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RESEARCH ARTICLE

Methodological features of the analysis of the fractal dimension of the heart rate

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Abstract
Objectives. The aim of the present work is to determine the fractal dimension parameter calculated for a sequence of R–R intervals in order to identify the boundaries of its change for healthy and sick patients, as well as the possibility of its use as an additional factor in the detection of cardiac pathology.
Methods. In order to determine the fractal dimension parameter, the Hurst-, Barrow-, minimum coverage area-, and Higuchi methods are used. For assessing the stationarity of a number of electrocardiography (ECG) intervals, a standard method is used to compare arithmetic averages and variances from samples of the total data array of ECG intervals. To identify differences in fractal dimensions of healthy and sick patients, this parameter was ranked. Using the Kolmogorov–Smirnov two-sample criterion, the difference between the distribution laws in the samples for healthy and sick patients is shown.
Results. Among the considered methods for calculating the fractal dimension, the Higuchi method demonstrates the smallest data spread between healthy patients. By ranking the calculated fractional dimension values, it was possible to identify the difference between this parameter for healthy and sick patients. The difference in the distribution of fractal dimension of healthy and sick patients is shown to be statistically significant for the coverage and Higuchi methods. At the same time, when using the traditional Hurst method, there is no reason to reject the null hypothesis that two groups of patients belong to the same general population.
Conclusions. Based on the obtained data, the difference between the fractal dimension indicators of the duration of R–R intervals of healthy and sick patients is shown to be statistically significant when using the Higuchi method. The fractal dimensions of healthy and sick patients can be effectively distinguished by ranking samples. The results of the research substantiate prospects for further studies aimed at using fractal characteristics of the heart rhythm to identify abnormalities of the latter, which can serve as an additional factor in determining heart pathologies.

Keywords: fractal, fractal dimension, coronary heart disease, chronic heart failure, Higuchi method, minimum coverage area method, Hurst method

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НАУЧНАЯ СТАТЬЯ

Методические особенности анализа фрактальной размерности сердечного ритма

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Резюме
Цель. Целью работы было определение параметра фрактальной размерности, рассчитанного для последовательности длительностей R-R интервалов, выявление границы его изменения для здоровых и больных пациентов, а также возможности его использования в качестве дополнительного фактора при выявлении патологии сердечной деятельности.
Методы. Для определения параметра фрактальной размерности используются методики Херста, Барроу, минимальной площади покрытия и Хигучи. При оценке стационарности ряда кардиоинтервалов применяется стандартный метод сравнения средних арифметических и дисперсий по выборкам общего массива данных кардиоинтервалов. Для выявления различий фрактальных размерностей здоровых и больных пациентов выполнено ранжирование данного параметра. С помощью использования двухвыборочного критерия Колмогорова – Смирнова показано различие законов распределения в выборках для здоровых и больных пациентов.
Результаты. Показано, что из рассмотренных методов расчета фрактальной размерности наименьший разброс данных между здоровыми пациентами демонстрирует метод Хигучи. Выполнено ранжирование рассчитанных значений фрактальной размерности, позволившее выявить различие данного параметра для здоровых и больных пациентов. Показано, что различие в распределении фрактальной размерности здоровых и больных пациентов является статистически значимым для методов покрытия и Хигучи. В то же время при использовании традиционного метода Херста нет основания отвергать нулевую гипотезу о принадлежности двух групп пациентов одной генеральной совокупности.
Выводы. На основании полученных данных было показано, что статистически значимое различие между показателями фрактальной размерности длительностей R-R интервалов здоровых и больных пациентов имеет место при применении метода Хигучи. Установлено, что ранжирование выборок позволяет эффективно различать фрактальные размерности здоровых и больных пациентов. Результаты работы показывают перспективность дальнейших исследований, направленных на использование фрактальных характеристик кардиоритма для выявления нарушений последнего, что может служить дополнительным фактором при определении патологии деятельности сердца.

Ключевые слова: фрактал, фрактальная размерность, ишемическая болезнь сердца, хроническая сердечная недостаточность, метод Хигучи, метод минимальной площади покрытия, метод Херста

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INTRODUCTION

Heart rate variability (HRV) has become one of the effective methods for assessing the neural regulation of the heart, analyzing the interaction between sympathetic and vagal fluctuations, as well as examining their effect on heart rate. Heart rate fluctuations exhibit various linear, non-linear, periodic and irregular oscillation patterns.

Thanks to an intensive study of heart rate variability carried out over several decades, it is now possible to correlate changes in the functioning of the cardiovascular system with the presence of pathologies. In Russia, fundamental research in this area carried out at the scientific institution founded by R.M. Baevsky[1], as well as other scientists [1–4], has made it possible to give a physiological interpretation of heart rate variability through the analysis of information about the state and functioning of systems that regulate heart rhythm.

