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RESEARCH ARTICLE

Optimal nonlinear filtering of MPSK signals against a background of harmonic interference with a random initial phase

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Abstract.
Objectives. The widespread use of radio data transmission systems using signals with multiposition phase shift keying (MPSK) is due to their high noise immunity and the simplicity of constructing the transmitting and receiving parts of the equipment. The conducted studies have shown that the presence of non-fluctuation interference, in particular, harmonic interference, in the radio channel significantly reduces the noise immunity of receiving discrete information. The energy loss in this case, depending on the interference intensity, can range from fractions of dB to 10 db or more. Therefore, interference suppression is an important task for such radio systems. The aim of the work is to synthesize and analyze an algorithm for optimal nonlinear filtering of MPSK signals against a background of harmonic interference with a random initial phase.
Methods. The provisions of the theory of optimal nonlinear signal filtering and methods of statistical radio engineering are used.
Results. The synthesis and analysis of the algorithm of optimal nonlinear filtering of MPSK signals against the background of harmonic interference with a random initial phase are carried out. The synthesized receiver contains a discrete symbol evaluation unit, two phase-locked frequency circuits of reference generators that form evaluation copies of the signal and interference, and cross-links between them. Analytical expressions are obtained that allow calculating the dependences of the bit error probability on the signal-to-noise ratio and the interference intensity μ. It is established that uncompensated fluctuations of the initial phase of the useful signal have a greater effect on the receiver noise immunity than similar fluctuations of the phase of harmonic interference, especially with low positional signals.
Conclusions. Comparison of the obtained results with the results obtained in the case when there are no harmonic interference compensation circuits shows that the use of the obtained phase filtering algorithms allows for almost complete suppression of harmonic interference. Thus, if $\mu = 0.5$ and the probability of error is 10^{-2}, the energy gain at $M = 2$ is about 2.5 dB, at $M = 4$ is about 6 dB, at $M = 8$ and $M = 16$ is at least 10 dB.

Keywords: multi-position phase manipulation, harmonic interference, optimal nonlinear filtering, noise immunity, bit error probability

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НАУЧНАЯ СТАТЬЯ

Оптимальная нелинейная фильтрация сигналов М-ФМ на фоне гармонической помехи со случайной начальной фазой

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Резюме.
Цели. Широкое распространение радиосистем передачи данных с использованием сигналов с многопозиционной фазовой манипуляции (М-ФМ) обусловлено их высокой помехоустойчивостью и простотой построения передающей и приемной частей аппаратуры. Проведенные исследования показали, что наличие в радиоканале, кроме шумовой, нефлуктуационных, в частности, гармонических помех существенно снижает помехоустойчивость приема дискретной информации, энергетический проигрыш в этом случае в зависимости от интенсивности помехи может составлять от долей дБ до 10 дБ и более, поэтому борьба с ними является важной задачей для таких радиосистем. Цель работы – синтез и анализ алгоритма оптимальной нелинейной фильтрации сигналов М-ФМ на фоне гармонической помехи со случайной начальной фазой.
Методы. Использованы положения теории оптимальной нелинейной фильтрации сигналов и методы статистической радиотехники.
Результаты. Проведены синтез и анализ алгоритма оптимальной нелинейной фильтрации сигналов М-ФМ на фоне гармонической помехи со случайной начальной фазой. Синтезированный приемник содержит блок оценки дискретного символа, две схемы фазовой автоподстройки частоты опорных генераторов, формирующих оценочные копии сигнала и помехи, и перекрестные связи между ними, получены аналитические выражения, позволяющие рассчитать зависимости вероятности битовой ошибки от отношения сигнал/шум и интенсивности помехи μ. Установлено, что нескомпенсированные флуктуации начальной фазы полезного сигнала оказывают большее влияние на помехоустойчивость приемника, чем аналогичные флуктуации фазы гармонической помехи, особенно при малой позиционности сигналов.
Выводы. Сравнение полученных результатов с результатами, полученными в случае отсутствия цепи компенсации гармонической помехи, показывает, что использование полученных алгоритмов фильтрации фаз позволяет обеспечить практически полное подавление гармонической помехи. Так, при $\mu = 0.5$ для вероятности ошибки $P_{eb} = 10^{-2}$ энергетический выигрыш при $M = 2$ составляет около 2.5 дБ, при $M = 4$ – около 6 дБ, при $M = 8$ и $M = 16$ – не менее 10 дБ.

Ключевые слова: многопозиционная фазовая манипуляция, гармоническая помеха, оптимальная нелинейная фильтрация, помехоустойчивость, вероятность битовой ошибки

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INTRODUCTION

The widespread use of multiple phase-shift keyed (MPSK) signals in modern digital information transmission systems is due to their high energy and spectral characteristics. However, the quality of communication was reported [1–7] to decrease significantly, if, in the radio channel, there is not only a noise interference, but also nonfluctuating interferences of various types. This is particularly so where such an interference is narrowband (harmonic) and has the same frequency as that of the desired signal [1, 2]. Depending on the interference intensity, the energy loss in this case can be fractions of decibel to 10 dB and more.

Such a decrease in the quality of reception is explained by the fact that demodulator algorithms are optimized to receive signals against the background of only white Gaussian noise and do not take into account the characteristics of nonfluctuating interferences. Introduction of units of compensation of such interferences to the demodulator circuit can considerably increase the noise immunity of signal reception. The purpose of this work was to synthesize and analyze an algorithm of optimal nonlinear filtrating of MPSK signals against the background of a harmonic interference with a random initial phase.

