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RESEARCH ARTICLE

Measurement of capillary waves with a laser wave recorder

Viktor V. Sterlyadkin, Konstantin V. Kulikovsky [®]*MIREA – Russian Technological University, Moscow, 119454 Russia*[®] Corresponding author, e-mail: constantinkk@mail.ru**Abstract**

Objectives. Capillary waves on the sea surface play an important role in remote sensing, both in the optical and microwave wavelength ranges. However, processes of electromagnetic radiation scattering on a rough sea surface cannot be studied in the absence of reliable monitoring of the parameters of these capillary waves under natural conditions. Therefore, the aim of the present work was to develop methods for such monitoring purposes and test them under laboratory and field conditions.

Methods. Novel laser-based methods for recording capillary waves at frequencies up to 100 Hz were developed in the laboratory. The proposed remote methods, which do not interfere with the sea surface, are based on the recording of scattered laser radiation using a video camera.

Results. Under laboratory conditions, spatial profiles, time dependences of heights for all points of a laser sweep trajectory, and frequency power spectra were obtained. It is shown that slopes in capillary waves can reach 30° and that the amplitude of capillary waves at frequencies above 25 Hz does not exceed 0.5 mm. A new version of a scanning laser wave recorder was tested under natural conditions on an offshore platform. The measurements confirmed the possibility of measuring the parameters of sea waves on spatial scales covering 3 orders of magnitude: from units of millimeters to units of meters.

Conclusions. The developed wave recorder can be used to carry out direct measurements of “instantaneous” sea surface profiles with a time synchronization precision of 10^{-4} s and a spatial accuracy of better than 0.5 mm. The method makes it possible to obtain large series (21000) of «instantaneous» wave profiles with a refresh rate of 60 Hz, which opens up opportunities for studying the physics of wave evolution and the influence of wave parameters on the scattering of electromagnetic waves. The advantage of the method is the direct nature of the measurement of applicates and other wave characteristics not only in time but also in space. The entirely remote method does not distort the properties of the surface and is not affected by wind, waves, or sea currents. The possibility of using the proposed method under natural conditions at any time of the day and in a wide range of weather conditions has been experimentally ascertained.

Keywords: altimetry, spectrum of sea waves, capillary waves, atmosphere-ocean interaction, laser wave recorder, remote sensing

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НАУЧНАЯ СТАТЬЯ

Измерение капиллярных волн лазерным волнографом

В.В. Стерлядкин, К.В. Куликовский @

МИРЭА – Российский технологический университет, Москва, 119454 Россия

@ Автор для переписки, e-mail: constantinkk@mail.ru

Резюме

Цели. Капиллярные волны на морской поверхности играют важную роль в задачах дистанционного зондирования как в оптическом, так и в микроволновом диапазонах длин волн. Однако исследовать процессы рассеяния электромагнитного излучения на взволнованной морской поверхности можно только при надежном контроле параметров этих капиллярных волн в натурных условиях. До настоящего времени не существовало методов измерения капиллярных волн в натурных условиях. Целью настоящей работы являлось создание таких методов и их проверка в лабораторных и натурных условиях.

Методы. В лаборатории были отработаны новые лазерные методы регистрации капиллярных волн на частотах до 100 Гц. Предложенные методы являются дистанционными, не искажающими поверхность. Они основаны на регистрации рассеянного лазерного излучения с помощью видеокамеры.

Результаты. В лабораторных условиях получены пространственные профили, временные зависимости высот для всех точек траектории лазерной развертки, частотные спектры мощности. Показано, что уклоны в капиллярных волнах могут достигать 30° , а амплитуда капиллярных волн на частотах выше 25 Гц не превышает 0.5 мм. В натурных условиях на морской платформе апробирована новая версия сканирующего лазерного волнографа. Измерения подтвердили возможность измерения параметров морского волнения на пространственных масштабах, охватывающих 3 порядка: от единиц миллиметров до единиц метров.

Выводы. Созданный волнограф позволяет проводить прямые измерения «мгновенных» профилей морской поверхности с временной синхронизацией в 10^{-4} с и пространственной точностью лучше 0.5 мм. Метод позволяет получать большие ряды (21 000) «мгновенных» профилей волнения с частотой обновления 60 Гц, что открывает возможности для исследования физики эволюции волнения, влияния параметров волнения на рассеяние электромагнитных волн. Достоинством метода является прямой характер измерения аппликат и всех характеристик волнения не только во времени, но и в пространстве. Метод полностью дистанционен, не искажает свойства поверхности, не подвержен влиянию ветра, волн и морского течения. Экспериментально в натурных условиях доказана возможность применения предложенного метода в любое время суток и в широком диапазоне погодных условий.

Ключевые слова: альтиметрия, спектр морского волнения, капиллярные волны, взаимодействие атмосферы и океана, лазерный волнограф, дистанционное зондирование

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Авторы заявляют об отсутствии конфликта интересов.

INTRODUCTION

Capillary waves play a fundamentally important role in problems associated with the scattering of electromagnetic waves on a rough sea surface when solving inverse problems of remote sensing of the Earth from space. The influence of capillary waves can be clearly observed over a calm water area: due to gusts of wind, the surface becomes covered with small ripples, changing its color and darkening due to an increase in the absorption coefficient. Similar effects of a sharp increase in the absorption coefficient with the appearance of capillary waves are observed not only in the optical, but also in the microwave wavelength range. For this reason, capillary waves often turn out to be the determining factor when reconstructing the meteorological parameters of the atmosphere and the underlying surface from radiometric and optical measurements of the Earth from spacecraft.

In this regard, determining the detailed relationship between the parameters of capillary waves and the scattering characteristics of the sea surface becomes an important fundamental and applied problem. However, the difficulty of conducting such studies lies in the fact that, until recently, there have been no direct methods for remote determination of the parameters of capillary waves under natural conditions. While remote sensing methods are often used from under water to measure the parameters of gravity waves using acoustic reflections or laser beam refraction [1, 2], these measurements do not allow capillary waves to be recorded. When conducting measurements from offshore platforms or aircraft, laser altimetry methods can be used. However, spatial resolution in these cases is either absent [3] (single-point measurements) or limited to a few centimeters [4, 5]. For this reason, such methods cannot be used to record capillary waves, whose amplitude is measured in fractions of millimeters. It is also worth mentioning remote methods for measuring wave parameters based on stereo imaging [6]. However, such methods are not direct; moreover, their sensitivity is not sufficient for measuring the parameters of capillary waves.

