

Mathematical modeling
Математическое моделирование

UDC 334.71: 656: 338.245
<https://doi.org/10.32362/2500-316X-2021-9-6-64-72>



RESEARCH ARTICLE

Method for assessing testing difficulty in educational sphere

Alexander S. Sigov,
Viktor Ya. Tsvetkov[@],
Igor E. Rogov

MIREA – Russian Technological University, Moscow, 119454 Russia
[@] Corresponding author, e-mail: cvj2@mail.ru

Abstract. The problem of testing in education is relevant for many countries. Testing solves three problems. The first task is to assess the quality of current training. The second task is to conduct a comparative analysis of learning outcomes. The third task is the management of the educational process in a particular educational institution and in the education sector. This determines the relevance of testing and the relevance of developing new methods for assessing test results. The article proposes a new method for assessing test results for different situations: “teacher–student,” computer test, virtual testing model, test on a mixed reality model and others. To solve the problem, a special quasi-sigmoidal function is introduced. It is analogous to the logistic function, but takes into account the peculiarities of real testing of students. The logistic function ranges from minus infinity to plus infinity. There are no negative assessments in education. The introduced function lies only in the positive range of the argument. It describes actual positive scores when testing students. The authors called this function the complexity function. With its help, the complexity of the subject is assessed according to the test results. To substantiate the method, the function of the logarithms of the odds, logistic regression and the resulting Rasch method are investigated. The article notes two shortcomings of the Rasch method. The testing principle has been defined for the new function, which is used to estimate complexity. The article introduces two new concepts: the test difficulty function and the integral test score. Integral assessment of testing is a smooth function and makes it possible to go from a stepwise dependence to a continuous one. The cumulative test score translates the point test results into a continuous function and creates a correlation between the scores. The results of an experiment with the participation of RTU MIREA students are presented. The experimental results are analyzed. The possibility of using the method in educational processes is shown. The method is an alternative to the Rasch method.

Keywords: algorithm, education, testing, complexity, software components, logits, logistic equation, Rasch model

• Submitted: 27.08.2021 • Revised: 04.10.2021 • Accepted: 11.10.2021

For citation: Sigov A.S., Tsvetkov V.Ya., Rogov I.E. Method for assessing testing difficulty in educational sphere. *Russ. Technol. J.* 2021;9(6):64–72. <https://doi.org/10.32362/2500-316X-2021-9-6-64-72>

Financial disclosure: The authors have no a financial or property interest in any material or method mentioned.

The authors declare no conflicts of interest.

НАУЧНАЯ СТАТЬЯ

Методы оценки сложности тестирования в сфере образования

**А.С. Сигов,
В.Я. Цветков[@],
И.Е. Рогов**

МИРЭА – Российский технологический университет, Москва, 119454 Россия

[@] Автор для переписки, e-mail: cvj2@mail.ru

Резюме. Проблема тестирования в сфере образования является актуальной для многих стран. Тестирование решает три задачи: проводит оценку качества текущего обучения, дает инструмент для сравнительного анализа результатов обучения, дает инструмент для управления образовательным процессом в отдельном учебном заведении и по отрасли. Это определяет актуальность тестирования и разработки новых методов оценки результатов тестирования. Статья предлагает новый метод оценки результатов тестирования для разных ситуаций: «преподаватель – учащийся», компьютерный тест, виртуальная тестирующая модель, тест на модели смешанной реальности и другие. Для решения этой задачи вводится квазисигмоидальная функция. Она является аналогом логистической функции, но учитывает особенности реального тестирования. Логистическая функция лежит в интервале от минус до плюс бесконечности. В образовании отрицательных оценок не бывает. Введенная функция лежит только в положительной области аргумента, то есть оценок тестирования. Эту функцию авторы называют функцией сложности. Предварительно исследуется функция логарифмов шансов, логистическая регрессия и вытекающий из этого метод Раша. Отмечены два недостатка метода Раша. Определяется принцип тестирования, который используется в функции сложности. Статья вводит два новых понятия: функция сложности тестирования и интегральная оценка тестирования. Интегральная оценка тестирования является интегральной функцией от точечных оценок. Она переводит точечные результаты тестирования в непрерывную функцию и создает корреляцию между оценками. Приводятся результаты эксперимента с участием студентов РТУ МИРЭА. Результаты эксперимента анализируются. Показана возможность применения метода в образовательных процессах. Метод является альтернативой методу Раша.

