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RESEARCH ARTICLE

Properties of objective functions and search algorithms in multi-objective optimization problems

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Abstract

Objectives. A frequently used method for obtaining Pareto-optimal solutions is to minimize a selected quality index under restrictions of the other quality indices, whose values are thus preset. For a scalar objective function, the global minimum is sought that contains the restricted indices as penalty terms. However, the landscape of such a function has steep-ascent areas, which significantly complicate the search for the global minimum. This work compared the results of various heuristic algorithms in solving problems of this type. In addition, the possibility of solving such problems using the sequential quadratic programming (SQP) method, in which the restrictions are not imposed as the penalty terms, but included into the Lagrange function, was investigated.

Methods. The experiments were conducted using two analytically defined objective functions and two objective functions that are encountered in problems of multi-objective optimization of characteristics of analog filters. The corresponding algorithms were realized in the MATLAB environment.

Results. The only heuristic algorithm shown to obtain the optimal solutions for all the functions is the particle swarm optimization algorithm. The sequential quadratic programming (SQP) algorithm was applicable to one of the analytically defined objective functions and one of the filter optimization objective functions, as well as appearing to be significantly superior to heuristic algorithms in speed and accuracy of solutions search. However, for the other two functions, this method was found to be incapable of finding correct solutions.

Conclusions. A topical problem is the estimation of the applicability of the considered methods to obtaining Pareto-optimal solutions based on preliminary analysis of properties of functions that determine the quality indices.

Keywords: multi-objective optimization, Pareto optimality, quality index, objective function, fitness landscape, heuristic algorithm, quadratic programming

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НАУЧНАЯ СТАТЬЯ

Свойства целевых функций и алгоритмов поиска в задачах многокритериальной оптимизации

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Резюме
Цели. Часто применяемый метод поиска оптимальных по Парето решений состоит в минимизации выбранного показателя качества при задании ограничений на остальные показатели, значения которых, таким образом, оказываются заранее определенными. При этом выполняется поиск глобального минимума скалярной целевой функции, в которую ограничиваемые показатели входят в виде штрафных слагаемых. Рельеф такой функции содержит участки быстрого роста, значительно затрудняющие поиск глобального минимума. В работе сравниваются результаты различных эвристических алгоритмов при решении задач этого типа. Кроме того, исследуется возможность использования алгоритма последовательного квадратичного программирования (SQP), в котором ограничения учитываются не через штрафные слагаемые, а включаются в функцию Лагранжа.
Методы. В экспериментах использовались две аналитически заданные целевые функции и две целевые функции, встречающиеся в задачах многокритериальной оптимизации характеристик аналоговых фильтров. Исследуемые алгоритмы были реализованы программами в среде *MATLAB*.
Результаты. Установлено, что единственным эвристическим алгоритмом, который нашел оптимальные решения для всех функций, оказался алгоритм роя частиц. Алгоритм SQP оказался применим для одной из аналитически определенных функций и для одной из целевых функций оптимизации фильтров, существенно превзойдя при этом эвристические алгоритмы по точности и скорости поиска решения. Но для двух других функций данный алгоритм оказался неспособным находить правильные решения.
Выводы. Актуальной является задача оценки применимости рассмотренных методов для поиска Парето-оптимальных решений на основе предварительного анализа свойств функций, определяющих показатели качества.

Ключевые слова: многокритериальная оптимизация, оптимальность по Парето, показатель качества, целевая функция, рельеф целевой функции, эвристический алгоритм, квадратичное программирование

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INTRODUCTION

The global extrema of multimodal objective functions (OFs) having many local extrema can be obtained using heuristic algorithms [1, 2]. Unlike classical optimization methods, heuristic algorithms have not yet been subjected to comprehensive theoretical analysis [2–4]. Their characteristics can be evaluated and compared experimentally using sets of test functions [5, 6]. Different algorithms show the best characteristics on different test functions, which leads to the problem of choosing the most appropriate algorithm for an OF with certain properties. Methods have recently been developed for automatic analysis of the properties of the OF relief (exploratory landscape analysis, ELA) and machine learning to select an algorithm and/or adjust its parameters according to the results of such an analysis [7, 8]. However, the complete solution of this problem is still far away.

In radio engineering and other sciences, of considerable interest are multi-objective optimization problems [9]. As a rule, it is impossible to simultaneously optimize all quality indices (QI) because improvement of some of the QIs leads to impairment of others. Therefore, the goal of multi-objective optimization is to find a set of Pareto-optimal solutions [1, 10]. A widely used way to search for them is to solve the problem

$$\mathbf{x}^* = \arg\min_{\mathbf{x}\in D'}\left(Q_k\left(\mathbf{x}\right)\right),$$
$$D' = \left\{\mathbf{x}\in D | Q_j\left(\mathbf{x}\right)\le Q_{jt};\, i=1,\ldots,M;\; j\ne k\right\}, \tag{1}$$

where $\mathbf{x}$ is the coordinate vector in the search space, D is the set (defined by inequality and/or equality restrictions) of allowable values of $\mathbf{x}$ in the search space, $Q_j(\mathbf{x})$ are functions that describe the QIs, M is the number of QIs, and $\mathbf{x}^*$ is the coordinate vector of the optimal solution. Without loss of generality, the problem of minimizing all the QIs was considered here.

