UDC 534-16
https://doi.org/10.32362/2500-316X-2023-11-2-92-99

RESEARCH ARTICLE

Identification of a longitudinal notch of a rod by natural vibration frequencies

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Abstract
Objectives. To study the direct and inverse problem of vibrations of a rectangular rod having a longitudinal notch, to analyze regularities of the behavior of natural frequencies and natural forms of longitudinal vibrations when changing the location and size of the notch, and to develop a method for uniquely identifying the parameters of the longitudinal notch using the natural frequencies of longitudinal vibrations of the rod.
Methods. The rod with a longitudinal notch is modeled as two rods, where the first one does not have a notch, while the second one does. For connection, conjugation conditions are used, in which longitudinal vibrations and deformations are equated. The solution of the inverse problem is based on the construction of a frequency equation under the assumption that the desired parameters are included in the equation. Substituting natural frequencies into this equation, the nonlinear system with respect to unknown parameters is derived. The solution of the latter is the desired notch parameters.
Results. Tables of eigenfrequencies and graphs of eigenforms are given for different notch parameters. The results for different boundary conditions are obtained and analyzed. A method for identifying notch parameters by a finite number of eigenfrequencies is presented. The inverse problem is shown to have two solutions, which are symmetrical about the center of the rod. The unambiguous solution requires eigenfrequencies of the same problem with different boundary conditions at the right end. By adding additional conditions at the ends of the rod, the inverse problem can be solved with new boundary conditions to construct the exact solution and develop an algorithm for checking the uniqueness of the solution.
Conclusions. The developed method can be used to solve the problem of identification of geometric parameters of various parts and structures modeled by rods.

Keywords: longitudinal vibrations, natural frequency, eigenform, rod, identification problem, direct problem, inverse problem, Sturm–Liouville problem, boundary conditions

• **Submitted:** 08.11.2021 • **Revised:** 29.11.2022 • **Accepted:** 22.01.2023

For citation: Utyashev I.M., Fatkhelislamov A.F. Identification of a longitudinal notch of a rod by natural vibration frequencies. *Russ. Technol. J.* 2023;11(2):92–99. https://doi.org/10.32362/2500-316X-2023-11-2-92-99

Financial disclosure: The authors have no a financial or property interest in any material or method mentioned.

The authors declare no conflicts of interest.

НАУЧНАЯ СТАТЬЯ

Идентификация продольного надреза стержня по собственным частотам колебаний

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Резюме
Цели. Цели работы: рассмотреть прямую и обратную задачу о колебании прямоугольного стержня с продольным надрезом; исследовать закономерности поведения собственных частот и собственных форм продольных колебаний при изменении места и размера надреза; разработать метод, позволяющий однозначно идентифицировать параметры продольного надреза с помощью собственных частот продольных колебаний стержня.
Методы. Стержень с продольным надрезом моделируется как два стержня, где первый не имеет надреза, а второй – имеет. Для соединения используются условия сопряжения, в которых приравниваются продольные колебания и деформации. Решение обратной задачи основано на построении частотного уравнения в предположении, что искомые параметры входят в уравнение. При подстановке собственных частот в это уравнение получим нелинейную систему относительно неизвестных параметров. Решение последнего есть искомые параметры надреза.
Результаты. Приведены таблицы собственных частот и графики собственных форм для разных параметров надреза. Получены и проанализированы результаты для различных краевых условий. Представлен метод идентификации параметров надреза по конечному числу собственных частот. Показано, что обратная задача имеет два решения, симметричных относительно центра стержня. Для однозначного решения требуются собственные частоты той же задачи с другими граничными условиями на правом конце. Добавление дополнительных условий на концах стержня позволило решить обратную задачу с новыми краевыми условиями, дающими возможность построить точное решение и разработать алгоритм проверки однозначности решения.
Выводы. Разработанный метод позволяет решить задачу идентификации геометрических параметров различных деталей и конструкций, моделируемых стержнями.

