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RESEARCH ARTICLE

Mathematical modeling of some social processes using game-theoretic approaches and making managerial decisions based on them

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Abstract. In this article, using game-theoretic approaches, the human community is modeled as a dynamic system, and the influence of such ethical norms of behavior as egoism and altruism, morality (on the example of the Kant imperative or the Golden Rule of Morality) on the state of this system is investigated, as well as the question of determining the effectiveness of the community depending on the prevailing worldview of its representatives. Using the example of a game model of social choice between two norms of behavior: one generally accepted, but outdated, and the other newone, not yet widespread, but more advanced and progressive, it is shown that communities, among whose representatives a predominantly egoistic worldview prevails, are less likely to innovate and abandon outdated norms of behavior. Conversely, those communities whose representatives share basic ethical principles are more confident and quickly moving to advanced and progressive norms. In conclusion, the paper examines the question of what advantages a community acquires in which purposeful educational and educational activities are conducted, designed to increase the level of morality and morality among its representatives. The results obtained can be used, firstly, as an integral part of the course on the mathematical base of ethics, which could perform the functions of educational work in higher and secondary educational institutions, and, secondly, for the purposes of evaluating the effectiveness of educational work and state planning in this area.

Keywords: game theory, conflict equilibria, behavioral economics

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НАУЧНАЯ СТАТЬЯ

Математическое моделирование некоторых социальных процессов с помощью теоретико-игровых подходов и принятие на их основе управленческих решений

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Резюме. В статье с помощью теоретико-игровых подходов моделируется человеческое сообщество как динамическая система и исследуется, какое влияние оказывают на состояние этой системы такие этические нормы поведения, как эгоизм и альтруизм, мораль (на примере императива Канта или Золотого правила нравственности), а также изучается вопрос определения эффективности сообщества в зависимости от превалирующего среди его представителей мировоззрения. На примере игровой модели социального выбора между двумя нормами поведения: одной общепринятой, но устаревшей, и другой новой, еще не распространенной, но более передовой и прогрессивной, показывается, что сообщества, среди представителей которых преобладает преимущественно эгоистическое мировоззрение, менее склонны к инновациям и отказу от устаревших норм поведения. И наоборот: те сообщества, представители которых разделяют базовые этические принципы, увереннее и быстрее переходят к более передовым и благоприятным для сообщества в целом поведенческим нормам. В заключении работы с помощью модели пороговых значений, определяющих коллективный выбор, исследуется вопрос, какие преимущества приобретает сообщество, в котором ведется целенаправленная воспитательная, просветительская деятельность, призванная повысить уровень морали и нравственности среди его представителей. Полученные результаты могут быть использованы, во-первых, в качестве составной части курса по математическим основам этики, который мог бы исполнять функции воспитательной работы в высших и средних учебных заведениях, а, во-вторых, для целей оценки эффективности проводимой воспитательной работы и государственного планирования в сферах воспитания и образования.

Ключевые слова: теория игр, конфликтные равновесия, моделирование социально-этических норм поведения

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INTRODUCTION

What is the world's driver? "*The world is set in motion by ideas, and the ideas are realized in the world through people*." Since the human community is an example of a dynamic system, its state is determined in each moment in time by some internal and external parameters. For example, while the state of vapour inside a steam engine is determined by pressure and temperature, the state of a human community is determined by dominating in the given community cultural and worldview principles.

These worldview principles are embedded in every member of a community mainly by education and upbringing. Therefore, to establish the state policy in the area of education and culture it is important to investigate and analyze the influence of moral and ethical values on the development of the community.

On September 1, 2020, the State Duma of the Russian Federation adopted amendments of law "On Education in Russian Federation" recommended by the President of the Russian Federation. As a result, the law has determined the concept of youth development as the activity targeted "to the development of an individual, creation of conditions for self-determination and socialization on the basis of sociocultural, moralethical values, and also "*the formation of patriotic feelings and education for citizenship, respectfulness to the memory of Motherland's defenders… to the law and order, to working and older people, mutual respectfulness, careful relation to cultural heritage and traditions of multinational people of Russian Federation, and to the nature*…" [1].

The law also requires middle, middle professional institutions and higher schools to make corresponding changes in their programs on youth development and related educational work during the period of one year (before September 2021).

The following questions arise, however:

- What specifically are these values that should be cultivated in the young generation?
- In what way will these values affect the development of the community as a whole at the time when students—having reached the age of maturity—become fully responsible members of the community?
- And most importantly, how we should evaluate the efficacy youth development that is conducted?

Philosophy and psychology are not the only two domains that can help to get answers to these and other arising questions. It may be surprising, but mathematics, particularly one of its applied branches—the game theory—can do the same.

In this paper, using approaches of the game theory we model the influence of such behavioural norms as egoism and altruism, morality (we understand the latter in terms of Kant Imperative and closely related to it the Golden Rule of Morality) on the process of making decisions by individuals in some human community. We develop a game model of choice between two norms of behaviour: the first of which is generally accepted but less effective, and the second one—new, poorly known but more favourable for the community as a whole once it spreads throughout the community. This model rather indicatively illustrates how dominating among members of a community moral and ethical norms can lead the community either to progress and wellbeing, or, in contrast, to disintegration and degradation.

In Conclusions, an attempt is made to model how youth development and educational work affects the process of making decisions by members of the community that results in a growth (according to a certain law) of the moral level of the community.

Let us briefly overview the results obtained in this area of research by Russian and foreign thinkers.

OVERVIEW OF MODELS OF SOCIOETHICAL BEHAVIOUR BASED ON THEORETICAL APPROACHES OF THE GAME THEORY

Since the time of Adam Smith [2], the founder of economics theory, it has been generally accepted that first and foremost it is an individualistic motif of personal well-being maximisation that is a driving force. Even the term *homo economicus* had appeared—the rational human.

However, even Adam Smith himself had doubted this. For example, in his work "The theory of moral senses" [3] he introduces the notion "sympathy"—the sense which is the attribute of people forcing them to behave sometimes exclusively against their interests.

In the 20th century there appeared a new area of research (*behavioral economics*) that studies the impact of psychological, moral and ethical, cognitive and cultural factors on making a decision. This analysis is highly demanded because it more realistically—than the generally accepted classical yet rather inaccurate *homo economicus* model—takes into account all aspects which affect making a decision by a human.

Because one of the branches of mathematics used for analysis of processes in economics is the game theory, a lot was devoted to model the processes and phenomena, which until recently have been the subject of sociology, philosophy, and psychology.

One of the first attempts to model moral-ethical behavioral norms employing a game theory approach was undertaken by prof. Braithwaite in his lecture in Cambridge in 1955 [4]; ever since similar studies have been conducted on regular basis by different authors.

