Modern radio engineering and telecommunication systems

Современные радиотехнические и телекоммуникационные системы

UDC 621.3.09
https://doi.org/10.32362/2500-316X-2022-10-4-38-43

RESEARCH ARTICLE

Parameterization of user functions in digital signal processing for obtaining angular superresolution

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Abstract
Objectives. One of the most important tasks in the development of goniometric systems is improving resolution in terms of angular coordinates. This can be achieved in two ways: firstly, by increasing the aperture, which is very expensive and often technically challenging to implement; secondly, with the help of digital signal processing methods. If the recorded signal sources are located close to each other and not resolved by the Rayleigh criterion, it can be impossible to determine their number, location and reflection characteristics. The aim of the present work is to develop a digital signal processing algorithm for obtaining angular superresolution.
Methods. Mathematical methods for solving inverse problems are used to overcome the Rayleigh criterion, i.e., obtain angular superresolution. These problems are unstable, since there is an infinite number of approximate solutions and false targets may occur. The search for the optimal solution is carried out by minimizing the standard deviation.
Results. A description of a mathematical model for a goniometric system is presented. A signal processing algorithm is developed based on existing methods according to the principle of parameterization of user functions. Results of numerical experiments for achieving superresolution by algebraic methods are given along with an estimation of solution stability. The accuracy and correspondence of the amplitude of the obtained objects to the initial parameters are measured. The degree of excess of the Rayleigh criterion by the obtained solution is estimated.
Conclusions. Algebraic methods can be used to obtain stable solutions with angular superresolution. The results obtained correctly reflect the location of objects with a minor error. Errors in the distribution of the signal amplitude are small, appearing false targets have negligible amplitude.

Keywords: computer simulation, super resolution, object search, simulation model

• **Submitted:** 15.12.2021 • **Revised:** 17.05.2022 • **Accepted:** 24.06.2022

For citation: Shchukin A.A., Pavlov A.E. Parameterization of user functions in digital signal processing for obtaining angular superresolution. *Russ. Technol. J.* 2022;10(4):38–43. https://doi.org/10.32362/2500-316X-2022-10-4-38-43

Financial disclosure: The authors have no a financial or property interest in any material or method mentioned.

The authors declare no conflicts of interest.

НАУЧНАЯ СТАТЬЯ

Параметризация пользовательских функций в цифровой обработке сигналов для получения углового сверхразрешения

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Резюме
Цели. Одна из важнейших задач развития угломерных систем – улучшение разрешающей способности по угловым координатам. Этого можно добиться двумя способами: во-первых, увеличением апертуры такой системы, что весьма дорого и часто технически трудновыполнимо; во-вторых, с помощью методов цифровой обработки сигналов. Если регистрируемые источники сигнала расположены близко друг к другу и не разрешаются по критерию Рэлея, то невозможно определить их количество, расположение и характеристики отражения. Цель работы – разработка алгоритма цифровой обработки сигналов для получения углового сверхразрешения.
Методы. Использованы математические методы решения обратных задач. Эти методы позволяют преодолеть критерий Рэлея, т.е. дают возможность получить угловое сверхразрешение. Данные задачи обладают неустойчивостью. Существует бесконечное количество приближенных решений, возможно возникновение ложных целей. Поиск оптимального решения проводится путем минимизации среднеквадратического отклонения.
Результаты. В статье приведено описание математической модели работы угломерной системы. На основе существующих методов разработан алгоритм обработки сигнала, использующий принцип параметризации пользовательских функций. Представлены результаты численных экспериментов по достижению сверхразрешения алгебраическими методами. Проведена оценка устойчивости решения. Измерены точность и соответствие амплитуды полученных объектов начальным параметрам. Проведена оценка степени превышения критерия Рэлея полученным решением.
Выводы. Показано, что алгебраические методы позволяют получать устойчивые решения с угловым сверхразрешением. Получаемые результаты правильно отражают расположение объектов с незначительной ошибкой. Ошибки в распределении амплитуды сигнала невелики, появляющиеся ложные цели имеют пренебрежимо малую амплитуду.

Ключевые слова: компьютерное моделирование, сверхразрешение, поиск объектов, имитационная модель

• **Поступила:** 15.12.2021 • **Доработана:** 17.05.2022 • **Принята к опубликованию:** 24.06.2022

Для цитирования: Щукин А.А., Павлов А.Е. Параметризация пользовательских функций в цифровой обработке сигналов для получения углового сверхразрешения. *Russ. Technol. J.* 2022;10(4):38–43. https://doi.org/10.32362/2500-316X-2022-10-4-38-43

Прозрачность финансовой деятельности: Авторы не имеют финансовой заинтересованности в представленных материалах или методах.