Ключевые слова: продольные колебания, собственная частота, собственная форма, стержень, задача идентификации, прямая задача, обратная задача, задача Штурма – Лиувилля, граничные условия

• **Поступила:** 24.07.2022 • **Доработана:** 06.09.2022 • **Принята к опубликованию:** 22.01.2023

Для цитирования: Утяшев И.М., Фатхелисламов А.Ф. Идентификация продольного надреза стержня по собственным частотам колебаний. *Russ. Technol. J.* 2023;11(2):92–99. https://doi.org/10.32362/2500-316X-2023-11-2-92-99

Прозрачность финансовой деятельности: Авторы не имеют финансовой заинтересованности в представленных материалах или методах.

Авторы заявляют об отсутствии конфликта интересов.

INTRODUCTION

Vibrations are one of the most common forms of motion. The study of vibrations is of great practical importance both in terms of utilizing their positive properties in engineering and technology, as well as avoiding undesirable effects of vibrations by limiting their level. Important practical problems of structural dynamics can be solved on the basis of the profound study of different kinds of vibrations only [1]. Studying natural frequencies of vibrations is of great scientific and applied interest in many engineering problems.

Many studies are devoted to the identification of cracks [2–14]. Starting from studies [3–5], transverse open cracks are typically modeled according to spring conjugation conditions. In the contemporary literature, other conjugation conditions for modeling transverse cracks are also proposed [2, 6, and 7]. However, longitudinal cracks cannot be modeled by conjugation conditions. The study [2] proposes to solve the rod identification problem on the basis of changes in the moments of inertia around the axes and cross-sectional areas at the notch. Another study [8] gives the simplest model of longitudinal vibrations of the rod with nascent transverse cracks, determining natural frequencies of vibrations along with coordinates and dimensions of cracks by experimental values of natural frequencies. In [9], a rod with a rigidly fixed left end is considered; here, the fixation on the right end can be either free, or elastic or rigid. The first three natural frequencies for different cross-section profiles are given. The closest problem in terms of the formulation is given in [10], where the evolution of characteristics of natural longitudinal vibrations and natural shapes of a circular rod when increasing its cross-section defect is considered. The study [14] deals with the method of solving inverse problems of defectoscopy for rods performing longitudinal vibrations. Using numerical modeling, the use of several low frequencies for satisfactorily determining defect properties is demonstrated. In [15], experimental data is compared with different theoretical models for describing the longitudinal vibrations of a rod. In the present paper, the result is obtained for the case when the longitudinal notch does not run along the entire length of a rectangular rod. The results of the study can find application in the acoustic diagnostics of various rods, such as I-beams, rails, frame bridges, etc.

The results show that natural frequencies of longitudinal vibrations can be used to find the start point of the longitudinal notch of the rod along with its depth and width. The dependence of the problem output data on the input data can be determined by analyzing the graphs of the vibration natural forms.

DIRECT PROBLEM

We consider a homogeneous isotropic rectangular rod of length $L = 1$, density ρ and cross-sectional area F (Fig. 1). The boundary conditions are as follows: the rod is embedded on the left end and free on the right. The cross section of the rod has height H and width B. A longitudinal notch of depth h and width b is located from point x_c to the right end.

In order to determine the influence of the size and location of the notch start point on these frequencies, it is necessary to determine the natural frequencies of longitudinal vibrations of the rod. For clarity of the solution, natural forms of vibrations are constructed.

The equation of longitudinal vibrations may be described by the following equation [11, p. 189]:

$$EF\frac{d^2U(x,t)}{dx^2}+\rho F\frac{d^2U(x,t)}{dt^2}=0, \quad (1)$$

where $U = U(x, t)$ is the longitudinal displacement; EF is the bending stiffness of the rod; ρ is the rod density; F is the cross-sectional area of the rod.

The solution of Eq. (1) is sought in the form of $U(x, t) = y(x)\cos\omega t$. Then (1) may be reduced to the following equation:

$$y''+\lambda^2 y=0, \quad (2)$$

where spectral parameter $\lambda^2=\dfrac{\rho F\omega^2}{E}$. Since the rod to the left and right of point x_c has a different cross-sectional shape, the equations of longitudinal vibrations to the left and right of point x_c may be written in the following form:

$$y_-''+\lambda^2 y_-=0, \quad y_+''+\lambda^2 y_+=0, \quad (3)$$

where y_-, y_+ are longitudinal displacements to the left and right of the point x_c.

