Russian Technological Journal. 2022;10(1):41–49

ISSN 2500-316X (Online)

Modern radio engineering and telecommunication systems

Современные радиотехнические и телекоммуникационные системы

UDC 621.391.072
https://doi.org/10.32362/2500-316X-2022-10-1-41-49

RESEARCH ARTICLE

Optimal nonlinear filtering of MPSK signals in the presence of a Doppler frequency shift

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Abstract

Objectives. Phase-shift keyed (PSK) signals are widely used in many telecommunication, communication, and cellular information transmission systems. Phase-shift keying provides a higher noise immunity than amplitude and frequency modulations do. An increase in the modulation order of such a signal leads not only to an increase in its spectral efficiency, but also to a certain decrease in the noise immunity of reception. To ensure a high noise immunity of reception of multiple phase-shift keyed (MPSK) signals, a demodulator should provide the coherence of the reference oscillation with the carrier. Ignorance of the frequency and phase of the received signal leads to significant energy losses. The purpose of this work was to synthesize and analyze algorithms for receiving MPSK signals with phase fluctuations caused by changes in the carrier frequency due to the Doppler effect against the background of white Gaussian noise.

Methods. The problem was solved using the apparatus of optimal nonlinear filtering and methods of statistical radio engineering.

Results. A demodulator was synthesized, which includes two interconnected units. One of them is a discrete symbol estimation unit, at the output of which a decision on the received symbol is issued, and the other is a phase-lock circuit. Analytical expressions were derived to estimate the characteristics of the receiver noise immunity as functions of the signal-to-noise ratio and fluctuation parameters. It was shown that the synthesized quasi-coherent algorithm compensates well for the MPSK signal phase fluctuations caused by the instability of the master oscillator and the Doppler effect.

Conclusions. Comparison of the results of this work with results obtained in the case of the absence of fluctuations in the initial phase showed that, at a high relative speed of the transmitter and the receiver (satellite radio channel), the energy loss is no more than 1 dB, and at lower speeds of the objects, it is about 0.2 dB and less.

Keywords: multiple phase-shift keying, Doppler effect, optimal nonlinear filtering, noise immunity, bit error probability

• **Submitted:** 08.10.2021 • **Revised:** 15.10.2021 • **Accepted:** 22.12.2021

For citation: Kulikov G.V., Do Tr.T., Usmanov R.R. Optimal nonlinear filtering of MPSK signals in the presence of a Doppler frequency shift. *Russ. Technol. J.* 2022;10(1):41–49. https://doi.org/10.32362/2500-316X-2022-10-1-41-49

Financial disclosure: The authors have no a financial or property interest in any material or method mentioned.

The authors declare no conflicts of interest.

НАУЧНАЯ СТАТЬЯ

Оптимальная нелинейная фильтрация сигналов М-ФМ при наличии доплеровского смещения частоты

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Резюме

Цели. Сигналы с фазовой манипуляцией (ФМ) широко применяются во многих телекоммуникационных, связных и сотовых системах передачи информации. По сравнению с амплитудной и частотной манипуляцией применение ФМ обеспечивает более высокую помехоустойчивость. Увеличение позиционности такого сигнала приводит к повышению его спектральной эффективности, но в то же время к некоторому снижению помехоустойчивости приема. Для обеспечения высокой помехоустойчивости при приеме многопозиционных сигналов (М-ФМ) в демодуляторе требуется обеспечить когерентность опорного колебания с несущей. Незнание частоты и фазы принимаемого сигнала приводит к существенным энергетическим потерям. Цель работы – синтез и анализ алгоритмов приема сигналов М-ФМ с флуктуациями фазы, вызванными изменениями его несущей частоты, связанными с эффектом Доплера, на фоне белого гауссовского шума.

Методы. Задача решается с применением аппарата оптимальной нелинейной фильтрации и методов статистической радиотехники.

Результаты. Синтезирован демодулятор, включающий в себя взаимосвязанные блок оценки дискретного символа, на выходе которого выдается решение о принимаемом символе, и блок фазовой автоподстройки частоты. Получены аналитические выражения, позволяющие оценить характеристики помехоустойчивости приемника в зависимости от отношения сигнал/шум и параметров флуктуаций. Показано, что синтезированный квазикогерентный алгоритм хорошо компенсирует флуктуации фазы сигнала М-ФМ, вызванные нестабильностью задающего генератора и доплеровским эффектом.

