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RESEARCH ARTICLE

Extremum in the problem of paired comparisons

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Abstract
Objectives. An analysis of the problem of evaluating alternatives based on the results of expert paired comparisons is presented. The importance and relevance of this task is due to its numerous applications in a variety of fields, whether in the technical and natural sciences or in the humanities, ranging from construction to politics. In such contexts, the problem frequently arises concerning how to calculate an objective ratings vector based on expert evaluations. In terms of a mathematical formulation, the problem of finding the vector of objective ratings can be reduced to approximating the matrices of paired comparisons by consistent matrices.
Methods. Analytical analysis and higher algebra methods are used. For some special cases, the results of numerical calculations are given.
Results. The theorem stating that there is always a unique and consistent matrix that optimally approximates a given inversely symmetric matrix in a log-Euclidean metric is proven. In addition, derived formulas for calculating such a consistent matrix are presented. For small dimensions, examples are considered that allow the results obtained according to the derived formula to be compared with results for other known methods of finding a consistent matrix, i.e., for calculating the eigenvector and minimizing the discrepancy in the log-Chebyshev metric. It is proven that all these methods lead to the same result in dimension 3, while in dimension 4 all results are already different.
Conclusions. The results obtained in the paper allow us to calculate the vector of objective ratings based on expert evaluation data. This method can be used in strategic planning in cases where conclusions and recommendations are possible only on the basis of expert evaluations.

Keywords: expert estimates, paired comparisons, inversely symmetric matrix, consistent matrix, metric, discrepancy minimization

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НАУЧНАЯ СТАТЬЯ

Экстремум в задаче о парных сравнениях

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Резюме
Цели. Рассмотрена задача оценки альтернатив на основе результатов экспертных парных сравнений. Важность и актуальность этой задачи обусловлены ее многочисленными применениями в самых разных областях – как в технических и естественных, так и в гуманитарных, от строительства до политики. Ставится задача вычисления вектора объективных рейтингов на основе экспертных оценок. В математической формулировке задача нахождения вектора объективных рейтингов сводится к аппроксимации матриц парных сравнений согласованными матрицами.
Методы. Используются аналитические методы анализа и высшей алгебры. Для некоторых частных случаев приведены результаты численных расчетов.
Результаты. В работе доказана теорема, утверждающая, что согласованная матрица, наилучшим образом аппроксимирующая заданную обратно-симметрическую матрицу в лог-евклидовой метрике, всегда существует и единственна. Кроме того, выведены формулы для вычисления такой согласованной матрицы. Для малых размерностей рассматриваются примеры, позволяющие сравнить результаты, полученные по выведенной формуле, с результатами для других известных способов нахождения согласованной матрицы – для вычисления собственного вектора и для минимизации невязки в лог-чебышевской метрике. Доказано, что в размерности 3 все эти способы приводят к одному и тому же результату, а уже в размерности 4 все результаты различны.
Выводы. Полученные в статье результаты позволяют вычислять вектор объективных рейтингов по данным экспертной оценки. Этот метод может быть использован в стратегическом планировании в тех случаях, когда выводы и рекомендации возможны только на основании экспертных суждений.

Ключевые слова: экспертные оценки, парные сравнения, обратно-симметрическая матрица, согласованная матрица, метрика, минимизация невязки

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INTRODUCTION

Since a person responsible for making decisions does not always have complete information, decisions can be taken on the basis of criteria that are not always objective. In cases where a large number of factors must be taken into account, an error may lead to disastrous outcomes. Such situations include, for example, strategic planning issues, particularly in construction, as well as medicine, politics, economics, and many other areas of human activity.

The science according to which a strategy is selected under conditions of incomplete information, as well as providing a rationale for such choices, is commonly referred to as decision-making theory. Such studies attract close attention of experts in various fields [1, 2]. Many aspects are discussed in the books of Thomas L. Saaty [3, 4], one of the founders of this theory.

The continued growth of research in this direction can confidently predicted due to the possibility of applying decision theory methods to machine learning. In fact, the use of inconsistent and inaccurate expert evaluations which nevertheless allow necessary information to be obtained when based on large datasets are broadly similar to situations that typically arise, for example, when training neural networks or constructing an ensemble of decision trees that comprises a random forest [5].

Numerous recent studies discuss the technique of comparing heterogeneous assets as applied to various problems from the field of information technology, in particular, to select storage formats for big data for various computing complexes, both local and distributed. For example, studies [6–11] are devoted to this topic.

A situation commonly arises when neither priori distributions nor prior statistics are available but a forecast or decision must be made on the basis of earlier forecasts and expert recommendations. Thus, the task of processing expert evaluations should be given a mathematical formulation.