The conducted studies led to the conclusion that the assessment of the overall heart rate variability helps to carry out early diagnosis of disorders of the cardiovascular system. There are a number of different approaches to the analysis of the heartbeat process. In particular, the analysis of variability based on the study of the statistical parameters of rhythmograms is successfully used. The spectral analysis of rhythmograms based on the fast Fourier transform and subsequent analysis of the spectral density distribution over the frequency range has also become widespread. As a result of the performed studies, it has been shown, for example, that the high-frequency region (0.15–0.4 Hz) is a marker of wandering modulation, while the low-frequency region (0.04–0.15 Hz) mainly reflects sympathetic tone and baroreflex activity. In common with a number of other methods, those based on frequency and time measurements are based on the assumption that HRV signals cannot reflect and quantify the dynamic structure of the signal due to their linear character. A number of methods have also been proposed to evaluate nonlinear properties, including fractal dimension, Lyapunov exponents, correlation dimension, approximate entropy, and downtrend analysis of fluctuations [5–9]. All these methods define the properties of HRV as a non-linear process that reacts to external disturbances in a nonlinear manner. Over the past 2–3 decades, attempts have been made to describe such nonlinear systems from the standpoint of deterministic chaos [10–12], which, in contrast to the everyday understanding of disorder as an absolutely random process, refers to processes characterized by limited randomness such as the heart rate. As noted in [13], a certain norm in terms of randomness is necessary for the normal functioning of an organism. Any significant deviation from the norm, both in the direction of greater order and in the direction of greater randomness, may indicate a disease of the body.

In this paper, we consider fractal dimension analysis, comprising one of the methods for nonlinear study of heart rhythm. Fractal dimension analysis is based on a coefficient that describes fractal structures or sets based on a quantitative assessment of their complexity.[2] This parameter is determined for the sequence of durations of R–R intervals of healthy patients and patients with chronic heart failure (CHF) and coronary heart disease (CHD).

METHODS FOR DETERMINING THE FRACTAL DIMENSION *D*

Benoit Mandelbrot [14] gives the following definition of fractals: "A fractal is a structure consisting of parts that are in some sense similar to the whole." The main characteristic of self-similar structures, which determines the degree of space indentation, is the fractal dimension D. There are a number of different ways to define it. One of the most popular is the Hurst method [15, 16], based on the ratio of the range of the accumulated deviation to the standard deviation. Another name for this technique is R/S-analysis.

Its essence is expressed by the following formula:

$$\frac{R}{S} = (aN)^H,$$

where a is some constant for a particular process; N is the amount of data; H is the Hurst exponent; S is the standard deviation of the series; R is the range of the accumulated deviation, i.e., the difference between the maximum and minimum values of the accumulated deviation from the average value of series Z in the interval $[1; u]$. In turn, u belongs to the interval from 1 to N.

Figure 1 shows changes in some value X, its accumulated deviation Z and the average value $X_{\text{av}u}$ in the interval $[1; u]$:

[1] Roman Markovich Baevsky, Dr. Sci. (Med.), Professor, Honored Scientist of the Russian Federation, Academician at the International Academy of Astronautics, Academician at the International Academy of Informatization. He is one of the founders of aerospace cardiology, space telemetry, and the concept of prenosological diagnostics, having personally carried out the development of a medical control system for Yu.A. Gagarin. In Baevsky's scientific school, three fundamental directions can be distinguished: ballistocardiography and cardiography; heart rate variability; space medicine and prenosological diagnostics.

[2] Fractal dimension. https://en.wikipedia.org/wiki/Fractal_dimension. Accessed April 23, 2022 (in Russ.).

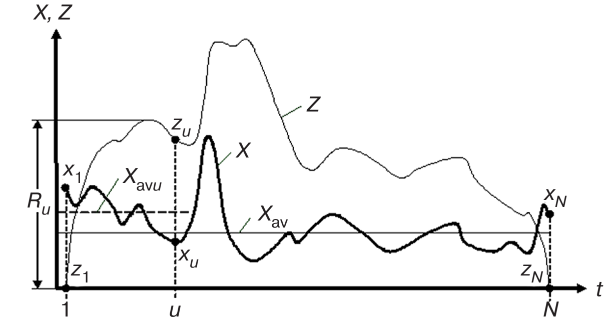

Fig. 1. Changes in the X value (thick line), its accumulated deviation Z (thin line), and the average value of X_{av} (R_u is the range of the accumulated deviation; X_{avu} is the average value of X in the interval; x_1 is the initial value of X; z_1 is the initial value of the accumulated deviation Z; z_u is the value of the accumulated deviation Z taken at the point u; x_u is the value of X, taken at the point u; x_N is the final value of X; z_N is the final value of the accumulated deviation Z)[3]

The fractal dimension is related to the Hurst exponent by the relation:

$$D = 2 - H$$

Another approach for determining the fractal dimension is the Barrow method[4], which consists in finding the average variance of increments W, calculated by the following formula:

$$W(\Delta N) = \frac{1}{N - \Delta N} \sum_{i=1}^{N-\Delta N} \left(x_{i+\Delta N} - x_i\right)^2,$$

where $1 \le \Delta N \le N - 1$.