SYNTHESIS OF AN ALGORITHM OF OPTIMAL NONLINEAR FILTERING

Let us consider the following receiver-input process over the time range $t \in (0, kT]$:

$$x(t) = s_\Sigma(\mathbf{C}_k, t, \varphi_{sig}) + s_{int}(t, \varphi_{int}) + n(t) = \\ = s_{sig,int}(\mathbf{C}_k, t, \varphi_{sig}, \varphi_{int}) + n(t). \quad (1)$$

It is an additive mixture of MPSK signal $s_\Sigma(\mathbf{C}_k, t, \varphi_{sig})$, which in a certain ($k$th) digit time slot has the form

$$s_k(C_k = i, t, \varphi_{sig}) = A_0 \cos(\omega_0 t + \varphi_i + \varphi_{sig}), \\ \varphi_i = i2\pi/M,\ t \in ((k-1)T, kT],\ i = 0, 1, \ldots, M-1; \quad (2)$$

a harmonic interference with a random initial phase:

$$s_{int}(t, \varphi_{int}) = \mu A_0 \cos(\omega_{int} t + \varphi_{int});$$

and noise interference $n(t)$ with the parameters:

$$\langle n(t) \rangle = 0; \langle n(t_1) n(t_2) \rangle = \frac{N_0}{2} \delta(t_2 - t_1).$$

Here, $\mathbf{C}_k$ is the vector of discrete information symbols.

Let us consider the MPSK signal as a discrete–continuous Markov process in which the states of a discrete parameter can change at certain times, multiple of T, and the random initial phases φ_{sig} and φ_{int} of the signal and interference, respectively, are Wiener processes [8]:

$$\dot{\varphi}_{sig}(t) = n_{\varphi_{sig}}(t), \quad \dot{\varphi}_{int}(t) = n_{\varphi_{int}}(t).$$

Here, $n_{\varphi_{sig}}(t)$ and $n_{\varphi_{int}}(t)$ are white Gaussian noises with zero means and single-sided spectral densities $N_{\varphi_{sig}}$ и $N_{\varphi_{int}}$, respectively. Such a representation of these random processes describes well the fluctuating phase shifts in self-excited oscillators [9]. In practice, especially in narrowband channels, the rates of the continuous processes $\varphi_{sig}(t)$ and $\varphi_{int}(t)$ typically vary slowly in comparison with the rates of change of information and channel symbols; i.e.,

$$\tau_{\varphi_{sig}} >> T, \ \tau_{\varphi_{int}} >> T,$$

where $\tau_{\varphi_{sig}}$ and $\tau_{\varphi_{int}}$ are the correlation times of these processes.

A priori information on each of the diffusion Markov processes, $\varphi_{sig}(t)$ and $\varphi_{int}(t)$, is given by the Fokker–Planck–Kolmogorov equation [8].

Thus, in view of all the above, the circuit diagram of a receiver of such discrete–continuous processes can be synthesized using a previously proposed filtrating algorithm [10]. In this case, the mixed *a posteriori* probability density of $s_{sig,int}(\mathbf{C}_k, t, \varphi_{sig}, \varphi_{int})$ containing the vector $\mathbf{C}_k$ is

$$p_{ps}(t, \mathbf{C}_k, \varphi_{sig}, \varphi_{int}) = \\ = w_{ps}(t, \varphi_{sig}, \varphi_{int}) p_{ps}(t, \mathbf{C}_k | \varphi_{sig}, \varphi_{int}),$$

where $w_{ps}(t, \varphi_{sig}, \varphi_{int})$ is the *a posteriori* probability density of the independent parameters φ_{sig} and φ_{int}, which is unconditional on the vector of discrete symbols; and $p_{ps}(t, \mathbf{C}_k|\varphi_{sig}, \varphi_{int})$ is the conditional *a posteriori* probability of the state of the vector of discrete symbols at fixed values of φ_{sig} and φ_{int}.

The conditional *a posteriori* probability $p_{ps}(t, \mathbf{C}_k|\varphi_{sig}, \varphi_{int})$ obeys the equation [10]

$$\dot{p}_{ps}(t,\mathbf{C}_k|\varphi_{sig},\varphi_{int}) = p_{ps}(t,\mathbf{C}_k|\varphi_{sig},\varphi_{int}) \times \left\{F(t,\mathbf{C}_k,\varphi_{sig},\varphi_{int}) - \left\langle F(t,\varphi_{sig},\varphi_{int})\right\rangle\right\}, \quad (3)$$

where

$$F(t,\mathbf{C}_k,\varphi_{sig},\varphi_{int}) = \sum_{j=1}^{k} F_j(t,\mathbf{C}_j,\varphi_{sig},\varphi_{int}),$$

$$F_j(t,\mathbf{C}_j,\varphi_{sig},\varphi_{int}) = -\frac{1}{N_0}[x(t) - s_{sig,int,j}(\mathbf{C}_j,t,\varphi_{sig},\varphi_{int})]^2, \quad (4)$$

$$\left\langle F(t,\varphi_{sig},\varphi_{int})\right\rangle = \sum_{C_1=0}^{M-1}\sum_{C_2=0}^{M-1}\dots\sum_{C_k=0}^{M-1} F(t,\mathbf{C}_k,\varphi_{sig},\varphi_{int}) \times p_{sig,int}(t,\mathbf{C}_k|\varphi_{sig},\varphi_{int}).$$

Under the assumption that the *a priori* probabilities of the values of information symbols (for MPSK) in radio communication are identical and equal to $1/M$, and also identical are the probabilities of the transition of a discrete symbol from one state to another, the solution of Eq. (3) at time $t = kT$ is written as

$$p_{ps}(kT,\mathbf{C}_k|\varphi_{sig},\varphi_{int}) = \frac{\exp\left[\sum_{j=1}^{k}\int_{(j-1)T}^{jT} F_j(\tau,C_j,\mathbf{C}_{j-1},\varphi_{sig},\varphi_{int})d\tau\right]}{\sum_{\mathbf{C}_k}\exp\left[\sum_{j=1}^{k}\int_{(j-1)T}^{jT} F_j(\tau,C_j,\mathbf{C}_{j-1},\varphi_{sig},\varphi_{int})d\tau\right]}.$$

Here, for the writing to be more informative, the discrete symbol C_j is separately indicated in the list of the arguments of the function $F_j(\times)$.