Let us deal with expert evaluations. For example, an expert compares an apple (A), orange (O), and banana (B). He compares the fruits in pairs. Suppose the following opinions are expressed:

- A banana is three times better than an apple;
- An orange is five times better than an apple;
- An orange is twice as good as a banana.

Based on these evaluations, the following comparison matrix wherein the first column and the first row correspond to the apple, the second to banana, and the third to orange can be constructed:

$$\mathbf{W} = \begin{pmatrix} 1 & 3 & 5 \\ 1/3 & 1 & 2 \\ 1/5 & 1/2 & 1 \end{pmatrix}.$$

At the intersection of the ith column and the jth row, there is a number equal to the ratio of the values of the ith and the jth fruit. For such a positive-definite symmetric matrix, all of whose elements are strictly positive, the following relation is satisfied:

$$a_{ij} = a_{ji}^{-1}.$$

However, the matrix is inconsistent. If O is 5 times better than A and O is 3 times better than B, then it would be appropriate to assume that B should be 5/3 times better than A, not 2 times better.

This situation arises commonly when carrying out expert evaluations. Moreover, there are also non-transitive evaluations, e.g., when A is better than B, B is better than C, but C is better than A. This occurs, for example, in tournaments when A beats B, B beats C, and C beats A.

For making an objective evaluation, we assume that there are objective ratings w_1, w_2, and w_3, for the evaluated items, and that the expert evaluations are the same ratings distorted by random errors. The problem then arises as to how to reconstruct these ratings based on the given expert evaluations.

If ratings w_i are found, then the comparison matrix elements may be written as follows:

$$x_{ij} = \frac{w_i}{w_j}.$$

We shall denote this matrix, whose rank is 1, by $\mathbf{W}_0$. Such matrix is called a consistent matrix.

Thus, the task of processing expert evaluations is reduced to that of finding the consistent matrix $\mathbf{W}_0$ approximating the inversely symmetric matrix $\mathbf{W}$ in the best way. Here, it turns out that the answer changes significantly depending on the metric which the difference between these matrices is calculated in.

The study by N.K. Krivulin and his students [12] proposes to calculate this difference in a log-Chebyshev metric. In particular, it is noted there that this results in the problem of processing expert evaluations becoming the problem from the field of so-called idempotent or tropical mathematics [13], new direction in modern mathematics that is rapidly developing. However, it is also noted there that this metric in high dimensions results in non-uniqueness of the solution.

The studies by Saaty [3, 4] propose to consider the correspondingly normalized eigenvector of matrix A corresponding to its maximal eigenvalue as the required rank vector. The well-known Perron–Frobenius theorem [14] states that any positive matrix (consisting only of positive numbers) has a single maximal modulo eigenvalue; the multiplicity of such a strictly positive matrix is equal to 1. However, the metric according to which the obtained solution is optimal is not specified in those studies.

Thus, the present work is aimed at finding the optimal solution in the log-Euclidean metric.

DERIVING OPTIMAL EVALUATION

We shall consider the comparison matrix of arbitrary dimension ($n \times n$). The discrepancy of the original comparison matrix $\mathbf{W} = (a_{ij})$ and its matched counterpart $\mathbf{W}_0 = (x_{ij})$ in the log-Euclidean metric under consideration may be calculated in the following way:

$$\Phi = \sum_{i,j=1}^{n} \log^2\left(\frac{x_{ij}}{a_{ij}}\right).$$

If we consider that the consistent matrix elements are expressed through the components of the matrix eigenvector in the form of $x_{ij} = w_i/w_j$, then the function Φ depends on n variables, as follows:

$$\Phi(w_1, ..., w_n) = \sum_{i,j=1}^{n} \log^2 \left(\frac{w_i}{w_j a_{ij}} \right).$$

The conditions of equality to zero of the derivative of the residual function on the *k*th component of the eigenvector w_k can give the following system of equations:

$$n \log w_k - \sum_{\beta=1}^{n} \log w_\beta = \sum_{\beta=1}^{n} \log a_{k\beta}.$$

The solution to this system of equations is the following:

$$w_k = N \prod_{\beta=1}^{n} (a_{k\beta})^{1/n},$$

where *N* is the arbitrary normalization factor.

It is considered that the product of all elements of the original inversely symmetric comparison matrix is equal to one as follows:

$$\prod_{\alpha,\beta=1}^{n} a_{\alpha\beta} = 1.$$

Thus, the following statement is proved.

Theorem. Consider an inversely symmetric matrix (a_{ij}). Then the components of the consistent matrix (x_{ij}) minimizing the residual function for the log-Euclidean metric have the following form:

$$x_{ij} = \frac{w_i}{w_j} = \prod_{\beta=1}^{n} (a_{i\beta} \cdot a_{\beta j})^{1/n}.$$

In other words, the matrix element x_{ij} is equal to the product of the geometric mean of the *i*th row and the *j*th column of the original matrix.