The dependence $W = f(\Delta N)$ is described by the following equation:

$$W = (a\Delta N)^B,$$

where a is some constant value for the given series of data, B is the Barrow exponent.

The fractal dimension is defined as follows:

$$D = 2 - B.$$

The method of minimum coverage area has also become widespread. It is used, in particular, in the field of economics, as well as having application in the analysis of meteorological series [17]. In this case, after determining the fractal dimension by dividing the data volume N into δ parts, the sum of the amplitude variations for each of the obtained parts is calculated. Then δ changes; following several repetitions of the algorithm, a graph is plotted on a logarithmic scale of the dependence of V on δ. The resulting set of points is approximated by a straight line, after which the slope k is calculated using the least squares method. The process of calculating the coverage area for various values of δ is illustrated by Fig. 2.

In this case, the fractal dimension is found by the formula:

$$D = k + 1.$$

In some studies related to the fractal analysis of biological processes, the Higuchi algorithm was used to

[3] Kobenko V.Yu. *Fractals in science and technology. Guidelines for performing laboratory work in the Microsoft Excel application*. Omsk: OmGTU; 2005. P. 6 (in Russ.).

[4] Ibid. P. 13–15.

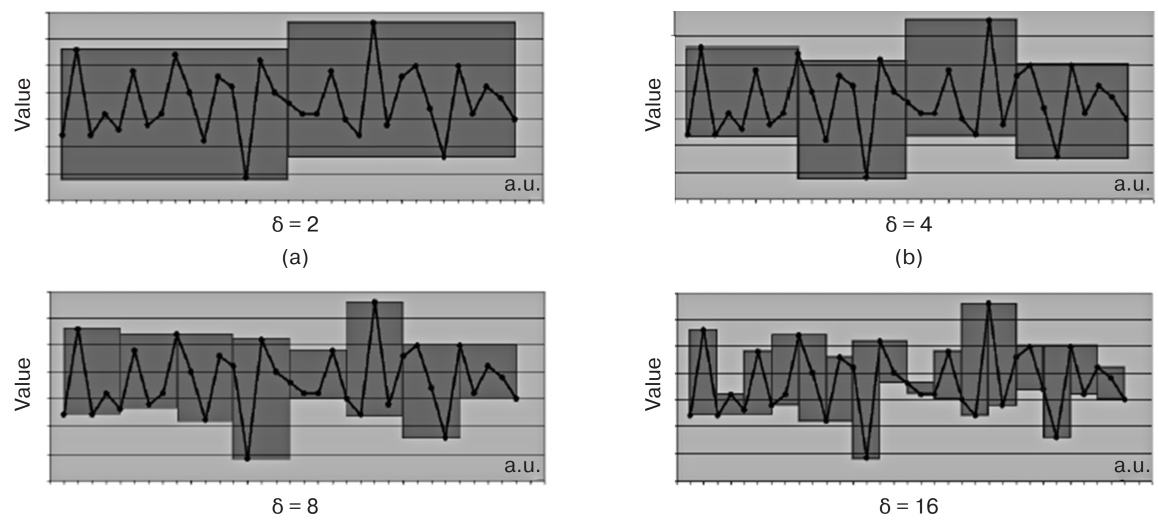

Fig. 2. Calculation of the coverage area for various values of δ [17]

estimate the fractal dimension [15, 18]. This technique will also be used in the present work.

To obtain the fractal dimension D, Higuchi examined a finite set of observations obtained at a regular interval:

$$X(1), X(2), X(3), ..., X(N).$$

A new series, X_k^m, is compiled from these time series as follows:

$$X_k^m : X(m),\ X(m+k), X(m+2k),..., X\left(m+\left[\frac{N-m}{k}\right]k\right),$$

where m = 1, 2, …, k; k and m are integers; m and k are the serial number of the sample and the size of the interval, respectively.

Within the framework of this method, the length of the curve associated with each time series X_k^m is determined as follows:

$$L_m(k) =$$
$$=\frac{1}{k}\left(\sum_{i=1}^{\left[\frac{N-m}{k}\right]}(X(m+ik)-X(m+(i-1)k))\right)\left(\frac{N-1}{\left[\frac{N-m}{k}\right]k}\right),$$

where $\frac{N-1}{\left[\frac{N-m}{k}\right]k}$ is the normalizing factor.