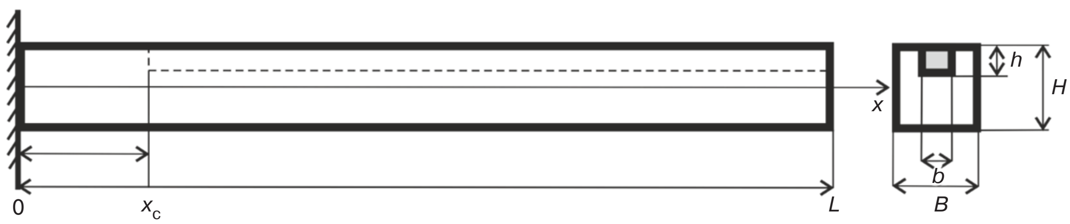

Fig. 1. Rod with a notch

The conjugation condition at point x_c for the rod sections may be written as follows:

$$EF_- \frac{dy_-}{dx} = EF_+ \frac{dy_+}{dx}. \quad (4)$$

We shall denote the ratio of areas $\frac{F_+}{F_-}$ by P:

$$F_+ = BH - bh, \quad F_- = BH, \quad P = \frac{F_+}{F_-}. \quad (5)$$

The conjugation conditions using P are written as follows:

$$y_-(x_c) = y_+(x_c), \quad y'_-(x_c) = y'_+(x_c)P. \quad (6)$$

Since the rod is embedded at the left end and free at the right end, the boundary conditions may be written in the following form:

$$y_-(0) = 0, \quad y'_+(1) = 0. \quad (7)$$

The general solution of equations (3) may be written as follows:

$$y_- = C_1^- \cos\lambda x + C_2^- \frac{\sin\lambda x}{\lambda},$$

$$y_+ = C_1^+ \cos\lambda x + C_2^+ \frac{\sin\lambda x}{\lambda}. \quad (8)$$

Substituting Eqs. (8) into (6) and (7), the following equation may be written:

$$y_-(0) = C_1^- 1 + C_2^+ 0 = C_1^- = 0. \quad (9)$$

From Eq. (9), $C_1^- = 0$.

$$y'_+(1) = -\lambda C_1^+ \sin\lambda + C_2^+ \cos\lambda = 0. \quad (10)$$

$$C_1^- + C_2^- \frac{\sin\lambda x_c}{\lambda} - C_1^+ \cos\lambda x_c - C_2^+ \frac{\sin\lambda x_c}{\lambda} = 0. \quad (11)$$

$$\begin{gathered} -\lambda C_1^- + C_2^- \cos\lambda x_c - \\ -P\left(-\lambda C_1^+ \sin\lambda x_c + C_2^+ \cos\lambda x_c\right) = 0. \end{gathered} \quad (12)$$

The system of Eqs. (10)–(12) for finding constants C_1^+, C_2^+, C_2^- has a nonzero solution if and only if the determinant of the system is zero, as follows:

$$D = \begin{vmatrix} 0 & -\lambda\sin\lambda & \cos\lambda \\ \frac{\sin\lambda x_c}{\lambda} & -\cos\lambda x_c & -\frac{\sin\lambda x_c}{\lambda} \\ \cos\lambda x_c & P\lambda\sin\lambda x_c & -P\cos\lambda x_c \end{vmatrix} = 0. \quad (13)$$

The result is the following equation for finding the eigenvalues (natural frequencies):

$$\begin{gathered} D = -\lambda\sin\lambda\cos\lambda x_c \sin\lambda x_c P + \cos\lambda\sin^2\lambda x_c + \\ +\sin\lambda\cos\lambda x_c \sin\lambda x_c + \cos^2\lambda x_c \cos\lambda = 0. \end{gathered} \quad (14)$$

Two kinds of problems—direct and inverse—can be solved on the basis of this equation.