The inversely symmetric matrices of small dimensions may be considered as an example.

Let $\mathbf{W} = (a_{ij})$ be the three-dimensional matrix of expert comparisons:

$$\mathbf{W} = \begin{pmatrix} 1 & a & b \\ 1/a & 1 & c \\ 1/b & 1/c & 1 \end{pmatrix}.$$

The matrix elements are positive $a_{ij} > 0$ and inversely symmetric $a_{ij} = a_{ji}^{-1}$. Note that a matrix is called consistent if the condition $c = b/a$ or $ac/b = 1$ is satisfied for its elements *a, b,* and $c > 0$. The similar parameter is called "tropical radius" and notated $R=(ac/b)^{1/3}$ in [12]; this notation is also used in the paper.

It is found in [15] that the eigenvalues of the comparison matrix for $n = 3$ are the following:

$$\lambda_1 = 1 + \left(R + \frac{1}{R} \right);$$

$$\lambda_2 = \lambda_3 = 1 - \frac{1}{2}\left(R + \frac{1}{R} \right) \pm \frac{i\sqrt{3}}{2}\left(R - \frac{1}{R} \right).$$

One of the roots of the characteristic equation is real, while the other two are complex-conjugate. The real root has the largest value in absolute value. For the consistent matrix, $R = 1$ and the eigenvalues are $\lambda_1 = 3$; $\lambda_2 = \lambda_3 = 0$. The largest nonzero eigenvalue coincides with the dimension of the comparison matrix in general.

The eigenvector of the original comparison matrix for the first eigenvalue is also easy to find. It has the following form (in normalizing $w_1 = 1$):

$$\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 1 \\ R/a \\ 1/bR \end{pmatrix}.$$

In this case, the elements of the consistent matrix may be found as follows: $\mathbf{W}_0 = (x_{ij}) = w_i/w_j$, and hence:

$$\mathbf{W}_0 = \begin{pmatrix} 1 & a/R & bR \\ R/a & 1 & c/R \\ 1/bR & R/c & 1 \end{pmatrix}.$$

The found eigenvector of the original matrix is also the eigenvector of the consistent matrix. It corresponds to the eigenvalue of this matrix equal to the dimension $\lambda = 3$.

For the following three-dimensional comparison matrix and its corresponding consistent matrix

$$\mathbf{W} = (a_{ij}) = \begin{pmatrix} 1 & a & b \\ 1/a & 1 & c \\ 1/b & 1/c & 1 \end{pmatrix};$$

$$\mathbf{W}_0 = (x_{ij}) = \begin{pmatrix} 1 & x & y \\ 1/x & 1 & y/x \\ 1/y & x/y & 1 \end{pmatrix}$$

the problem in the log-Euclidean metric is reduced to finding the minimum of the residual function $\Phi(x, y)$ of two variables included in the following consistent matrix:

$$\Phi(x,y)=\sum_{i,j=1}^{3}(\log a_{ij}-\log x_{ij})^2,$$

which may be written in the following form with provision for the explicit form of matrices:

$$\Phi(x,y)=2\log^2\left(\frac{x}{a}\right)+2\log^2\left(\frac{y}{b}\right)+2\log^2\left(\frac{y}{cx}\right).$$

The function extremum (minimum) is reached at $x = a/R$, $y = bR$. The minimum value of the function $\min\Phi = 6\log^2 R$ depends on the inconsistency of the original matrix of expert judgments.

The results for the three-dimensional case can be also obtained in another way.

As before, the inversely symmetric matrix in the three-dimensional case is following:

$$\mathbf{W}=\begin{pmatrix}1 & a & b\\ 1/a & 1 & c\\ 1/b & 1/c & 1\end{pmatrix}.$$

When logarithmic, it becomes cosymmetric:

$$\mathbf{H}=\begin{pmatrix}0 & u & v\\ -u & 0 & w\\ -v & w & 0\end{pmatrix}.$$

Here, $u = \log a$, $v = \log b$, and $w = \log c$.

The consistent matrix has the following form:

$$\mathbf{W}_0=\begin{pmatrix}w_1/w_1 & w_2/w_1 & w_3/w_1\\ w_1/w_2 & w_2/w_2 & w_3/w_2\\ w_1/w_3 & w_2/w_3 & w_3/w_3\end{pmatrix}.$$

After logarithmizing, it has the following form:

$$\mathbf{L}=\begin{pmatrix}0 & y_1 & y_2\\ -y_1 & 0 & y_3\\ -y_2 & -y_3 & 0\end{pmatrix}.$$

Here, $y_1 = \log w_2 - \log w_1$; $y_2 = \log w_3 - \log w_1$; $y_3 = \log w_3 - \log w_2$.

