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RESEARCH ARTICLE

Synthesis of matched multichannel processing of orthogonal MIMO radar signals

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Abstract
Objectives. In contemporary radar studies, increasing attention is being paid to MIMO (Multiple-Input, Multiple-Output) radars, which are predicted to offer significant advantages over traditional phased-array radars. MIMO radars increase spatial degrees of freedom by simultaneously transmitting coherent, orthogonal radio waveforms in parallel. The main challenge for MIMO radars is to optimize space-time processing (STP) of the entire set of echo signals against a white Gaussian noise background according to the Neyman–Pearson criterion. In order to ensure optimal detection performance, the work statistically synthesizes STP in MIMO radars to derive variants of STP structural schemes for identifying their features as compared to STP in traditional phased-array radars, and analyze the angular resolution of the MIMO radar with the resulting STP structure.
Methods. Statistical optimization methods for signal detection against interference using the Neyman–Pearson criterion were used in accordance with approaches informed by the theory of beamforming in multibeam antenna arrays.
Results. Two STP variants were synthesized to generate optimal pre-threshold statistics for a MIMO radar detector with an *M*-element transmitting and *N*-element receiving antenna array. The structural diagrams of these variants were found to differ in their sequences of spatial and temporal processing. Due to the increased size of the equivalent aperture of the receiving antenna array, the angular resolution of the MIMO radar is demonstrated to be significantly better than that of a traditional radar at the same signal-to-noise ratio.
Conclusions. The structural diagrams of the synthesized MIMO radar detectors confirm their statistically optimal detection performance.

Keywords: MIMO radar, multichannel space-time processing, fast Fourier transform algorithm, multi-beam antenna array, detector pre-threshold statistics, angular resolution

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НАУЧНАЯ СТАТЬЯ

Синтез согласованной многоканальной обработки ортогональных сигналов MIMO радиолокационных станций

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Резюме

Цели. В современной теории радиолокации все большее внимание уделяется MIMO (англ. «много входов – много выходов») радиолокационных станций (РЛС), для которых прогнозируются важные преимущества перед традиционными РЛС с фазированными антенными решетками (ФАР). MIMO РЛС имеют дополнительную степень свободы за счет одномоментного излучения системы когерентных ортогональных радиосигналов, что однозначно определяет параллельный обзор пространства. Для MIMO РЛС возникает актуальная задача синтеза оптимальной по критерию Неймана – Пирсона пространственно-временной обработки (ПВО) всей совокупности эхо-сигналов на фоне белого гауссова шума. Целями данной статьи являются: статистический синтез ПВО в MIMO РЛС, обеспечивающий оптимальные показатели качества обнаружения; получение вариантов структурных схем ПВО и выявление их особенностей по сравнению с ПВО традиционных РЛС с ФАР; анализ углового разрешения MIMO РЛС с полученной структурой ПВО.

Методы. Использованы методы статистической оптимизации процедуры обнаружения сигналов на фоне помех по критерию Неймана – Пирсона, а также методы теории диаграммообразования в многолучевых антенных решетках (АР).

Результаты. Синтезированы два варианта ПВО, формирующие оптимальную предпороговую статистику обнаружителя MIMO РЛС с M-элементной передающей и N-элементной приемной АР. Показано, что структурные схемы этих вариантов различаются последовательностями пространственной и временной обработок. Доказано, что за счет увеличенного размера «эквивалентной» апертуры приемной АР разрешающая способность MIMO РЛС по угловым координатам существенно лучше, чем у традиционной РЛС при одинаковом отношении сигнал/шум.

Выводы. Показано, что структурные схемы синтезированных обнаружителей MIMO РЛС обеспечивают статистически оптимальные показатели качества обнаружения.