Выводы. Сравнение полученных результатов с результатами в случае отсутствия флуктуации начальной фазы, показывает, что энергетический проигрыш при большой относительной скорости движения передатчика и приемника (спутниковый радиоканал) составляет не более 1 дБ, при меньших скоростях движения объектов – около 0.2 дБ и менее.

Ключевые слова: многопозиционная фазовая манипуляция, эффект Доплера, оптимальная нелинейная фильтрация, помехоустойчивость, вероятность битовой ошибки

• **Поступила:** 08.10.2021 • **Доработана:** 15.10.2021 • **Принята к опубликованию:** 22.12.2021

Для цитирования: Куликов Г.В., До Ч.Т., Усманов Р.Р. Оптимальная нелинейная фильтрация сигналов М-ФМ при наличии доплеровского смещения частоты. *Russ. Technol. J.* 2022;10(1):41–49. https://doi.org/10.32362/2500-316X-2022-10-1-41-49

Прозрачность финансовой деятельности: Никто из авторов не имеет финансовой заинтересованности в представленных материалах или методах.

Авторы заявляют об отсутствии конфликта интересов.

INTRODUCTION

One of the most known classes of complex signals used widely in digital information transmission systems are multiple phase-shift keyed (MPSK) signals [1, 2]. In so modulated signals, the phase of the carrier changes stepwise, depending on the transmitted signal. The maximum gain in noise immunity in reception of such signals is reached by using a coherent receiver, in which the reference oscillation is frequency- and phase-locked with the carrier frequency oscillation [3]. The absence of information on the phase of the received signal inevitably leads to losses of noise immunity [4–9]. This can be caused by the instability of the frequency of the basic oscillator, the Doppler shift of the signal carrier frequency, and others. In these cases, the problem of signal reception is solved using the apparatus of optimal nonlinear filtering [10–16].

The purpose of this work was to develop an algorithm of reception of MPSK signals based on optimal nonlinear filtering theory in the presence of a Doppler frequency shift and to estimate the noise immunity of the synthesized quasi-coherent receiver.

SYNTHESIS OF AN ALGORITHM OF THE OPTIMAL NONLINEAR FILTERING OF MPSK SIGNALS

Let the received signal be the sum of two components:

$$x(t) = s_{\sum}(\mathbf{C}_k, t, \varphi) + n(t);\ t \in (0, kT];$$
$$\mathbf{C}_k = \{C_1, \ldots, C_k\}, \tag{1}$$

where $s_{\sum}(\mathbf{C}_k, t, \varphi)$ is an MPSK signal with random initial phase φ that, in the kth digit-time slot, has the form

$$\begin{gathered} s_k(C_k = i, t, \varphi) = A_0 \cos(\omega_0 t + \varphi_i + \varphi), \\ \varphi_i = \frac{i2\pi}{M},\ t \in \left((k-1)T, kT\right],\ i = \overline{0, M-1}; \end{gathered} \tag{2}$$

and $n(t)$ is a noise interference with parameters

$$\langle n(t) \rangle = 0; \langle n(t_1)n(t_2) \rangle = \frac{N_0}{2}\delta(t_2 - t_1). \tag{3}$$

We assume that the random initial phase φ of the signal is determined by the instability $\varphi_{\text{mo}}(t)$ of the basic oscillator and the Doppler shift of the signal carrier frequency [10]:

$$\varphi(t) = \int_0^t (\omega(\tau) - \omega_0)d\tau + \varphi_{\text{mo}}(t).$$

The processes $\omega(t)$ and $\varphi(t)$ can be described by the system of the stochastic differential equations [11]

$$\left.\begin{aligned} \dot{\varphi} &= \omega - \omega_0 + n_{\varphi_{\text{mo}}}(t), \\ \dot{\omega} &= -\alpha_\omega(\omega - \omega_0) + n_\omega(t), \end{aligned}\right\}, \tag{4}$$

where α_ω is the Doppler frequency fluctuation spectrum band; and $n_{\varphi_{\text{mo}}}(t)$ and $n_\omega(t)$ are mutually independent processes with delta-function correlation, zero means, and single-sided spectral densities $N_{\varphi_{\text{mo}}}$ and N_ω, respectively.

