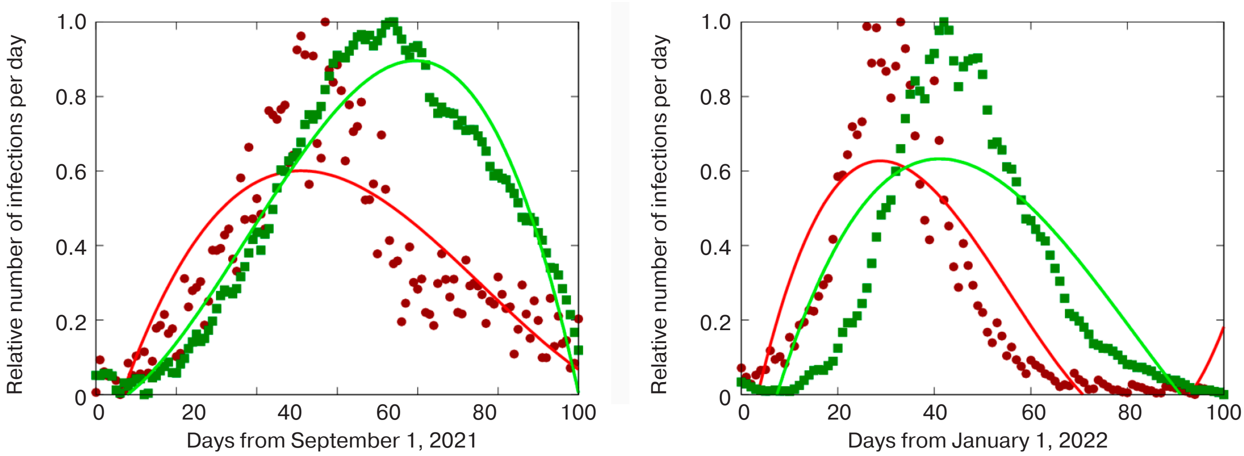


Fig. 9. Development of the pandemic in Moscow (a) and Russia without Moscow indicators (b) up to early March



(a) (b)

Fig. 10. Distribution of the number of infections for the fourth (a) and fifth (b) pandemic waves in Moscow (red lines) and Russia without regard to Moscow indicators (green lines)

to Moscow after the peak of infections of the fourth wave. For the fifth wave, the plots constructed with the help of LSM have a similar amplitude. However, the time shifts of the local maxima for these different waves are approximately the same.

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Along with the values determining the shift of maximum waves in different spatial points of the country, we can also try to identify patterns in the form of different pandemic waves in individual points—e.g., in the city of Moscow—taking into account changes in the nature of infection in new epidemic waves.

The construction of some forms realized in successive pandemic waves is an important task, which partly overlaps with the topic of works [1–3].

To do this, we use the patterns obtained earlier. Figure 11 shows the expected infection curves in the third wave, taking into account the data on the second and the first waves. We can judge from them how accurate the forecast will be. A curve based on the method of least squares was plotted according to the real data for the second pandemic wave. This line is then moved to the right side of the figure and superimposed on the beginning of the infection points in the third wave. This sets the prediction of the maximum infection that was expected by about July 15, 2021. Here we also plotted the LSM curve using the new data obtained for the third wave of the pandemic. According to this graph, the expected maximum should also be reached around July 15. Figure 11 shows the corresponding lines; here however, the points are plotted up to July 15. We can conclude that these obtained curves approximately correspond to the forecasts. Although the shape of the curves is different, they have certain features in common.

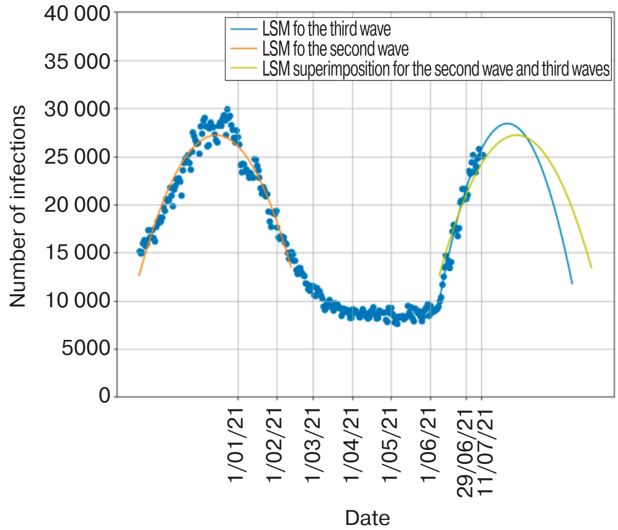

Fig. 11. Superimposition of the curve obtained for the second wave on the expected curve of the third wave up to June 15