For example, in his work "Models of Game Theory and Making Decisions in Ethics" [5] Nobel Prize winner J. Harsanyi argues that ethical (moral) behavior is based on the notion of collective rationale that goes beyond the frameworks of a traditional for the game theory concept of maximization of individual or cooperative income: "*The theory of rational behavior in social medium can be divided into game theory and ethics. Game theory applies to two or more individuals, who often have different interests, and who attempt to maximize their (selfish or selfless) interests in a rational way against all other individuals, who also attempt to maximize their (selfish or selfless) interests*" [5].

Harsanyi, in his work "Utilitarianism of rules and the theory of making decisions" [6], applies a fundamental concept of utilitarianism for the creation of a more realistic model of making decisions by individuals in a community. Utilitarianism is the branch of ethics, according to which moral and ethical values of any act are determined by combined utility brought by this act to all individuals for who this act has the influence [7]. In this respect, Harsanyi introduces a function of a social utility, which value for each participant in every point (of each behavioral strategy) is determined by the average value of all participants: $W_i(s) = \frac{1}{N}\sum_{i=1}^{N} U_i(s)$ [5]. The theory of utility is discussed in more detail in [8].

This approach was significantly developed by many experts on behavioral economics and game theory [9–11].

We have to draw attention to the so-called evolutionary game theory, which is the game theory application for the investigation of the development of populations in biology as well as sociology. The feature of this theory is that it analyses, as a rule, repeatable games; therefore, each strategy is evaluated on the basis of whether it is evolutionary stable that is capable of being verified by the time. For example, if applied to biology, different strategies represent genetic traits—inherited by descendants—which determine the behavior of species. Based on evolutionary game theory it was possible to justify—often observable in nature, particularly for social species,—"gentlemen's" and even altruistic behavior that is a behavior for the benefit of species. This in no way agrees with Darwinian assumption that natural selection happens at individual level [12, 13].

With regard to researches conducted by Russian scientists, we refer to work Yu.B. Germeier and I.A. Vatel' "Games with hierarchical vector of interests" [14]. In this work the authors to analyze a problem of distribution of resources between individual and societal needs introduce a notion of "egoism" in relation to the needs of the given community for the case when a participant prefers to spend all means at his possession exclusively for personal objectives ignoring societal interests.

Some ideas proposed by Germeier and Vatel' laid the foundation for a model of compliance of communal and private interests (CCPI-model) [15, 16]. In this model a two-level community is discussed, and similar to [14] the problem of resource distribution between private and community needs is investigated. In [16] participants are divided into two classes depending on whether they prefer to spend resources for personal or communal objectives; these are *individualists* and *collectivists*.

In 2017 in game theory-oriented journal "Games" (Basel, Switzerland) a special edition was issued under the title "Ethics, morality and game theory" [10]. In this edition, a collection of articles of different authors were presented; these articles covered the problem of modeling moral-ethical norms of behavior and their impact on decision making by participants of the game problem.

In "Behavioral strategy of moralists and altruists" [11] in addition to already mentioned types of behavior, based on individualism and collectivism, a third class of participants is introduced. These participants when choosing their own strategy follow Kant Imperative, according to which "*a human has to strive so that his or her goal is to become a part of general law*" [7] or a Golden rule of morale: "*treat people the way you would like they treat you*" [17]. The essence of such behavior if applied to a game theory model means that before choosing a strategy, every participant assumes that with a certain probability all of the participants would choose the same strategy. Therefore, it is the assumption that must be taken into account when making a decision and acting.

Analogously to *homo economicus—rational human* (to name the first class of *participants–individualists*, who are guided exclusively by achieving the maximum of their personal income), the participants of the third class are named in [11] *homo-moralis—moral human*.

This type of behavior can relatively successfully be used to model a dynamic model of social choice between two norms of behavior: the first one which is traditional but less favorable and effective and the second one which is not applied yet by most participants. However, employing the new norm by the vast majority of members of the community under consideration would enable the community as a whole to attain much better results. It is shown that specifically the participants of *homo-moralis* class are able to a certain degree to serve as an example of how to employ the new behavioral norm even though being initially a minority and losers, and thereby leading the community to a fundamentally new qualitative level.

According to [11], since the transit to the new norm may not occur under natural conditions, an educational model is also considered. This model supposes that the level of morale and "consciousness" in the community as a result of some educational activities is enhanced in accordance with a certain law. As a result, a greater number of individuals accept new behavioral norms; the latter is becoming generally accepted by the community and is leading to undoubted progress.

THE MODEL

In this paper, a gaming model with N participants is considered. It is supposed that all participants choose their own strategies from the same set of permissible strategies.

Assumption 1. Let Q be a metric space, G—a compact set:

$$G \overset{\Delta}{=} Q^N = \underbrace{Q \times \ldots \times Q}_{N}.$$

Let continuous functions (a functional) $J_i(q), i = \overline{1,N}, q = (q_1,\ldots,q_N) \in G$ are determined in set G, where q_i is the strategy of the ith player, $q_i \in Q, q^i \overset{\Delta}{=} (q_1,\ldots,q_{i-1},q_{i+1},\ldots,q_N)$ are the strategies of the rest $N-1$ players with a fixed strategy, q_i of the ith player, $q^i \in Q^{N-1}$. $J_i(q)$ is the *payoff function* (*functional*) of player i, which determines the size of some benefit or resource gained by the ith participant when choosing strategy q_i while the rest participants choose strategy q^i. Under these conditions $J_i(q), i = \overline{1,N}$ are supposed to be transferable, which means that they can be split and distributed in any way between the players.

Let $J(q) \overset{\Delta}{=} \sum_{k=1}^{N} J_k(q)$ is the total payoff function of all players, $J^i(q) \overset{\Delta}{=} \sum_{k \neq i} J_k(q)$ is the total payoff function of all players but the ith player.

Definition 1. *We will call a game problem for which Assumption 1 is valid a classic game (or Game)* G^{he} *if each of the players when choosing strategy* $q_i \in Q$, *aims to get maximal payoff function* $J_i(q_i, q^i)$.

This a classic problem statement in game theory that models the behavior based on getting exclusively personal benefits. In order to stress that every player maximizes only own payoff function and distinguish it from the model as determined below, we will also call it the model of *participants–individualists* or the *homo economicus* model as it is called in [11].

Alternatively, a class of game problem is considered in which every player supposedly takes into account (with some weighting coefficient) interests of other participants. This statement is modeled by a transition from initially stated problem with a set of payoff functions $\{J_i, i = \overline{1,N}\} = \{J_i\}$, to an auxiliary problem determined by a parametric family of *utility functions* $\{U_i(J_k, \alpha)\} = \{U_i\}$.