Ключевые слова: алгоритм, образование, тестирование, сложность, программные компоненты, логиты, логистическое уравнение, модель Раша

• Поступила: 27.08.2021 • Доработана: 04.10.2021 • Принята к опубликованию: 11.10.2021

Для цитирования: Сигов А.С., Цветков В.Я., Рогов И.Е. Методы оценки сложности тестирования в сфере образования. *Russ. Technol. J.* 2021;9(6):64–72. <https://doi.org/10.32362/2500-316X-2021-9-6-64-72>

Прозрачность финансовой деятельности: Никто из авторов не имеет финансовой заинтересованности в представленных материалах или методах.

Авторы заявляют об отсутствии конфликта интересов.

INTRODUCTION

The problem of assessing the complexity of testing and testing programs [1–3] is one of the most important in the field of education. At the same time, the complexity of the software components [4] as a computational object and the complexity of the test as a technological object, taking into account cognitive factors, are distinguished. Testing algorithms are qualitatively different from computational ones. They are designed to include a person in the information processing chain

in the “person–test” system. Testing algorithms include two groups: algorithms for informational interaction “student–test” and algorithms for processing test results. There is a difference between the two. Algorithms of information interaction “student–test” are interactive or the algorithms of the second kind. Algorithms for processing test results are direct or the algorithms of the first kind. Testing itself can be viewed as human–machine interaction. Accordingly, the test structure or system can be considered as a human–machine system (HMS). Currently, the HMS comprises various systems,

including an informational HMS. An example of the informational HMS is a geographic information system, in which information is interactively processed, and the solution to the problem is carried out by the methods of human and computer iterations. The testing system can be considered as an information man-machine system. Human resources and cognitive components are especially important for testing systems designed to work with students of a certain level of knowledge and certain forms of cognitive perception. Therefore, it is important in the process of using testing systems to evaluate the complexity of tests, to adapt them to certain levels of students' knowledge. Such adaptation is possible based on the analysis of testing results and the development of methods for comparative assessment of the complexity of testing programs based on the results of direct testing. This article is devoted to this issue.

1. RESEARCH METHODOLOGY

The research is based on logical, statistical, qualitative, and comparative analysis. The materials used were publications in the development of testing algorithms and the application of testing methods in education.

2. RESULTS OF STUDIES

2.1. Theoretical justification

In 1934, Chester Itner Bliss used the cumulative normal distribution function to map it and called it "probit" [7] as a kind of probability unit. In 1944, Joseph Berkson used the log of odds and called the function "logit," an abbreviation for "log istic un it," after the analogy of "probit." This unit has a logarithmic connotation and can be viewed as a logarithmic unit. In statistics, the logit function or log-odds is the logarithm of the odds or logarithmic odds, and the ratio [8] $p/(1-p)$ is called the chance, where p is the probability of an event occurring, $(1-p)$ is the probability non-occurrence of the event. Chance is a probabilistic characteristic, therefore logit connects the logarithmic and probabilistic components.

Logit-function (log-odds) plays an important role in logistic regression. By finding the $p/(1-p)$ relation, each probability can easily be transformed to logit. Notwithstanding a simple transformation, the physical meaning of log-odds can not always be comprehended. Jaccard, [9, p.10] called them "...illogical and complicated for interpretation, particularly in cases when large amount of statistical data is not available. However, corresponding formulas for their calculation are relatively simple, even though the results of calculation are difficult for deciphering."

Chance is the ratio of the likelihood of success to the likelihood of failure. As an equation, this is $p(A)/p(-A)$, where $p(A)$ is the probability of event A , and $p(-A)$ is the probability of "not A " (that is, the complement to A). The log odds (logit) give us the log odds of A , which can be written as

$$\log(A) = \log(p(A)/p(-A)) \quad (1)$$

Generically, we can describe logarithmic chances (logit) in the form:

$$\text{logit-odds} = \log[p/(1-p)]. \quad (2)$$

Mathematically, logit is the inversion of a standard logistic function $\sigma(x) = 1/(1 + e^{-x})$, therefore, logit is defined as follows:

$$\text{logit}(p) = \sigma^{-1}(p) = \log[p/(1-p)] \quad (3)$$

for $p \in (0, 1)$.

Thus, logit-function $f(x)$ is a kind of function (Fig. 1), which maps probability values from $(0, 1)$ on real numbers in set $(-\mu, +\mu)$ [10].

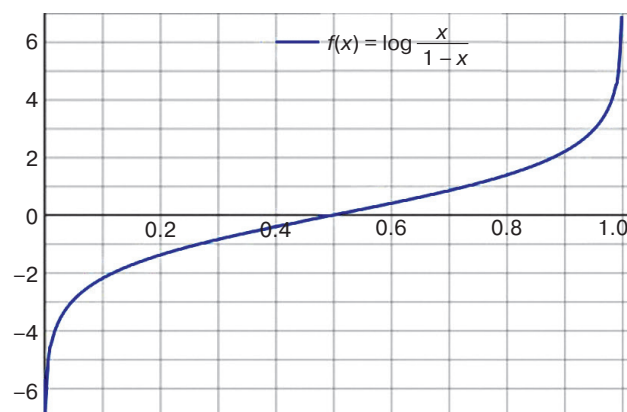


Fig. 1. Plot of the logit function

In Fig. 1, probabilities are placed on the horizontal axis. Real numbers on a conventional scale are shown on the vertical axis. The scale is chosen according to the conditions of a specific task.