The mixed *a posteriori* probability density of $s_{\sum}(\mathbf{C}_k, t, \varphi)$ is

$$p_{\text{ps}}(t, \mathbf{C}_k, \varphi) = w_{\text{ps}}(t, \varphi)\, p_{\text{ps}}(t, \mathbf{C}_k|\varphi)$$

where $w_{\text{ps}}(t, \varphi)$ is the *a posteriori* probability density of the parameter φ, and $p_{\text{ps}}(t, \mathbf{C}_k|\varphi)$ is the conditional (on φ) *a posteriori* probability of the vector $\mathbf{C}_k$.

The conditional *a posteriori* probability of $p_{\text{ps}}(t, \mathbf{C}_k|\varphi)$ is found from the equation [12]

$$\begin{aligned} &\dot{p}_{\text{ps}}(t, \mathbf{C}_k | \varphi) = \\ &= p_{\text{ps}}(t, \mathbf{C}_k | \varphi)\left\{F(t, \mathbf{C}_k, \varphi) - \langle F(t, \varphi) \rangle\right\}, \end{aligned} \tag{5}$$

where

$$\begin{gathered} F(t, \mathbf{C}_k, \varphi) = \sum_{j=1}^{k} F_j(t, \mathbf{C}_j, \varphi), \\ F_j(t, \mathbf{C}_j, \varphi) = \frac{2}{N_0} x(t) s_{\Sigma j}(\mathbf{C}_j, t, \varphi), \\ \langle F(t, \varphi) \rangle = \sum_{C_1=0}^{M-1} \sum_{C_2=0}^{M-1} \ldots \sum_{C_k=0}^{M-1} F(t, \mathbf{C}_k, \varphi) p_{\text{ps}}(t, \mathbf{C}_k | \varphi). \end{gathered} \tag{6}$$

We assume that the *a priori* probabilities of the values of the channel symbols for MPSK are equal and are $1/M$. If it is also taken into account that the probabilities of the transition of a symbol from one state to another are also equal, then the solution of Eq. (5) at time $t = kT$ can be represented as

$$\begin{aligned} &p_{\text{ps}}(kT, \mathbf{C}_k | \varphi) = \\ &= \frac{\exp[\sum_{j=1}^{k} \int_{(j-1)T}^{jT} F_j(t, C_j, \mathbf{C}_{j-1}, \varphi)dt]}{\sum_{\mathbf{C}_k} \exp[\sum_{j=1}^{k} \int_{(j-1)T}^{jT} F_j(t, C_j, \mathbf{C}_{j-1}, \varphi)dt]}. \end{aligned} \tag{7}$$

The *a posteriori* probability of the value of the symbol C_k is determined by averaging relation (7) over M possible values $C_1, C_2, \ldots, C_{k-1}$ under the assumption of a good quality of the signal reception. As a result, the symbols received before the time $t = (k-1)T$ are replaced by their estimated values, and the terms independent of C_k can be combined into coefficient K. This coefficient will contain all the time history, which allows one to simplify relation (7):

$$\begin{aligned} &p_{\text{ps}}(T, C_k | \varphi) = \\ &= K \frac{\exp[\int_0^T F_k(t, C_k, \varphi)dt]}{\sum_{C_k=0}^{M-1} \exp[\int_0^T F_k(t, C_k, \varphi)dt]}. \end{aligned}$$

A channel symbol estimation rule is found from the condition of maximum of the *a posteriori* probability at time $t = T$:

$$(C_k = i) \Rightarrow \max\{p_{\text{ps}}(T, C_k \mid \varphi)\}. \tag{8}$$

Let us introduce the notation

$$J_0 = \int_0^T F_k(t, C_k = 0, \varphi)dt,$$
$$\ldots \tag{9}$$
$$J_{M-1} = \int_0^T F_k(t, C_k = M-1, \varphi)dt,$$

where the integrands are determined from expressions (6).