Definition 2. *We will call a game problem, satisfying Assumption 1, game* G^{α} *if every player aims to realize maximum of his utility function* U_i, *which is expressed through a payoff function of the given player* $J_i(q)$ *and a total payoff function of the rest players* $J^i(q)$, *as follows:*

$$U_i(q) = (1-\alpha)J_i(q) + \frac{\alpha}{N-1}J^i(q),$$
$$q \in G, \alpha \in \mathbb{R}, \alpha \in \left[0, \frac{N-1}{N}\right], i = \overline{1,N}. \tag{1}$$

Let us use substitution $\beta \overset{\Delta}{=} \alpha\frac{N}{N-1}$. As $\alpha \in \left[0, \frac{N-1}{N}\right]$, $\beta \in [0,1]$, and utility function $U_i(q)$ can be written in the following form:

$$U_i(q) = (1-\beta)J_i(q) + \frac{\beta}{N}J(q), \beta \in [0,1]. \tag{2}$$

The model, written in this form, determined by utility functions (2), can be considered as a public goods game, in which functions $\beta J_i(q)$ determine a contribution of the ith participant to some community needs. Term $(1-\beta)J_i(q)$ determines a part of resources, which a participant holds for his own needs, while sum $\frac{\beta}{N}J(q)$ determines what he gets from the community.

Unlike the first model (of *participants–individualists*) the model given by Definition 2, supposes that there is no direct antagonism between participants, and even the interest of other participants is taken to some degree into account that follows from the form of function (1). Therefore, this model can be called a model of *participants–collectivists.* Note that in a number of publications (for example, in [2, 4, 6]) similar models yet with somewhat different forms of utility functions U_i called the models of *participants–altruists.*

A number of works accomplished by the author are devoted to this model. For example, in [18] it is shown that in the class of *participants–collectivists* G^{α} under a certain degree of cooperation between participants that is modeled by parameter α, total payoff function J becomes the strongest game equilibrium.

When considering the third model, which is predominantly discussed in this paper, note that there is something common in behavioral and decision making patterns, given by Definitions 1 and 2. Both *individualists* and *collectivists* (or *altruists* as they are called in a number of papers) do not care to some degree of means: if the former pursue exclusively a personal interest, the latter with some weighting coefficient care about community's good. Because according to Assumption 1 all participants can use the same set of possible strategies (actions) Q, the players from both classes when choosing a strategy do not take into account what may happen if the rest participants choose the same strategy. However, the participants of the third class—*homo moralis*—as called in [11, 19], do analyze what may happen.

In the basis of the behavioral pattern that corresponds to this class lies a well-known ethical principle—Kant's categorical imperative: "Act so that maxima of your will could be a universal law [20]." A close in its sense principle is known in ethics under the name "Golden rule of morality:" "Treat others as you want to be treated by others" [21].

In [11, 19] this principle is suggested to be modeled in the following way. Let the ith participant supposes that every other player with probability $k_i \in [0,1]$ will choose the same strategy as he does, and with probability $(1 - k_i)$, it is a different strategy. Thus, every player, when choosing strategy $q_i \in Q$, gets known from the probability theory Bernoulli scheme of $N - 1$ trials (corresponding to the rest players). The scheme has two outcomes for the jth trial, $j = \overline{1, N-1}$: jth participant has chosen strategy $q_j = q_i$ or strategy $(q_j \neq q_i)$. Under these assumptions, instead of initial payoff functions, the game is conducted on utility functions, which for every participant represents a mathematical expectation described by binomial distribution.

Definition 3. *We will to call a game problem, which meets Assumption 1, game* G^{hm} *(the game in homo moralis class), every player instead of his initial payoff function* J_i *pursues maximum of utility function* W_i*, defined as mathematical expectation of random value* $J_i(q_i, \tilde{q}^i)$:

$$W_i(q_i, q^i) = \mathbb{E}_{k_i}[J_i(q_i, \tilde{q}^i)],$$
$$q_i \in Q, k_i \in \mathbb{R}, k_i \in [0,1], i = \overline{1, N}, \qquad (3)$$

where $\tilde{q}^i$ is a random $(N - 1)$-dimensional vector with values taken from Q^{N-1}, having the following distribution: exactly $m \in \{0, \ldots, N-1\}$ number of its component with probability $k_i^m (1 - k_i)^{N-m-1}$ gets value equal to q_i, the rest components keeping their initial values.

Note that for each m there are $\binom{N-1}{m} \overset{\Delta}{=} C_{N-1}^m$ ways to choose m out of $(N - 1)$ components of q^i.

We also note that for $k_i = 0$ only one random vector gets a value with a non-zero (equal to unity—that is total) probability. It means that a random vector gets the only value, namely the one which corresponds to the argument of function W_i. In this case $W_i(q_i, q^i) \equiv J_i(q_i, q^i)$, that is the players from class *homo moralis* with coefficients $k_i = 0$ are actually the *participants–individualists* of the first class G^{he}. It will be demonstrated more clearly in the model of social choice to be considered below.

For example, for a game with three participants the utility function (3) has the form:

$$W_i(q_i, q_j, q_k) = (1 - k_i)^2 J_i(q_i, q_j, q_k) + k_i(1 - k_i) \times$$
$$\times J_i(q_i, q_i, q_k) + k_i(1 - k_i) J_i(q_i, q_j, q_i) + k_i^2 J_i(q_i, q_i, q_i).$$

SOCIAL MODEL OF CHOICE BETWEEN TWO BEHAVIORAL NORMS

Let us illustrate the difference introduced in the previous section between three behavioral patterns using an example of a coordination game. A coordination game implies a class of game problems with pure strategies, in which participants obtain substantially higher gain if they choose equal or corresponding to each other strategy than if they choose different strategies. This class of game problems models life situations for which some new, progressive behavioral norms if employed by few do not have significant influence on community life. Let us assume that separate garbage collection by a small group of enthusiasts does not have a noticeable ecological effect on the environment in the region. However, when such a behavioral strategy becomes a norm and employed by a majority of community members, this kind of garbage utilization can substantially reduce pollution of the environment. If we consider this situation as a game model, then the two strategies emerge before every community member: to act in the old way or to use new behavioral models.

As an example of a coordination game with two participants, let us consider the following problem taken from the paper of Edna Ullmann-Margalit "The Emergence of Norms" [22]. Let two gunners in the course of a battle have to choose whether to run away from the enemy or stay and continue to fight. Their gun is in a strategically important mountain pass. If they both stay, the enemy may take the pass, overtake them and take them. If one of the gunners stays but the second one runs away, the brave gunner will be killed and his partner, the aimer, will have enough time to escape for his good. Supposing that the both will attempt to survive, both soldiers have reasons to run away. So each of them has a choice: to run or stay and fight.

Table 1. Payoff matrix of the problem

	Fight	Run
Fight	(2, 2)	(0, 3)
Run	(3, 0)	(1, 1)

Note that coordination games have a lot of applications in economics, described in [5].