There is a significant number of publications on methods for measuring artificial capillary waves in pools. A good analysis of these is given in [7, 8]. However, a common disadvantage of pool measurements is the presence of resonant edge effects that lead to distorted results. Although the theory of turbulence of gravity and capillary waves has been developed in many works¹ [9–13], we were not able to find any studies

whose results included remote space-time measurements of the characteristics of capillary waves carried out under natural conditions.

The present work is aimed at creating a remote laser method for measuring waves from gravitational to capillary scales. The developed laser wave recorder model was tested both under laboratory and natural conditions. The paper provides a description of the method and measurement results.

In previous works [14, 15], we proposed novel laser methods for measuring the “instantaneous” shape of a rough sea surface. The basis of such methods involves laser scanning of the water surface with the beam trajectory recorded at the interface using a video camera. In this case, the beginning of the beam scan is synchronized with the start of recording the image on the sensor of the video camera. These methods can be used to remotely determine the detailed structure of all types of gravity and gravity-capillary waves across a wide range of weather conditions. The achieved accuracy of the method for measuring the interface height was 1 mm, while the accuracy of “tying” the entire trajectory to a single point in time was between 4 and 10 s. However, experimental verification of the proposed methods under natural conditions showed that their further development was required in order to be able to measure the purely capillary component of the waves. This was firstly due to the capillary component of the waves becoming significant for frequencies above 16 Hz, the traditional video sample rate of 25 Hz is not adequate for recording them: the recording frequency should be at least twice as high as the profile being recorded. Secondly, since the amplitude of capillary waves has a scale comparable to or less than 1 mm, the sensitivity of the method by which these waves will be recorded should be at the level of 0.5 mm or less. Thus, in the present work, the method of remote laser measurements of waves is improved to the level required for separately measuring the capillary component of the waves.

GENERATION OF CAPILLARY WAVES UNDER LABORATORY CONDITIONS AND THE TECHNIQUE OF LASER MEASUREMENTS

Laboratory measurements comprise an important step in creating a method for measuring capillary waves under marine conditions. As well as providing remote measurements of the parameters of the generated waves, Figure 1 shows a schematic of a laboratory setup that makes it possible to generate capillary waves of a given amplitude and frequency. Vibrations generated by a low frequency woofer and transmitted by a light rod to a round platform comprise a source of disturbances

¹ Filatov S.V. Nonlinear wave and vortex motions on the surface of a liquid. Cand. Sci. Thesis (Phys.-Math.). Chernogolovka; 2020. 82 p. (in Russ.). <https://docplayer.com/170397192-Filatov-sergey-vasilevich-nelineynye-volnovye-i-vihrevye-dvizheniya-na-poverhnosti-zhidkosti.html>

on the water surface. A ruler serves for recording the wavelength of the generated waves in the photographs. The stroboscope allows the image of capillary waves to be “stopped” if its frequency is consistent with that of the signal supplied to the woofer in order to obtain a clear image of the waves on the water surface in the reservoir and thus reliably measure the expanse of a set of several waves.

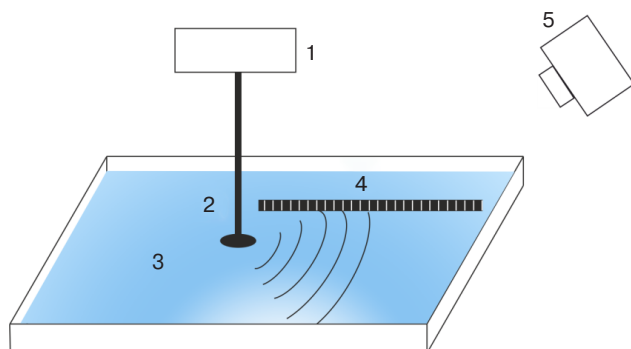


Fig. 1. Schematic of generating capillary waves in laboratory conditions:

1 – low-frequency woofer, 2 – vibrating platform, 3 – water surface, 4 – ruler, 5 – stroboscope

Photographs of the produced waves at different generator frequencies f are presented in Figs. 2a and 2b. In Fig. 2a, the frequency of produced waves $f = 16$ Hz corresponds to a gravity-capillary wave, while the frequency $f = 60$ Hz in Fig. 2b relates to purely capillary oscillations.



Fig. 2. Images of surface waves at generation frequency $f = 16$ Hz (a) and at $f = 60$ Hz (b)

The created setup made it possible to find the dispersion relation between the angular frequency of the wave ω and the wavelength λ . The theoretical expressions for the dispersion and the phase velocity of waves have the form [16]:

$$\omega = \sqrt{kg + \frac{k^3\sigma}{\rho}}, \quad (1)$$

Table. Laboratory measurements

Frequency f , Hz	8	10	12	14	16	18	20	24	28	35	40	50	60	70	80	100
Wavelength λ , mm	27.5	22.0	18.3	16.3	13.7	12.0	11.0	9.2	8.0	7.0	6.2	5.2	4.3	3.75	3.6	2.9

$$U = \frac{\omega}{k} = \sqrt{\frac{g}{k} + \frac{k\sigma}{\rho}}, \quad (2)$$

where $k = \frac{2\pi}{\lambda}$ is the wavenumber, g is the gravitational acceleration, and σ is the surface tension coefficient of the liquid.

In this equation, the first term under the root determines the gravitational part, while the second term is responsible for the capillary component. For pure water, $\sigma = 72$ mN/m, and the conditional boundary of the transition from predominantly gravitational to predominantly capillary waves corresponds to the equality of these terms. In this case, $k = \frac{\rho g}{\sigma} = 369 \text{ m}^{-1}$, and the wavelength corresponding to the boundary of the transition to capillary waves is $\lambda_{\text{bound}} = 17$ mm. The results of laboratory measurements are presented in Table.

Figure 3 shows the dispersion for surface waves. Here it should be noted that no special measures were taken to purify water from surfactants, resulting in a lowered value of σ . Optimization of the theoretical dependence for this parameter with respect to experimental measurements showed that the best approximation corresponds to $\sigma = 45$ mN/m. Note also that in laboratory measurements, the conditional wavelength λ_{bound} corresponding to the transition to the capillary component decreases in proportion to the surface tension coefficient.