Ключевые слова: MIMO РЛС, многоканальная пространственно-временная обработка, алгоритм быстрого преобразования Фурье, многолучевая антенная решетка, предпороговая статистика обнаружителя, разрешающая способность по угловым координатам

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INTRODUCTION

We consider a multiple-input, multiple-output (MIMO) radar system based on an equidistant antenna array (AA) configuration with M-element transmitters and N-element receivers. The distances between the antennas are denoted as d_t and d_r (index t for transmit and r for receive), respectively. According to MIMO principles, the AA transmits M component vector of orthogonal coherent signals, $\mathbf{U}(t) = \{U_m(t)\}_{m=1}^{M}$, with partial complex amplitudes, $U_m(t)$, where t is time [1, 2]. As demonstrated in [3, 4], this approach defines a unique, parallel space survey in the $\Delta\theta_{svy}$ sector, which is consistent with the directional pattern (DP) beamwidth of the AA. The system operates in target acquisition mode (Fig. 1).

The special case of using a single weakly directional antenna (WDA) with $N = 1$ for reception, which is transformed by aperture synthesis into a virtual subarray (VSA), is considered in [5]. By synthesizing a separable STP, this study demonstrates the equivalence of target detection results against uncorrelated white Gaussian noise (WGN) backgrounds in MIMO radar with parallel space surveys in WDA reception and AA transmission, as well as in traditional radar with WDA transmission and AA reception.

The results obtained in [5] are generalized in the present work to a multichannel ($N > 1$) MIMO structure. This involves radar reception without altering the transmission AA structure or the parameters of the ranging signals. Here, while the use of MIMO radar can deliver a significant improvement in performance, no equivalence should be expected in the results of observing targets in the compared variants; rather, the improvement primarily applies to the angular coordinate resolution at a fixed energy potential. In the compared radars, STP synthesis is employed to detect a point target with an a priori unknown three-dimensional vector of informative parameters: $\boldsymbol{\alpha} = \{R_{TRG}, V_{rTRG}, \theta_{TRG}\}$. Here, $R_{TRG} = c\tau_{TRG}/2$ is the range capacity; V_{rTRG} is the radial velocity; θ_{TRG} is the angular coordinate of the target; c is the speed of light; τ_{TRG} is the time delay of the echo signal.

The predicted advantages of the MIMO radar systems over conventional radars are primarily due to the additional degrees of freedom, by which means the dimensionality of the spatial coordinate space is increased to improve the accuracy and precision of the system. In conventional radars, the transmitting WDA (or AA) has