Let us now consider a coordination game, representing a model of social choice in the problem of many participants, described in [2]. Let N participants of some community make independently from each other a choice between two behavioral norms (strategies) A and B; norm A being more effective than norm B in the sense that if all individuals make a transit to norm A, the well-being (in a broad sense) of each participant will be higher than in the case when all participants choose norm B. However, norm B is commonly adopted, which is why at the beginning of the social model all participants choose norm B, while A is a new norm for them.

For example, we often see that young people—when in the process of socialization find them in new social groups (classmates, friends, etc.)—takes over from some members of these groups habits which are not always useful. But sometimes we have opposite examples. Suppose a group of acquainted individuals dependent on a harmful habit. If somebody from this group has managed to get rid of this habit, he or she initially experiences discomfort since he or she becomes kind of "a white crow." However, gradually other members of the group begin to follow the example of that individual, and starting from some critical fraction of those who got rid off, others who are still subjected to the harmful habit feel "disapproval." Gradually, the community as a whole begins to change relation to this harmful habit: banning advertisements in mass media is introduced; selling to youngsters is also being banned, etc. This change of attitude in a community and increase in restrictions makes the lives of followers of harmful habits more and more difficult as long as a healthy way of life becomes a norm. This, in its turn, results in the decrease in occurrence of various illnesses, births of healthier children, and strengthening the gene pool. In other words, the transition to a new norm of behavior has a rather positive effect on the development of the community as a whole. A lot of other similar examples can be given to support the above said

Let us clarify under what conditions a community is able to have a transit from less effective old norm B to more effective new norm A. In order to make this transition we will formulate the described model in terms of a game problem. First, let us consider a steady-state case, and then study dynamic behavior of the model.

Let $q_i \in Q = \{0,1\}$ is the choice of the ith participant, where q_i means that norm A is chosen, and if $q_i = 0$, then norm B is chosen. If the ith participant chooses norm A and other n_A participants also choose this norm, then the payoff function of the ith participant takes the value of $a \cdot n_A$. On the other hand, if a participant chooses B and n_B other participants acts the same way, then the value of his utility function equals $b \cdot n_B$. We will suppose that $0 < b < a$.

In the model of *participants-individualist*s G^{he}, the payoff functions have the following form:

$$J_i(q_i, q^i) = aq_i \sum_{\substack{j=1,\\ j\neq i}}^{N} q_j + \\ + b(1-q_i)\sum_{\substack{j=1,\\ j\neq i}}^{N}(1-q_j), q_i \in Q, q^i \in Q^{N-1}. \quad (4)$$

For the model of *participants–collectivists* G^{α} , the payoff function takes the form:

$$U_i(q_i, q^i) = (1-\alpha)J_i(q_i, q^i) + \\ + \frac{\alpha}{N-1}\sum_{\substack{k=1,\\ k\neq i}}^{N} J_k(q_k, q^k), q_i \in Q, q^i \in Q^{N-1}, \quad (5)$$

where J_1 and J_2 are determined by formula (4), $\alpha \in \left[0, \frac{N-1}{N}\right]$ is the parameter that determines to what degree each individual prefers community interests. For $\alpha = 0$ functions (4) and (5) are equivalent to each other: $J_i \equiv U_i$. It is not difficult to realize that both for the players of the first and the second classes independently of the value of coefficient α the problem has (according to Nash) two situations with equal weights—either all participants choose norm A: $q = (1,\ldots,1)$, or B: $q = (0,\ldots,0)$.

Thus, if norm B is considered as generally accepted and each player supposes that the rest will do choose this norm, while the number of players is high enough and a direct cooperation between them is impossible, then in the case of participants pursuing exclusively personal interests, norm B remains to be an equilibrium since by acting alone in choosing A the player would get nothing.

Similar situation is in players' class G^{α} that takes into account interests of other participants. Even at low values of coefficient α, when $U_i(q) = \frac{1}{N}J(q) = \frac{1}{N}\sum_{k=1}^{N} J_k$, that means that the utility function—which is maximized by every player—is directly proportional to a total payoff function, neither of players wish to step away from less effective norm B since the community as a whole will get less if a participant makes transition to norm A.

However, the situation changes radically for the players of the third class (*homo moralis*). Utility functions, the maximums of which the players of this class wish to attain, according to Definition 3 have the form of mathematical expectation: $W_i(q) = \mathbb{E}_{k_i}[J_i(q_i, \tilde{q}^i)]$, where $\tilde{q}^i$ is a random vector with such distribution that with probability $\tilde{q}^i$ $k_i^m(1-k_i)^{N-m-1}$ exactly its $m \in \{0,\ldots,N-1\}$ components takes the value equal to q_i, other components retaining their initial values. This distribution looks like the well known from the probability theory binomial distribution, $B_{k_i}^{N-1}$, however the condition applied to the latter is different: namely, $(N-m-1)$ of its component must have their initial values (that is values in point $q \in G$, in which the values of function $W_i(q)$ are determined) unchanged.

Thus, the values of $W_i(q_i, q^i)$ are determined by the expression:

$$W_i(q_i,q^i) = \sum_{m=0}^{N-1}\binom{N-1}{m}k_i^m(1-k_i)^{N-m-1} \times \Big[\underbrace{aq_i \cdot \Big(mq_i + \frac{N-1-m}{N-1}\cdot\sum_{\substack{j=1,\\ j\neq i}}^{N} q_j\Big)}_{I} + \underbrace{b(1-q_i)\Big(m(1-q_j) + \frac{N-1-m}{N-1}\cdot\sum_{\substack{j=1,\\ j\neq i}}^{N}(1-q_j)\Big)}_{II}\Big], \quad (6)$$

where term I corresponds to the case when $q_i = 1$, and term II—when $q_i = 0$. The term with coefficient $\frac{N-1-m}{N-1}$ reflects the situation that the rest players except for those m players, whose strategies are considered equal to q, keep their strategies unchanged.

Once again formula (6) clearly shows a feature that we already mentioned: for $k_i = 0$, $W_i(q_i,q^i) \equiv J_I(q_i,q^i)$, i.e., participants belonging to *homo moralis* with nonzero level of coefficient k_i become *players–individualists*.