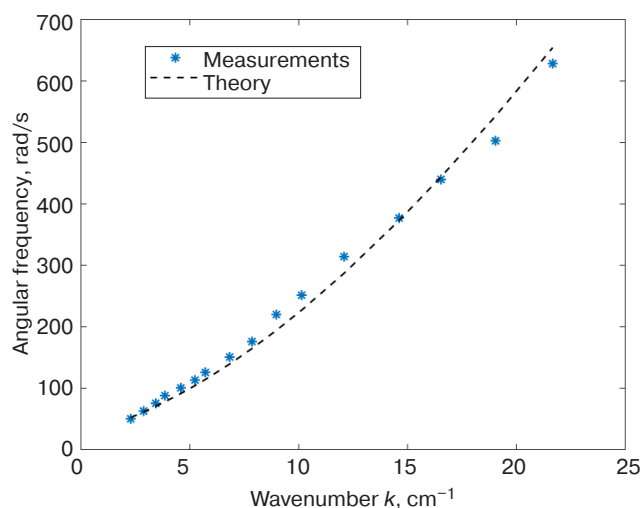


Fig. 3. Measured dispersion of gravity-capillary and capillary waves ($\sigma = 45$ mN/m)

It can be seen from the graph that the measured dispersion dependence is in good agreement with the theoretical dependence (1).

The next step of the experiment was to register the amplitude of the emerging gravity-capillary waves. To do this, the surface was illuminated by laser radiation, which scanned along the surface with short stops at separate points. In a photograph, shown in Fig. 4, the boundary where the laser beams are scattered is clearly visible. Registration of surface movement during wave propagation is carried out using a video camera placed laterally with a recording frequency of 60 Hz. Laser beams scatter in water, when hit it, creating an almost vertical beam. The upper boundary of each beam corresponds to the water-air interface.

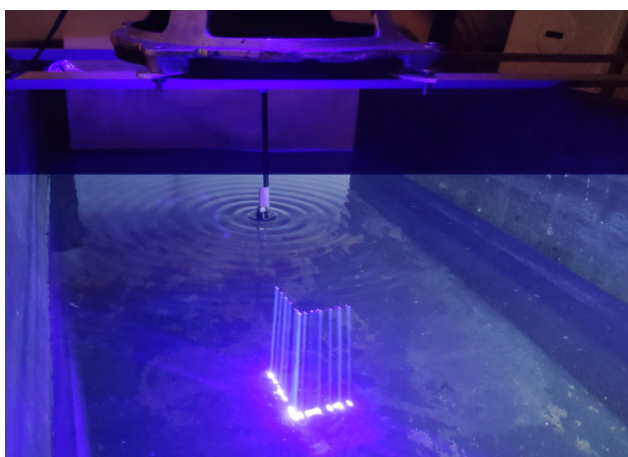


Fig. 4. Laser beams, when scattered in water, form a clear edge on the surface

However, such a measurement scheme did not allow recording the cross section of the surface and its profile. To solve this problem, the measurement scheme was changed. For this, a setup was assembled schematically shown in Fig. 5.

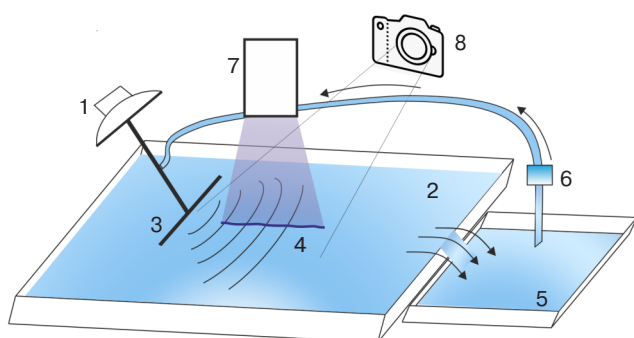


Fig. 5. Schematic of laboratory optical measurements of capillary waves: 1 – woofer, 2 – water surface, 3 – vibrating plate, 4 – interface illuminated by a scanning laser, 5 – lower reservoir, 6 – pump, 7 – electronically controlled scanner, 8 – video camera

In this experiment, the wave generator comprised a vibrating plate partially submerged in water. Vibrations from the woofer cone were transmitted to

the water surface via a plate rigidly attached to the woofer. The woofer was powered via an amplifier using a sound generator, which allowed the amplitude and frequency of the plate vibrations to be changed over a wide range. Plane waves appearing on the water surface as a result of the displacement of the plate were illuminated by a scanning laser beam. To reduce the amount of dust particles settling on the surface, water was drained over the edge of the upper reservoir into the lower reservoir from which the water was returned to the upper bath by means of a pump. Laser radiation with a wavelength of 450 nm and power of about 1 W was swept on the surface along a straight line about 80 mm long using an electronically controlled scanner. The beam scattered at the interface was recorded from a laterally positioned video camera. In this case, the time of the beam sweep was close to the exposure time of video frames. An example of a video frame in which laser radiation is scattered onto a disturbed interface is shown in Fig. 6. The boundary of the water surface clearly visible in each frame can be converted into the height of the disturbance at each point of the ray trajectory taking the calibration of the video camera sensor into account. The presence of dust particles on the surface led to bright spots at their locations; however, draining the water significantly reduced their number to speed up the processing of frames.

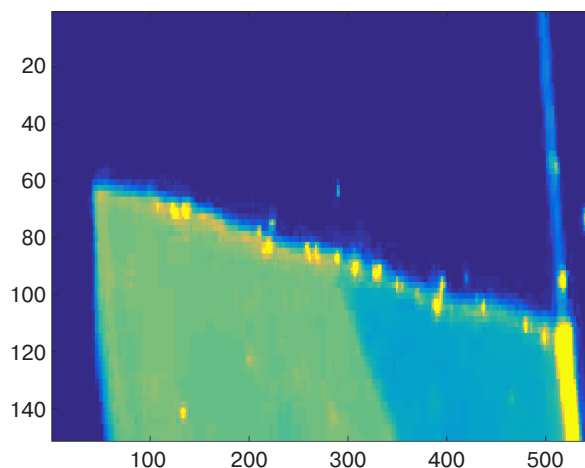


Fig. 6. Video frame of the wave surface, along which the capillary wave propagates

Video frames were processed for each column of the image. Figure 7 shows a graph of changes in the intensity of illumination along one of the columns of the frame. It can be seen that the boundary of the interface corresponding to the jump in intensity to a given level has a length of one pixel. Therefore, the height resolution of this method corresponds to the scale of one pixel, which in our experiment was 0.15 mm. The wave profile was recorded with sequential processing of all columns of the video frame.