In problem (1), all the QIs, except Q_k, are restricted from above, while Q_k is minimized. If the QIs are competing, then the minimum of Q_k is at the point $\mathbf{x}^*$ at which the other QIs reach the restrictions Q_{jt} imposed on them. This makes it possible to obtain solutions in which all the coordinates in the space of QIs, except the kth, are fixed at the objective values of Q_{jt}. It is known that this method can find any Pareto-optimal solution [10].

One of the methods to solve problem (1) is based on minimizing a scalar OF of the form

$$f\left(\mathbf{x}\right) = \sum_{j=1}^{M}\left(W_j\cdot\max\left(\frac{\left(Q_j\left(\mathbf{x}\right)-Q_{jt}\right)}{Q_{jt}^{\,q_j}},0\right)\right), \tag{2}$$

where $Q_j(\mathbf{x})$ and Q_{jt} are the current and objective values of the jth QI, respectively; and W_j is the weighting factor of the jth QI [2, 10]. If it is necessary to normalize the deviation of a QI from the objective value to reduce the terms in (2) to the same range of values, the exponent $q_j = 1$. If such a normalization is unnecessary, then the exponent $q_k = 0$. At $x > 0$, $\max(x, 0) = x$; otherwise, $\max(x, 0) = 0$.

The objective value Q_{kt} of the QI being minimized is given sufficiently low, e.g., equal to its minimum possible value, and the weighting factor is taken to be $W_k = 1$. The terms containing the other QIs are penalties for violation of restrictions imposed on them. The weighting factors at them should meet the conditions $W_j >> 1$ for such QI to be fixed near the objective values.

Problems of multi-objective optimization of the characteristics of analog and digital filters by the scalar OF method were considered earlier [11, 12].

The relief of OF (2) can be complex. If the objective values of the QIs to be fixed are exceeded, the OF value rapidly increases; for this reason, the relief has areas hereinafter referred to as “walls.” In the available works aimed at analyzing the properties of OFs and selecting optimization algorithms, such OF properties were not studied. In view of the importance of multi-objective optimization, this gap needs to be addressed.

Another approach to solving problem (1) is based on nonlinear programming methods, in which restrictions on QIs are not imposed as penalty terms, but included in the Lagrange function; as applied to problem (1), this function has the form

$$L(\mathbf{x},\boldsymbol{\lambda}) = Q_k(\mathbf{x}) + \sum_{i=1}^{M-1}\lambda_i g_i\left(\mathbf{x}\right),$$
$$g_i\left(\mathbf{x}\right) = Q_j(\mathbf{x}) - Q_{jt},\; i=1,\ldots,M-1,\; j=1,\ldots,M,\; j\ne k, \tag{3}$$
$$\mathbf{x}\in D,$$

where λ_i are the Lagrange multipliers. The minimum of function (3) is found, in particular, using sequential quadratic programming (SQP) algorithm [3], provided that the Lagrange function $L(\mathbf{x}, \boldsymbol{\lambda})$ has continuous second derivatives.

Population algorithms, which comprise an alternative to the scalar OF method, can give an approximation of the Pareto set containing a given number of elements within one search cycle [1]. The main advantage of these methods is their significantly accelerated search. However, there are difficulties in obtaining solutions with given values of some of the QIs. Moreover, with increasing number of QIs, the quality of the found approximations of the Pareto set may decrease. For example, by comparing the results of solving the problem of multi-objective optimization of the characteristics of electric filters using population algorithms and the scalar OF method, it was shown that, at the number of QIs $M = 2$, population algorithms are advantageous not only in terms of search speed, but also in quality of the solutions obtained. At the same time, at $M = 3$, they are inferior in quality to the approximation of the Pareto set to the scalar function method [13]. The prospects for the use of population algorithms for multi-objective optimization require further research and will not be considered here.

The purpose of the present work was to study the characteristics of various optimization algorithms of searching for the global minimum of scalar OF of type (2) whose relief has walls, as well as to develop recommendations for choosing algorithms for solving such problems. In addition, we studied the possibilities of applying methods for solving problem (1) that do not use penalty terms.

METHODS OF INVESTIGATION

First of all, let us define OFs with necessary properties, i.e., with walls. The first two OFs are defined analytically:

$$f_1\left(\mathbf{x}\right) = \sum_{i=1}^{ND}x_i + W_1\cdot\max\left(\sum_{i=1}^{ND}x_i^2 - a_1, 0\right), \tag{4}$$

$$f_2(\mathbf{x}) = \sum_{i=1}^{ND} x_i +$$
$$+W_2 \cdot \max\left(ND + \sum_{i=1}^{ND}\left(x_i^2 - \cos 2\pi x_i\right) - a_2, 0\right). \quad (5)$$

In these expressions, ND is the dimension of the search space. In both functions, the QI being minimized is calculated as the sum of the coordinates of the vector $\mathbf{x}$. In function $f_1(\mathbf{x})$, the QI being fixed is defined as the sum of the squares of the coordinates; while in function $f_2(\mathbf{x})$, it is represented by the known Rastrigin test function [5]. The parameters a_1 and a_2 are the objective values of the QI being fixed, while the parameters W_1 and W_2 are the weighting factors of the penalty terms.