Associated with the concept of logits is the concept of logistic regression. Simple logistic regression is ideologically close to linear regression. There are two differences. Linear regression uses a linear relationship between two measurements (x, y). Logistic regression uses a logistic relationship or logistic function between one measurement x and its probability p .

The logistic function can be one-parameter, two-parameter, three-parameter, and four-parameter [11]. A one-parameter function is used more often, which is in good agreement with the theory of logits.

The logistic function belongs to the class of sigmoidal functions. Sigmoid or sigmoidal function is usually called a smooth monotonic function [12], shaped like the letter “S,” which has two asymptotes and describes the reaction and saturation (Fig. 1). The canonical example of a sigmoid is the one-parameter logistic function

$$f(x) = \frac{1}{1 + e^{-x}}. \quad (4)$$

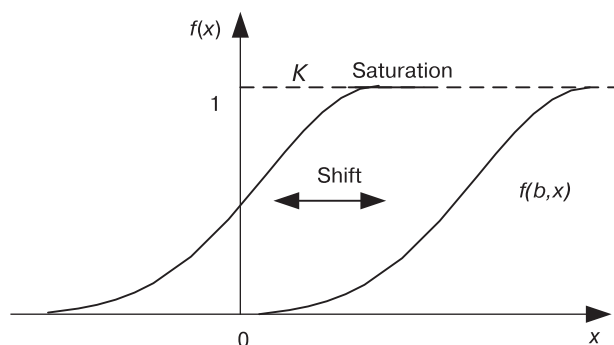


Fig. 2. Ascending sigmoid

The sigmoid is bounded by two horizontal asymptotes (Fig. 2), to which it approaches as the argument approaches to $\pm\infty$. Usually, these asymptotes (for an one-parameter function) are “0” (at $x = -\infty$) and some constant (at $x = +\infty$). In many cases, the constant at $x = +\infty$ is 1. This simplifies the relationship between probability and logistic functions. In Rasch’s assessment theory, the model in Fig. 2 is used to evaluate the performance of groups. This type of sigmoid can be called ascending. Such a sigmoid has positive function values for negative argument values. To shift it, you need to enter a special parameter b (Fig. 2). There is another type of sigmoid, which is called descending (Fig. 3). For this sigmoid, the reaction shows a transition from the upper asymptote to the lower one.

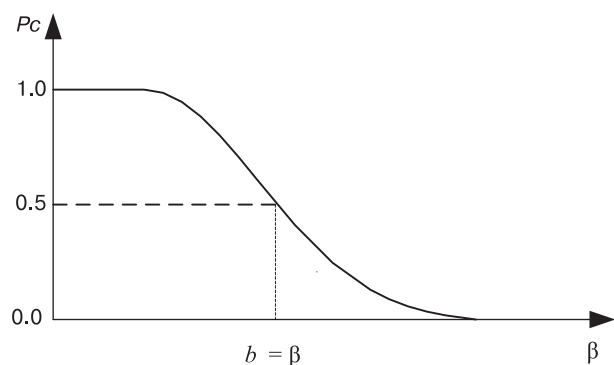


Fig. 3. Descending sigmoid

Figure 3 shows a descending sigmoid for which the probability values are plotted along the vertical axis and the test complexity values—along the horizontal axis.

The derivative of any sigmoid is a “Gaussian-like” curve with a maximum at zero, asymptotically approaching to zero at $x = \pm\infty$. The sigmoid family includes arctangent, hyperbolic tangent, and other functions.

If we compare the logit function (Fig. 1) and the descending sigmoid (Fig. 3), then the rotation of Fig. 1 by 90° clockwise will lead to complete similarity of the graphs in Fig. 1 and Fig. 3. This is what we mean when talking about the inversion of the logit function. The decreasing sigmoid (Fig. 3) is usually used to assess the difficulty of testing.

Thus, to assess the difficulty of testing, functions are used that have two asymptotes (upper and lower) and a smooth section connecting them.