If all players choose the strategy A, then the ith participant also gets $(N-1)a$ by choosing A, but if he decides to choose B, his utility function equals to:

$$W_i(0,q^i = (1,\ldots,1)) = b\sum_{m=0}^{N-1}\binom{N-1}{m}k_i^m(1-k_i)^{N-m-1}m. \quad (7)$$

Let us simplify expression (7). Because for $m = 0$ the corresponding term of the series also equals 0, the summation can be performed starting with $m = 1$. As

$$\binom{N-1}{m}m = \frac{(N-1)!}{m!(N-1-m)!}m = \frac{(N-1)(N-2)!}{(m-1)!(N-2-(m-1))!} = (N-1)\binom{N-2}{m-1},$$

expression (7) can be rewritten in the following form:

$$W_i(0,q^i = (1,\ldots,1)) = b(N-1)\sum_{m=1}^{N-1}\binom{N-2}{m-1}k_i k_i^{m-1}(1-k_i)^{N-2-(m-1)} =$$

$$= \{\text{Substitution: } m-1 \overset{\Delta}{=} l\} = b(N-1)k_i\sum_{l=0}^{N-2}\binom{N-2}{l}k_i^l(1-k_i)^{N-2-l} =$$

$$= \{\text{Newton's Binomial Formula}\} = b(N-1)k_i(k_i + (1-k_i))^{N-2} = b(N-1)k_i. \quad (8)$$

If all players choose B, then acting as everyone the ith participant gets $W_i(0,\ldots,0) = (N-1)b$, but when choosing A alone, he gets

$$W_i(1,q^i = (0,\ldots,0)) = a\sum_{m=0}^{N-1}\binom{N-1}{m}k_i^m(1-k_i)^{N-m-1}m = a(N-1)k_i. \quad (9)$$

Thus, for $k_i > \frac{b}{a}$ it happens that $W_i(1,q^i = (0,\ldots,0)) > W_i(0,\ldots,0)$, that is a player with high enough level of coefficient k_i is ready to make a transition to the more effective norm A even if he is alone. It is worth to note that in a homogeneous community, in which all participants have the same level of the coefficient $k_i > \frac{b}{a}$, situation $q = (1,\ldots,1)$, meaning that all participants choose A, appears the only equilibrium as defined by Nash.

A more realistic scenario, however, is the so-called heterogeneous case, when coefficients k_i of all members of the community in question can be different.

THRESHOLD VALUES IN THE MODEL OF HETEROGENEOUS COMMUNITIES

Let us introduce a concept of a *threshold value* to study such heterogeneous communities. Under the threshold value θ_i of the ith participant we will imply the least fraction (of the total number of other participants of the community that have made a transition to norm A) required that the ith participant would have also made a choice in favor of norm A. For example, the ith participant makes a transition to the norm A if he believes that it will be chosen by half of the community, and the jth participant makes so if one third of the community chooses norm *A*. In this case $\theta_i = \frac{1}{2}$, and $\theta_i = \frac{1}{2}$,

We can determine a threshold value $i \in \{1, N\}$ for each number on the basis of the following reasoning. Let the *i*th participant supposes that $\tilde{n} \in \{0, \dots, N-1\}$ other participants will choose norm *A*. Then, the participant's utility function for the case of choosing *B* will take the form:

$$W_i(0, q^i) = b\sum_{m=0}^{N-1}\binom{N-1}{m}k_i^m \times$$
$$\times (1-k_i)^{N-1-m}[\frac{N-1-m}{N-1}(N-\tilde{n}-1)+m] =$$
$$= \overbrace{b\sum_{m=0}^{N-1}\frac{(N-1)!}{m!(N-1-m)!}\cdot\frac{N-1-m}{N-1}k_i^m(1-k_i)^{N-1-m}(N-\tilde{n}-1)}^{I} +$$
$$+ \underbrace{b\sum_{m=0}^{N-1}\binom{N-1}{m}k_i^m(1-k_i)^{N-1-m}m}_{II}.$$

For $m = N - 1$, the corresponding term of series *I* equals 0, therefore the upper limit of the summation can be substituted with $m = N - 2$. According to formula (8) term *II* equals $b(N-1)k_i$, therefore

$$W_i(0, q^i) = b\overbrace{\sum_{m=0}^{N-2}\binom{N-2}{m}k_i^m(1-k_i)^{N-2-m}}^{I} \times$$
$$\times (1-k_i)(N-\tilde{n}-1) + b(N-1)k_i =$$
$$= \{\text{Newton's Binomial Formula applied to } I\} =$$
$$= b\cdot[(1-k_i)(N-\tilde{n}-1)+(N-1)k_i] =$$
$$= b\cdot[(N-\tilde{n}-1)+\tilde{n}k_i].$$

If under the same conditions the *i*th participant chooses norm *A*, he gets

$$W_i(1, q^i) = b\sum_{m=0}^{N-1}\binom{N-1}{m}k_i^m \times$$
$$\times (1-k_i)^{N-1-m}[\frac{N-1-m}{N-1}\cdot\tilde{n}+m] =$$
$$= a\cdot[(1-k_i)\tilde{n}+(N-1)k_i] = a\cdot[\tilde{n}+(N-\tilde{n}-1)k_i].$$

Thus, the *i*th participant will make a choice in favour of norm *A* if $W_i(1, q^i) > W_i(0, q^i)$: $a\cdot[\tilde{n}+(N-\tilde{n}-1)k_i] \geq b\cdot[(N-\tilde{n}-1)+\tilde{n}k_i]$. This condition is equivalent to the following one:

$$\frac{\tilde{n}}{N-1} \geq \frac{b-k_i a}{(a+b)(1-k_i)} \overset{\Delta}{=} \theta_i, \qquad (10)$$

where θ_i is the threshold value, i.e., the minimal fraction of participants who have chosen norm *A*, at which the *i*th participant is also ready to make a choice in favor of norm *A*. Note that when $k_i > \frac{b}{a}$, the threshold value θ_i is negative; this can be interpreted so that for a sufficiently large value of coefficient k_i (determining the level of morality as interpreted in [2]), the *i*th participant is ready to make a transition to a new norm even if he is alone in this decision.

Also note that players with the lowest acceptable level of coefficient $k_i = 0$, the threshold value $\theta_i = \frac{b}{a+b}$. It means that if the fraction of community members who have made a transition to norm *A* exceeds this level, even *players–individualists* make a transition to norm *A*.

To model inhomogeneity of a community relatively a coefficient k_i and correspondingly the threshold value θ_i of each individual, let us consider a distribution function $F(x): \mathbb{R} \to [0,1]$, its values equal to the fraction of the total number of the community members whose threshold value θ_i does not exceed *x*.

If the threshold value θ of some member of the community consider as a random value, that takes some value within interval $\left(-\infty, \frac{b}{a+b}\right]$, then we can consider the function *F*(*x*) as a function of the distribution of the given random value: $F(x) = \mathbf{P}(\theta < x)$, where **P** is a corresponding probability equal to a fraction of the total number of those community members whose threshold value does not exceed *x*.

To find numerical parameters of the distribution of a threshold value at which members of the community are ready to make a transition to a new behavioral norm, we can rely on a special area of statistical research called moral statistics.