In addition, the condition $y_1 - y_2 + y_3 = 0$ is satisfied. Thus, the problem is reduced to finding the point **Q** on the plane $y_1 - y_2 + y_3 = 0$ being the closest to the given point $\mathbf{P}(u, v, w) \in R^3$. The solution to this problem depends on the metric.

For the Euclidean metric, draw a line perpendicular to the plane through point **P** and find the intersection point:

$$(u + t) - (v - t) + (w + t) = 0.$$

We obtain:

$$t=-\frac{1}{3}(u-v+w).$$

Hence:

$$y_1=u+t=\frac{2}{3}u+\frac{1}{3}v-\frac{1}{3}w,\quad y_2=v-t=\frac{1}{3}u+\frac{2}{3}v+\frac{1}{3}w,$$

$$y_3=w+t=-\frac{1}{3}u+\frac{1}{3}v+\frac{2}{3}w.$$

Let us assume without loss of generality that $w_1 = 1$. Then

$$w_2=e^{y_1}=a^{2/3}b^{1/3}c^{-1/3},$$
$$w_3=e^{y_2}=a^{1/3}b^{2/3}c^{1/3}.$$

These are the elements of the first row of the matrix. The first column contains their inverse elements. The first column is equal to

$$\mathbf{V}=\begin{pmatrix}1\\ a^{-2/3}b^{-1/3}c^{1/3}\\ a^{-1/3}b^{-2/3}c^{-1/3}\end{pmatrix},$$

which coincides with the earlier obtained result

$$\mathbf{V}=\begin{pmatrix}1\\ R/a\\ 1/bR\end{pmatrix}.$$

This column (as well as the other two) is an eigenvector of the original matrix **A**. This can be easily checked by direct calculation. However, unfortunately, this calculation method is not generalized to higher dimensions.

We shall also consider other metrics. It seems most natural to consider the most common log-Manhattan and log-Chebyshev metrics.

It is easy to demonstrate that the solution is not unique in the log-Manhattan metric even in dimension 3. Indeed, in geometrical terms, the solution to the problem of the minimum distance from a point to a plane is reduced to constructing a sphere centered at this point and touching this plane. However, in Manhattan metrics, the "sphere" is an octahedron, one of whose facets lies just on the plane $y_1 - y_2 + y_3 = 0$. All points of this facet are solutions. In addition, the solution in Euclidean metric belongs to the same facet, i.e., it is one of the solutions in Manhattan metric.

In the Chebyshev metric in dimension 3, the solution is singular and coincides with the solution in the Euclidean metric. Indeed, the "ball" in this metric is actually a cube. The vector **PQ** has coordinates $(t, -t, t)$

and is half of the diagonal of the cube, so the cube also touches the plane at a single point, point **Q**.

In [12], another approach is used to minimize the discrepancy in the eigenvector in the log-Chebyshev metric in dimension 3.

Thus, all the described ways of computing the matrix consistent in dimension 3—computing the eigenvector and computing the vector by minimizing the discrepancy—lead to the same result (although this solution is not the only one in the log-Manhattan metric).

In dimension 4, these solutions are already different.

In [12], the numerical example with such a matrix is considered for dimension 4:

$$\mathbf{D} = \begin{pmatrix} 1 & 2 & 4 & 1 \\ 1/2 & 1 & 1/2 & 1/3 \\ 1/4 & 2 & 1 & 2 \\ 1 & 3 & 1/2 & 1 \end{pmatrix}.$$

The above paper proves that any vector belonging to the segment **AB** is optimal in the log-Chebyshev metric, where

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1/4 \\ 1/2 \\ 1/2 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 1 \\ 1/3 \\ 1/2 \\ 1/2 \end{pmatrix}.$$

Thus, in the log-Chebyshev metric in dimension 4, the solution is not unique in general.

The results of the method of calculating the eigenvector are as follows: the eigenvalue is $\lambda_{max} = 4.5056$, while the eigenvector in normalization when its first coordinate is equal to 1 may be written as follows:

$$\mathbf{V} = \begin{pmatrix} 1.0000 \\ 0.2837 \\ 0.5818 \\ 0.6110 \end{pmatrix}.$$

Calculating the solution giving the minimum discrepancy in the log-Euclidean metric, in accordance with the theorem proved above, results in the following:

$$\mathbf{V} = \begin{pmatrix} 1.0000 \\ 0.3195 \\ 0.5946 \\ 0.6580 \end{pmatrix}.$$

Thus, already in dimension 4, the methods of calculating the rating vector described above lead to different results.

CONCLUSIONS

The theorem proved in the paper can be used to process expert opinions by reducing them to the form of a ranking list. It is shown to give the best evaluation in the log-Euclidean metric. Examples demonstrate that this evaluation in high dimensions may not coincide with those obtained by other methods. Thus, the selection of the desired method should be related to the specifics of the problem under consideration.

Authors' contribution. All authors equally contributed to the research work.

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