2.2. Experimental research

Experimental researches were conducted in the study groups of the Institute of Information Technologies of the RTU MIREA. The shortcoming of the Rasch model when assessing the complexity of testing is that its argument exists from $-\infty$ to $+\infty$, while in real testing there are no negative values of the argument. This is mathematically explicable if we analyze the logit function (Fig. 1). This leads to the fact that most of the asymptote (Fig. 3) lies in the negative region of the argument, and the shift of the curve itself in Fig. 3 is used as a comparative characteristic of the test difficulty.

The basic concept of the Rasch model is that functions are needed that have two asymptotes and a transition region between them. To implement this idea, the authors introduce the “complexity function” $f_c(x)$ or “test complexity function,” which exists in the positive domain of the argument and has an asymptotic complexity constraint only from below. One can speak of a quasi-sigmoidal function, since it has a restricted domain of the argument, only in the positive region. Its initial value at $x = 0$ is equal to the constant “ c .” This function allows us to assess the difficulty of different subjects based on the results of testing one group. There are many such functions that can be built. The authors have investigated a number of functions and chosen a simple form of the complexity function

$$f_c(x) = c \frac{1}{1 + kx}. \quad (5)$$

In Eq. (5) x is a real argument, $f_c(x)$ is an assessment function. For $x > 0$, $k > 1$, $c \geq 1$. For the normalized function, f_c is equal to 1 at $x = 0$ and asymptotically approaches to 0 as $x \rightarrow +\infty$. This function is a quasi-sigmoid in the positive domain of the argument.

The ordered number of the student in the study group was chosen as an argument. This is a strictly point

value. The function was determined by the assessment of the given student. The ordered student number means that the students in the group were pre-ranked by their grade level in ascending order from lowest to highest. Conditionally, these can be the numbers that correspond to real names. If we compare the ordered numbers of students to their grades, then we get an approximate picture, shown in Fig. 4.

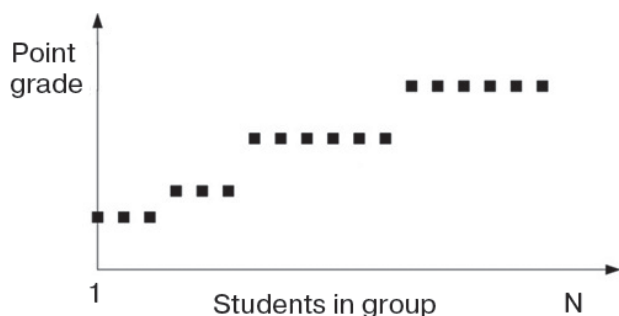


Fig. 4. Grouping grades within a group

Figure 4 shows the discrete values of the grades for the discrete value of the argument (student number in the group). There is no need to talk about any continuity in such a situation. Students can have the same grade, which corresponds to the rows of fixed grades on the graph. In the course of the research, it turned out that for sustainable assessment it is advisable to choose a function in the form of a cumulative or integral value. The formation of x is carried out as follows.

1. The grades of students in group z , obtained during testing, are recorded. Then, the statistics in the form of a “comb” is collected.
2. Grades z are ranked, ranked values z^* and stepwise increasing statistics are obtained (Fig. 4).
 $z \rightarrow z^*$.
3. After that, the integral grades x are formed according to the rule

$$x_i = \sum_{j=1}^i z_j^*.$$

Figure 5 shows integral and point grades.

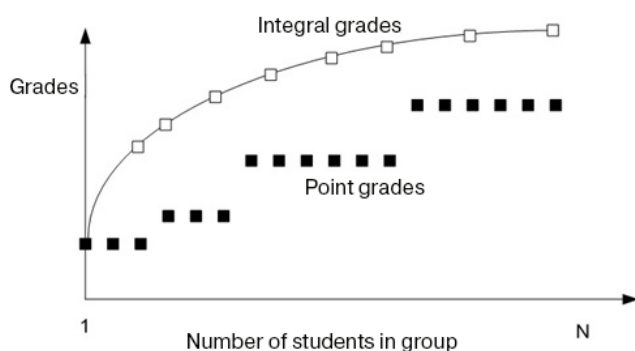


Fig. 5. Integral and point grades

In Fig. 5, integral grades (selectively) are shown by open squares. The formation of integral grades, as a method, to some degree resembles the smoothening of discrete values by the moving average method. Even the same point grades have different meanings in the integral grade. This creates correlation and continuity in the formation of grades.

The complexity function is constructed according to expression (5). The name of the function is due to the fact that it characterizes the complexity of the test or subject for a given study group. Table 1 shows the initial data of point grades for four subjects of students “Subject 1–Subject 4.” In groups, 21 students were selected.