Moral statistics covers a broad area of problems related to negative phenomena in a society, such as different kinds of criminality as well as violation of social order and violation of moral-ethical norms. Positive phenomena that characterize morality of the population are also studied by moral statistics; participation of citizens in public organizations on preserving the environment, free donation, participation in rescue services, etc. [23].

For example, if we assume that there is free blood donation at some enterprise or university, then each employee, the enterprise or student has two strategies: to participate in donation (norm *A*), or not (norm *B*). Since it is difficult to formalize the sense of moral satisfaction experienced by a person participating in these activities, finding numerical values of

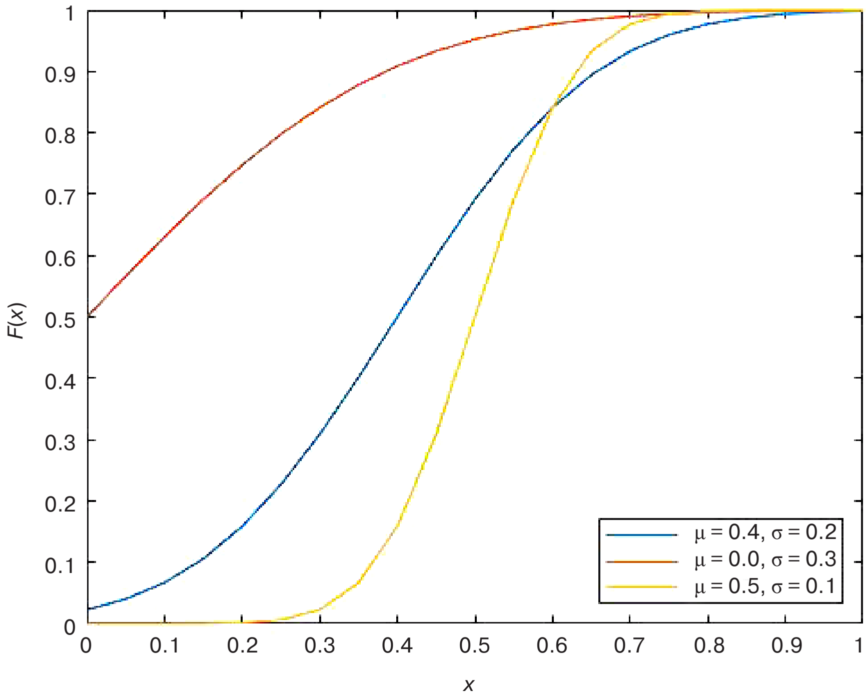

Fig. 1. Distribution function $F(x)$ for threshold value θ_i

coefficients a and b is impossible. However, threshold values, corresponding to the transition from A to B can be found numerically.

To make this, a sociological study among those who came up to donate can be conducted in order to clarify and evaluate the number of their acquaintances who had participated in the donation of blood before they decided to do the same. This will enable us to determine a threshold value for each participant.

Of course, the particular form of a distribution function will be different for each problem. However, because the considered social model is supposedly rely on high enough number of participants, we can take Gaussian function of normal distribution with mathematical expectation μ and dispersion σ^2, where μ and σ are the parameters characterizing a community:

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-(u-\mu)^2/(2\sigma^2)} du.$$

In the above example of blood donation, the average threshold values for all interviewed participants enable to determine mathematical expectation, while mean square deviation determines the dispersion.

The graphs of distribution functions for different parameters μ and σ are shown in Fig. 2. Note that $F(x) = 1$ for $x \geq \frac{b}{a+b}$.

Here, the normal distribution serves as an approach, since in the real world when analyzing social processes, we should take into account a human factor; this is because humans are capable of self-organizing and because they have memory.

A number of contemporary authors (D.O. Zhukov, T.Yu. Khvatova, and others [26, 27]), have researched stochastic dynamics in the social systems based on a cellular automaton; a memory system of participants is taken into account. The memory system is the dependence of a state, in which each participant is present, on the same state in previous moments of time. This model enables via giving initial parameters of a system (for example, the number of contacts between community members) to find a distribution function of threshold values required for the community as a whole to make a transition from one state to another.

Let us analyze the dynamics of a transition between norms A and B.

THE DYNAMICS OF A SOCIAL MODEL

We will analyze the dynamics of a transition of the community members in a certain time interval $[t_0, T]$. We begin our analysis from a model with a discrete time increment Δt, and then will make Δt approaching to zero. Let $N_A(t)$ is the number of community members who choose norm A at time t. We have the following condition: $N_A(t_0) = 0$. Then $\frac{N_A(t)}{N-1}$ gives the fraction of participants making a transition to A at time t. According to the definition of $F(x)$, $F\left(\frac{N_A(t)}{N-1}\right)$ is the fraction of the total number of individuals, whose the threshold value does not exceed $\frac{N_A(t)}{N-1}$. Therefore, the number of individuals making a transition to A at the next moment in time is determined by the following relation: $N_A(t+\Delta t) = F\left(\frac{N_A(t)}{N-1}\right)\cdot N$. If we suppose that the community is large enough, then $N-1 \approx N$. By denoting the fraction of all individuals who have made a transition to norm A at time t as $x(t) \overset{\Delta}{=} \frac{N_A(t)}{N}$, we obtain

$$x(t+\Delta t) = F(x(t)) \tag{11}$$

or

$$x(t+\Delta t) - x(t) = F(x(t)) - x(t). \tag{12}$$

It follows from the last expression that if $F(x) > x$, then $x(t)$ and correspondingly $N_A(t)$ increases with time, and if $F(x) < x$, then $N_A(t)$ decreases. If in equality (11) $\Delta t \to 0$, then we obtain a condition for equilibrium: $x(t) = F(x(t))$, at which the number of individuals who have made a transition to norm A stabilizes. The states of the equilibrium correspond to a fixed point in the graph of the function F. These states, however, can be both stable and unstable. To illustrate this feature, let us consider an example.

THE STABILITY OF EQUILIBRIUM STATES

Let us consider a community with the distribution function $F(x)$ of a threshold value θ, shown in Fig. 2. First, we analyze a model with discrete time. According to the initial condition $N_A(t_0) = 0$. Individuals with the negative value of a threshold will be the first to make a transition to norm A, therefore $N_A(\Delta t) = F(0)\cdot N$. In the next moment in time a transition will be made by individuals whose threshold value does not exceed the fraction of participants who chose to make a transition in the previous moment in time. That is $N_A(2\cdot\Delta t) = F\left(\frac{N_A(\Delta t)}{N}\right) = F(F(0))$, and so on. Once $\Delta t \to 0$, we get a continuous process.

Function F, displayed in Fig. 2 has three fixed points and corresponding equilibrium states: point L near zero, points M and P near unity.

The feature of points L and P is that they are stable: if the fraction of the individuals who made a transition to norm A is close to θ_L or θ_P, then it will oscillate closely about these values. Indeed, as shown above, for $x < \theta_L$ $F(x) > x$, therefore, $N_A(t)$ is increasing. And *vice versa*, if $x > \theta_L$. $N_A(t)$ is decreasing.