Figure 1 presents the graphs of the OFs $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$ at $ND = 2$, $a_1 = 1$, $a_2 = 2$, $W_1 = W_2 = 100$, $-2 \le x_i \le 2$, and $i = 1, 2$. In the graphs, the values of the functions are bounded from above at the levels $f_1(\mathbf{x}) = 1$ and $f_2(\mathbf{x}) = 2$. The reliefs of both functions contain pronounced walls. The number of local extrema of $f_2(\mathbf{x})$ increases exponentially with increasing dimension ND.

Let us further define two OFs that are encountered in the problems of multi-objective optimization of the characteristics of analog electric filter [11]. The first of them has the form

$$f_3(\mathbf{x}) = \frac{DTd(\mathbf{x}) - DTd_t}{DTd_t} +$$
$$+ WHp \cdot \max\left(\frac{DHp(\mathbf{x}) - DHp_t}{DHp_t}, 0\right) +$$
$$+ WHs \cdot \max\left(\frac{HS_t - Hs(\mathbf{x})}{Hs_t}, 0\right) + \quad (6)$$
$$+ WHt \cdot \max\left(DHt(\mathbf{x}) - DHt_t, 0\right).$$

Here, DHp is the passband attenuation ripple (dB), Hs is the stopband attenuation (dB), DHt is the excess of the frequency response over the permissible level in the transition band between the passband and the stopband (dimensionless), and DTd is the relative passband delay time ripple (%). The coordinate vector $\mathbf{x}$ consists of the real and imaginary coordinates of the poles and zeros of the transfer function (one each from the complex conjugate pair). Methods to calculate the listed QIs were described in the literature [11, 14].

OF $f_3(\mathbf{x})$ (6) is obtained in the problem of minimizing the DTd QI under restrictions on the other three QIs. Since the Hs value should be maximized, this QI is subtracted from its objective value. In the experiments below, the values of the following quantities were given: the number of filter poles, $NP = 6$; the number of zeros, $NZ = 0$; the objective values of QI, $DHp_t = 0.5$, $Hs_t = 40$, $DHt_t = 1$, and $DTd_t = 10$; and the weighting factors, $WHp = 20$, $WHs = 500$, and $WHt = 1000$. The search space in all the coordinates was bounded by the inequalities $-3 \le x_i \le -0.01$.

The last OF has the form

$$f_4(\mathbf{x}) = \frac{Tss(\mathbf{x}) - Tss_t}{Tss_t} +$$
$$+ WTfr \cdot \max\left(\frac{Tfr(\mathbf{x}) - Tfr_t}{Tfr_t}, 0\right) +$$
$$+ WUm \cdot \max\left(\frac{Um(\mathbf{x}) - Um_t}{Um_t}, 0\right) + \quad (7)$$
$$+ WHs \cdot \max\left(\frac{HS_t - Hs(\mathbf{x})}{Hs_t}, 0\right) +$$
$$+ WHt \cdot \max\left(DHt(\mathbf{x}) - DHt_t, 0\right).$$

Here, Tss is the transient-process time; Tfr is the rise time of the transient front; and Um is the maximum

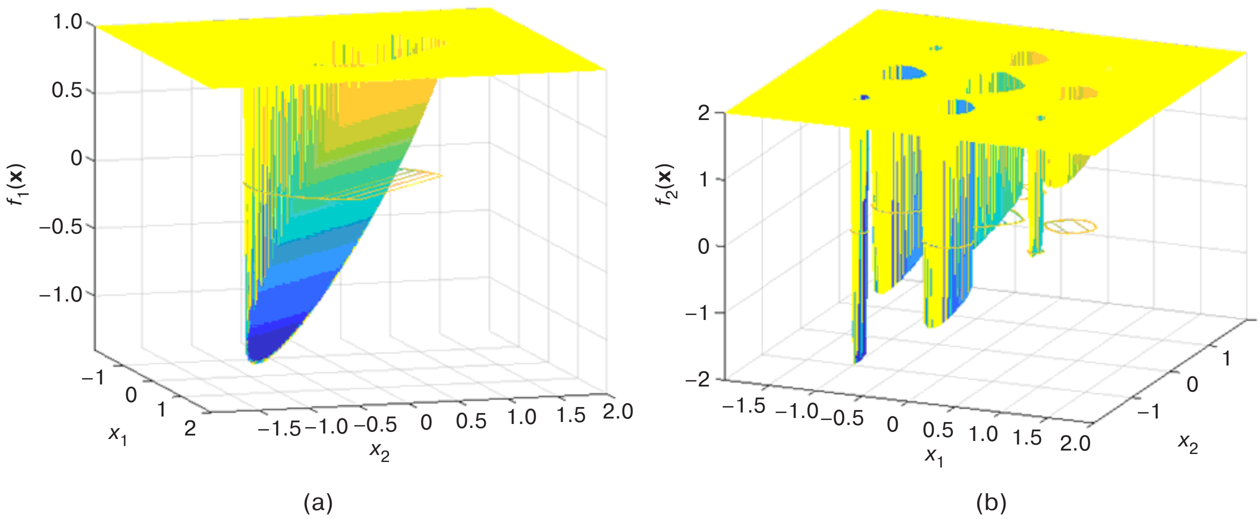

Fig. 1. Graphs of the test functions (a) $f_1(\mathbf{x})$ and (b) $f_2(\mathbf{x})$ at the dimension $ND = 2$

value (surge) of the transient voltage. The time intervals and the voltage are measured on normalized scales and are expressed in dimensionless quantities. The other QIs are defined above.