Table 1. Initial grades z^* in ascending order for four subjects in one group

Number	Subject 1	Subject 2	Subject 3	Subject 4
1	4	4.5	5	5.5
2	5	5.5	6	6.5
3	5.5	6	6.5	7
4	6	6.5	7	7.5
5	6	6.5	7	7.5
6	6	6.5	7	7.5
7	6.5	7	7.5	8
8	7	7.5	8	8.5
9	7	7.5	8	8.5
10	7.5	8	8.5	9
11	7.5	8	8.5	9
12	8	8.5	9	9.5
13	8	8.5	9	9.5
14	8	8.5	9	9.5
15	8.5	9	9.5	10
16	8.5	9	9.5	10
17	8.5	9	9.5	10
18	9	9.5	10	10
19	9	9.5	10	10
20	9	9.5	10	10
21	9	9.5	10	10

Students were assessed on a 10-point system. A 20-point assessment and even a 100-point one, as recommended by Khlebnikov [1], is acceptable. Our research has shown that it is better to normalize the grades, that is, to set the grade interval for processing from 0 to 1. Table 2 shows integral normalized grades for the same groups.

Table 2. Normalized integral grades for four subjects

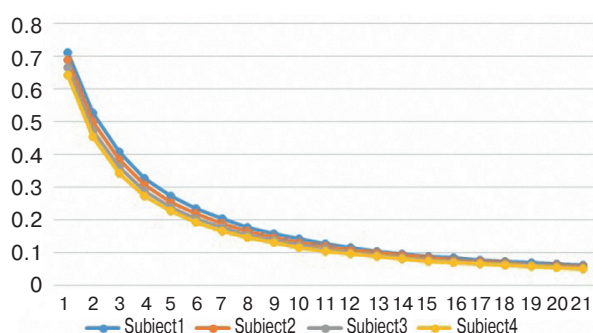
Number	Subject 1	Subject 2	Subject 3	Subject 4
1	0.4	0.45	0.5	0.55
2	0.9	1	1.1	1.2
3	1.45	1.6	1.75	1.9
4	2.05	2.25	2.45	2.65
5	2.65	2.9	3.15	3.4
6	3.25	3.55	3.85	4.15
7	3.9	4.25	4.6	4.95
8	4.6	5	5.4	5.8
9	5.3	5.75	6.2	6.65
10	6.05	6.55	7.05	7.55
11	6.8	7.35	7.9	8.45
12	7.6	8.2	8.8	9.4
13	8.4	9.05	9.7	10.35
14	9.2	9.9	10.6	11.3
15	10.05	10.8	11.55	12.3
16	10.9	11.7	12.5	13.3
17	11.75	12.6	13.45	14.3
18	12.65	13.55	14.45	15.3
19	13.55	14.5	15.45	16.3
20	14.45	15.45	16.45	17.3
21	15.35	16.4	17.45	18.3

Table 3. Values of the complexity function for four subjects

Number	Subject 1	Subject 2	Subject 3	Subject 4
1	0.714286	0.689655	0.666667	0.645161
2	0.526316	0.5	0.47619	0.454545
3	0.408163	0.384615	0.363636	0.344828
4	0.327869	0.307692	0.289855	0.273973
5	0.273973	0.25641	0.240964	0.227273
6	0.235294	0.21978	0.206186	0.194175
7	0.204082	0.190476	0.178571	0.168067
8	0.178571	0.166667	0.15625	0.147059
9	0.15873	0.148148	0.138889	0.130719
10	0.141844	0.13245	0.124224	0.116959
11	0.128205	0.11976	0.11236	0.10582
12	0.116279	0.108696	0.102041	0.096154
13	0.106383	0.099502	0.093458	0.088106
14	0.098039	0.091743	0.086207	0.081301
15	0.090498	0.084746	0.079681	0.075188
16	0.084034	0.07874	0.074074	0.06993
17	0.078431	0.073529	0.069204	0.065359
18	0.07326	0.068729	0.064725	0.06135
19	0.068729	0.064516	0.06079	0.057803
20	0.064725	0.06079	0.057307	0.054645
21	0.061162	0.057471	0.054201	0.051813

Table 3 shows the calculation of the complexity function for four subjects taken by the same group.

The values given in Table 3 are used to build the function shown below (Fig. 6).

**Fig. 6.** Plots of the complexity function for 4 subjects

A feature of real testing (Fig. 6) is that the graph displays only a part of the sigmoid (Fig. 3), since RTU MIREA students do not receive zero grades and low grades of type 1 and 2. A value of $f_c = 1$ corresponds to a zero result in the test assessment. In reality, students

always gain some points, and therefore the value of the function $f_c < 1$ for learning conditions at RTU MIREA (Fig. 6).

In the research, it was shown that the graphical proximity of the complexity functions to each other is not clear. Therefore, the true comparison is based on tabular data. Tables such as Table 3 allow us to find numerical characteristics featuring the comparative complexity of objects for a given group. There are two such characteristics: integral and point. The integral grade is defined as the integral under the curves. The point grade is determined as the average of the columns in Table 3.