Calibration of the video image and conversion of pixels to height in millimeters was carried out taking into account the position of the undisturbed surface and scaling according to the known image (*mire*). At the next stage, smoothing the profile with a sliding window turned out to be useful. Figure 8 shows examples of the wave profile for gravity-capillary and

capillary waves of various frequencies, taking into account smoothing.

Although the vibration amplitude of the woofer cone and the rod disturbing the water surface in the reservoir did not change, the amplitude of the waves at frequencies above 16 Hz, which decreased with increasing frequency, did not exceed one millimeter. This indicates an increase

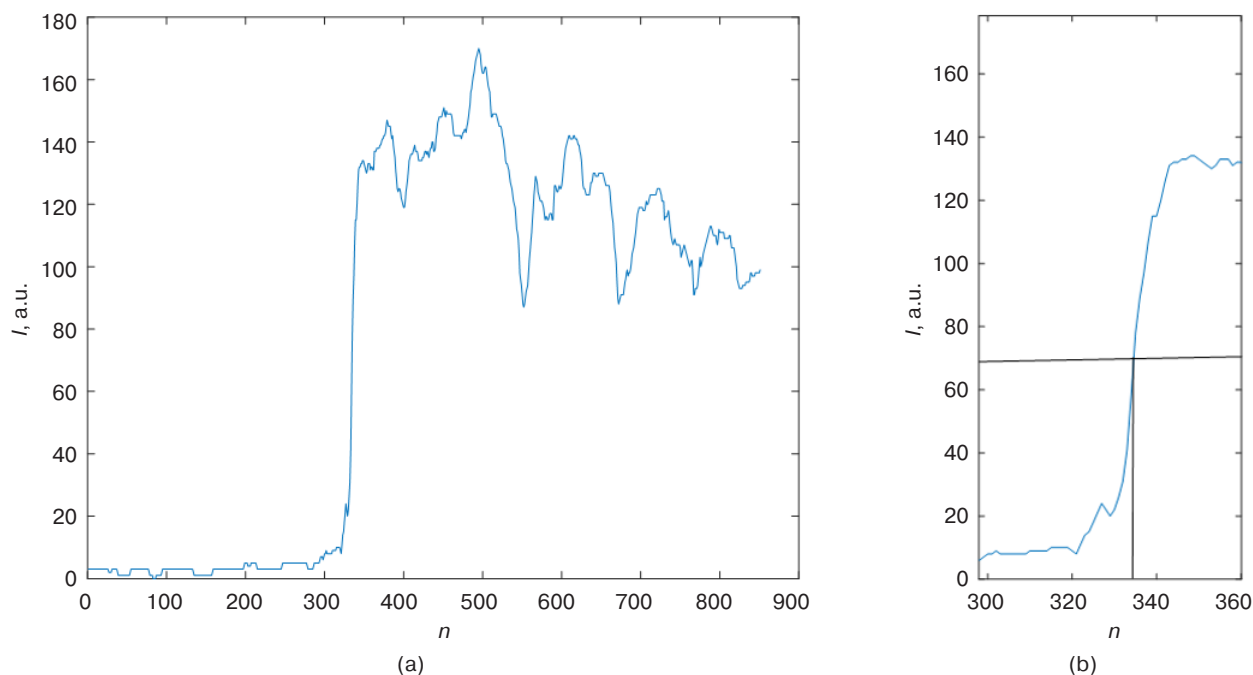


Fig. 7. Image intensity against the row number along one of the columns of the video frame in Fig. 5 (a) and the border of illumination clearly visible on an enlarged scale (b)

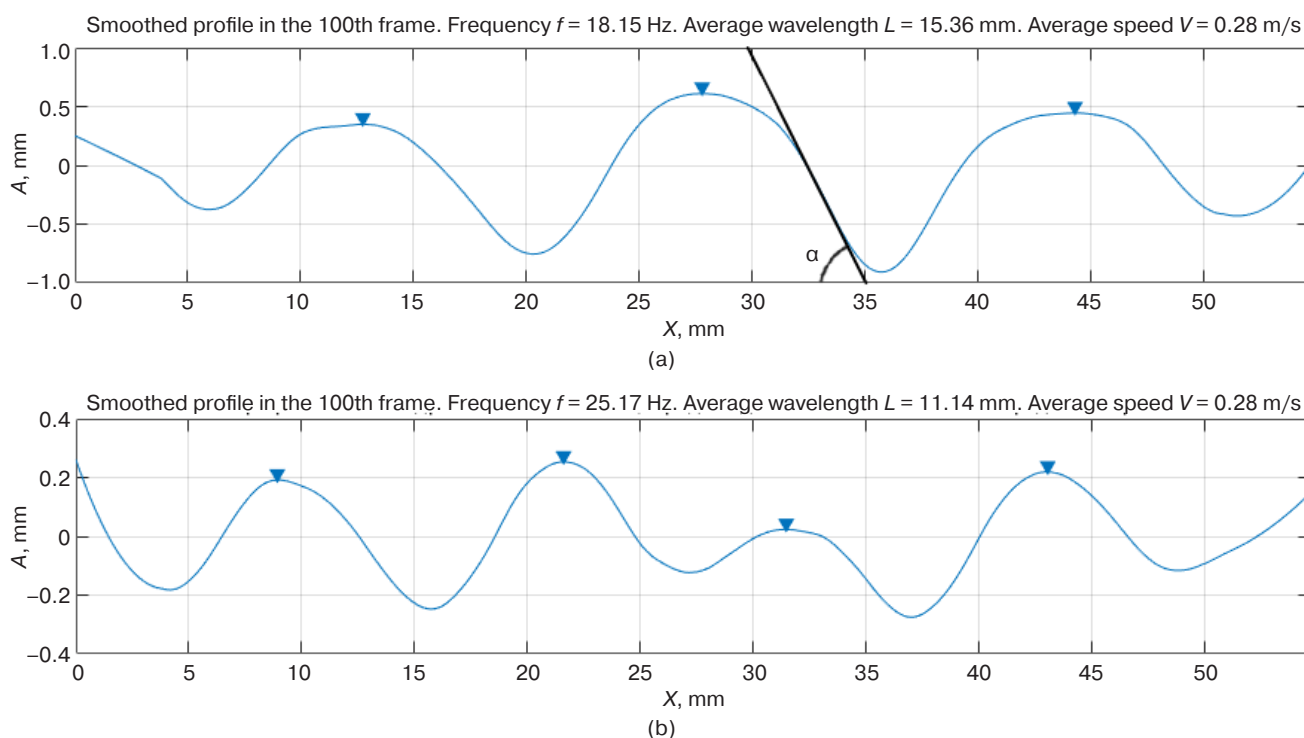


Fig. 8. Profile of capillary waves for a separate image frame: wave frequency 18 Hz (a), wave frequency 25 Hz (b)