The average value $\langle L(k)\rangle$ of the lengths associated with the time series determines the fractal dimension D. At the same time, the following relation takes place:

$$<L(k)> \propto k^{-D}.$$

FRACTIONAL DIMENSION OF HEART RATE VARIABILITY IN HEALTHY AND ILL PATIENTS

To calculate the fractal dimension of HRV in healthy patients, records of R–R intervals from the database "RR interval time series from healthy subjects" were used. For sick patients corresponding data were taken from the databases "Congestive Heart Failure RR Interval Database" and "St. Petersburg INCART 12-lead Arrhythmia Database." The bases are presented in the Physionet[5] open resource of medical signals. The durations of R–R intervals were obtained using the Show RR intervals as text provided by this database, which automatically determines this parameter at a given time interval. At the beginning of the study, a time interval of 450 R–R values was considered, which corresponded to approximately a 5-minute recording. All further calculations were carried out in Microsoft Excel. Algorithms for determining the fractal dimension by the Hurst, Barrow, and coverage methods are described in detail in [15–18]. The procedure for constructing a fractal plane by the Higuchi method consisted of the following steps:

1. Compilation of time series, $k \in [2; 10]$.
2. Calculation of the length of the curve $L_m(k)$ of each series.
3. Determining the average value $\langle L(k)\rangle$ of the lengths associated with the time series for each set of observations.
4. Plotting the dependence of $\langle L(k)\rangle$ on k on a logarithmic scale and determining the value of the fractal dimension D by the least squares method.

The fractal planes determined by the four methods used for one healthy patient are shown in Fig. 3.

The coordinates of points for fractal planes of the R/S method, the Barrow method, the area of least coverage method, and the Higuchi method are highlighted in blue, red, green, and purple, respectively. The black dotted line indicates the approximating trend line, whose slope is used to calculate the value of the fractal dimension. As follows from the graphs, the values of the fractal dimension determined by the last two methods have a significantly lower determination error due to the smaller scatter of the data to be approximated. Using the methods described above, fractal dimensions were found for ten healthy patients, as well as patients with diagnoses of chronic heart failure (CHF) and coronary heart disease (CHD). The obtained values are presented in Table 1.

Since the technique proposed by Barrow uses a similar approach to the traditional Hurst method, it will not be considered further. The fractal dimension D obtained by the Higuchi method shows a smaller spread of data between healthy patients. In this regard, the results obtained by this method will be considered in more detail.

The fact that most of the values of the fractal dimension fall within the interval from 1.5 to 2 is an indicator of the antipersistence of the series. This concept indicates a more frequent change in the direction of the vector of the system development than would be the case with a random sequence. Approximation of the fractal dimension parameter to D = 2, which indicates an increase in the variability of the series, is typical for a situation where the studied sequence tends to completely fill the fractal plane. Values approaching 1.5 indicate that the process tends to complete randomness, i.e., white noise. The fact that the values of the fractal dimension fall into the interval (1.5–2.0) indicates the ergodicity of the process, a special property of certain dynamic systems, which consists in the fact that during development any state with rare exceptions have a certain probability to pass near each other state of the system[6].

[5] https://physionet.org/. Accessed February 14, 2022.

[6] Ergodicity. https://en.wikipedia.org/wiki/Ergodicity. Accessed April 17, 2022 (in Russ.).