OF $f_4(\mathbf{x})$ (7) is obtained in the problem of minimizing the transition-process time *Tss* under restrictions on the front rise time *Tfr*, the maximum transient voltage *Um*, the stopband attenuation *Hs*, and the excess of the frequency response in the transition band *DHt*. In the experiments, the values of the following quantities were given: number of poles, $NP = 6$; number of zeros, $NZ = 0$; objective values of QIs, $Tss_t = 0.1$, $Tfr_t = 0.5$, $Um_t = 1.1$, $Hs_t = 40$, and $DHt_t = 0$; the weighting factors, $WTfr = 100$, $WUm = 10$, $WHs = 100$, and $WHt = 1000$. The boundaries of the search space are the same as for $f_3(\mathbf{x})$. Information on the methods to calculate the QIs is available in the literature [11, 14].

The graphs of these OFs are not presented here since solving the problems of optimizing OFs (6) and (7) at $ND = 2$, i.e., for filters of the second order, is of no interest; moreover, at higher dimensions, the graphical representation is complicated.

Let us further list the studied optimization algorithms. All of them were implemented in the *MATLAB* software environment[1]. For each algorithm, an abbreviated notation is introduced, as presented below:

- SS (Step Search)—a simple coordinate search algorithm with the step size bound from above [2, 3].
- PS—*patternsearch*(..) function from the *Global Optimization Toolbox* in *MATLAB*. An improved coordinate search with the possibility of transition between the domains of attraction of local extrema [2].
- MS—*fminsearch*(..) function from the *Optimization Toolbox*. A search for a minimum of an OF using the Nelder–Mead simplex algorithm [2].
- SA—*simulannealbnd*(..) function from the *Global Optimization Toolbox*. A search for the global minimum of an OF using the simulated annealing algorithm [1, 2].
- GA—*ga*(..) function from the *Global Optimization Toolbox*. A search for the global minimum of an OF using a genetic algorithm [1, 2].
- PSO—*particleswarm*(..) function from the *Global Optimization Toolbox*. A search for the global minimum of an OF by the particle swarm optimization algorithm [1, 2].
- CS—a function realizing the cuckoo search algorithm for searching for the global minimum of an OF [1]. The function is not included in the *MATLAB* toolboxes and is written based on a published example [15].
- MC1—*fmincon*(..) function that is included in the *Optimization Toolbox* and realizes the SQP algorithm. In this case, an OF is minimized with penalties (4)–(7), and restrictions are imposed only on the coordinates of the search space.
- MC2—*fmincon*(..) function from the *Optimization Toolbox*, too. But in this case, a selected QI is minimized, and functions used to calculate the fixed QIs are introduced in the arguments of *fmincon*(..) as inequality restrictions.

Each algorithm was run $NT = 100$ times for the functions $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$ and $NT = 40$ times for $f_3(\mathbf{x})$ and $f_4(\mathbf{x})$. The starting points in the search space for each run of the non-population algorithms were given using the *lhsdesign*(*NT*, *ND*) function, which returns a Latin hypercube sample matrix; random starting positions of agents of the population algorithms GA, PSO, and CS were given by their realizing functions. The algorithms included in the *MATLAB* toolbox were set by default. The end condition of the search was the absence of changes in the function being minimized that exceeded the $DGF_{\min}$ level, which was one of the settings and, as noted above, was set by default. For the CS algorithm, the population size $Npop = 20$ and the number of generations $maxgenN = 400$ were given. The search was ended after all the generations had been sought.

RESULTS AND DISCUSSION

Table 1 presents the results of the experiments with the OF $f_1(\mathbf{x})$. The first and second subcolumns of each column show the minimum and maximum values, respectively, of the found solutions over 100 search cycles at the dimensions $ND = 2$, 4, and 8. The third subcolumns present the numbers *Neval* of calculations of the OF in the course of the search. The top row shows the analytically found exact values of the global minimum.

Additional information on the operation of the algorithms is provided by the maps of the positions of the starting points of the search cycles, which are connected by straight line segments to the corresponding end points; examples are given in Fig. 2. For the population algorithms, in particular, for PSO, only the positions of the found optimal solutions are shown, since each agent in the population has its own starting position.