The solution of the direct problem with different initial data allows the dependence of natural frequencies of vibrations on the rod parameters to be analyzed to derive main conclusions on the problem's solution. Therefore, the next stage of the work consists in analyzing the results of the direct problem.

The measurement values are dimensioned for the convenience of calculations.

Initial data: $H = B = 0.1$; $h = b = 0.01$.

It is necessary to find natural frequencies of longitudinal vibrations.

Consider the dependence of the first five natural frequencies on the position of the notch start point of the rod.

In values x_c equidistant from the middle (1, 2, 5, and 7 in Table 1), the same values of longitudinal vibration frequencies can be seen. Hence, the solution of the inverse problem is dual, i.e., there are two solutions symmetric to the middle of the rod. The solution duality can be visualized by plotting the first three eigenforms of the function (Figs. 2–4).

Table 1. Natural frequencies of longitudinal vibrations when changing the notch location

No.	Position x_c	λ_1	λ_2	λ_3	λ_4	λ_5
1	0.1	1.57389	4.72052	7.86408	11.00378	14.14031
2	0.25	1.57791	4.71956	7.84681	10.98846	14.14428
3	0.3	1.57895	4.71552	7.84388	10.99868	14.14536
4	0.5	1.58090	4.70229	7.86408	10.98547	14.14727
5	0.75	1.57791	4.71956	7.84681	10.98846	14.14428
6	0.8	1.57670	4.72201	7.85398	10.98595	14.13126
7	0.9	1.57389	4.72052	7.86408	11.00378	14.14031

Note: λ_n is natural frequencies of longitudinal vibrations of the rod found numerically in *Maple* software[1].

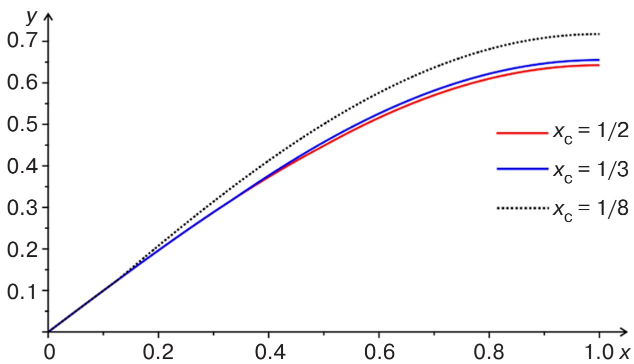

Fig. 2. First eigenform

[1] https://www.maplesoft.com/. Accessed January 01, 2023.

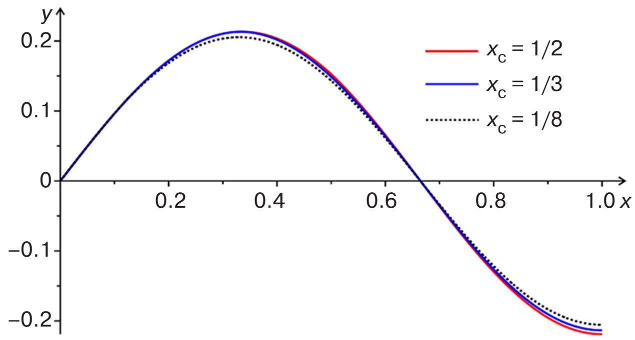

Fig. 3. Second eigenform

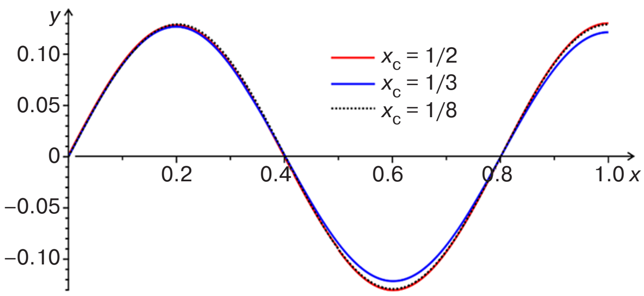

Fig. 4. Third eigenform

The graphs of eigenforms (Figs. 2–4) show that the problem of identifying the notch start point is unstable and strongly depends on the error of the input data.