Rule (8) can be written differently, using the monotonicity of the exponential function:

$$(C_k = i) \Rightarrow \max(J_i). \tag{10}$$

Transformation of integrals (9) using expressions (1)–(3) gives

$$J_0 = \frac{2}{N_0}\int_0^T x(t)s_k(t, C_k = 0, \varphi)dt;$$
$$\ldots \tag{11}$$
$$J_{M-1} = \frac{2}{N_0}\int_0^T x(t)s_k(t, C_k = M-1, \varphi)dt.$$

The structure of channel symbol estimation of a receiver that implements algorithm (10) is classical for MPSK signals and is characteristic of reception against the background of white Gaussian noise [1, 2]. This receiver is a multichannel correlator, which determines the degree of similarity between the received process $x(t)$ and the reference signals corresponding to all the possible values of the channel symbol C_k.

Equations of the optimal nonlinear filtering of the random quantities φ and ω are derived by the Gaussian approximation of their *a posteriori* probability densities [13, 14]. Equations for the expected values (marked with asterisks *) in the steady state form the system

$$\left.\begin{aligned} \dot{\varphi}^* &= \omega^* - \omega_0 + \overline{K_{\varphi\varphi}} < F_k >_\varphi, \\ \dot{\omega}^* &= -\alpha_\omega(\omega^* - \omega_0) + \overline{K_{\omega\varphi}} < F_k >_\varphi, \end{aligned}\right\} \tag{12}$$

and the *a posteriori* variances of the approximating distributions can be found from the relations

$$\left.\begin{aligned} &\frac{1}{2}N_{\varphi_{\text{mo}}} + 2\overline{K_{\varphi\omega}} + \left(\overline{K_{\varphi\varphi}}\right)^2 \cdot \overline{< F_k >_{\varphi\varphi}} = 0, \\ &\frac{1}{2}N_\omega + \left(\overline{K_{\varphi\omega}}\right)^2 \cdot \overline{< F_k >_{\varphi\varphi}} - 2\alpha_\omega \cdot \overline{K_{\omega\omega}} = 0, \\ &\overline{K_{\omega\omega}} - \alpha_\omega \cdot \overline{K_{\varphi\omega}} + \overline{K_{\varphi\varphi}} \cdot \overline{K_{\varphi\omega}} \cdot \overline{< F_k >_{\varphi\varphi}} = 0, \end{aligned}\right\} \tag{13}$$

where $\left\langle F_k \right\rangle_\varphi = \dfrac{\partial\left\langle F_k(t,\varphi^*)\right\rangle}{\partial\varphi^*}$, $\left\langle F_k \right\rangle_{\varphi\varphi} = \dfrac{\partial^2\left\langle F_k(t,\varphi^*)\right\rangle}{\partial\varphi^{*2}}$, and

$$< F_k(t,\varphi^*) > = F_k(t, C_k = 0, \varphi)\frac{\exp J_0}{\sum_{i=0}^{M-1}\exp J_i} + F_k(t, C_k = 1, \varphi)\frac{\exp J_1}{\sum_{i=0}^{M-1}\exp J_i} + F_k(t, C_k = 2, \varphi)\frac{\exp J_2}{\sum_{i=0}^{M-1}\exp J_i} + \ldots + F_k(t, C_k = M-1, \varphi)\frac{\exp J_{M-1}}{\sum_{i=0}^{M-1}\exp J_i}.$$

In view of expressions (6),

$$< F_k >_\varphi = \frac{2A_0}{N_0}x(t)\sum_{i=0}^{M-1} s_k^h(t, C_k = i, \varphi^*)\frac{\exp J_i}{\sum_{l=0}^{M-1}\exp J_l},$$

here,

$$s_k^h(t, C_k = i, \varphi^*) = \frac{ds_k(t, C_k = i, \varphi^*)}{d\varphi^*} = -A_0\sin(\omega_0 t + \varphi_i + \varphi^*).$$

At a high signal-to-noise ratio, the latter expressions can be simplified. If the parity and symmetry of the constellation diagram of the MPSK signal are also taken into account, then,

$$< F_k >_\varphi \approx \frac{2A_0}{N_0}x(t)\sum_{i=0}^{M/2-1} s_k^h(t, C_k = i, \varphi^*)\,\text{th}\, J_i.$$