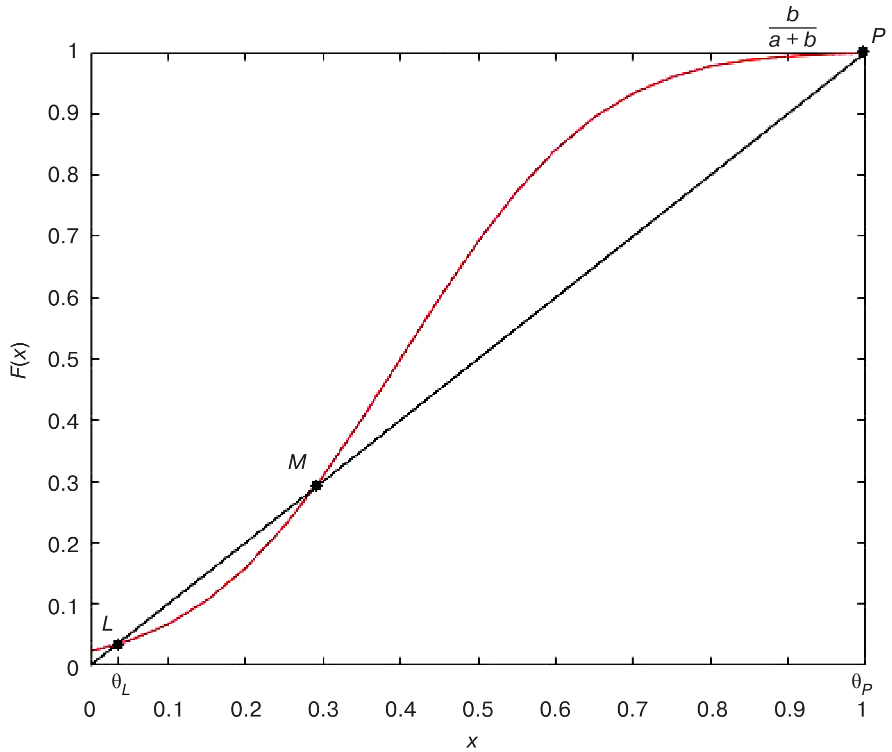

Fig. 2. Distribution function of threshold values with marked points at stable states

The equilibrium point M is unstable: if a fraction of individuals who made a transition to the norm A exceeds θ_M by any negligibly small amount, then $F(x) > x$, and $N_A(t)$ will be growing until the fraction stabilizes at a level corresponding to the nearest equilibrium point $\theta_P = 1$, that will indicate that the community has in general made the transition to the norm A. And vice versa, if fraction $x(t) = \frac{N_A(t)}{N}$ is arbitrary smaller than θ_M, it will continue decreasing until it reaches a stable position near θ_L, that means that the community has "rolled" back to the ineffective norm B. Application of the theory of stability of fixed points to a number of economical, social and biological processes is considered in [28].

Note that for a continuous function of the distribution, the fixed points, in which the equilibrium is reached, will be the points corresponding to changes in the concavity and the convexity of the function. If in a fixed point the function is concave from the left, then the point is stable, if it is convex, it is unstable.

As $F(x) = 1$ at $x > \frac{b}{a+b}$, then the function $F(x)$ is convex for $x \to 1 - 0$. Therefore the point $x = -1$, corresponding to a scenario when the entire community has made a transition to a new norm A, will always be stable. But if the distribution function F is such that there exists a fixed point with a value less than unity that represents a stable equilibrium, then the community as a whole will never make a transition to a more effective norm, and will be stuck in the vicinity of the nearest to zero equilibrium point.

EDUCATIONAL MODEL

Let us suppose in addition that in a community there is an education program in place. As a result, the moral level in the community increases.

For example, the author knows the fund "For Morality"—The Fund for revival and the development of culture and morality of citizens. The Fund's volunteers along with experienced teachers and scientists with the expertise in education, developed a course of lectures "Morality is the Nation's Strength;" corresponding text book for middle school students was released [29].

The course was approved and supported by experts with reviews from the members of educational enterprises and government of 40 regions across Russia [30], and was also used in facultative classes in middle schools of many regions in Russia [31]. It illustrates a constructive cooperation of the state and society.

One section of the text book titled "Moral traditions of the past is the foundation of modern society" has a subtitle "You will harvest what you seeded". In this section moral-ethical traditions of many peoples are generalized. The essence of these traditions is the necessity for an individual to comprehend a causal relationship between individuals' own actions and their consequences. In other words, before making an action an individual has to think: what happens if others will act towards him or her the same way as the individual is going to act. Will be it good?

Thus, we will consider that behavioral and educational activities contribute in a way that coefficient k_i for each community member increases with time. Of course, the value of coefficient k_i is difficult to formalize, and it is difficult to predict in advance which low it will follow (linearly or nonlinearly). It depends both on the kind of educational activity and on every particular member of a community.

However, we can indirectly estimate the efficacy of educational activity and, correspondingly, the rate of growth of coefficient k_i, based on the rate of change of threshold values θ_i which can be determined by statistical methods that is shown above in the example about blood donation.

Indeed, by differentiating θ_i with respect to k_i in expression (10), we obtain:

$$\frac{\partial \theta_i}{\partial k_i} = \frac{(a+b)(b-a)}{(1-k_i)^2 (a+b)^2} < 0,$$

as we assume that $a > b > c$.

Thus, as the coefficient k_i increases, corresponding threshold value θ_i of the ith participant decreases. In other words, the higher the moral level of an individual, the sooner he is ready to a more effective norm A.

This process can be represented in the following form: $\theta_i(t) = \theta_i(t_0) - \phi(t)$, $\phi(t) > 0$, $\frac{\partial \phi}{\partial t} > 0$, $t \in [t_0, T]$. We assume that the function $\phi(t)$ is the same for community members.

We recall that for a fixed state function F is defined as a function of the distribution of a random value θ, which is a threshold value of a randomly chosen community member: $F \stackrel{\Delta}{=} P(\theta \equiv \theta(t_0) < x)$.

As for the case of the dynamics with education, the function F depends on time; moreover, it is related to its statistical analog in the following way:

$$F(x,t) = P(\theta(t_0) - \phi(t) < x) =$$
$$= P(\theta(t_0) < x + \phi(t)) = F(x + \phi(t)).$$

As $\phi(t) > 0$, the graph of $F(x,t)$ is obtained in every moment in time $t \in [t_0, T]$ from the graph of $F(x)$ through the shifting to the left by a none-negative value $\phi(t)$. This process is illustrated in Fig. 3.