in the attenuation coefficient with an increase in the frequency of capillary waves. In addition to the spatial image of capillary waves, the experiment made it possible to obtain the temporal variability of the applicate (height) at any point of the trajectory. Figure 9a shows the time dependence of the perturbation height. The power spectrum of the wave process is obtained via Fourier analysis of this dependence (Fig. 9b). The spectrum clearly registers the main frequency of the excited capillary waves, which corresponds to the vibration frequency of the woofer cone.

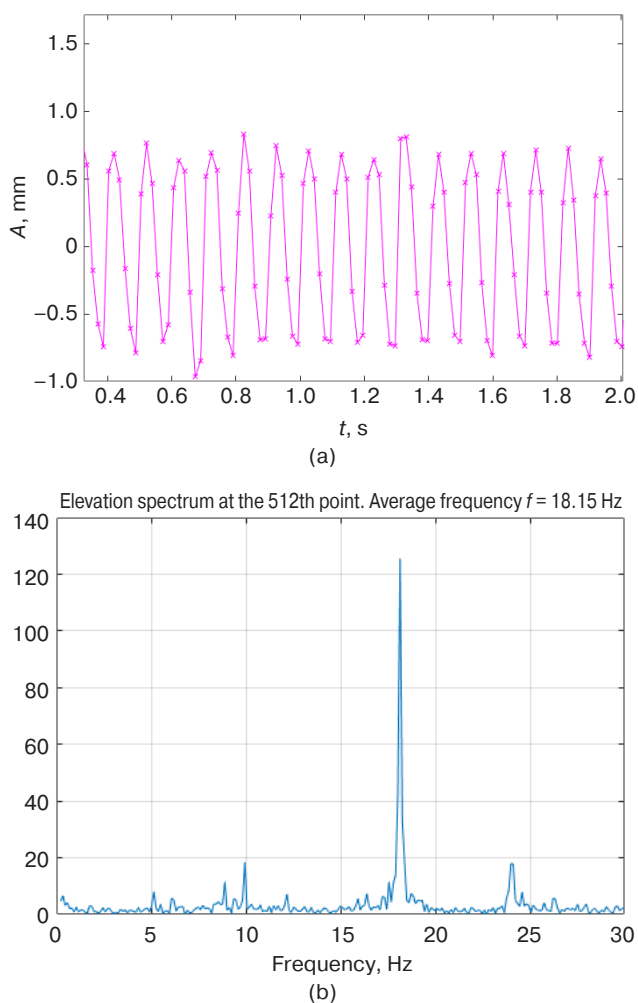


Fig. 9. Dependence of the height of the interface at one point of the trajectory on time (a) and the power spectrum in the 512th point (b)

The conducted laboratory studies of capillary waves made it possible to directly measure their main features and take these features into account when designing a marine wave recorder enabling to record the capillary component of sea waves. As a result of the analysis, a number of conclusions can be drawn. First, from the waveform in Fig. 8a it follows that the slopes in capillary waves can reach the level $\xi = \operatorname{tg}(\alpha) = 2/5$, which is 23° . Under natural conditions, capillary waves are superimposed onto gravitational waves, increasing the overall slope of the surface area. Under marine

conditions, the resulting slopes can be expected to reach 30° or more. In this case, a laser beam incident vertically down on the sea surface would deviate by more than 60° .

At frequencies of 25–30 Hz, it is not possible to create capillary waves with an amplitude greater than 0.5 mm. From this it follows that equipment used to measure the capillary component of sea waves under natural conditions at frequencies from 16 Hz to 30 Hz will require sensitivity at 0.5 mm or higher.

FIELD MEASUREMENTS OF CAPILLARY WAVES

In August 2021, the first field measurements were carried out using a scanning laser wave recorder, which made it possible to record capillary waves. The measurements were carried out from the sea pier of the Marine Hydrophysical Institute and from the stationary oceanographic platform of the Marine Hydrophysical Institute of the Russian Academy of Sciences, located in the town of Katsiveli in Crimea. Unlike the previous version of the wave recorder described in [14], the video recording frequency was increased from 25 Hz to 60 Hz. This made it possible to measure wave spectra up to 30 Hz, by which frequency the capillary component is already dominant. Recall that for pure water, starting from frequencies above 16 Hz, the capillary forces exceed the gravitational ones. A second improvement concerned the use of a zoom lens on a video camera, which increased the spatial resolution to 0.3–0.6 mm.

An example of a video frame received on August 26, 2021, at 20:47 Moscow time at the sea pier is shown in Fig. 10. The shape of the wave surface is clearly recorded in the image. For each 6-minute scanning session, it is possible to record 21 600 “instantaneous” spectra. These can be used as a basis for calculating both temporal and spatial wave spectra, one-dimensional and two-dimensional slopes on any scale, as well as the variability of wave parameters when the meteorological situation changes.