To find the *a posteriori* probability of one or another value of the symbol C_k, the latter expression should be averaged over M possible values $C_1, C_2, \dots, C_{k-1}$. This gives

$$p_{ps}(kT,C_k|\varphi_{sig},\varphi_{int}) = \sum_{\mathbf{C}_{k-1}}\left\{\frac{\exp\left[\sum_{j=1}^{k}\int_{(j-1)T}^{jT} F_j(\tau,C_j,\mathbf{C}_{j-1},\varphi_{sig},\varphi_{int})d\tau\right]}{\sum_{\mathbf{C}_k}\exp\left[\sum_{j=1}^{k}\int_{(j-1)T}^{jT} F_j(\tau,C_j,\mathbf{C}_{j-1},\varphi_{sig},\varphi_{int})d\tau\right]}\right\}.$$

The expression can be significantly simplified under the additional assumption that the quality of reception is good. In this case, the symbols that have already been decided by the time $t = kT$ can be replaced by their estimated values; i.e., the vector $\mathbf{C}_{k-1}$ can be replaced by $\mathbf{C}^*_{k-1} = \left\{C^*_1, C^*_2, \dots, C^*_{k-1}\right\}$. Combining the terms independent of C_k into coefficient K and taking into account the independence of symbols in the information sequence of the corresponding MPSK signal transmissions, one can obtain the formula

$$p_{ps}(T,C_k|\varphi_{sig},\varphi_{int}) = K\frac{\exp\left[\int_0^T F_k(\tau,C_k,\varphi_{sig},\varphi_{int})d\tau\right]}{\sum_{C_k=0}^{M-1}\exp\left[\int_0^T F_k(\tau,C_k,\varphi_{sig},\varphi_{int})d\tau\right]}.$$

An algorithm to estimate a discrete information symbol follows from the condition of the maximum of this *a posteriori* probability at time $t = T$:

$$(C_k = i) \Rightarrow \max\{p_{ps}(t, \mathbf{C}_k|\varphi_{sig}, \varphi_{int})\}. \quad (5)$$

Let us introduce the notation

$$J_0 = \int_0^T F_k(\tau, C_k = 0, \varphi_{sig},\varphi_{int})d\tau,$$
$$\dots \quad (6)$$
$$J_{M-1} = \int_0^T F_k(\tau, C_k = M-1, \varphi_{sig},\varphi_{int})d\tau.$$

The integrands of these formulas are found from expressions (4). Then, algorithm (5) can be rewritten in the form

$$(C_k = i) \Rightarrow \max\{\exp(J_i)\}, \quad (7)$$

or

$$(C_k = i) \Rightarrow \max\{J_i\}. \quad (8)$$

Let us transform integrals (6), taking into account expressions (1) and (2):

$$J_0=\frac{2}{N_0}\int_0^T[x(t)-s_{\text{int}}(t,\varphi_{\text{int}})]s_k(t,C_k=0,\varphi_{\text{int}})dt,$$
$$\ldots \tag{9}$$
$$J_{M-1}=\frac{2}{N_0}\int_0^T[x(t)-s_{\text{int}}(t,\varphi_{\text{int}})]s_k(t,C_k=M-1,\varphi_{\text{sig}})dt.$$

Algorithm (8) is structurally similar to the MPSK signal processing algorithm for the case where signals are received against the background of only white Gaussian noise [11]. The difference consists in the presence of a procedure of subtraction from the received mixture $x(t)$ a copy of the interference that is formed on the receiver side. The circuit contains a set of correlators (9), which determine the degree of similarity of the compensated mixture with reference signals corresponding to all the possible values of the information symbol C_k.

The reference signals and the copy of the interference contain information on the initial phases φ_{sig} and φ_{int}. The true values of these quantities can be replaced by their estimated values φ^*_{sig} and φ^*_{int}. Let us develop algorithms to make these estimates.

Let us solve this problem by the Gaussian approximation [8, 12] of the *a posteriori* probability density of these random parameters. This method is valid at high signal-to-noise ratios and long observation times. In this case, such an assumption is valid, which makes it possible to transition from difficult-to-solve differential equations for probability densities, which follow from the Fokker–Planck–Kolmogorov equation [8], to approximate relations for the expected values φ^*_α and the *a posteriori* variances $K_{\alpha\beta}$ of the approximating multidimensional (of dimension z) Gaussian distribution:

$$\dot{\varphi}^*_\alpha=a_\alpha(\varphi^*)+\sum_{\beta=1}^{z}K_{\alpha\beta}\frac{\partial<F(t,\varphi^*)>}{\partial\varphi^*_\beta},$$
$$\dot{K}_{\alpha\beta}=\sum_{\nu=1}^{z}\left[\frac{\partial a_\alpha(\varphi^*)}{\partial\varphi^*_\nu}K_{\nu\beta}+\frac{\partial a_\beta(\varphi^*)}{\partial\varphi^*_\nu}K_{\alpha\nu}\right]+$$
$$+b_{\alpha\beta}(\varphi^*)+\sum_{\nu=1}^{z}\sum_{\gamma=1}^{z}K_{\alpha\nu}\frac{\partial^2<F(t,\varphi^*)>}{\partial\varphi^*_\nu\partial\varphi^*_\gamma}K_{\gamma\beta}.$$