Making statistical averaging and assuming that, in the steady-state mode, at small filtering error,

$$\cos(\varphi - \varphi^*) \approx 1,$$

we obtain

$$\overline{< F_k >_{\varphi\varphi}} \approx -\frac{A_0^2}{2N_0}\left(1 + \text{th}\frac{A_0^2 T}{2N_0}\right). \tag{14}$$

Solving the system of Eqs. (13) gives [11]

$$\left.\begin{aligned} \overline{K_{\varphi\varphi}} &= -\frac{\alpha_\omega\left(\sqrt{1+2G+L} - 1\right)}{\overline{< F_k >_{\varphi\varphi}}}, \\ \overline{K_{\varphi\omega}} &= -\frac{\alpha_\omega^2\left(1 + G - \sqrt{1+2G+L}\right)}{\overline{< F_k >_{\varphi\varphi}}}, \\ \overline{K_{\omega\omega}} &= -\frac{\alpha_\omega^3\sqrt{1+2G+L}\left(1 + G - \sqrt{1+2G+L}\right)}{\overline{< F_k >_{\varphi\varphi}}}, \end{aligned}\right\} \tag{15}$$

where

$$L = -\frac{N_{\varphi_{\text{mo}}}}{2\alpha_\omega^2}\overline{<F_k>_{\varphi\varphi}} = \frac{A_0^2 N_{\varphi_{\text{mo}}}}{4N_0\alpha_\omega^2}\left(1+\text{th}\frac{A_0^2 T}{2N_0}\right),$$

$$G = \sqrt{-\frac{N_\omega + \alpha_\omega^2 N_{\varphi_{\text{mo}}}}{2\alpha_\omega^4}\overline{<F_k>_{\varphi\varphi}}} = \sqrt{\frac{A_0^2(N_\omega + \alpha_\omega^2 N_{\varphi_{mo}})}{4N_0\alpha_\omega^4}\left(1+\text{th}\frac{A_0^2 T}{2N_0}\right)}.$$

Thus, algorithms (12) of filtrating of the random phase of the signal in the presence of a Doppler frequency shift have the form

$$\bar{\varphi}^* = SA_0\left(K_1 + \frac{K_2}{1+T_\omega D}\right)x(t)\times \times \sum_{i=0}^{M/2-1} s_k^h(t, C_k = i, \varphi^*)\,\text{th}\,J_i, \qquad (16)$$

where S is the transconductance of the control element (CE) in the phase-lock channel, $K_1 = \frac{2\overline{K_{\varphi\varphi}}}{N_0 S}$, $K_2 = \frac{2\overline{K_{\varphi\omega}}}{N_0 S\alpha_\omega}$, $T_\omega = \frac{1}{\alpha_\omega}$, and $D = \frac{d}{dt}$.

Figure 1 presents the circuit diagram of the receiver in which interrelated algorithms (10), (11), and (16) are implemented. The synthesized receiver contains two main units: a channel symbol estimator, a phase-lock unit of reference oscillator O, and cross couplings between them. The phase-lock unit includes a proportionally integrating filter with transfer function $K_1 + \frac{K_2}{1+T_\omega D}$. The linear branch of the filter tracks phase fluctuations. The second (integrating) component of the filter tracks frequency fluctuations. The integration time constant should be chosen depending on the average rate of frequency change due to the Doppler effect.