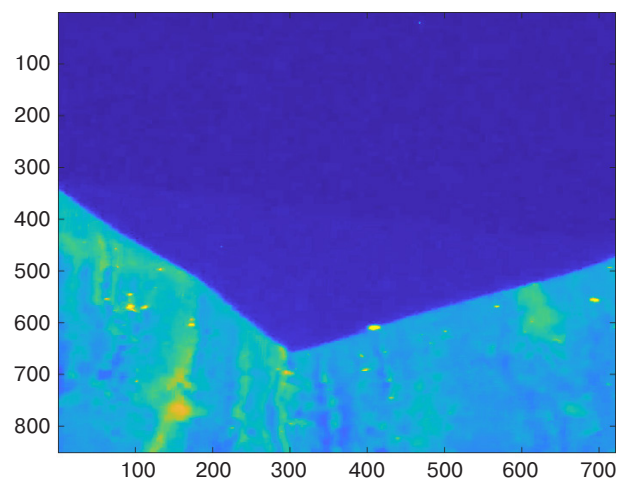


Fig. 10. Typical video frame of disturbance (values along axes are in pixels). Sampling frequency is 60 Hz

Figure 11 shows an example of a scanning realization dated August 26, 2021, for the dependence of the height on the frame number for one point of the trajectory (the total number of resolved points on the trajectory of the laser beam scan is 700). Figure 11a shows the entire scan, and Fig. 11b demonstrates a small segment of the scan on an enlarged scale.

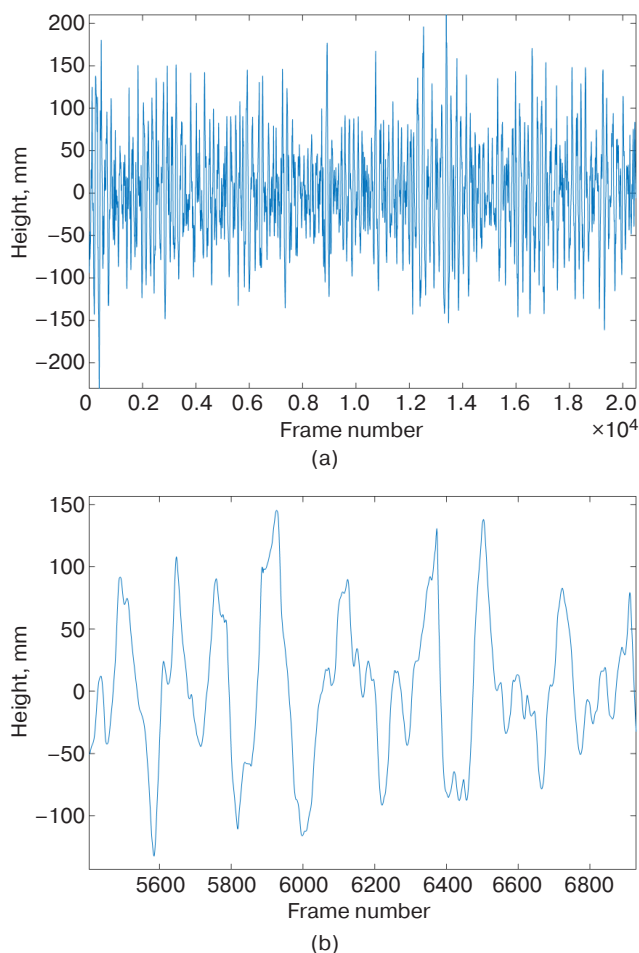


Fig. 11. An example of the dependence of the wave height on the frame number for one point on the scan trajectory (a), and a section of the recording on an enlarged scale (b)

The time spectrum of the resulting realization is of particular interest (Fig. 12). Traditionally, it is represented on a double logarithmic scale. There are three segments on the spectrum, whose slope characterizes various forms of waves. Here, the lowest frequency region of the spectrum (long wavelength) has a slope of -4.5 ± 0.3 , while the middle region is responsible for gravitational waves and has a slope of -4.0 ± 0.2 . The much less steep slope of the third region, which already takes capillary waves into account, is -2.6 ± 0.3 . The obtained exponential dependence indices are in good agreement with the known theoretical and experimental data. Thus, the power dependence with a slope of -4.0 corresponds to the spectrum of V.E. Zakharova and

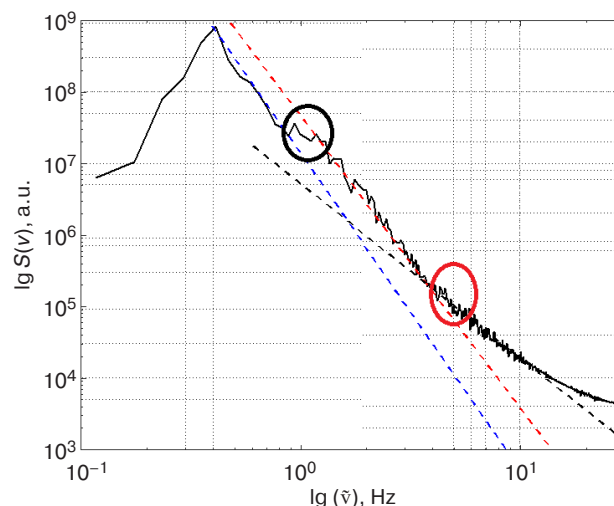


Fig. 12. Elevation spectrum at one point of the profile on a logarithmic scale for developing waves (August 26, 2021, Katsiveli, Crimea). The circles indicate the regions of transition from one spectrum slope to another

N.N. Filonenko [10], while the section with a slope of -4.5 forms a transitional regime to the Phillips spectrum $E \sim f^{-5}$ [12].

CONCLUSIONS

At this phase of research, the authors did not set the task of interpreting the spectral or other characteristics of waves. The purpose of this work was to create and test a new laser wave recorder under both laboratory and field conditions, in order to evaluate its capabilities for measuring the full spectrum of gravitational, gravity-capillary, and capillary waves under natural marine conditions. Under laboratory conditions, laser methods for recording capillary waves were tested, and the characteristics of waves at high frequencies were obtained: spatial profiles, time dependencies of heights for all points of the laser scan trajectory, as well as frequency power spectra. It is concluded that the slopes in capillary waves can reach 30° , while the amplitude at frequencies above 25 Hz does not exceed 0.5 mm. An important stage of the work was the testing of a new version of the scanning laser wave recorder under natural marine conditions. The measurements confirmed the feasibility of using the proposed laser method to measure the parameters of sea waves on spatial scales covering 3 orders: from millimeters to meters. We are not aware of world analogs having such characteristics. The method is patented [15]. The creation of a wave recorder that makes it possible to conduct direct measurements of “instantaneous” sea surface profiles synchronized within 10^{-4} s with a spatial accuracy of better than 0.5 mm opens up wide opportunities for studying the physics of wave evolution and the effect of waves on the scattering of electromagnetic waves. This is the first time such

opportunities have been opened up. The advantage of the method is that it provides direct measurements of applicates and all wave characteristics not only in time but also in the spatial domain. This makes it possible to obtain large series (21 000) of “instantaneous” wave profiles for each 6-minute scanning session. The described remote method, which does not distort the properties of the surface, is unaffected by wind, waves and sea currents. The possibility of using the proposed method to measure the proportion of foam on the

sea surface at any time of the day and under a wide range of weather conditions has been experimentally demonstrated under field conditions.