The difference between the dynamics of the complexity function and the function in the Rasch model is that the Rasch model is shifted horizontally, but the complexity function is shifted vertically; the more difficult the subject, the lower the curve. Unlike the logit-based Rasch model, the complexity function is calculated with a minimum of computational resources. Using the data of Table 3, the grades for a group performance were obtained (Table 4).

Table 4. Final comparative characteristics of the complexity of 4 subjects for one test group

Indicators	Subject 1	Subject 2	Subject 3	Subject 4
Integral indicator of complexity	4.139	3.904	3.695	3.510
Relative increment of integral indicator	n/a	0.235	0.443	0.629
Point complexity indicator	0.197	0.186	0.176	0.167
Relative increment of point indicator	n/a	0.011	0.021	0.030

It follows from Table 4 that the most difficult subject for this experiment turned out to be Subject 1. The easiest subject for testing for RTU MIREA students is Subject 4 (Mathematical logic). These indicators are comparative and can be used for different subjects, but for a given group within one university. Each group is a bearer of a collective intelligence. It is a self-organizing system in which group members “pull” each other up to a common level. For another group, a qualitative correlation between the complexity of subjects is possible, but it may have quantitative difference. This method is an alternative to the Rasch model and serves as the basis for constructing fairly simple software components and algorithms for processing test results. This technique allows conducting comparative analysis with robust results.

3. DISCUSSION

A functional approach [13] and a statistical approach can be distinguished in testing. An example of a statistical approach is the application of the Rasch model [14, 15]. A significant shortcoming of this model in assessing the complexity of testing is that in it the concept of probability is incorrectly used, that is incomprehensible for specialists who do not know statistics. And it is unknown whether Rasch himself knew it. Many humanitarians use the formula without understanding its meaning and limitations. In statistics, only the frequency of observations is usually equated with probability. Usually, a statistical value is equated to a statistical value. But in the Rasch model, the probability is determined not by the statistics of observations (interval grade), but by one (point) student's grade. In this approach, the probability is considered to be 1 if a student (one-time grade) gets the highest grade, that is, a certain standard is chosen that equates

to a probability equal to 1. The ratio of one student's grade to the normative grade is declared a probability and the Rasch model is applied. In fact, the grade characterizes the intellectual level of the student, taking into account his mental state (anxiety, physical fatigue, mental fatigue, etc.). In a different state, a student can receive different grades at the same intellectual level. Accordingly, the application of the Rasch method to the same group will give different grades, meaning that it is a statistically unstable characteristic. The difference in this assessment is explained by the presence of an objective causal relationship, and not by a probabilistic characteristic, as in the Rasch model. Therefore, the Rasch's model is simple and good, but completely unreliable.

The obtained results have shown that the complexity function is applicable for assessing the complexity of subjects and is easy to use. Application of the function gives relative grades [16] and group grades [17].

CONCLUSIONS

The proposed method is applicable for the majority of tests of the following kinds: “teacher–student,” “software program–student,” “virtual test simulator–student,” “mixed reality simulator–student,” and others. Testing assessment quality increases with the increase of the grading scale up to a 10-score system and higher. This method differs from the majority of the assessment methods by introduction of an integral grade that relates to the grades in a group thereby reducing random errors. The method and the algorithm based on this method are fairly clear if compared to the Rasch model which is conditional. In the Rasch model, a single-parameter logistic function is used that has one asymptote at the zero value of the function and another asymptote at the value of the function equal to 1. Such a function can suitably be related with probabilities. However, the meaning of the sigmoid slope value stays behind the frames of this work. The study conducted by Arnold [18] has shown that in some cases the slope can change depending on the value of a parameter of the two-parametric logistic model, and characterizes the rate of the resource consumption. The proposed method includes simple algorithms of ranging, computation of the integral variable, computation of the complexity function, and statistical processing of the results. The method is simple and available for the majority of teachers/professors in universities. At the same time, the method is accessible for researching other complexity functions.