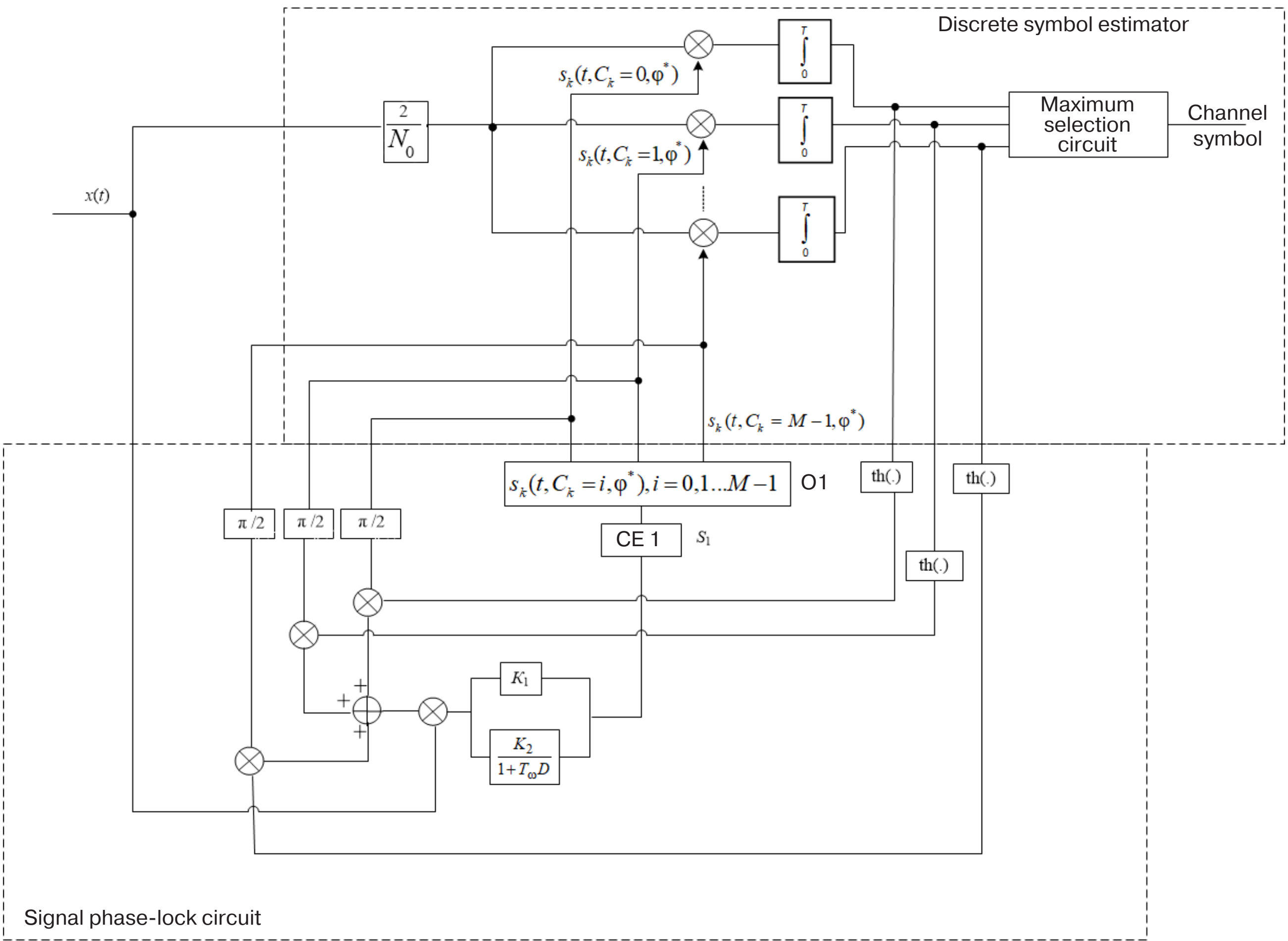

Fig. 1. Circuit diagram of a quasi-coherent receiver of an MPSK signal in a presence of a Doppler frequency shift

ANALYSIS OF THE NOISE IMMUNITY OF THE SYNTHESIZED RECEIVER

Let us analyze the noise immunity of the synthesized quasi-coherent receiver of MPSK signals using a published procedure [7]. The symbol and bit error probabilities P_{se} and P_{be}, respectively, which are conditional on the phases φ and φ^* and the frequency ω, can be found by algorithm (10) using the symmetry of the constellation diagram (e.g., provided that the transmitted signal has a phase of $\varphi_i = 0$ (2)):