The non-population algorithms SS, PS, MS, and MC1 move toward a point of minimum until they reach a wall. Their results over the search cycles have a significant scatter. The population algorithms PSO and CS turned out to be significantly better. Since most of the solutions they found are in the immediate vicinity of the global minimum, it does not take many iterations of the search to get a good result. The best results were shown by the MC2 algorithm. This is understandable because $f_1(\mathbf{x})$ has no local minima. In this case, both the QI to be minimized and the restricted QI are described by twice

[1] http://www.mathworks.com. Accessed December 14, 2021.

Table 1. Results of the experiments with the OF $f_1(\mathbf{x})$

Algorithm	$ND = 2$, $\min(f_1) = -1.414$			$ND = 4$, $\min(f_1) = -2.000$			$ND = 8$, $\min(f_1) = -2.828$		
	$\min(f_1)$	$\max(f_1)$	*Neval*	$\min(f_1)$	$\max(f_1)$	*Neval*	$\min(f_1)$	$\max(f_1)$	*Neval*
SS	−1.414	−1.017	84844	−2.000	−1.396	320100	−2.717	−2.327	1280948
PS	−1.414	−1.126	14503	−1.998	−1.540	30330	−2.780	−1.833	70876
MS	−1.414	−1.052	23623	−2.000	0.053	60650	−2.806	1.010	147627
SA	−1.414	−1.386	192039	−1.998	−1.873	381223	−2.782	−2.340	752386
GA	−1.414	−1.372	631387	−2.000	−1.946	1456566	−2.827	−2.821	5748740
PSO	−1.414	−1.412	187060	−2.000	−1.996	541760	−2.828	−2.821	1508800
CS	−1.414	−1.413	1677900	−2.000	−1.984	1677900	−2.825	−2.793	1677900
MC1	−1.414	−0.756	29340	−1.999	−0.542	72418	−2.828	−0.440	100494
MC2	−1.414	−1.414	3465	−2.000	−2.000	7201	−2.828	−2.828	16211

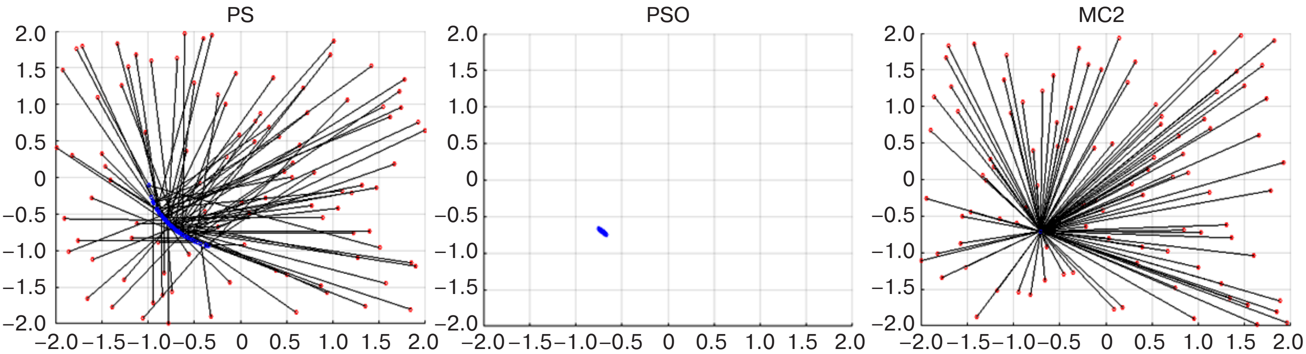

Fig. 2. Starting points of search and the positions of solutions (blue) for $f_1(\mathbf{x})$ at $ND = 2$

differentiable functions. Under such conditions, the SQP algorithm quickly and accurately finds the optimal solution in each run at all the dimensions of the search space.

Table 2 presents the results of the experiments with the OF $f_2(\mathbf{x})$. The estimated values of the global minimum in the top row were obtained by an additional search within the restricted neighborhoods of the best of the found solutions. Figure 3 gives the examples of the position maps of the starting and end points of search. Figure 4 presents the examples of the histograms of the found optimal solutions at $ND = 8$.

The algorithms SS, MS, and MC1 end the search at the local minimum closest to the starting point (Fig. 3). To obtain the global minimum or at least a minimum close to it, a significant number of search cycles should be performed. At the same time, the algorithms SA, GA, PSO, and CS end all the search cycles either at the global minimum or at two local minimums closest to it. This was to be expected, since these algorithms are intended for global optimization.

At the dimension $ND = 8$, the exact value of the global minimum is only found using the PSO algorithm. However, as the histogram in Fig. 4 shows, this value is found once in a hundred attempts. At the same time, the CS algorithm repeatedly ended the search near the global minimum, while, in the other cases, it hit local minima closest to the global one.

The results of the MC2 algorithm should be noted. Among them are many values that are significantly smaller than the global minimum of $f_2(\mathbf{x})$ (Fig. 4). A check shows that these solutions violate the restrictions imposed on the fixed QI. Thus, although both QIs are described by twice differentiable functions in the vicinity of any of its local minima, the MC2 algorithm turned out to be unsuitable for optimizing this function.