We shall consider changes in the natural frequency from height h and width b of the rod notch.

Tables 2 and 3 show that a change in the notch size results in a more significant alteration to the natural frequencies of vibrations than a change in the notch location.

INVERSE PROBLEM

We shall consider the inverse problem, where it becomes necessary to find the notch start point not located in the middle of the rod by a finite number of eigenfrequencies.

Let eigenvalues $\lambda_1 = 1.57952$, $\lambda_2 = 4.71238$, and $\lambda_3 = 7.84524$, rod length $L = 1$, width and height $H = 0.1$, $B = 0.1$, and notch width and depth $b = 0.01$, $h = 0.01$, respectively, be known. It is necessary to find the start point of the notch x_c.

Substituting the ratio of areas (5) and known values into the frequency Eq. (14), the following may be written:

$$\begin{aligned} D = -\lambda_n \sin\lambda_n \cos\lambda_n x_c \sin\lambda_n x_c P + \\ + \cos\lambda_n \sin^2\lambda_n x_c + \\ + \sin\lambda_n \cos\lambda_n x_c \sin\lambda_n x_c + \\ + \cos^2\lambda_n x_c \cos\lambda_n = 0. \end{aligned} \quad (15)$$

Equation (15) gives two valid solutions (Fig. 5), symmetric about the middle of the segment L:

$$x_{c1} = 0.25,\ x_{c2} = 0.75.$$

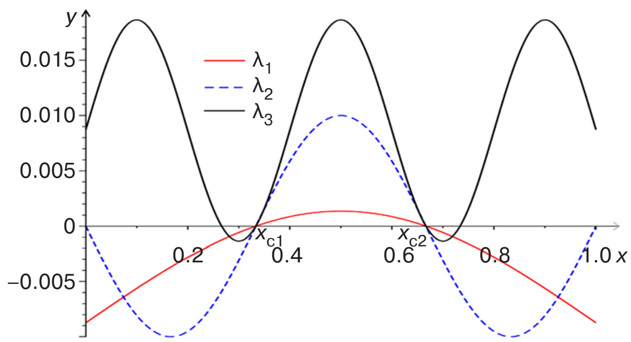

Fig. 5. The inverse problem solution

It follows from the solution of the inverse problem that an unambiguous solution would not be obtained by identifying the rod by means of longitudinal vibrations only. The exact solution requires adding supplementary

Table 2. Change in the frequency of natural vibrations from the notch height increase

No.	Height h	λ_1	λ_2	λ_3	λ_4	λ_5
1	0.01	1.57582	4.70736	7.85901	10.9905	14.14219
2	0.02	1.58090	4.70229	7.86408	10.9854	14.14727
3	0.04	1.59120	4.69198	7.87439	10.9752	14.15758
4	0.05	1.59644	4.68675	7.87963	10.9699	14.16281
5	0.06	1.60173	4.68146	7.88491	10.9646	14.16810
6	0.08	1.61247	4.67071	7.89566	10.9539	14.17884
7	0.09	1.61793	4.66525	7.90112	10.9484	14.18430

Table 3. Change in the frequency of longitudinal vibrations from the position and height of the rod notch

No.	Position x_c	Height h	λ_1	λ_2	λ_3	λ_4	λ_5
1	0.9	0.09	1.58485	4.74968	7.90108	11.03455	14.15227
2	0.1	0.01	1.57234	4.71644	7.85901	10.99965	14.13873
3	0.5	0.05	1.59644	4.68675	7.87963	10.96993	14.16281

conditions, e.g., changing the boundary condition at one of its ends.

We shall add an elastic element to the right end of the rod. Accordingly, the Robin condition appears.

The boundary conditions (Robin conditions) for a rod of unit length ($L = 1$) may be written in the following way:

$$y_x'(1) - Ky(1) = 0, \tag{16}$$

where K is the stiffness of the spring at the end.