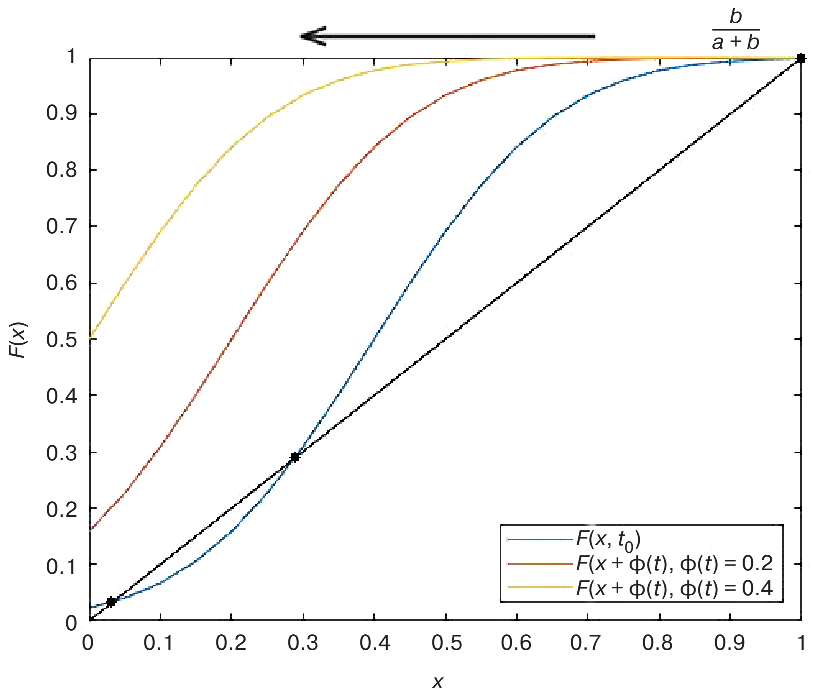

Fig. 3. Change of the distribution function of a threshold value with time in the model with education

Hence, if the distribution $F(x)$ has stable fixed points $x^* < 1$, we can choose such moment in time t', that function $F(x,t')$ will have only one fixed point $x = 1$. Thus, the community, in general, will successfully make a transition to a new norm A.

DISCUSSION

The model of a behavior of individuals, following the principle of morality in the sense of Kant Imperative, developed and presented in a number of papers (for example, [11, 32]), shows an essential difference between the behavior of individuals who we call *homo moralis* and *homo economicus* traditionally studied in papers on the game theory.

Another approach described in the literature is the modeling of collectivism or altruism which supposedly takes into account (with some weighting coefficient) interests of other participants. In a number of publications. For example, in [9, 11], collectivism is modeled so that in a problem with two participants each of them seeks for making maximum of not his initial payoff function $J_i(q)$ but a special utility function $U_i(q) = (1-\alpha)J_i(q) + \alpha J^i(q), \alpha \in [0,1]$, or (as it was generalized for an arbitrary number of participants in [18]) $U_i(q) = (1-\alpha)J_i(q) + \frac{\alpha}{N}\sum_{k=1}^{N} J_k(q), \alpha \in [0,1]$. For a particular case of such a function at $\alpha = 1$ Harsanyi [5] suggested a function in the form $U_i(q) = \frac{1}{N}\sum_{k=1}^{N} J_k(q)$.

There is an essential difference between *homo moralis*, the so-called *individualists* (*homo economicus*), and even *altruists*: while the former (*homo moralis*), when evaluating advantages of the transition of all community members to a new norm of behavior are able to become kind of a catalysis of the process, pioneers, neither *participants–individualists*, no *altruists*, are able play such a role.

This feature allows us to realize that there is some evolutionary stability in this model of behavior that, apparently, can be indirectly confirmed employing methods of evolutionary game theory. As was already mentioned, in this theory repeatable games are accepted for the analysis; and each behavioral strategy can be tested against a success not in one game but in the long run of a number of game situations.

It is exactly the approach that was "taken into service" by American game designer Nicky Case, who created an interactive game that illustrates how different behavioral strategies act in the processes of a repeatable dilemma of a prisoner [33]. The essence of the dilemma is that two players have a choice: to cooperate with or betray a friend. If both players choose to cooperate, they both are on the plus side. But each of them experiences a temptation, because if deception is successful, the one who deceived would get even more than if they cooperated, but the one who was deceived would lose. If both players are tempted and choose to deceive each other, they are punished and get the least favorable game situation.

Nicky Case considered as strategies such behavioral pattern as "naïve"—a type of players who try to continue to cooperate even when they are deceived, "rogues"—who deceive even though others try to cooperate with them, and "imitators"—who begin from cooperation and then just repeat the behavior of the opponent. Then Nicky Case models a society with the help of the so called cellular automaton, in which each cell employs only one of the listed strategies. Interestingly enough that it was cleared out that exactly the last type of behavior—characterized by word "mutuality" which, as Confucius believed, determines the essence of ethical teaching—appears to be the most evolutionary stable.

Nevertheless, based on the above reasoning about the stability of the equilibrium in heterogeneous communities we conclude that under natural conditions a new, more advanced behavioral model may never become a commonly accepted norm. In this case a society is "stuck" in a less effective model of behavior if additional measures are not accepted which favor the growth of a moral level (increase of the coefficient k_i in our model). Such measures, in particular, are educational and social-educational work.

Note, that there is a drawback in the analyzed model: it is difficult to formalize the parameters that were used in the model (for example, the coefficients k_i) that makes it difficult to determine their numerical value necessary for applications.

However, employing different statistical methods [23] will enable us to solve this problem. This makes it possible to use the presented theoretical material, for example, for the evaluation of the efficacy of state-guided work in education of young people and youth development. These are the topics the author is planning to cover in his future works.

CONCLUSIONS

In Conclusions, the investigated social model of the choice between two norms of the behavior enabled to obtain a nontrivial result that the higher the moral level of an individual, the higher the readiness of this individual to make a transition to a more favorable for a community in general behavioral norm. This feature distinguishes such individuals greatly from both the individuals (*individualists*) who seek exclusively for enhancing their personal well-being as well as those (*altruists*) who also take into accountsocietal but momentary interests.

The participants of the *homo moralis* class when choosing their behavioral strategy analyze what happens if the rest members will act the same way as they do. It gives them the opportunity, even though they initially lose, to foresee the advantages of accepting new behavioral patterns as new behavioral norms.

Therefore, we cannot disagree with T.N. Mickushina and M.L. Skuratovskaya [34] arguing that "*the states built on ethical and moral principles had always had economical and political advantage that resulted in prosperity and economic growth*."

With regard to the above said, governmental policy in the area of education and uprising can have significant impact on the rate of economic development because young people educated by employing the best cultural traditions will more effectively cope with challenges and bring new, more advanced communities into life of the society.

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