Let us turn to the results of optimizing the OF $f_3(\mathbf{x})$ (6) and $f_4(\mathbf{x})$ (7). These problems belong to Black Box Optimization problems, for which neither the exact solutions nor any information on the properties of the OF is known before the start of the search. Tables 3 and 4 present the best solutions for each algorithm, respectively. Significant violations of the restrictions imposed on the OF are marked in italics. The best results are highlighted in bold. Figures 5 and 6 show the examples of the histograms of the OF values for the found solutions. For the MC2 algorithm, the scale of the horizontal axis presents the values of the QI being minimized, rather than the scalar OF.

Table 2. Results of the experiments with the OF $f_2(\mathbf{x})$

Algorithm	ND = 2, min(f_2) = −2.000			ND = 4, min(f_2) = −2.040			ND = 8, min(f_2) = −2.097		
	min(f_2)	max(f_2)	*Neval*	min(f_2)	max(f_2)	*Neval*	min(f_2)	max(f_2)	*Neval*
SS	−2.000	560.2	24313	−2.016	1036.3	94421	86.4	1892.6	376213
PS	−1.999	−0.475	15173	−2.034	−0.545	34606	−1.559	−0.690	87668
MS	−2.000	564.0	10383	−1.998	1042.0	20267	−2.082	1896.4	88995
SA	−2.000	−0.924	190021	−2.039	1.765	455552	−2.089	372.8	819478
GA	−2.000	−1.269	535649	−2.040	−0.633	1400355	−1.593	−0.895	6594051
PSO	−2.000	−1.277	208501	−2.040	−0.647	593081	−2.097	0.304	1603361
CS	−2.000	−1.279	31960	−2.040	−1.404	31960	−2.091	−1.566	31960
MC1	−1.998	278.1	15445	−2.036	942.1	20946	−0.609	1608.6	36057
MC2	−2.848	1.897	130078	−5.696	5.696	329294	−11.386	5.701	577958

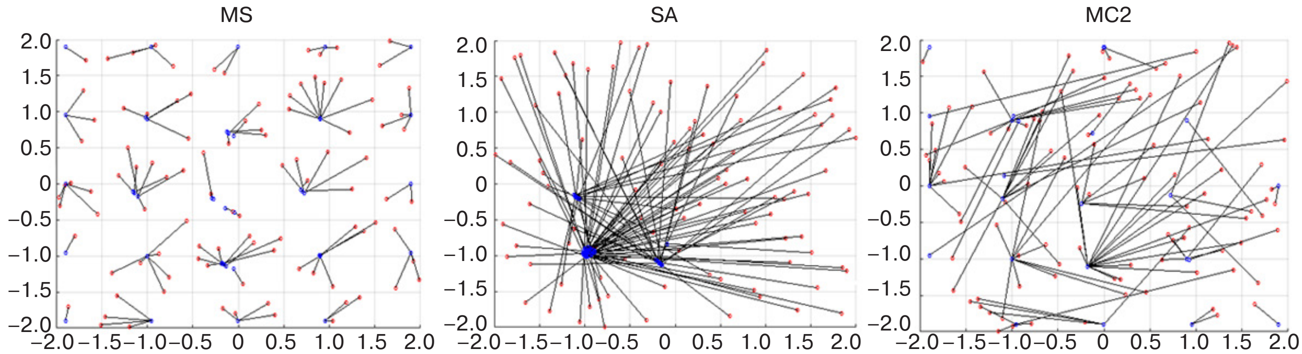

Fig. 3. Starting points of search and the positions of solutions (blue) for $f_2(\mathbf{x})$ at ND = 2

Table 3. Results of the experiments with the OF $f_3(\mathbf{x})$

Algorithm	min(OF)	*DHp*, dB	*Hs*, dB	*DHt*	*DTd* %	*Neval*
SS	5.070	0.500	40.040	0.000	60.699	332488
PS	5.615	0.510	40.000	0.000	62.103	46699
MS	6.708	0.500	40.000	0.000	77.084	33887
SA	2.552	0.500	40.009	0.000	35.515	280390
GA	2.253	0.500	40.881	0.000	32.530	3009450
PSO	**2.127**	0.500	40.000	0.000	**31.266**	1211160
CS	62.124	*1.752*	39.559	0.000	75.268	1278400
MC1	2.913	0.500	40.533	0.000	39.128	40771
MC2	**31.126**	0.500	40.000	0.000	**31.126**	39828

The problem of optimizing $f_3(\mathbf{x})$ consisted in minimizing the *DTd* QI under the restrictions $DHp \leq 0.5$ dB, $Hs \geq 40$ dB, and $DHt \leq 0$. These restrictions are satisfied by all the algorithms, except CS. The minimum value of *DTd* was found by the MC2 method, which also outperformed the other algorithms in number of solutions coinciding with the best one (Fig. 5). But at the same time, among the results of the search by this method, there are several solutions violating the restrictions. Because of this, the *DTd* QI turned out to be less than the correct optimal value.