Авторы заявляют об отсутствии конфликта интересов.

INTRODUCTION

Goniometer systems have many applications. The determining criteria for the implementation of such a system are accuracy and speed. There are various ways to obtain angular superresolution—the Capon method, the thermal noise method, MUSIC (MUltiple SIgnal Classification), ESPRIT (Estimation of Signal Parameters via Rotational Invariant Techniques), etc. Methods such as MUSIC and ESPRIT use narrowband signals and are inefficient when applied to broadband and ultra-wideband signals (UWB). Many of these methods are not sufficiently effective and universal, because at a signal-to-noise ratio of less than 20 dB, errors occur in the solution [1–8]. In addition, not all of the listed methods allow one to solve two-dimensional problems [9–11].

In this study, an algorithm for the approximate determination of the angular location of closely spaced targets has been developed. The algebraic methods considered in the article make it possible to obtain a solution with low computational costs. The presented algorithm has a high speed, which allows it to be used in real time. To assess the quality of the method, the degree of excess of the Rayleigh criterion by the obtained solution is measured.

PROBLEM STATEMENT

The model of the signal received by the surveillance system is a two-dimensional integral (1):

$$U(x,y) := \int_{\Omega} F(x-q, y-r) I(q,r) dr dq, \tag{1}$$

where $F(x, y)$ is the radiation pattern (RP) of the observation system; Ω is the two-dimensional region of the source location; $I(q, r)$ is the angular distribution of the amplitude of the signal source, which is to be determined [12, 13].

The purpose of the study is to develop a digital signal processing algorithm for obtaining angular superresolution. To accomplish this, it is necessary to restore the angular distributions of the reflected signal amplitude from the measured signal $U(x, y)$ and the known characteristics of the goniometric system. This can be achieved through obtaining an approximate solution of the linear Fredholm integral equation of the first kind of the convolution type (1).

Obtaining angular superresolution using digital signal processing $U(x, y)$ is a solution to the inverse problem.

SOLUTION METHOD

The search for a solution to the inverse problem is carried out based on the parametrization [14, 15], when instead of the unknown function $I(q, r)$, the expansion of the desired distribution of amplitudes with respect to a given system of orthogonal functions is used. Then the solution can be represented in the form:

$$I(q,r) = \sum_{i=1}^{\infty} a_i g_i(q,r) \approx \sum_{i=1}^{N} a_i g_i(q,r), \tag{2}$$

where a_i are unknown coefficients; $g_i(q, r)$ are functions specified by the user. The coefficients a_i can be found by minimizing the standard deviation of the solution in the region $\theta > \Omega$ from the original signal:

$$\delta^2 =$$

$$= \int_{\theta} \left[U(x,y) - \sum_{i=1}^{\infty} a_i \int_{\Omega} F(x-q, y-r) g_i(q,r) dr dq \right]^2 dy dx, \tag{3}$$

where θ is the two-dimensional region of scanning.

To do this, the partial derivatives of δ^2 with respect to a_i are equated to zero. The result is a system of linear algebraic equations (SLAE) of N equations:

$$a = GV, \tag{4}$$

where

$$G_{j,i} := \int_{\theta} \psi_j(x,y) \psi_i(x,y) dy dx,$$

$$\psi_i = \int_{\Omega} F(x-q, y-r) g_i(q,r) dr dq, \tag{5}$$

$$V_j := \int_{\theta} U(x,y) \psi_j(x,y) dy dx. \tag{6}$$

Although the angular superresolution increases when using a larger number of functions $g_i(q, r)$, the dimension of the SLAE also increases along with the consequent instability of the solution, which manifests itself in the form of false signal sources and distortions in the resulting solution. This is due to the characteristic problem of inverse problems that minor changes in the initial data—for example, random variables (noise)—can lead to significant errors, i.e., instability. While low levels of noise in well-posed problems can lead to small errors in the solution, the resulting solution in inverse problems may differ from the true one by several orders of magnitude.

RESULTS OF NUMERICAL EXPERIMENTS

The resulting solutions of inverse problems were studied in the course of numerical experiments on mathematical models. The source objects and the received signal are shown in Fig. 1, where $\theta_{0.5}$ is the RP half-width. Three-point objects were specified. All objects are located sufficiently close to each other and are not resolved by the Rayleigh criterion.