REFERENCES

1. Neiman Yu.M., Khlebnikov V.A. *Vvedenie v teoriyu modelirovaniya i parametrizatsii pedagogicheskikh testov (Introduction to the theory of modeling and parametrization of pedagogical tests)*. Moscow: Prometei; 2000. 168 p. (in Russ.). ISBN 5-7042-1068-6
2. Safiulin R.Z. Technology development of testing in education. *Upravlenie obrazovaniem: teoriya i praktika = Education management review*. 2015;1(17):139–149 (in Russ.). Available from URL: <https://emreview.ru/index.php/emr/issue/view/17>
3. Kolster R. Structural ambidexterity in higher education: excellence education as a testing ground for educational innovations. *Eur. J. High. Educ.* 2021;11(1):64–81. <https://doi.org/10.1080/21568235.2020.1850312>
4. Kudzh S.A., Tsvetkov V.Ya., Rogov I.E. Life cycle support software components. *Rossiiskii tekhnologicheskii zhurnal = Russian Technological Journal*. 2020;8(5):19–33 (in Russ.). <https://doi.org/10.32362/2500-316X-2020-8-5-19-33>
5. Mgbemena E. Man-machine systems: A review of current trends and applications. *FUPRE J. Sci. Ind. Res. (FJSIR)*. 2020;4(2):91–117.
6. Bogoutdinov B.B., Tsvetkov V.Ya. Application of the model of complementary resources in investing activities. *Vestnik Mordovskogo universiteta = Engineering Technologies and Systems*. 2014;24(4):103–116 (in Russ.). <https://doi.org/10.15507/VMU.024.201403.103>
7. Bliss C.I. The method of probits. *Science*. 1934;79(2037):38–39. <https://doi.org/10.1126/science.79.2037.38>
8. Viera A.J. Odds ratios and risk ratios: what's the difference and why does it matter? *South. Med. J.* 2008;101(7):730–734. <https://doi.org/10.1097/smj.0b013e31817a7ee4>
9. Jaccard J. Interaction Effects in Logistic Regression. USA: Sage Publishing; 2001. V. 135. 80 p. <https://dx.doi.org/10.4135/9781412984515>
10. Cramer J.S. The origins and development of the logit model. In book: *Logit models from economics and other fields*. Cambridge University Press; 2003. Ch. 9. <https://doi.org/10.1017/CBO9780511615412.010>
11. Jokar M., Subbey S., Gjøsaeter H. A logistic function to track time-dependent fish population dynamics. *Fisheries Research*. 2021;236:105840. <https://doi.org/10.1016/j.fishres.2020.105840>
12. Muñoz García D.A., Cardona Gómez D.C., Hoyos Mendez Y.C. Vólculo del sigmoide: revisión narrative. *Revista Facultad Ciencias de la Salud: Universidad del Cauca*. 2020;22(1):36–44. <https://doi.org/10.47373/rfcs.2020.v22.1575>
13. Tsvetkov V.Ya., Rogov I.E. Functional approach to the assessment of academic performance of study groups. *Obrazovatel'nye resursy i tekhnologii = Educational Resources and Technologies*. 2021;1(34):61–68 (in Russ.).
14. Seif N.A.S.M.A. *Building and standardizing a school readiness battery for kindergarten children in light of classical theory and item response theory (A comparative study)*. MA Thesis. Fayoum University; 2018. Available from URL: <http://www.fayoum.edu.eg/english/Education/Psychology/pdf/MsnadaMscE.pdf>

СПИСОК ЛИТЕРАТУРЫ

1. Нейман Ю.М., Хлебников В.А. *Введение в теорию моделирования и параметризации педагогических тестов*. М.: Прометей; 2000. 168 с. ISBN 5-7042-1068-6
2. Сафиулин Р.З. Развитие технологий тестирования в образовании. *Управление образованием: теория и практика*. 2015;1(17):139–149. URL: <https://emreview.ru/index.php/emr/issue/view/17>
3. Kolster R. Structural ambidexterity in higher education: excellence education as a testing ground for educational innovations. *Eur. J. High. Educ.* 2021;11(1):64–81. <https://doi.org/10.1080/21568235.2020.1850312>
4. Кудж С.А., Цветков В.Я., Рогов И.Е. Поддержка жизненного цикла программных компонент. *Российский технологический журнал*. 2020;8(5):19–33. <https://doi.org/10.32362/2500-316X-2020-8-5-19-33>
5. Mgbemena E. Man-machine systems: A review of current trends and applications. *FUPRE J. Sci. Ind. Res. (FJSIR)*. 2020;4(2):91–117.
6. Богоутдинов Б.Б., Цветков В.Я. Применение модели комплементарных ресурсов в инвестиционной деятельности. *Вестник Мордовского университета*. 2014;24(4):103–116. <https://doi.org/10.15507/VMU.024.201403.103>
7. Bliss C.I. The method of probits. *Science*. 1934;79(2037):38–39. <https://doi.org/10.1126/science.79.2037.38>
8. Viera A.J. Odds ratios and risk ratios: what's the difference and why does it matter? *South. Med. J.* 2008;101(7):730–734. <https://doi.org/10.1097/smj.0b013e31817a7ee4>
9. Jaccard J. Interaction effects in logistic regression. USA: Sage Publishing; 2001. V. 135. 80 p. <https://dx.doi.org/10.4135/9781412984515>
10. Cramer J.S. The origins and development of the logit model. In book: *Logit models from economics and other fields*. Cambridge University Press; 2003. Ch. 9. <https://doi.org/10.1017/CBO9780511615412.010>
11. Jokar M., Subbey S., Gjøsaeter H. A logistic function to track time-dependent fish population dynamics. *Fisheries Research*. 2021;236:105840. <https://doi.org/10.1016/j.fishres.2020.105840>
12. Muñoz García D.A., Cardona Gómez D.C., Hoyos Mendez Y.C. Vólculo del sigmoide: revisión narrative. *Revista Facultad Ciencias de la Salud: Universidad del Cauca*. 2020;22(1):36–44. <https://doi.org/10.47373/rfcs.2020.v22.1575>
13. Цветков В.Я., Рогов И.Е. Функциональный подход к оценке успеваемости учебных групп. *Образовательные ресурсы и технологии*. 2021;1(34):61–68.
14. Seif N.A.S.M.A. *Building and standardizing a school readiness battery for kindergarten children in light of classical theory and item response theory (A comparative study)*. MA Thesis. Fayoum University; 2018. URL: <http://www.fayoum.edu.eg/english/Education/Psychology/pdf/MsnadaMscE.pdf>
15. Цветков В.Я., Войнова Е.В. Модификация модели Раша для оценки свободного тестирования. *Вестник Рязанского государственного радиотехнического университета*. 2018;1(63):90–94. <https://doi.org/10.21667/1995-4565-2018-63-1-90-94>
16. Кудж С.А. Методы сравнительного анализа. *Славянский форум*. 2019;3(25):140–150.