The drift and diffusion coefficients for the independent Wiener processes $\varphi_{\text{sig}}(t)$ and $\varphi_{\text{int}}(t)$ are the following [8, 13]:

$$a(\varphi_{\text{sig}})=a(\varphi_{\text{int}})=0,$$
$$b(\varphi_{\text{sig}})=\frac{N_{\varphi_{\text{sig}}}}{2};\ b(\varphi_{\text{int}})=\frac{N_{\varphi_{\text{int}}}}{2},$$

and $K_{\varphi_{\text{sig}}\varphi_{\text{int}}}=K_{\varphi_{\text{int}}\varphi_{\text{sig}}}=0.$

Assuming that the phase lock time is less than T, we obtain the equations of optimal nonlinear filtering of the parameters φ^*_{sig} and φ^*_{int}:

$$\dot{\varphi}^*_{\text{sig}}=K_{\varphi_{\text{sig}}\varphi_{\text{sig}}}\frac{\partial\left\langle F_k(t,\varphi^*_{\text{sig}},\varphi^*_{\text{int}})\right\rangle}{\partial\varphi^*_{\text{sig}}},$$
$$\dot{\varphi}^*_{\text{int}}=K_{\varphi_{\text{int}}\varphi_{\text{int}}}\frac{\partial\left\langle F_k(t,\varphi^*_{\text{sig}},\varphi^*_{\text{int}})\right\rangle}{\partial\varphi^*_{\text{int}}}, \tag{10}$$

where the values of the *a posteriori* variances $K_{\varphi_{\text{sig}}\varphi_{\text{sig}}}$ and $K_{\varphi_{\text{int}}\varphi_{\text{int}}}$ are determined from the system of equations

$$\dot{K}_{\varphi_{\text{sig}}\varphi_{\text{sig}}}=\frac{N_{\varphi_{\text{sig}}}}{2}+K^2_{\varphi_{\text{sig}}\varphi_{\text{sig}}}\frac{\partial^2\left\langle F_k(t,\varphi^*_{\text{sig}},\varphi^*_{\text{int}})\right\rangle}{\partial{\varphi^*_{\text{sig}}}^2},$$
$$\dot{K}_{\varphi_{\text{int}}\varphi_{\text{int}}}=\frac{N_{\varphi_{\text{int}}}}{2}+K^2_{\varphi_{\text{int}}\varphi_{\text{int}}}\frac{\partial^2\left\langle F_k(t,\varphi^*_{\text{sig}},\varphi^*_{\text{int}})\right\rangle}{\partial{\varphi^*_{\text{int}}}^2}, \tag{11}$$

and

$$<F_k(t,\varphi^*_{\text{sig}},\varphi^*_{\text{int}})>=$$
$$=F_k(t,C_k=0,\varphi^*_{\text{sig}},\varphi^*_{\text{int}})\frac{\exp J_0}{\sum_{i=0}^{M-1}\exp J_i}+$$
$$+F_k(t,C_k=1,\varphi^*_{\text{sig}},\varphi^*_{\text{int}})\frac{\exp J_1}{\sum_{i=0}^{M-1}\exp J_i}+$$
$$+F_k(t,C_k=2,\varphi^*_{\text{sig}},\varphi^*_{\text{int}})\frac{\exp J_2}{\sum_{i=0}^{M-1}\exp J_i}+\ldots+$$
$$+F_k(t,C_k=M-1,\varphi^*_{\text{sig}},\varphi^*_{\text{int}})\frac{\exp J_{M-1}}{\sum_{i=0}^{M-1}\exp J_i}.$$

In view of expressions (4),

$$\frac{\partial<F_k(t,\varphi^*_{\text{sig}},\varphi^*_{\text{int}})>}{\partial\varphi^*_{\text{sig}}}=\frac{2A_0}{N_0}[x(t)-s_{\text{int}}(t,\varphi^*_{\text{int}})]\times$$
$$\times\sum_{i=0}^{M-1}s^h_k(t,C_k=i,\varphi^*_{\text{sig}})\frac{\exp J_i}{\sum_{l=0}^{M-1}\exp J_l},$$

$$\frac{\partial < F_k(t,\varphi^*_{sig},\varphi^*_{int}) >}{\partial\varphi^*_{int}} = -\frac{2\mu A_0}{N_0}\sin(\omega_{int}t+\varphi^*_{int})\times$$
$$\times\left[x(t)-\sum_{i=0}^{M-1} s_k(t,C_k=i,\varphi^*_{sig})\frac{\exp J_i}{\sum_{l=0}^{M-1}\exp J_l}\right].$$

Here,

$$s^h_k(t,C_k=i,\varphi^*_{sig}) = \frac{ds_k(t,C_k=i,\varphi^*_{sig})}{d\varphi^*_{sig}} =$$
$$= -A_0\sin(\omega_0 t+\varphi_i+\varphi^*_{sig}).$$