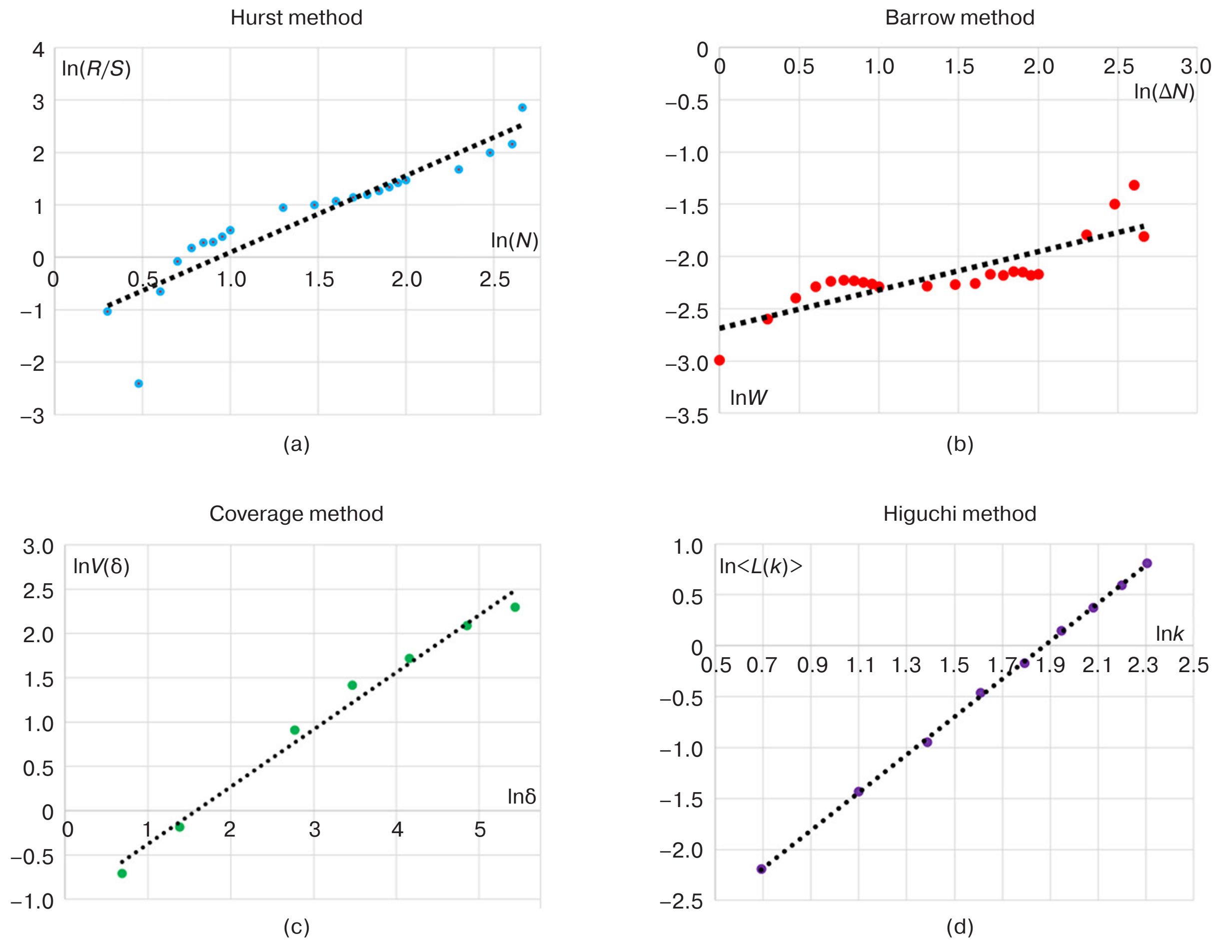

Fig. 3. Fractal plane of a healthy patient:
(a) Hurst method, (b) Barrow method, (c) area of least coverage method, (d) Higuchi method

Table 1. Fractal dimensions *D* of ten healthy patients and patients with CHF and IHD, calculated by various methods

Patient No.	Fractal dimension of healthy patients	Fractal dimension of sick patients (IHD)	Fractal dimension of sick patients (CHF)	Method
1	1.896	1.499	1.754	Hurst method
	1.791	1.502	1.753	Barrow method
	1.612	1.601	1.572	Coverage method
	1.638	1.695	1.685	Higuchi method
2	1.937	1.933	1.787	Hurst method
	1.794	1.886	1.943	Barrow method
	1.722	1.595	1.606	Coverage method
	1.757	1.822	1.981	Higuchi method
3	1.944	1.815	1.432	Hurst method
	1.845	1.767	1.871	Barrow method
	1.665	1.746	1.619	Coverage method
	1.702	1.910	1.805	Higuchi method
4	1.944	1.449	1.518	Hurst method
	1.845	1.790	1.944	Barrow method
	1.665	1.494	1.627	Coverage method
	1.702	1.654	1.797	Higuchi method

Table 1. Continued

Patient No.	Fractal dimension of healthy patients	Fractal dimension of sick patients (IHD)	Fractal dimension of sick patients (CHF)	Method
5	1.529	1.783	1.770	Hurst method
	1.463	1.275	1.994	Barrow method
	1.608	1.598	1.821	Coverage method
	1.630	1.665	1.982	Higuchi method
6	1.536	1.685	1.709	Hurst method
	1.633	1.579	1.994	Barrow method
	1.646	1.528	1.683	Coverage method
	1.767	1.538	2.000	Higuchi method
7	1.459	1.979	1.964	Hurst method
	1.615	1.722	1.863	Barrow method
	1.627	1.814	1.766	Coverage method
	1.751	2.000	1.928	Higuchi method
8	1.627	1.895	1.909	Hurst method
	1.721	1.912	1.776	Barrow method
	1.663	1.738	1.703	Coverage method
	1.708	1.956	1.803	Higuchi method
9	1.665	1.739	1.735	Hurst method
	1.988	1.747	1.699	Barrow method
	1.698	1.573	1.591	Coverage method
	1.772	1.608	1.560	Higuchi method
10	1.559	1.598	1.702	Hurst method
	1.787	1.665	1.949	Barrow method
	1.605	1.648	1.600	Coverage method
	1.675	1.692	1.989	Higuchi method

At the same time, in certain time intervals the statistical characteristics coincide. The alternation of such intervals is due to the presence of a latent periodicity of the process, which is a characteristic, in particular, for the heart rhythm.

DETERMINATION OF SAMPLE SIZE

Sample size is an important feature of any empirical study that aims to draw inferences about a population of parameters from sample observations. The chance of detecting statistically significant differences depends on the sample size and the magnitude of the true difference between the compared indicators [19].

The values of the Higuchi fractal dimension obtained for a healthy patient with an increase in the sample size from 200 to 1500 R–R intervals are presented in Table 2.

Based on the data of Table 2, the fractal dimensions D calculated by the Higuchi method were determined on a sample size equal to 1000 values of the R–R interval.

It should be noted that the fractal dimension calculated by the coating method does not show any dependence on the recording duration for both sick and healthy patients. The D values calculated by this method for samples of 450 and 1000 values are presented in Table 3.