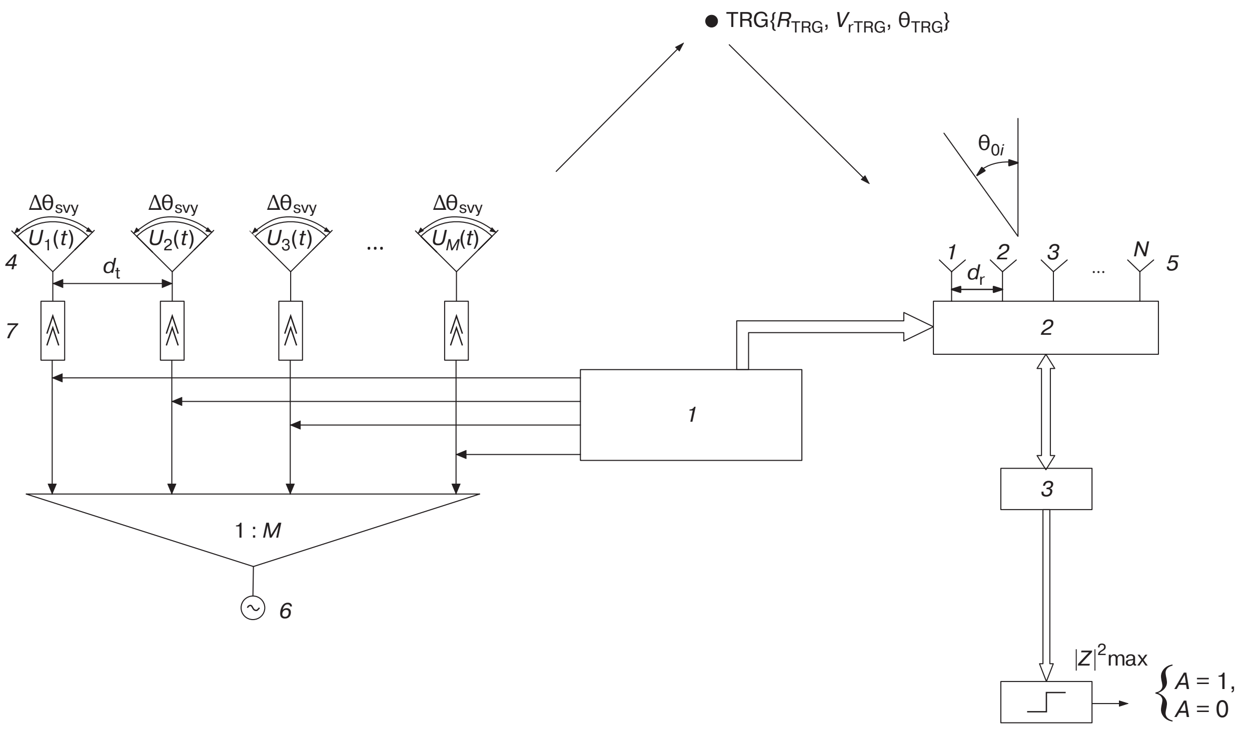

Fig. 1. Structure of MIMO radar with parallel space survey: (*1*) orthogonal ranging signal generator; (*2*) space-time processing (STP); (*3*) pre-threshold statistics generator; (*4*) transmitting antenna array (AA); (*5*) receiving AA; (*6*) driving oscillator; and (*7*) power amplifier. TRG{R_{TRG}, V_{rTRG}, θ_{TRG}} is the target; A is the threshold value used to make a decision regarding the presence or absence of a signal; $|Z|^2$max represents the maximum pre-threshold statistics value

a single phase center for emitting a coherent ranging signal, $\dot{\mathbf{u}}_0(t)$. In MIMO radars, conversely, each partial signal with a complex amplitude $\dot{U}_m(t)$, $m \in \overline{1, M}$ has a precisely known phase center with coordinates (x_m, y_m), resulting in a phase "coloring" of the signals, which can lead to improved performance. When combined with an N-element reception AA, this allows for a significant improvement in angular resolution as compared with an N-element conventional radar system having WDA transmission and AA reception, which is inversely proportional to the product $M \times N$.

In order to determine the resolution of the radar in terms of angular coordinates, the present work sets out to synthesize a consistent multichannel signal processing system for detecting and analyzing spatiotemporal signals reflected from targets and received by MIMO radars. The system is compared with conventional radar detectors against the WGN background.

When solving this problem, we kept the framework of the narrow-band system assumption for ranging signals to ensure orthogonality in either the time or frequency domain [6, 7]. In the first case, all partial signals may be generated using orthogonal codes for the initial phases of discrete signals τ_{dscr} at the carrier frequency f_0 $(1/\tau_{\text{dscr}} \ll f_0)$ analogous to a phase code modulated within transmitting AA channels. In the second case, orthogonality may be ensured by separating carrier frequencies by Δf between adjacent ranging signals within the transmitting AA. This may be considered as analogous to a stepped-frequency linear frequency modulated (SF-LFM) signal for which $(M-1)\Delta f \ll f_0$. Assuming a narrow-band probe signal system simplifies the process, as it allows for the following:

- Separating the processing into spatial and temporal components (non-separable STP for broadband signals is considered in [7]).
- Assuming that the MIMO radar operates at a specific wavelength of $\lambda_0 = c/f_0$, and thus, the Doppler frequency shift $F_{\text{D}} = 2V_{\text{rTRG}}/\lambda_0$ is approximately the same for all components of the vector signal.
- Calculating the delay time of the ranging signal at the aperture of the transmitting AA:

$$T_{\text{an.t}} = \frac{(M-1)d_{\text{t}}}{c} \ll \tau_{\text{eq}},$$

where τ_{eq} is the equivalent pulse duration corresponding to the bandwidth $\Delta f = 1/\tau_{\text{eq}}$ (for the receiving AA of the MIMO radar).