At a high signal-to-noise ratio, the latter expressions are simplified. If the parity and symmetry of the constellation diagram of the MPSK signal are also taken into account, then,

$$\frac{\partial < F_k(t,\varphi^*_{sig},\varphi^*_{int}) >}{\partial\varphi^*_{sig}} \approx \frac{2A_0}{N_0}[x(t)-s_{int}(t,\varphi^*_{int})]\times$$
$$\times\sum_{i=0}^{M/2-1} s^h_k(t,C_k=i,\varphi^*_{sig})\,\text{th}\,J_i,$$

$$\frac{\partial < F_k(t,\varphi^*_{sig},\varphi^*_{int}) >}{\partial\varphi^*_{int}} \approx -\frac{2\mu A_0}{N_0}\sin(\omega_{int}t+\varphi^*_{int})\times$$
$$\times[x(t)-\sum_{i=0}^{M/2-1} s_k t,C_k=i,\varphi^*_{sig})\,\text{th}\,J_i].$$

Making statistical averaging and assuming that, in the steady-state mode, at small filtering error,

$$\cos(\varphi_{int}-\varphi^*_{int})\approx 1,\ \cos(\varphi_{sig}-\varphi^*_{sig})\approx 1,$$

we obtain

$$\overline{\frac{\partial^2 < F_k(t,\varphi^*_{sig},\varphi^*_{int}) >}{\partial{\varphi^*_{sig}}^2}} \approx -\frac{A_0^2}{2N_0}\left(1+\text{th}\frac{A_0^2T}{2N_0}\right),$$

$$\overline{\frac{\partial^2 < F_k(t,\varphi^*_{sig},\varphi^*_{int}) >}{\partial{\varphi^*_{int}}^2}} \approx -\frac{\mu^2A_0^2}{N_0}.$$

In the steady-state mode of filtering of the continuous parameters φ_{sig} and φ_{int}, the solution of Eqs. (11) tends to the steady-state values $\overline{K_{\varphi_{sig}\varphi_{sig}}}$ and $\overline{K_{\varphi_{int}\varphi_{int}}}$, the derivatives of which are zero. Hence,

$$\overline{K_{\varphi_{sig}\varphi_{sig}}} = \sqrt{\frac{N_{\varphi_{sig}}N_0}{A_0^2\left(1+\text{th}\frac{A_0^2T}{2N_0}\right)}},$$

$$\overline{K_{\varphi_{int}\varphi_{int}}} = \sqrt{\frac{N_{\varphi_{int}}N_0}{2\mu^2A_0^2}}.$$

Finally, algorithms (10) of filtering of the random initial phases of the signal and the interference take the form

$$\dot\varphi^*_{sig} = S_1K_1A_0\left[x(t)-s_{int}(t,\varphi^*_{int})\right]\times$$
$$\times\sum_{i=0}^{M/2-1} s^h_k(t,C_k=i,\varphi^*_{sig})\,\text{th}\,J_i, \quad (12)$$

$$\dot\varphi^*_{int} = -S_2K_2\mu A_0\sin(\omega_{int}t+\varphi^*_{int})\times$$
$$\times[x(t)-\sum_{i=0}^{M/2-1} s_k(t,C_k=i,\varphi^*_{sig})\,\text{th}\,J_i], \quad (13)$$

where S_1 and S_2 are the transconductances of control elements (CEs) in each phase-lock channel, $K_1=\frac{2\overline{K_{\varphi_{sig}\varphi_{sig}}}}{N_0S_1}$, and $K_2=\frac{2\overline{K_{\varphi_{int}\varphi_{int}}}}{N_0S_2}$.

Figure 1 presents the circuit diagram of a quasi-coherent receiver in which interrelated algorithms (7)–(9), (12), and (13) are implemented. The receiver contains an estimator of discrete symbol C^*_k, two phase-lock circuits of reference oscillators O1 and O2, and cross couplings between them.

Oscillators O1 and O2 generate estimation copies of the signal and the interference, which are phase-locked with the corresponding oscillations contained in the received mixture $x(t)$. At the input of the receiver, two subtractors are installed. Subtractor S1 compensates the harmonic interference in the received mixture; after S1, the signal is transmitted to a demodulator to make a decision on the received symbol. Subtractor S2 compensates the signal in the received mixture, and the obtained oscillation in the ideal case contains only the harmonic interference and noise. This oscillation is used by the phase-lock circuit of the interference channel.

ANALYSIS OF THE NOISE IMMUNITY OF THE SYNTHESIZED RECEIVER

Let us analyze the noise immunity of the synthesized quasi-coherent receiver of MPSK signals using a