Table 2. Comparison of fractal dimensions for a healthy patient obtained by the Higuchi method for samples of 200, 450, 900, 1000, 1200, and 1500 values of the R–R interval

Fractal dimension calculated by the Higuchi method for different sizes of input data						
Sample size	200	450	900	1000	1200	1500
Healthy patient	1.673	1.702	1.710	1.709	1.707	1.714

Table 3. Comparison of fractal dimensions of 3 patients obtained by the coating method for samples of 450 and 1000 values of R–R intervals

Fractal dimension by the coverage method at interval value		Patient
450	1000	
1.72	1.75	Healthy
1.74	1.72	IHD
1.57	1.59	CHF

Based on the data obtained, it follows that the fractal dimension *D* calculated by the coverage method is practically independent of the sample size. Therefore, for this method, the sample size is not critical in the considered range of values. Taking into account the obtained results presented in Table 2, we will use a sample with an input data volume of 1000 values of R–R intervals in future studies.

STATIONARITY ESTIMATE

While fractals are closely related to the concept of dynamic chaos, reference is also occasionally made to the concept of deterministic chaos. One of the conditions for the existence of such chaos is non-linearity. These concepts are described in detail in [20]. In this section, the issues of stationarity of a sample of 1000 R–R intervals are considered.

As is known, stationarity is the invariability of the characteristics of a random process over time: the average value and variance of a stationary process remain constant regardless of time, and the autocorrelation function depends only on the difference between the time points at which it is determined. Table 4 shows the average values determined for successive samples consisting of 100 durations of R–R intervals. As can be seen from the presented data, the average values change by 11–25%; meanwhile, the dispersion value also varies significantly from ~50% for healthy patients to 300% for CHF patients. Thus, a number of values of the durations of R–R intervals are not stationary, whether in the presence of pathologies or in their absence.

This fact is an additional confirmation of the expediency of using the concepts of deterministic chaos to describe the array of R–R interval durations.

Using all the previously mentioned algorithms and conclusions, we compiled a table containing the fractal dimensions *D* for the array of 1000 R–R intervals. The fractal dimensions were calculated using the Hurst-, Minimum Area Coverage- and Higuchi methods.

It should be noted that the used database of patients with CHF contains information on R–R intervals for more than 20 patients. Therefore, the sample size for this pathology was increased to 20 patients. The results obtained are presented in Table 5.

It can be seen that the average values of the fractal dimension for all three methods are not sufficient to

Table 4. Average values and variances for samples of 100 intervals taken from an array of 1000 R–R intervals

Parameter	1–100	100–200	200–300	300–400	400–500	500–600	600–700	700–800	800–900	900–1000	Patient
Average value, s	0.754	0.785	0.779	0.754	0.725	0.639	0.610	0.638	0.660	0.734	CHF
Variance, $s^2 \cdot 10^3$	1.568	3.241	9.201	3.334	1.221	2.159	3.101	7.528	1.116	5.211	
Average value, s	0.799	0.807	0.801	0.755	0.722	0.750	0.742	0.748	0.783	0.753	Healthy
Variance, $s^2 \cdot 10^3$	1.857	1.744	2.400	2.006	2.465	2.608	1.868	3.088	2.253	2.180	

Table 5. Fractal dimensions calculated by three different methods for the array of input data equal to 1000 R–R intervals

Patient No.	Fractal dimension calculated with the Higuchi method			Fractal dimension calculated with the coverage method			Fractal dimension calculated with the Hurst method		
	CHF	Healthy	ISH	CHF	Healthy	ISH	CHF	Healthy	ISH
1	1.621	1.720	1.755	1.587	1.650	1.645	1.728	1.863	1.629
2	1.966	1.783	1.674	1.650	1.746	1.576	2.000	1.979	1.902
3	1.834	1.709	1.897	1.651	1.717	1.719	1.679	1.948	2.000
4	1.832	1.717	1.650	1.665	1.717	1.577	1.519	1.948	1.722
5	1.983	1.703	1.659	1.816	1.648	1.598	2.000	1.718	1.713

Table 5. Continued

Patient No.	Fractal dimension calculated with the Higuchi method			Fractal dimension calculated with the coverage method			Fractal dimension calculated with the Hurst method		
	CHF	Healthy	ISH	CHF	Healthy	ISH	CHF	Healthy	ISH
6	2.028	1.714	1.573	1.693	1.646	1.547	2.000	1.824	1.674
7	1.848	1.752	2.008	1.704	1.676	1.873	1.946	1.704	2.000
8	1.756	1.746	1.903	1.733	1.711	1.720	2.000	1.707	1.816
9	1.488	1.758	1.659	1.641	1.676	1.588	1.884	1.704	1.709
10	1.965	1.679	1.747	1.637	1.630	1.709	1.865	1.691	1.705
11	1.811	–	–	1.710	–	–	1.939	–	–
12	1.783	–	–	1.619	–	–	1.430	–	–
13	1.949	–	–	1.677	–	–	1.786	–	–
14	1.818	–	–	1.668	–	–	1.969	–	–
15	1.879	–	–	1.542	–	–	2.000	–	–
16	1.739	–	–	1.687	–	–	1.669	–	–
17	1.579	–	–	1.632	–	–	1.703	–	–
18	1.739	–	–	1.414	–	–	1.883	–	–
19	1.510	–	–	1.552	–	–	1.507	–	–
20	1.513	–	–	1.616	–	–	1.801	–	–
Average value ± interval of validity	1.782 ± 0.077	1.728 ± 0.022	1.752 ± 0.100	1.645 ± 0.039	1.682 ± 0.028	1.655 ± 0.072	1.866 ± 0.121	1.808 ± 0.084	1.827 ± 0.151