Authors' contribution

The ideas of the proposed methods, hardware implementation, and assembly of the device, as well as the development of software for processing data were implemented by Professor V.V. Sterlyadkin and Senior Lecturer K.V. Kulikovskiy, the staff of the Department of Physics at the MIREA – Russian Technological University.

REFERENCES

- Holthuijsen L.H. *Waves in oceanic and coastal waters*. Cambridge University Press; 2010. 404 p. <https://doi.org/10.1017/CBO9780511618536>
- Hashimoto N. Analysis of the directional wave spectrum from field data. In: *Advances in Coastal and Ocean Engineering*. Liu P.L.-F. (Ed.). Singapore: World Scientific. 1997;3:103–143. https://doi.org/10.1142/9789812797568_0004
- Grare L., Lenain L., Melville W.K. Vertical profiles of the wave-induced airflow above ocean surface waves. *J. Phys. Oceanogr.* 2018;48(12):2901–2922. <https://doi.org/10.1175/JPO-D-18-0121.1>
- Hwang P.A., Wang D.W., Walsh E.J., Krabill W.B., Swift R.N. Airborne measurements of the wave number spectra of ocean surface waves. Part I: Spectral slope and dimensionless spectral coefficient. *J. Phys. Oceanogr.* 2000;30(11):2753–2767. [https://doi.org/10.1175/1520-0485\(2001\)031<2753:AMOTWS>2.0.CO;2](https://doi.org/10.1175/1520-0485(2001)031<2753:AMOTWS>2.0.CO;2)
- Allender J., Audunson T., Barstow S.F., Bjerken S., Krogstad H.E., Steinbakke P., Vartdal L., Borgman L.E., Graham C. The WADIC project; a comprehensive field evaluation of directional wave instrumentation. *Ocean Eng.* 1989;16(5–6):505–536. [https://doi.org/10.1016/0029-8018\(89\)90050-4](https://doi.org/10.1016/0029-8018(89)90050-4)
- Banner M.L., Jones I.S., Trinder J. Wavenumber spectra of short gravity waves. *J. Fluid Mech.* 1989;198:321–344. <https://doi.org/10.1017/S0022112089000157>
- Falcon E., Mordant N. Experiments in surface gravity-capillary wave turbulence. *Annual Rev. Fluid Mech.* 2022;54:1–25. <https://doi.org/10.1146/annurev-fluid-021021-102043>
- Brazhnikov M.Yu., Levchenko A.A., Mezhev-Deglin L.P. Excitation and detection of nonlinear waves on a charged surface of liquid hydrogen. *Instruments and Experimental Techniques*. 2002;45(6):758–763. <https://doi.org/10.1023/A:1021418819539>
- Zakharov V.E. Weak turbulence in media with a decay spectrum. *J. Appl. Mech. Tech. Phys.* 1965;6(4):22–24. <https://doi.org/10.1007/BF01565814>
[Original Russian Text: Zakharov V.E. Weak turbulence in media with a decay spectrum. *Prikladnaya mekhanika i tekhnicheskaya fizika*. 1965;4:35–39 (in Russ.). Available from URL: <https://www.sibran.ru/upload/iblock/2f5/2f55daccf31e1bdf4c6c44e436b4166.pdf>]

СПИСОК ЛИТЕРАТУРЫ

- Holthuijsen L.H. *Waves in oceanic and coastal waters*. Cambridge University Press; 2010. 404 p. <https://doi.org/10.1017/CBO9780511618536>
- Hashimoto N. Analysis of the directional wave spectrum from field data. In: *Advances in Coastal and Ocean Engineering*. Liu P.L.-F. (Ed.). Singapore: World Scientific. 1997;3:103–143. https://doi.org/10.1142/9789812797568_0004
- Grare L., Lenain L., Melville W.K. Vertical profiles of the wave-induced airflow above ocean surface waves. *J. Phys. Oceanogr.* 2018;48(12):2901–2922. <https://doi.org/10.1175/JPO-D-18-0121.1>
- Hwang P.A., Wang D.W., Walsh E.J., Krabill W.B., Swift R.N. Airborne measurements of the wave number spectra of ocean surface waves. Part I: Spectral slope and dimensionless spectral coefficient. *J. Phys. Oceanogr.* 2000;30(11):2753–2767. [https://doi.org/10.1175/1520-0485\(2001\)031<2753:AMOTWS>2.0.CO;2](https://doi.org/10.1175/1520-0485(2001)031<2753:AMOTWS>2.0.CO;2)
- Allender J., Audunson T., Barstow S.F., Bjerken S., Krogstad H.E., Steinbakke P., Vartdal L., Borgman L.E., Graham C. The WADIC project; a comprehensive field evaluation of directional wave instrumentation. *Ocean Eng.* 1989;16(5–6):505–536. [https://doi.org/10.1016/0029-8018\(89\)90050-4](https://doi.org/10.1016/0029-8018(89)90050-4)
- Banner M.L., Jones I.S., Trinder J. Wavenumber spectra of short gravity waves. *J. Fluid Mech.* 1989;198:321–344. <https://doi.org/10.1017/S0022112089000157>
- Falcon E., Mordant N. Experiments in surface gravity-capillary wave turbulence. *Annual Rev. Fluid Mech.* 2022;54:1–25. <https://doi.org/10.1146/annurev-fluid-021021-102043>
- Brazhnikov M.Yu., Levchenko A.A., Mezhev-Deglin L.P. Excitation and detection of nonlinear waves on a charged surface of liquid hydrogen. *Instruments and Experimental Techniques*. 2002;45(6):758–763. <https://doi.org/10.1023/A:1021418819539>
- Захаров В.Е. Слабая турбулентность в средах с распадным спектром. *Прикладная механика и техническая физика*. 1965;4:35–39. URL: <https://www.sibran.ru/upload/iblock/2f5/2f55daccf31e1bdf4c6c44e436b4166.pdf>
- Захаров В. Е., Филоненко Н. Н. Спектр энергии для стохастических колебаний поверхности жидкости. *Докл. АН СССР*. 1966;170(6):1292–1295. URL: <http://www.mathnet.ru/links/ec2b951f99ebc10ab5bb4c2bf4fe5948/dan32646.pdf>