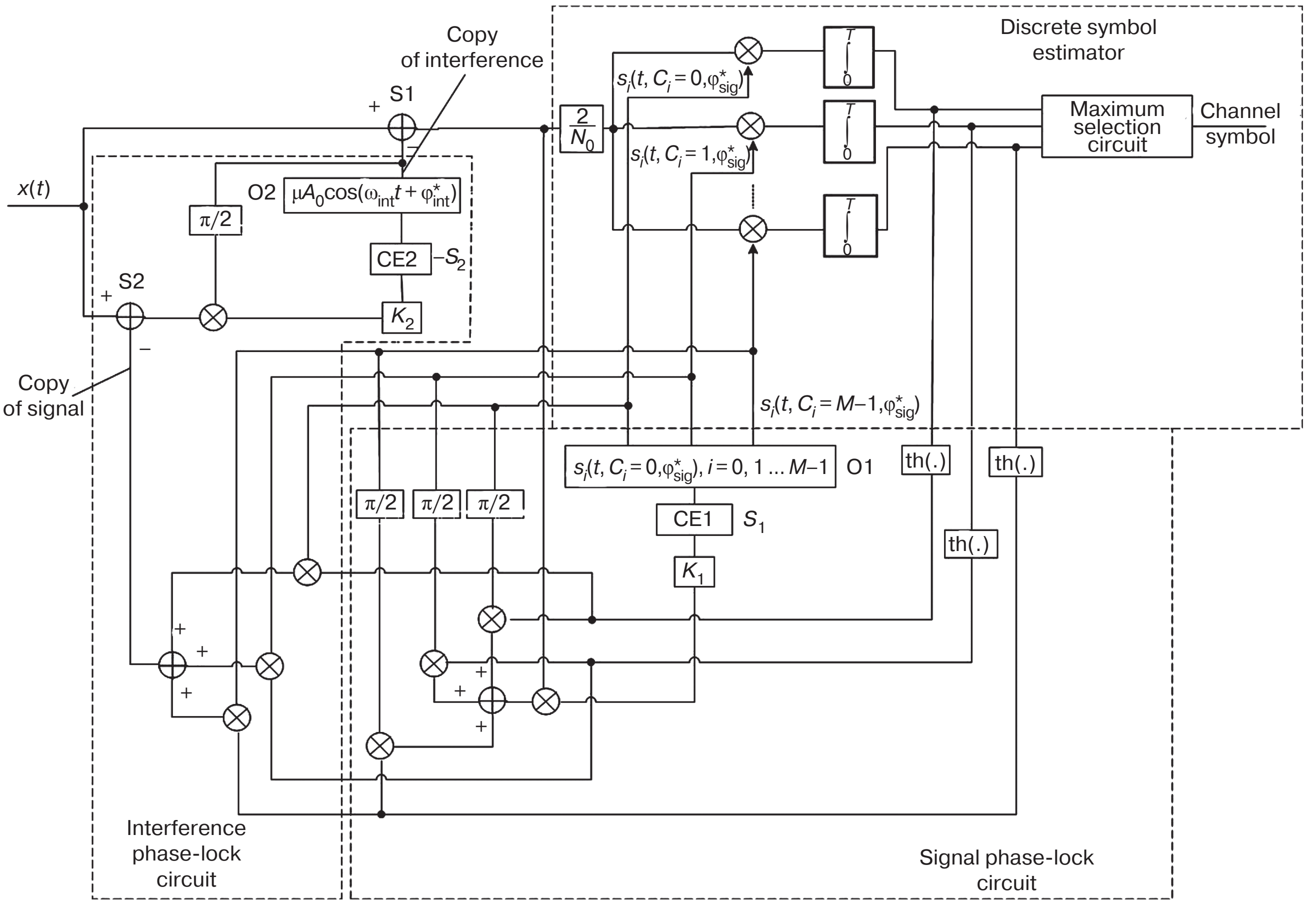

Fig. 1. Circuit diagram of a quasi-coherent receiver of an MPSK signal against a background of a harmonic interference with a random initial phase

published procedure [2]. The symbol and bit error probabilities can be defined as follows:

$$P_{se} = 1 - \prod_{i=1}^{M-1} p(u_i = J_0 - J_i > 0)\big|_0,$$

$$P_{be} = P_{se}/\log_2 M,$$

where $p(u_i = J_0 - J_i > 0)\big|_0 = 1 - \Phi\left(\frac{m_{ui}}{\sqrt{D_{ui}}}\right)$, and $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-t^2/2}dt.$

Using (1), (2), and (9), we obtain

$$m_{ui} = \langle J_0 - J_i \rangle = \frac{2E_s}{N_0}[\cos(\varphi_{sig} - \varphi^*_{sig}) \times$$
$$\times (1 - \cos(i2\pi/M)) - \sin(\varphi_{sig} - \varphi^*_{sig})\sin(i2\pi/M)] +$$
$$+ \mu\frac{2E_s}{N_0}\frac{\sin x}{x}[(\cos\eta_{int} - \cos\eta^*_{int})(1 - \cos(i2\pi/M)) -$$
$$- (\sin\eta_{int} - \sin\eta^*_{int})\sin(i2\pi/M)],$$

$$D_{ui} = \frac{4E_s}{N_0}(1 - \cos(i2\pi/M)),$$

where $x = \Delta\omega_{int}T/2$, $\Delta\omega_{int} = \omega_{int} - \omega_0$, $\eta_{int} = \Delta\omega_{int}T/2 + \varphi_{int} - \varphi^*_{sig}$ and $\eta^*_{int} = \Delta\omega_{int}T/2 + \varphi^*_{int} - \varphi^*_{sig}$.

The parameters m_{ui} and D_{ui} are conditional on the values of φ_{sig}, φ^*_{sig}, φ_{int}, and φ^*_{int}. To obtain the unconditional error probabilities, one should make the corresponding averaging under the assumption that the *a posteriori* probability densities of the random phases φ_{sig} and φ_{int} are Gaussian:

$$w(\varphi_{sig}) = \frac{1}{\sqrt{2\pi \overline{K_{\varphi_{sig}\varphi_{sig}}}}}\exp\left[-\frac{(\varphi_{sig} - \varphi^*_{sig})^2}{2\overline{K_{\varphi_{sig}\varphi_{sig}}}}\right],$$

$$w(\varphi_{int}) = \frac{1}{\sqrt{2\pi \overline{K_{\varphi_{int}\varphi_{int}}}}}\exp\left[-\frac{(\varphi_{int} - \varphi^*_{int})^2}{2\overline{K_{\varphi_{int}\varphi_{int}}}}\right],$$

and that the quantities φ^*_{sig} and φ^*_{int} are uniformly distributed in the range $(0, 2\pi]$. Moreover, in this case, one can use the approximate formula [8]

$$<< p(u_i > 0)\Big|_0 >>_{\varphi_{\text{sig}}\varphi_{\text{int}}} = 1 - \Phi\left(<< \frac{m_{ui}}{\sqrt{D_{ui}}} >>_{\varphi_{\text{sig}}\varphi_{\text{int}}} \right) = $$
$$= 1 - \Phi\left(\frac{<< m_{ui} >>_{\varphi_{\text{sig}}\varphi_{\text{int}}}}{\sqrt{D_{ui}}} \right).$$