$$P_{\text{se}} = 1 - \prod_{i=1}^{M-1} p(J_0 - J_i > 0)\big|_0,$$

$$P_{\text{be}} = P_{\text{se}}/\log_2 M,$$

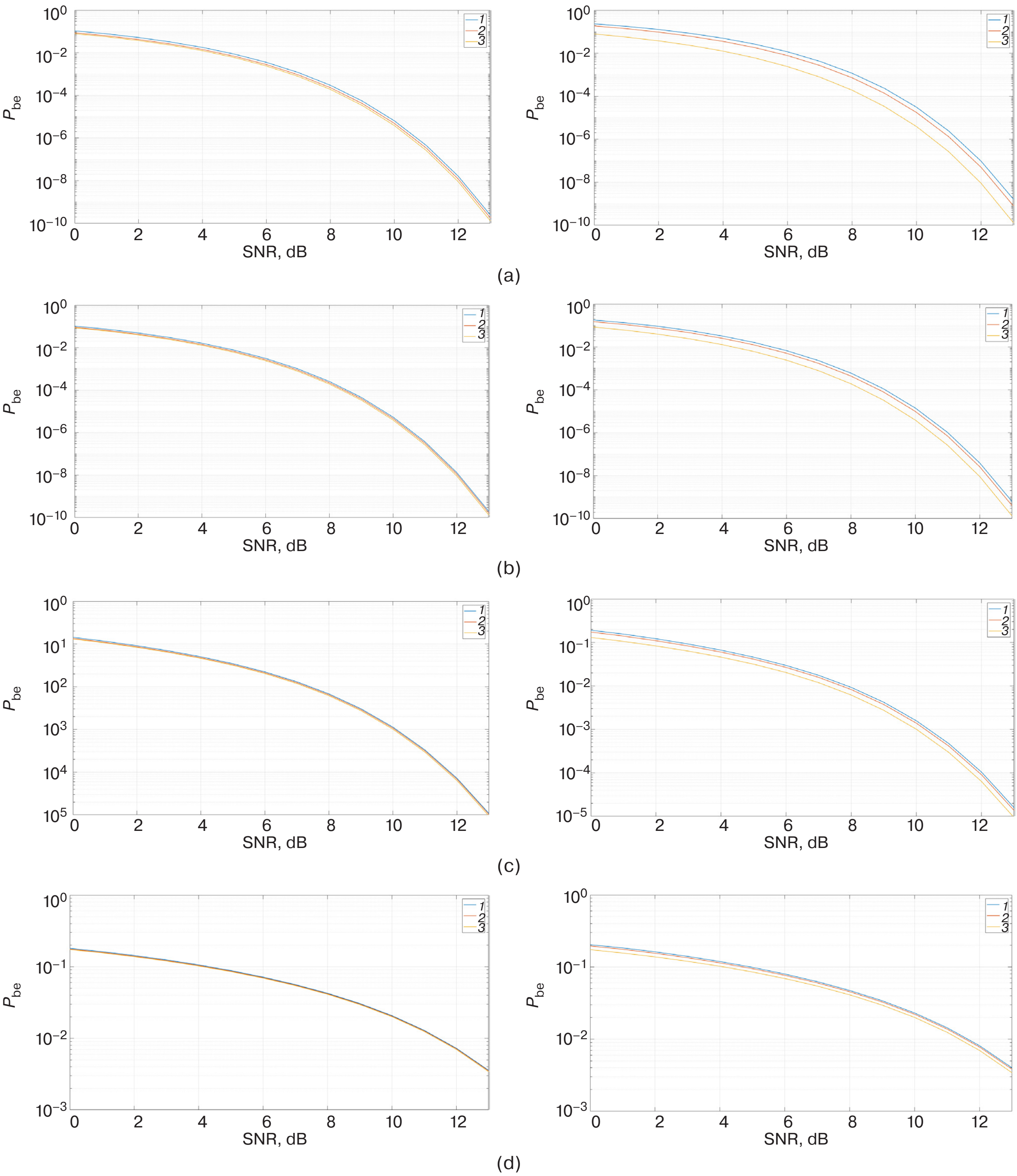

Fig. 2. Dependences of the bit error probability (P_{be}) on the signal-to-noise (SNR) ratio at M = (a) 2, (b) 4, (c) 8, and (d) 16

where

$$p(J_0 - J_i > 0)\big|_0 = 1 - \Phi\left(\frac{m_i}{\sqrt{D_i}}\right),$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-t^2/2}dt.$$