15. Tsvetkov V.Ya., Voinova E.V. Modification of Rasch model for free testing assessment. *Vestnik Ryazanskogo gosudarstvennogo radiotekhnicheskogo universiteta = Vestnik of RSREU*. 2018;1(63):90–94 (in Russ.). <https://doi.org/10.21667/1995-4565-2018-63-1-90-94>
16. Kudzh S.A. Methods for comparative analysis. *Slavyanskii forum = Slavic Forum*. 2019;3(25):140–150 (in Russ.).
17. Kudzh S.A. Assessment of group cognitive complexity. *Slavyanskii forum = Slavic Forum*. 2018;2(20):36–43 (in Russ.).
18. Arnol'd V.I. Catastrophe theory. *Itogi nauki i tekhniki. "Seriya Sovremennye problemy matematiki. Fundamental'nye napravleniya"*. 1986;5:219–277 (in Russ.).
17. Кудж С.А. Оценка групповой когнитивной сложности. *Славянский форум*. 2018;2(20):36–43.
18. Арнольд В.И. Теория катастроф. *Итоги науки и техники. Серия «Современные проблемы математики. Фундаментальные направления»*. 1986;5:219–277.

About the authors

Alexander S. Sigov, Academician of RAS, Dr. Sci. (Phys.–Math.), Professor, President, MIREA – Russian Technological University (78, Vernadskogo pr., Moscow, 119454 Russia). E-mail: sigov@mirea.ru. ResearcherID L-4103-2017, Scopus Author ID 35557510600.

Viktor Ya. Tsvetkov, Dr. Sci. (Eng.), Dr. Sci. (Econ.), Professor, MIREA – Russian Technological University (78, Vernadskogo pr., Moscow, 119454 Russia). E-mail: cvj2@mail.ru. Scopus Author ID 56069916700.

Igor E. Rogov, Director of the Institute of Pre-University Training, MIREA – Russian Technological University (78, Vernadskogo pr., Moscow, 119454 Russia). E-mail: rogov@mirea.ru.

Об авторах

Сигов Александр Сергеевич, академик РАН, д.ф.-м.н., профессор, президент ФГБОУ ВО «МИРЭА – Российский технологический университет» (119454, Россия, Москва, пр-т Вернадского, д. 78). E-mail: sigov@mirea.ru. ResearcherID L-4103-2017, Scopus Author ID 35557510600.

Цветков Виктор Яковлевич, д.т.н., д.э.н., профессор, советник ректората ФГБОУ ВО «МИРЭА – Российский технологический университет» (119454, Россия, Москва, пр-т Вернадского, д. 78). E-mail: cvj2@mail.ru. Scopus Author ID 56069916700.

Рогов Игорь Евгеньевич, директор Института довузовской подготовки ФГБОУ ВО «МИРЭА – Российский технологический университет» (119454, Россия, Москва, пр-т Вернадского, д. 78). E-mail: rogov@mirea.ru.

Translated by E. Shklovskii