In addition, we assume that there are no reflections or sources of external radiation within the survey sector $\pm\theta_{\text{svy}}/2$. Under these conditions, we can consider the radiators of the receiving AA to be excited by an M-dimensional vector signal $\mathbf{U}_{\text{TRG}}$, which is reflected from a point target:

$$\mathbf{U}_{\text{TRG}}(R_{\text{TRG}}, V_{\text{rTRG}}, \theta_{\text{TRG}}) = \\ = \left\{ U_m(\tau_{\text{TRG}}, V_{\text{rTRG}}) \exp\left(j\frac{2\pi d_{\text{t}}}{\lambda_0}(m-1)\sin\theta_{\text{TRG}} \right) \right\}_{m=1}^{M}. \quad (1)$$

It is further assumed that the components of the vector $\boldsymbol{\alpha}$ are a priori unknown. Following excitation of the N-element receiving AA, the M-dimensional vector (1) transforms into an $N \times M$ vector of the echo signal. This can be expressed as a Kronecker product, as follows:

$$\mathbf{U}_{\text{r}}(\tau_{\text{TRG}}, t', \theta_{\text{TRG}}) = \boldsymbol{\beta}^{\text{H}}(\theta_{\text{TRG}}) \otimes \mathbf{U}_{\text{TRG}}(t, \boldsymbol{\alpha}), \quad (2)$$

where $\boldsymbol{\beta}^{\text{H}}(\theta_{\text{TRG}}) = \left\{ \exp\left[j\frac{2\pi d_{\text{t}}}{\lambda_0}(m-1)\sin\theta_{\text{TRG}} \right] \right\}_{n=1}^{N}$ is the spatial phase progression vector between the receiving AA radiators, while H is the sign of Hermitian conjugacy (transposition, T, and complex conjugation, *).

Based on the accepted conditions and assumptions, we consider the conventional statistical approach to the STP synthesis, which is optimal according to the Neumann–Pearson criterion.

STATISTICAL APPROACH TO DETERMINING THE WEIGHT VECTOR OF OPTIMAL PROCESSING IN MIMO RADAR

The problem of synthesizing multichannel STP at the output of the MIMO radar's receiving AA is considered against the background of the δ-correlated WGN using the conventional formulation based on the likelihood ratio [8, 9]. The sampled input $\mathbf{y}$ is represented by the N-dimensional vector $\mathbf{u}$ of the interference (noise) mixture $\mathbf{p}$ and possibly useful signal ($A = 1$) as follows:

$$\mathbf{y} = A\mathbf{u} + \mathbf{p},\ \mathbf{y} = \{y_i\}_{i=1}^{N},\ A = [1, 0]. \quad (3)$$

According to the well-established results from the theory of optimal detection based on the Neumann–Pearson criterion [10], the multichannel processing weight vector can be expressed as follows:

$$\mathbf{r} = \boldsymbol{\varphi}^{-1}\mathbf{u}^*(\alpha_0),\ \varphi = \{\varphi_{ij}\}_{i,j}^{N} = \overline{\mathbf{p}\mathbf{p}^{\text{H}}}, \quad (4)$$

where $\boldsymbol{\varphi}^{-1}$ is an $N \times N$ matrix that is the inverse correlation matrix (CM) of the interference $\boldsymbol{\varphi}$, while the overbar denotes statistical averaging.