The problem of optimizing $f_4(\mathbf{x})$ consisted in minimizing the *Tss* QI under the restrictions $Hs \geq 40$ dB, $DHt \leq 0$, $Tfr \leq 0.5$, and $Um_t \leq 1.1$. The restrictions are only satisfied with acceptable accuracy by the population algorithms GA and PSO (Table 4), for which the best results of searching for the minimum of *Tss* are

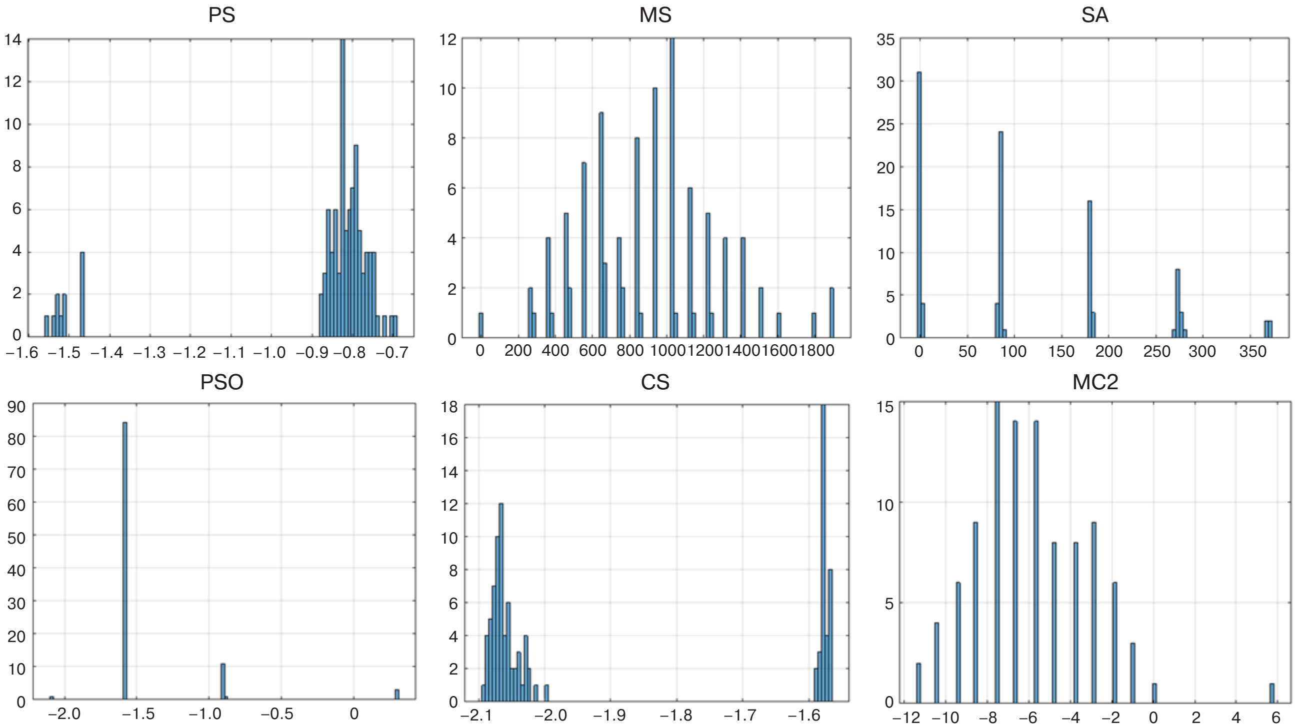

Fig. 4. Histograms of the found solutions for $f_2(\mathbf{x})$ at $ND = 8$

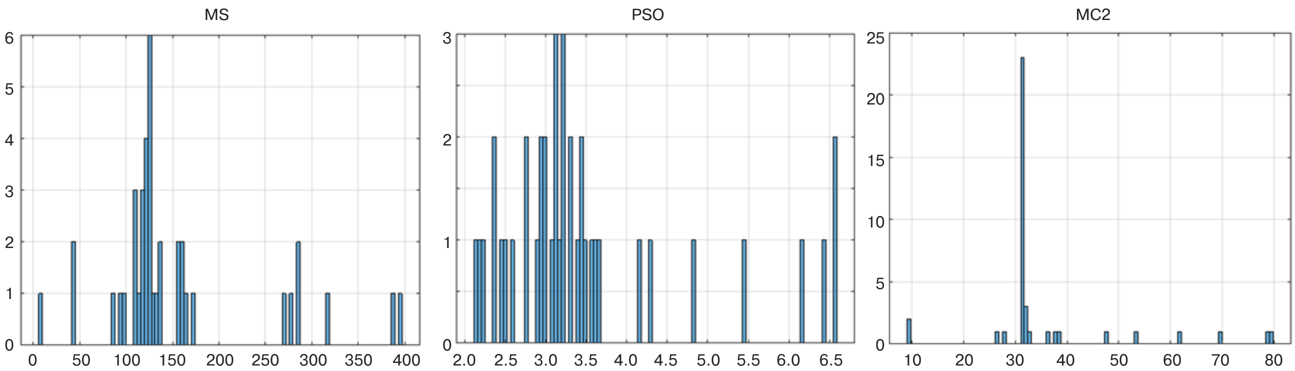

Fig. 5. Histograms of the found solutions for $f_3(\mathbf{x})$

Table 4. Results of the experiments with the OF $f_4(\mathbf{x})$

Algorithm	min(OF)	*Hs*, dB	*DHt*	*Tfr*	*Tss*	*Um*	*Neval*
SS	38.283	*33.821*	0.000	*0.528*	1.681	*1.235*	75904
PS	11.888	40.247	0.000	0.500	1.273	*1.116*	41477
MS	16.378	40.000	0.000	0.500	1.569	1.269	30440
SA	12.747	40.003	0.000	0.500	1.324	*1.151*	227064
GA	**11.287**	40.001	0.000	0.500	**1.229**	1.100	1512630
PSO	**11.301**	40.000	0.000	0.500	**1.229**	1.101	548280
CS	20.796	*35.942*	0.000	0.500	1.165	1.080	1278400
MC1	33.355	40.267	0.000	*0.551*	2.391	*1.132*	15107
MC2	1.480	43.663	0.000	*1.044*	1.480	1.006	19902

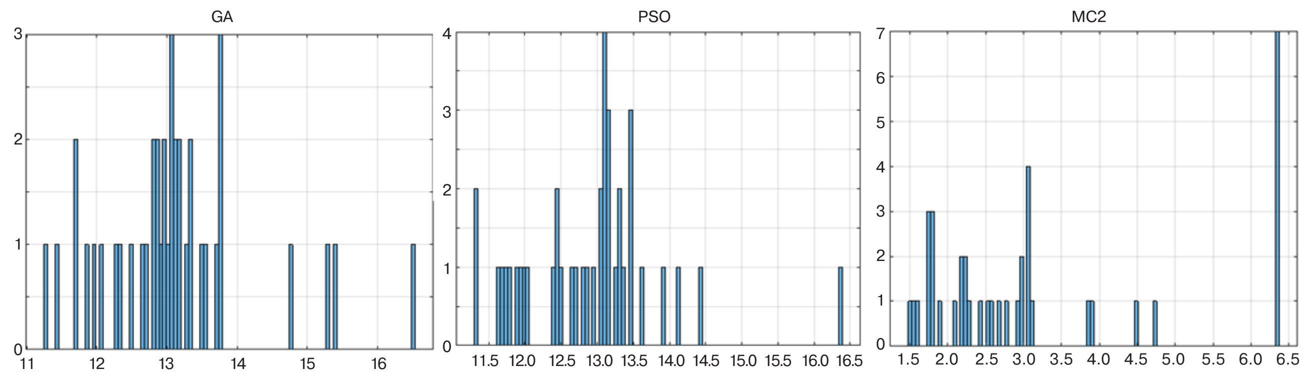

Fig. 6. Histograms of the found solutions for $f_4(\mathbf{x})$

the same. According to the statistics of the results, the two algorithms are also equivalent (Fig. 6), and in the required number of computational operations, the PSO algorithm turned out to be noticeably better than GA. The MC2 algorithm proved to be completely unsuitable for solving this problem.

CONCLUSIONS

The obtained results showed that the search for Pareto-optimal solutions by solving problem (1) can be performed by different methods, which should be selected based on the properties of the functions contained in (1). Since, out of the considered algorithms, the best solutions for all functions were only obtained by the heuristic algorithm PSO, this can be recommended as the main method to solve problems of type (1).