reliably identify the presence of pathology. It should be also noted that the average values of the fractal dimension *D* of healthy patients, determined by the coverage and Higuchi methods, correspond to the data of [21], while the results by the Hurst method give overestimated values. Since the tabular presentation of data is quite difficult for perception and analysis, the data in Table 5 were subjected to additional processing in this work.

RANKING AND STATISTICAL PROCESSING OF THE RESULTS

The ranking method used in the present work consists in arranging objects or phenomena in descending or ascending order of a certain feature inherent in each of them. In this case, the ranking is performed in ascending order of the fractal dimension *D*. Thus, it is necessary to determine the number of ranks and the range of *D* values to which each rank will correspond.

In this work, 10 ranks were used. Despite the fact that the fractal dimension is determined in the range of values from 1 to 2, the vast majority of *D* values is concentrated in the range from 1.5 to 2. This area is divided into 10 ranks through 0.05 with the boundaries of each rank of ±0.025.

After analyzing the data presented in Table 5, it was noticed that most of the values of the fractal dimensions *D* of healthy patients fell into the range of values from 1.675 to 1.825, corresponding to ranks 4 and 5. The distribution by ranks of the fractal dimension of healthy and sick patients is presented in Table 6.

As can be seen from the table, 90% of the dimension values for healthy patients calculated by the Higuchi method have a rank of 4–5. For CHF patients, only 3 out of 20 people have this rank (15%), while for IHD patients this value is 30% (3 out of 10 people). For the dimensions determined by the coverage and Hurst methods, the picture of the distribution of ranks between healthy and sick patients becomes more blurred.

To assess the statistical significance of the division into ranks of healthy and sick patients, the Kolmogorov–Smirnov criterion was used to test the homogeneity of the distribution of two samples [22]. This criterion is based on a comparison of empirical distribution functions that are determined for two samples. The calculated values of the criterion, which are compared with the table values at a significance level of 0.01, are presented in Table 7.

The first line shows the tabular value of the Kolmogorov–Smirnov criterion for 10 healthy and

Table 6. Ranking of the results of Table 5

Patient No.	Ranking for the Higuchi method			Ranking for the coverage method			Ranking for the Hurst method		
	CHF	Healthy	IHD	CHF	Healthy	IHD	CHF	Healthy	IHD
1	2	4	5	2	4	3	4	7	2
2	9	6	4	3	5	2	10	10	8
3	7	4	8	3	4	4	4	9	10
4	7	4	3	3	4	2	0	8	4
5	10	4	3	6	3	2	10	4	4
6	10	4	2	4	3	1	10	6	3
7	7	5	10	4	4	7	9	4	10
8	5	5	8	5	4	4	10	4	6
9	0	5	3	3	4	2	8	4	4
10	9	4	5	3	3	4	6	4	4
11	6			4			9		
12	6			2			0		
13	9			4			6		
14	6			3			9		
15	7			1			10		
16	5			4			3		
17	2			3			4		
18	5			0			8		
19	0			1			0		
20	0			2			6		

20 patients with CHF, the second line is for 10 healthy patients and 10 with coronary artery disease.

The data in Table 7 indicate that the difference in the distribution of the fractal dimension between healthy and sick patients is statistically significant ($D_{\max\text{calc}} > D_{\max\text{tab}}$) for the Higuchi method. At the same time, when using the Hurst and coverage methods, there is no reason to reject the null hypothesis concerning the absence of differences in the distribution of results for the two groups of patients, i.e., two samples may belong to the same general population. We also note that the previously reached conclusion, i.e., that the fractal dimension calculated by the Higuchi method is apparently the most preferable in detecting the pathology of the heart, is confirmed by the results of the present study.

Figure 4 shows a diagram of the process of implementing the method of dividing patients into groups according to the presence and absence of pathology based on the value of the fractal dimension of the heart rate.