Using expressions (1)–(3) and (11), we obtain

$$m_i = \langle J_0 - J_i \rangle =$$

$$= \frac{2E_s}{N_0}[\cos(\varphi - \varphi^*)(1 - \cos(i2\pi/M)) -$$

$$-\sin(\varphi - \varphi^*)\sin(i2\pi/M)],$$

$$D_i = \frac{4E_s}{N_0}(1 - \cos(i2\pi/M)).$$

To obtain the unconditional probabilities, including the bit error probability, the corresponding expressions should be averaged under the assumption that the *a posteriori* probability density of the random phase φ is Gaussian:

$$w(\varphi) = \frac{1}{\sqrt{2\pi \overline{K_{\varphi\varphi}}}} \cdot e^{-\frac{(\varphi-\varphi^*)^2}{2\overline{K_{\varphi\varphi}}}}.$$

In such an averaging, one can use the approximate formula [14]

$$< p(J_0 - J_i > 0) > \big|_{\text{on } \varphi} = 1 - \Phi\left(\frac{< m_i > \big|_{\text{on } \varphi}}{\sqrt{D_i}}\right),$$

where [17]

$$< m_i > = \frac{2E_s}{N_0}\left[\left(1 - \cos\frac{i2\pi}{M}\right) \cdot \exp\left(-\frac{\overline{K_{\varphi\varphi}}}{2}\right)\right].$$

The *a posteriori* phase variance $\overline{K_{\varphi\varphi}}$ is found from the first of expressions (15) and depends on both the phase variance $\sigma^2_{\varphi_{mo}} = \frac{N_{\varphi_{mo}} T}{2}$, and the frequency variance $\sigma^2_{\omega} = \frac{N_{\omega}}{4\alpha_{\omega}}$.

Figure 2 presents the curves constructed by the above formulas for the dependences of the bit error probability P_{be} on the signal-to-noise ratio E_b/N_0 for the quasi-coherent receiver of MPSK signals at various M in the presence of a Doppler frequency shift. The curves in the left panels of Fig. 2 were constructed at (*1*) $\sigma^2_{\varphi_{mo}} = 0.01$, $\alpha_{\omega}T = 0.1$, and $\sigma_{\omega}T = 0.25$; and (*2*) $\sigma^2_{\varphi_{mo}} = 0.01$, $\alpha_{\omega}T = 0.1$, and $\sigma_{\omega}T = 0.01$. Such parameter values are characteristic of aircraft radio communication (at a relative speed of objects to 3000 km/h). The curves in the right panels of Fig. 2 were constructed at (*1*) $\sigma^2_{\varphi_{mo}} = 0.01$, $\alpha_{\omega}T = 0.1$, and $\sigma_{\omega}T = 5$; and (*2*) $\sigma^2_{\varphi_{mo}} = 0.01$, $\alpha_{\omega}T = 0.1$, and $\sigma_{\omega}T = 2.5$. This is characteristic of satellite radio communication (at a relative speed of objects to 10 km/s). For comparison, Fig. 2 presents curves *3* constructed in the case of a deterministic signal.

The curves in Fig. 2 show that the synthesized quasi-coherent algorithm compensates well for the MPSK signal phase fluctuations caused by the instability of the basic oscillator and the Doppler effect. At a high relative speed of the transmitter and the receiver (satellite radio channel), the residual energy loss in comparison with the deterministic case is no more than 1 dB, and at lower speeds of the objects, it is about 0.2 dB and less.

CONCLUSIONS

In this work, an algorithm was synthesized for the optimal nonlinear filtering of MPSK signals in the presence of a Doppler frequency shift. The algorithm compensates well for the Doppler effect on the noise immunity of reception of discrete information at both low and high relative speeds of the transmitter and the receiver.

Authors' contribution. All authors equally contributed to the research work.

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[Original Russian Text: Yarlykov M.S. Optimal and quasi-optimal algorithms for receiving and processing BOC signals in promising global navigation satellite systems. *Radiotekhnika i elektronika*. 2021;66(1):39–61 (in Russ.). https://doi.org/10.31857/S0033849421010101]

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Translated by V. Glyanchenko
The abstract was edited for English language and spelling by Q. Scribner, Awatera