In Eq. (4), the N-dimensional vector $\mathbf{u}(\alpha_0)$ is a signal with an expected three-dimensional vector of informative parameters, $\boldsymbol{\alpha}_0 = \{\tau_0, V_{\text{r}0}, \theta_0\}$. The CM $\boldsymbol{\varphi}$ with $\varphi_{ij}(t,s) = \overline{\boldsymbol{\varphi}_i(t)\boldsymbol{\varphi}_j^*(s)}$ elements exhibits cross-correlation between the ith and jth reception channels at times t and s. For the existence of an inverse matrix, $\boldsymbol{\varphi}^{-1}$, the CM should be Hermitian and positive-definite [11].

The STP synthesis for the given problem involves searching for the vector **r** in (4) that provides sufficient pre-threshold statistics in the form of a weighted scalar sum [12] as follows:

$$\xi = \mathbf{y}^{\mathrm{T}}\boldsymbol{\varphi}^{-1}\mathbf{u}(\boldsymbol{\alpha}_0) = \sum_{i=1}^{N} y_i r_i, \tag{5}$$

where the energy detection parameter is the following:

$$q = [\mathbf{u}^{\mathrm{H}}(\boldsymbol{\alpha}_0)\varphi^{-1}\mathbf{u}(\boldsymbol{\alpha}_0)]^{1/2}. \tag{6}$$

Provided the processing is linear, such pre-threshold statistics can be used to optimize the detection quality indicators by enabling the use of known detection characteristics [8, 9].

The method for calculating the random variable ξ in Eq. (5) determines the structural design of optimal processing consistent against the WGN background.

Due to the narrow bandwidth, the parameters of the signal reflected from a target with coordinates $\boldsymbol{\alpha}_{\mathrm{TRG}} = \{\tau_0, V_{\mathrm{rTRG}}, \theta_{\mathrm{TRG}}\}$ can be separated into time-frequency and spatial components:

$$\mathrm{u}(t, \boldsymbol{\alpha}_{\mathrm{TRG}}) = \left\{u_m(t, \tau_{\mathrm{TRG}}, V_{\mathrm{rTRG}})\mathrm{e}^{j\varphi_m(\theta_{\mathrm{TRG}})}\right\}_{m=1}^{M}, \tag{7}$$

where $\varphi_m(\theta_{\mathrm{TRG}}) = \dfrac{2\pi}{\lambda} d_{\mathrm{t}}(m-1)\sin\theta_{\mathrm{TRG}}$ is the spatial phase progression of the mth component of the signal.

Thus, the processing of the components can be divided into time-frequency and spatial sequences.

We define the types of CM $\boldsymbol{\varphi}$ included in Eqs. (4) and (5) and note its characteristics for STP against WGN in the MIMO radar of the considered design (Fig. 1). It would be reasonable to assume that the noise in the N-element receiving AA channels is independent $\mathbf{p}\mathbf{p}_j^{\mathrm{H}} = 0$ at $i \neq j$, $i, j = \overline{1, N}$, while it is δ-correlated in time. For an arbitrary ith channel having a spectral noise power density of P_{0i}, CM has the following form:

$$\boldsymbol{\varphi}_m(t, s) = P_{0i}\mathbf{I}\delta(t-s), \tag{8}$$

where $\mathbf{I} = \mathrm{diag}\left\{\varphi_{ii}\right\}_{i=1}^{N}$ is the identity matrix.

Provided that the orthogonal components $\mathbf{u}(\boldsymbol{\alpha}_{\mathrm{TRG}})$ pass through the corresponding matched filters (MF), it is additionally possible to assume the noise in time-frequency channels is independent and δ-correlated [13].

To facilitate the CM description, we assign a double index $y_{(t)} = \left\{y_n^m\right\}_{n,m\neq 1}^{N,M}$ to the input samples. Here, n represents the channel index of the receiving AA, while m represents the index of signal components corresponding to the channel index in the transmitting AA.