Substituting the general solution of (8) into (16), the following may be written:

$$M = \begin{vmatrix} 0 & K\cos\lambda_n - \lambda_n \sin\lambda_n & \cos\lambda_n + \dfrac{\sin\lambda_n}{\lambda_n}K \\ \dfrac{\sin\lambda_n x_c}{\lambda_n} & -\cos\lambda_n x_c & -\dfrac{\sin\lambda_n x_c}{\lambda_n} \\ \cos\lambda_n x_c & P\lambda \sin\lambda_n x_c & -P\cos\lambda_n x_c \end{vmatrix} = 0.$$

Find the determinant

$$\begin{aligned} D = \frac{1}{\lambda_n}\big(&K\cos\lambda_n \cos(\lambda_n x_c)\sin(\lambda_n x_c)P + \\ &+K\sin\lambda_n \sin(\lambda_n x_c)^2 P + \\ &+\cos\lambda_n \sin(\lambda_n x_c)^2 P\lambda_n - \\ &-\sin\lambda_n \cos\left(\lambda_n x_c\right)\sin\left(\lambda_n x_c\right)P\lambda_n - \\ &-K\cos\left(\lambda_n x_c\right)\sin\left(\lambda_n x_c\right) + \\ &+K\sin\left(\lambda_n\right)\cos(\lambda_n x_c)^2 + \\ &+\lambda_n \cos(\lambda_n x_c)^2 \cos\lambda_n + \\ &+\sin(\lambda_n)\cos(\lambda_n x_c)\sin(\lambda_n x_c)\lambda_n\big) = 0. \end{aligned} \tag{17}$$

The solution (17) gives the eigenvalues of λ_n.

We shall solve the inverse problem and plot graphical solutions of the equation.

Let the first three natural frequencies $\lambda_1 = 2.2972263$, $\lambda_2 = 5.0846367$, and $\lambda_3 = 8.0885773$, cross-section area ratio $P = 0.98$, and spring stiffness factor $K = 2$ be known. It is necessary to find the notch location x_c. We shall substitute by order natural frequencies of longitudinal vibrations in (17). The solution of the equation is shown in Fig. 6.

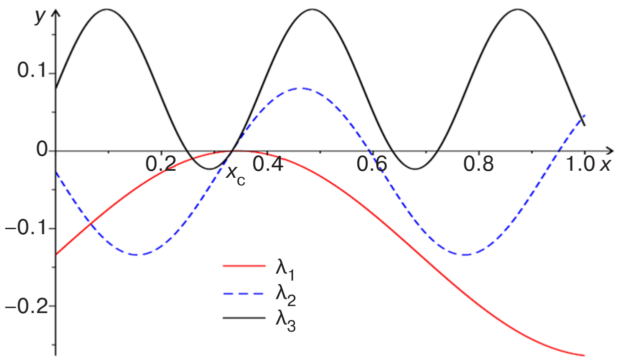

Fig. 6. Solutions to the problem with elastic anchoring

It is seen from Fig. 6 that the problem has an exact solution at the point $x_c = 0.25$ under the elastic anchoring condition. It follows that, in order to solve the inverse problem unambiguously, it will be necessary to apply elastic anchoring at one of the ends of the rod.

CONCLUSIONS

With boundary conditions (7), the solution of the inverse problem of identifying the longitudinal notch by natural frequencies of longitudinal vibrations is unstable since natural shapes are close to each other at the points symmetric from the middle of the notch; moreover, in order to solve the problem unambiguously, it is necessary to replace the fixing one of the rod ends by an elastic condition. The plotted graphs and given tables of the direct problem solution have shown the dependence of natural frequencies on the initial data. An example of the inverse problem solution is given showing that two natural frequencies obtained for different boundary conditions are sufficient for unambiguous determination of the start of the notch. This method may be applied to solving problems on identifying geometric parameters of parts modeled by the rod.

Authors' contribution. All authors equally contributed to the research work.

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Translated from Russian into English by Kirill V. Nazarov
Edited for English language and spelling by Thomas A. Beavitt