Figure 12 shows the curves for the relative number of infections as a function of time. Real data with characteristic statistical outliers (a) and lines averaged by the LSM method (b) are shown. These are shifted so that the maxima correspond to zero in time. For each wave, the day with the maximum number of infections and 60 days before and after (except the fifth wave) are taken. In the scaling for each time interval, the minimum turned out to be 0, while the maximum was 1. Here, while the wave profiles correspond fairly closely, we note the exception of the fifth wave, which may be attributed to the strong intensity of infection spread.

The identified similarity of wave forms indicates a certain universalism in the development of this infection in Russia, reflecting the situation in Moscow, allowing the cautious prediction of subsequent waves, taking into account the varying intensity of strains and vaccination.

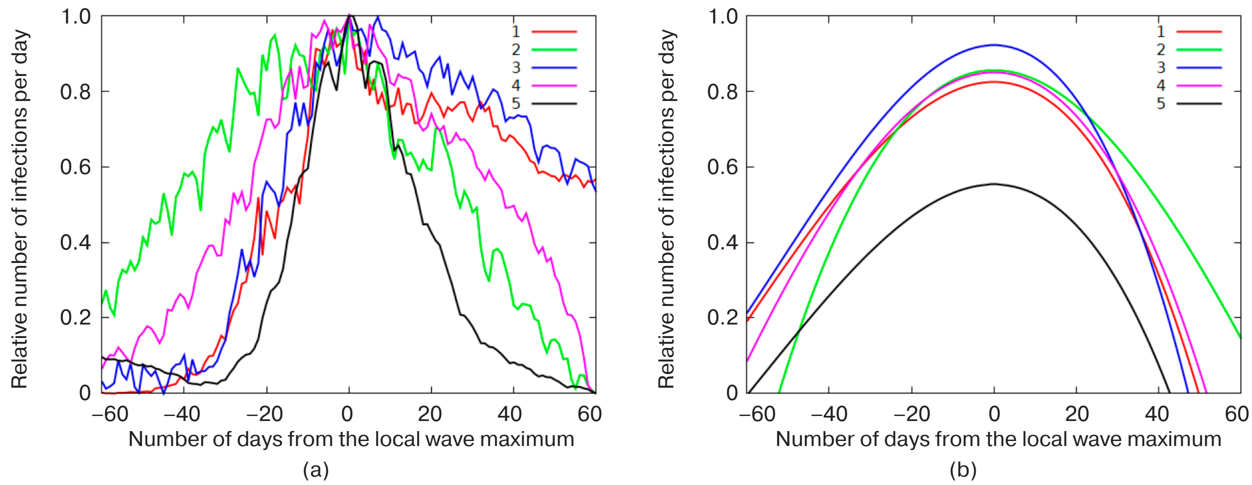

Fig. 12. Real data for all five waves (a) and smoothing curves for them of the third degree of LSM (b)

CONCLUSIONS

The present work has demonstrated the suitability of the previously proposed model to describe the propagation of modern pandemic waves over the territory of Russia. Despite differences in the features of the succeeding pandemic waves, the basic regularities apply. Thus, the model can be used to predict the parameters of future possible waves. Assumptions made in August 2022 for the new sixth wave were fully confirmed: the sixth wave developed similarly to the previous ones, including the lag of about 2.5 weeks between Moscow and Russia as a whole. The maximum infection rate per day was reached by the middle of the third decade of August, while that for Russia as a whole was clearly recorded by the middle of September.

At present, the authors are studying a two-dimensional problem in terms of the corresponding numerical scheme, which will allow other main centers of infection to be taken into account, primarily Saint Petersburg.

Authors' contribution. All authors equally contributed to the present work.

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