10. Zakharov V.E., Filonenko N.N. Energy spectrum for stochastic oscillations of the surface of a liquid. *Dokl. Akad. Nauk SSSR*. 1966;170(6):1292–1295 (in Russ.). Available from URL: <http://www.mathnet.ru/links/ec2b951f99ebc10ab5bb4c2bf4fe5948/dan32646.pdf>
11. Zakharov V.E., Filonenko N.H. Weak turbulence of capillary waves. *J. Appl. Mech. Tech. Phys.* 1967;8: 37–40. <https://doi.org/10.1007/BF00915178>
[Original Russian Text: Zakharov V.E., Filonenko N.H. Weak turbulence of capillary waves. *Prikladnaya mekhanika i tekhnicheskaya fizika*. 1967;5:62–67 (in Russ.). Available from URL: <https://www.sibran.ru/upload/iblock/24e/24ea0a63fb235c70765e3dd7e3eadeea.pdf>]
12. Badulin S.I., Zakharov V.E. Phillips spectrum and a model of wind wave dissipation. *Theoret. and Math. Phys.* 2020;202(3): 309–318. <https://doi.org/10.1134/S0040577920030034>
[Original Russian Text: Badulin S.I., Zakharov V.E. Phillips spectrum and a model of wind wave dissipation. *Teoreticheskaya i matematicheskaya fizika*. 2020;202(3): 353–363 (in Russ.). <https://doi.org/10.4213/tmf9801>]
13. Lukaschuk S., Nazarenko S., McLelland S., Denissenko P. Gravity wave turbulence in wave tanks: Space and time statistics. *Phys. Rev. Lett.* 2009;103(4):044501. <http://doi.org/10.1103/PhysRevLett.103.044501>
14. Sterlyadkin V.V., Kulikovskii K.V., Kuzmin A.V., Sharkov E.A., Likhacheva M.V. Scanning laser wave recorder with registration of «instantaneous» sea surface profiles. *J. Atmos. Oceanic Technol.* 2021;38(8): 1415–1424. <https://doi.org/10.1175/JTECH-D-21-0036.1>
15. Sterlyadkin V.V. *Scanning laser recorder recording “instant” shape of surface*: RU Pat. 2749727. Publ. 16.06.2021 (in Russ.).
16. Landau L.D., Lifshits E.M. *Teoreticheskaya fizika: Uchebnoe posobie*. V 10 t. T. VI. *Gidrodinamika (Theoretical Physics: Textbook)*. In 10 v. V. VI. *Hydrodynamics*. Moscow: Nauka; 1986. 736 p. (in Russ.).
11. Захаров В.Е., Филоненко Н.Н. Слабая турбулентность капиллярных волн. *Прикладная механика и техническая физика*. 1967;5:62–67. URL: <https://www.sibran.ru/upload/iblock/24e/24ea0a63fb235c70765e3dd7e3eadeea.pdf>
12. Бадудин С.И., Захаров В.Е. Спектр Филлипса и модель диссипации ветрового волнения. *Теоретическая и математическая физика*. 2020;202(3):353–363. <https://doi.org/10.4213/tmf9801>
13. Lukaschuk S., Nazarenko S., McLelland S., Denissenko P. Gravity wave turbulence in wave tanks: Space and time statistics. *Phys. Rev. Lett.* 2009;103(4):044501. <http://doi.org/10.1103/PhysRevLett.103.044501>
14. Sterlyadkin V.V., Kulikovskii K.V., Kuzmin A.V., Sharkov E.A., Likhacheva M.V. Scanning laser wave recorder with registration of «instantaneous» sea surface profiles. *J. Atmos. Oceanic Technol.* 2021;38(8): 1415–1424. <https://doi.org/10.1175/JTECH-D-21-0036.1>
15. Стерлядкин В.В. *Сканирующий лазерный волнограф с регистрацией «мгновенной» формы поверхности*: Пат. RU № 2749727. Заявка № RU2020134068А; заявл. 16.10.2020; опубл. 16.06.2021.
16. Ландау Л.Д., Лифшиц Е.М. *Теоретическая физика: Учебное пособие*. В 10 т. Т. VI. *Гидродинамика*. М.: Наука. Гл. ред. физ.-мат. лит.; 1986. 736 с.

About the authors

Viktor V. Sterlyadkin, Dr. Sci. (Phys.-Math.), Professor, Department of Physics, Institute for Advanced Technologies and Industrial Programming, MIREA – Russian Technological University (78, Vernadskogo pr., Moscow, 119454 Russia). E-mail: sterlyadkin@mirea.ru. Scopus Author ID 6505940691, ResearcherID D-7125-2017, <https://orcid.org/0000-0002-1832-8608>

Konstantin V. Kulikovskiy, Senior Lecturer, Department of Physics, Institute for Advanced Technologies and Industrial Programming, MIREA – Russian Technological University (78, Vernadskogo pr., Moscow, 119454 Russia). E-mail: constantinkk@mail.ru. Scopus Author ID 57223241696, <https://orcid.org/0000-0001-9296-6424>

Об авторах

Стерлядкин Виктор Вячеславович, д.ф.-м.н., профессор, кафедра физики Института перспективных технологий и индустриального программирования ФГБОУ ВО «МИРЭА – Российский технологический университет» (119454, Россия, Москва, пр-т Вернадского, д. 78). E-mail: sterlyadkin@mirea.ru. Scopus Author ID 6505940691, ResearcherID D-7125-2017, <https://orcid.org/0000-0002-1832-8608>

Куликовский Константин Владимирович, старший преподаватель, кафедра физики Института перспективных технологий и индустриального программирования ФГБОУ ВО «МИРЭА – Российский технологический университет» (119454, Россия, Москва, пр-т Вернадского, д. 78). E-mail: constantinkk@mail.ru. Scopus Author ID 57223241696, <https://orcid.org/0000-0001-9296-6424>

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