Averaging over the parameters φ_{sig} and φ_{int} gives [14, 15]

$$<< m_{ui} >>_{\varphi_{\text{sig}}\varphi_{\text{int}}} =$$
$$= \frac{2E_s}{N_0}[(1 - \cos(i2\pi/M))\exp(-\overline{K_{\varphi_{\text{sig}}\varphi_{\text{sig}}}}/2) +$$
$$+ \mu \frac{\sin x}{x}(\exp(-\overline{K_{\varphi_{\text{int}}\varphi_{\text{int}}}}/2) - 1)((1 - \cos(i2\pi/M)) \times$$
$$\times \cos \eta^*_{\text{int}} - \sin(i2\pi/M)\sin \eta^*_{\text{int}}))].$$

Analytical averaging over the parameter η^*_{int} failed; therefore, this averaging was performed numerically.

Figure 2 illustrates the dependences of the bit error probability P_{be} on the signal-to-noise ratio E_{b}/N_0 for the quasi-coherent receiver of MPSK signals at various M against the background of a harmonic interference with a random initial phase, $\Delta\omega_{\text{int}}$, and a relative intensity of $\mu = 0.5$. Here, the parameters are the *a priori* variances of the progressions of the phases φ_{sig} and φ_{int} in one digit time slot T:

$$\sigma^2_{\varphi_{\text{sig}}} = \frac{N_{\varphi_{\text{sig}}} T}{2}, \ \sigma^2_{\varphi_{\text{int}}} = \frac{N_{\varphi_{\text{int}}} T}{2}.$$

For comparison, Fig. 2 presents curves *1* constructed at $\mu = 0$. The other curves are constructed at the following parameters: (*2*) $\sigma^2_{\varphi_{\text{int}}} = 0.25$ and $\sigma^2_{\varphi_{\text{sig}}} = 0$, (*3*) $\sigma^2_{\varphi_{\text{int}}} = 0$ and $\sigma^2_{\varphi_{\text{sig}}} = 0.25$, and (*4*) $\sigma^2_{\varphi_{\text{int}}} = 0.25$ and $\sigma^2_{\varphi_{\text{sig}}} = 0.25$.

Analysis of the graphs shows the validity of the obtained algorithms. It is seen that the uncompensated fluctuations of the initial phase of the desired signal affect more strongly the interference immunity of the receiver than similar fluctuations of the phase of the harmonic interference. This is particularly clear at small M. This is due to the fact that it is the phase structure of the MPSK signal that contains information on the discrete symbol.

Figure 3 describes the effect of the interference μ on the bit error probability.

In Fig. 3, one of the curves is constructed without using interference compensation circuits, and the two others are built by simultaneously using algorithms (12) and (13). One can see that, in the former case, with increasing μ, the error probability increases significantly (by one to several