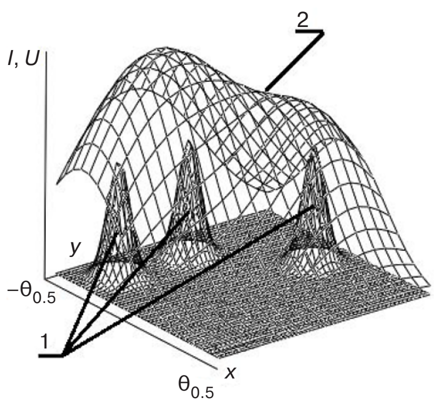


Fig. 1. Model of the received signal and the source objects:
1 – original objects; 2 – received signal

The form of the signal $U(x, y)$ does not allow objects to be observed separately, in particular, three original objects merge into one (mesh surface 2).

In order to search for a solution, nine functions $g_i(q, r)$ are specified to cover the entire region Ω under study, three of which are shown in Fig. 2.

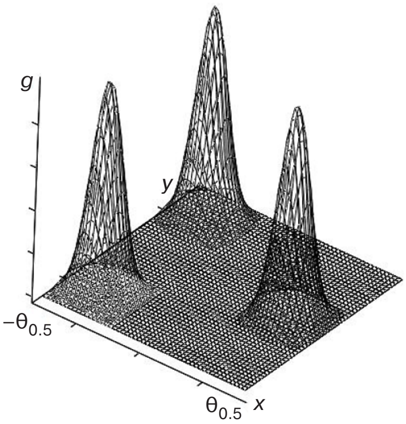
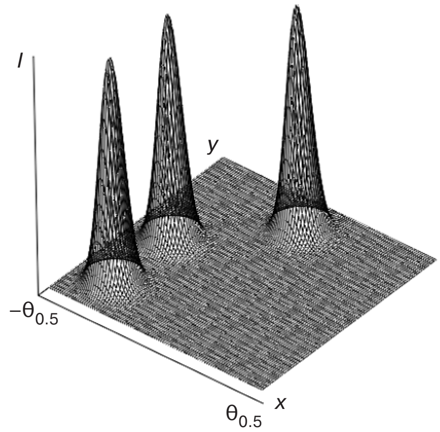


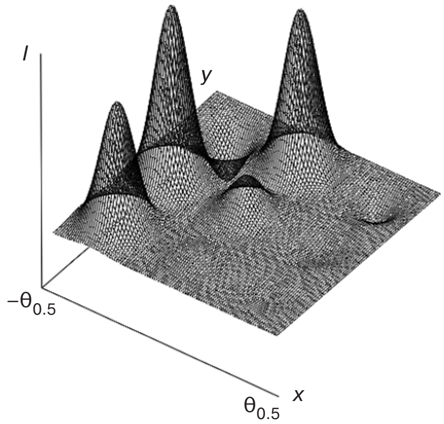
Fig. 2. View of three user-defined functions

The original objects and results of solving the SLAE system (4)–(6) are shown in Figs. 3a and 3b, respectively.

As a result of the experiment, a stable solution with angular superresolution was obtained. The angular coordinates of all three objects are determined with good accuracy, with the found amplitudes of the central and right object appearing close to the original (≈ 80%), while the amplitude of the left object is slightly lower than the original (≈ 60%). The resulting angular superresolution significantly exceeds the Rayleigh criterion (the angular distance between objects is 0.3θ).



(a)



(b)

Fig. 3. Original objects (a) and result (b)

CONCLUSIONS

The possibility of using algebraic methods to obtain stable solutions of inverse problems with angular superresolution, which reflect the location of the original objects with a slight error (the coordinates of the original objects and the resulting solution coincide), has been demonstrated. The errors in the distribution of the signal amplitude are small (the amplitude of the obtained solution is 60% to 80% of the amplitude of the original objects). A false target appears between the central and right objects, but with negligibly small amplitude <10% of the amplitude of the true objects (Fig. 3b, in the center of the region).

Authors' contributions

A.A. Shchukin—preparing algorithms and graphic materials, writing the text of the article.

A.E. Pavlov—preparing algorithms and graphic materials, writing the text of the article.

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Translated from Russian into English by E. Shklovskii
Edited for English language and spelling by Thomas Beavitt