For the functions $f_1(\mathbf{x})$ and $f_3(\mathbf{x})$, the nonlinear programming algorithm MC2 turned out to be applicable. For these functions, this method not only finds the optimal solution in most of the search cycles, but also requires much less computation resources for the search than the heuristic algorithms that optimize scalar OF (2). However, for the functions $f_2(\mathbf{x})$ and $f_4(\mathbf{x})$, the MC2 algorithm proved to be unsuitable. For the function $f_2(\mathbf{x})$, this can be explained in terms of its multimodality. For the function $f_4(\mathbf{x})$, there is still no explanation because nothing can be said about the properties of functions contained in (7).

One of the approaches for assessing the applicability of nonlinear programming methods of the MC2 type to solving problem (1) is to preliminarily study the properties of the functions that determine the values of the QIs contained in (1). However, this approach also entails certain problems, for example, in terms of how to check the multimodality of an OF. The commonly used approach is based on multiple searches for local extrema from uniformly distributed starting points. In this case, it is recommended to use the Nelder–Mead algorithm (referred to as MS above) [7]. However, if the relief of the OF has “valleys,” i.e., areas in which the rate of change in the function in one direction is much lower than in others, then the local search algorithms will stop at various points of the bottom of the valley, which are not local minima [10]. Methods for detecting walls, valleys, and plateaus in the relief of an OF, as well as those that search for extrema in the presence of such areas, are still poorly developed.

Thus, the issue of assessing the applicability of MC2-type nonlinear programming algorithms to solving problem (1) requires additional research. In the meantime, the following course of action can be proposed. First, it is necessary to run a certain number of search iterations for a solution to problem (1) using the MC2 method. If most of the solutions are close to each other and the imposed restrictions are not violated, then the best of these solutions can be taken as the desired optimum. Moreover, if the solutions have a significant scatter and the majority have violated restrictions, then one should proceed to the optimization of scalar OF (2) using the PSO algorithm.

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