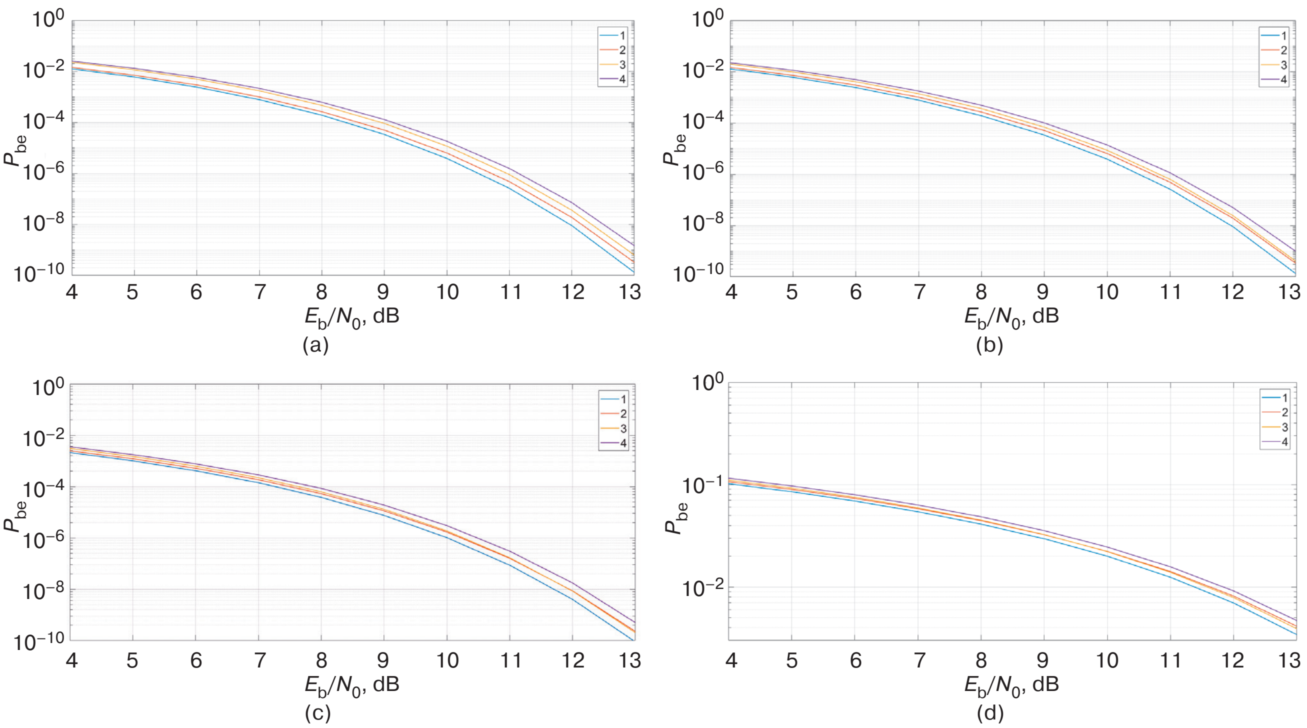

Fig. 2. Dependences of the bit error probability on the signal-to-noise ratio in the reception of MPSK signals against a background of a harmonic interference with a random initial phase at M = (a) 2, (b) 4, (c) 8, and (d) 16

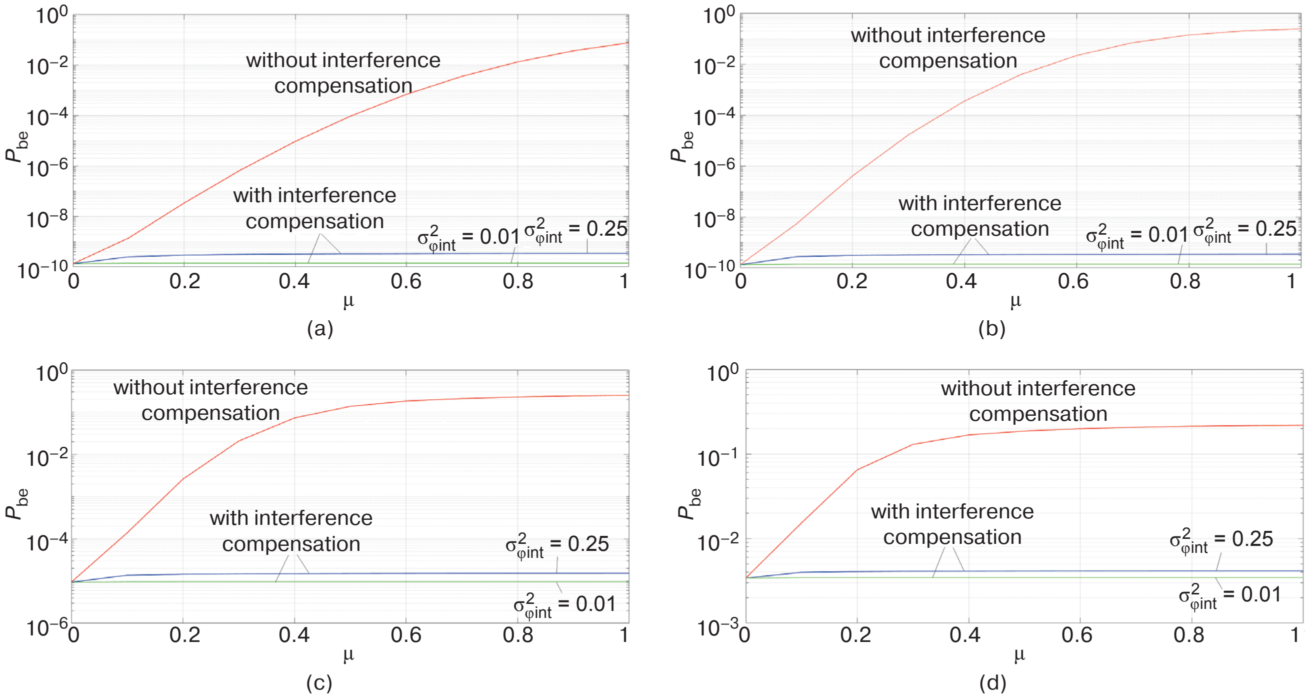

Fig. 3. Dependence of the bit error probability on the relative interference intensity at E_b/N_0 = 13 dB and M = (a) 2, (b) 4, (c) 8, and (d) 16

orders of magnitude), and in the latter cases, the harmonic interference is virtually completely suppressed at any μ.

Comparison of the results of this work with the previously obtained data [2] for the case of the absence of harmonic interference compensation circuits showed that the use of the above-derived phase-filtering algorithms considerably improves the noise immunity of the receiver. For example, at $\mu = 0.5$ and an error probability of $P_{be} = 10^{-2}$, the energy gain at $M = 2$ is about 2.5 dB; at $M = 4$, about 6 dB; and at $M = 8$ and $M = 16$, no less than 10 dB.

CONCLUSIONS

The synthesized algorithm of the optimal nonlinear filtering of MPSK signals against the background of harmonic interference with a random initial phase considerably improves the noise immunity of discrete information reception under complex interference conditions and ensures a significant energy gain of the radio system. At the same time, of note is the significant structural complexity of such a receiver. Furthermore, on the receiver side, the interference frequency and level should be known *a priori*, which is difficult to ensure in practice. Therefore, the determined characteristics should be regarded as potentially achievable, the ones at which the development of interference compensation devices should be targeted.

Authors' contribution. All authors equally contributed to the research work.

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