The diagram in Fig. 4 reflects some of the processing steps. The input data of R–R intervals were obtained on the basis of already analyzed Holter records of healthy and sick patients presented in the Physionet open resource. Holter monitoring data were recorded for

Table 7. Comparison of calculated and tabulated values of the two-sample Kolmogorov–Smirnov D_{max} criterion for healthy and sick patients

Tabular value of criterion D_{max} (for 30 values)	0.290
Tabular value of criterion D_{max} (for 20 values)	0.352
Calculated D_{max} value of Higuchi ranking (CHD and healthy)	0.500
Calculated D_{max} value of Higuchi ranking (IHD and healthy)	0.400
Calculated D_{max} value of coverage ranking (CHF and healthy)	0.350
Calculated D_{max} value of coverage ranking (IHD and healthy)	0.500
Calculated D_{max} value of Hurst ranking (CHF and healthy)	0.200
Calculated D_{max} value of Hurst ranking (IHD and healthy)	0.200

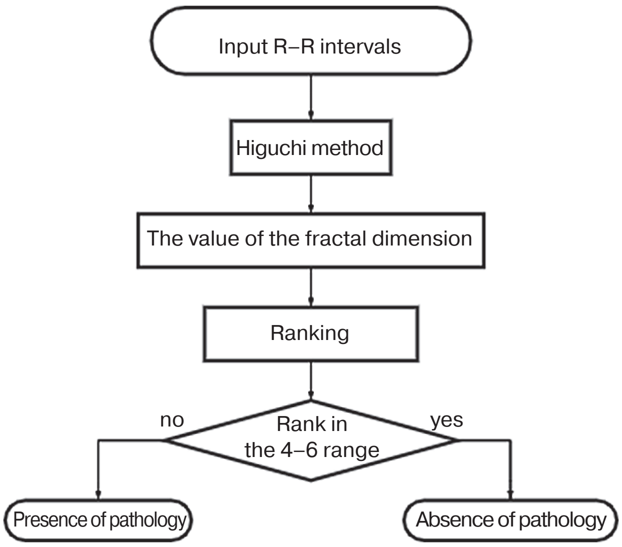

Fig. 4. Algorithm for the process of dividing patients into groups according to the presence and absence of pathology based on the value of the fractal dimension of the heart rate

24 h using DMS300-7 and DMS300-3A digital three-channel recorders, as well as Galix recorders using 3M electrodes. The Galix recorders had programmable read sampling rates of 512 and 1024 Hz and write sampling rates of 128 Hz. The DMS recorders had a sampling rate of 1024 Hz per channel for electrocardiography (ECG) analysis with an averaged signal, a read sampling rate of 512 Hz, and a recording sampling rate of 128 Hz otherwise. Signals were analyzed using Galix software and CardioScan 10.0, 11.0 software[7] for DMS recorders. The error in determining the R–R interval was approximately 8 ms (2 times more than the error in determining the R-wave). After automatic detection and classification using the *Holter* software[8], cardiac events in the records were then checked and corrected by two cardiologists. The recordings were then analyzed beat by beat to identify and correct as many R-waves as possible. Thus, the number and duration of artifacts in the signal was reduced.

The input R–R intervals then need to be processed using the Higuchi method presented above.

To determine the rank of the obtained value *D*, it is necessary to use Table 8, which presents the rank numbers, the ranges of their values and the average values of each.

Table 8. Ordinal numbers of ranks, their ranges and average values

Rank No.	Range of *D* values	Average value of *D*
0	1.500–1.525	1.5125
1	1.525–1.575	1.55
2	1.575–1.625	1.6
3	1.625–1.675	1.65
4	1.675–1.725	1.7
5	1.725–1.775	1.75
6	1.775–1.825	1.8
7	1.825–1.875	1.85
8	1.875–1.925	1.9
9	1.925–1.975	1.95
10	1.975–2.000	1.9875

If the rank value falls within the interval 4–6, then we can conclude that the input data of the R–R intervals belong to a healthy patient, otherwise there is reason to believe that the patient has pathology.

CONCLUSIONS

Mathematical approaches based on a study of the fractality of the non-stationarity or irregularity (in the geometric sense) of processes represent one of the

[7] https://vdd-pro.ru/ru/usb-kardiograf/programmnoe-obespechenie/. Accessed February 14, 2022 (in Russ.).

[8] https://dms-at.ru/products/programs/programmnoe-obespechenie-kholter/. Accessed February 14, 2022 (in Russ.).

possible approaches for their assessment. If considered from the point of view of amplitude changes in the magnitude of the electric potential over time, the graphic record of the ECG also represents a curve that has a strongly irregular shape. If we look at the rhythmogram in time, then we can come to the conclusion that in some way it has the properties of a fractal.

In the present work, we have considered several methods for obtaining the fractal dimension of a sequence of cardio interval durations, namely, the Hurst, Barrow, minimum coverage area and Higuchi methods.

Based on the data obtained, it is shown that the difference between the fractal dimensions of the durations of the R–R intervals of healthy and sick patients is statistically significant at a significance level of 0.01 when using the Higuchi method. The ranking of samples has demonstrated the possibility of effectively distinguishing between the fractal dimensions of healthy and sick patients. The results of the work demonstrate the promise of further research aimed at using the fractal characteristics of the heart rhythm to identify its failure, which can serve as an additional factor in determining the pathology of the heart.

Authors' contributions

M.O. Bykova—collection and analysis of information for the article, calculations and analysis of fractal dimension obtained by various methods.

V.A. Balandin—conceptual idea, discussion and analysis of the obtained results.

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