Given the aforementioned assumptions and considering the filtering properties of the δ-function, the CM for the stationary WGN will be as follows:

$$\begin{aligned}\boldsymbol{\varphi}(t,s) &= \delta(t-s)\overline{\mathbf{y}(t)\mathbf{y}^*(s)} = \left\{y_n^m(t)y_p^q(t)\right\} = \\ &= \mathrm{diag}\left\{y_{n,n}^{m,m}\right\}_{n,m=1}^{N,M} = \boldsymbol{\varphi}.\end{aligned} \tag{9}$$

In (9) it is assumed that the CM elements are zero for $n \neq p$ and $m \neq q$.

The sequence of stages in the STP process is determined by the order in which the $N \times M$ diagonal elements are grouped by indices, resulting in two variants of the $\boldsymbol{\varphi}$ matrix [14].

Variant 1. CM $\boldsymbol{\varphi}$ is represented by an $N \times M$ diagonal block matrix

$$\varphi_1 = \begin{Vmatrix} \mathbf{B}_{01}^{(m)} & & & & \\ & \mathbf{B}_{02}^{(m)} & & & \\ \ldots & & & & \\ & & \mathbf{B}_{0n}^{(m)} & & \\ \ldots & & & & \\ & & & & \mathbf{B}_{0N}^{(m)} \end{Vmatrix}. \tag{10}$$

Here, each of the N blocks is also represented by the diagonal CM noise between the time-frequency channels generated by the M MFs for the mth signal component in the nth channel of the receiving AA

$$\mathbf{B}_{0n}^{(m)} = \mathrm{diag}\left\{p_{0n}^{(m)}\right\}_{m=1}^{M}, \ \mathbf{B}_{0n} = p_{0n}\mathbf{I},$$

$$\text{at } p_{0n}^{(1)} = p_{0n}^{(2)} = \ldots = p_{0n}^{(M)} = p_{0n}. \tag{11}$$

Variant 2. CM $\boldsymbol{\varphi}$ is represented by an $M \times N$-diagonal block matrix

$$\varphi_2 = \begin{Vmatrix} \mathbf{B}_{01}^{(n)} & & & & \\ & \mathbf{B}_{02}^{(n)} & & & \\ \ldots & & & & \\ & & \mathbf{B}_{0m}^{(n)} & & \\ \ldots & & & & \\ & & & & \mathbf{B}_{0M}^{(n)} \end{Vmatrix}. \tag{12}$$

In this representation, each of the M CM blocks is the diagonal CM of noise between the N spatial AA channels for one mth signal component

$$\mathbf{B}_{0m}^{(n)} = \mathrm{diag}\left\{p_{0m}^{(n)}\right\}_{n=1}^{N}, \ \mathbf{B}_{0m} = p_{0m}\mathbf{I},$$

$$\text{at } p_{0m}^{(1)} = p_{0m}^{(2)} = \ldots = p_{0m}^{(M)} = p_{0m}. \tag{13}$$

As follows from (10) and (12), the statistical approach to synthesizing STP for MIMO radars differs from the conventional methods involving CM dimensionality of interference $M \times N$ (instead of an N-dimensional vector), as well as the ability to combine time-frequency and spatial processing in any order.

ANALYSING SUFFICIENT STATISTICS

For a sufficient detector of pre-threshold statistics, Eq. (5) can be represented in analogue form using the correlation integral of the complex envelopes of the received mixture and the expected signal $Z = \int \mathbf{Y}^{\mathrm{H}}(t, \boldsymbol{\alpha}_{\mathrm{TRG}})\dot{\mathbf{U}}(t, \boldsymbol{\alpha}_0)dt$. To ensure reliable detection characteristics, it is assumed that the echo signal has a random Rayleigh amplitude and a uniform initial phase ranging from 0 to 2π. Taking into account (11) and (12), the sufficient pre-threshold statistics against WGN background can then be written in terms of the squared modulus of the correlation integral [10, 12] as follows:

$$\xi = |Z|^2 = \sum_{n=1}^{N}\sum_{m=1}^{M}\frac{1}{P_{0n,m}^{(m)(n)}}\left|\int_{-\infty}^{\infty}\dot{U}_m(t,\tau_{\mathrm{TRG}},\theta_{\mathrm{TRG}})\dot{U}_m(t,\tau_0,\theta_0)\exp[j2\pi(f_0+\Delta f_{\mathrm{D}m})(t-t_{0m})]dt\right| \times$$
$$\times \exp\left[j\frac{2\pi d_{\mathrm{t}}(m-1)}{\lambda}(\sin\theta_{\mathrm{TRG}}-\sin\theta_0)\right]\exp\left[j\frac{2\pi d_{\mathrm{r}}(n-1)}{\lambda}(\sin\theta_{\mathrm{TRG}}-\sin\theta_0)\right], \quad (14)$$

where $\Delta f_{\mathrm{D}m}$ is the Doppler frequency shift for the mth signal component; t_{0m} is the constant delay in the temporal accumulation filter for the mth signal component; $\tau_{\mathrm{TRG}} = 2R_{\mathrm{TRG}}/c$ is the time delay for the expected echo signal.

Equation (14) can be considered as a four-dimensional ambiguity function in range–speed–angle coordinates. In this context, the double summation can be reduced to the product of the transmitting and receiving AA multipliers, which is a characteristic feature of MIMO radars [15]. Here, the DP refers to the cross section of the ambiguity function when the time-frequency parameters τ_0 and $V_{\mathrm{r}0}$ are fixed.

For simplicity, it is assumed that the Doppler frequency shift will be approximately the same for all signal components due to their narrow bandwidth ($\Delta f_{\mathrm{D}m} \approx \Delta f_{\mathrm{D}}$), and that the time delay in all accumulation filters is also equal ($t_{0m} = t_0$). In this context, the MIMO radar is considered to be in target detection mode, meaning that the only value of interest is the maximum value ξ applied to the threshold. It is also assumed that temporal accumulation is achieved through M MFs whose maximum response is independent of the signal delay, $\tau_{\mathrm{TRG}} = 2R_{\mathrm{TRG}}/c$. At time t_0, the maximum value of the integral in Eq. (14) is reached to obtain the optimal SNR at the MF output.

In this case, the maximum MF response value for the mth component of the signal is proportional to its energy, E_m:

$$\dot{U}_{m\ \mathrm{mf}}(0) = \int_{-\infty}^{\infty}|U_m(t,0,0)|^2\,dt = kE_m, \quad (15)$$

where k is the proportionality coefficient.

Regarding Eq. (14), the concept of a DP is meaningful in terms of the instantaneous cross-section of the ambiguity function at the instantaneous time $t_0 = 0$, i.e., when a maximum appears at the MF output.

Equation (14) shows that the calculation method of pre-threshold statistics determines the structure of the detector, which depends on the order of summation by the "m" and "n" indices, i.e., the sequence of temporal and spatial accumulation. In the first case, temporal accumulation occurs before spatial accumulation, while in the second case, the order is reversed. In this case, the CM of noise corresponds to Eqs. (11) and (12). Taking the above comments and simplifications into account, we can now consider variants for constructing STP structural schemes.

STRUCTURAL DIAGRAMS OF MIMO RADAR DETECTORS

Variant 1 (temporal + spatial accumulation)

In this STP variant, formula (14) first performs coherent summation by the "m" index, followed by summation by the "n" index.

Rewriting formula (14) in terms of the condition $p_{0n}^{(m)} = p_{0n}$ gives the following form:

$$\xi = \left|\left\{\sum_{m=1}^{M}U_{m\ \mathrm{mf}}(0)\exp\left[j\frac{2\pi d_{\mathrm{t}}(m-1)}{\lambda}(\sin\theta_{\mathrm{TRG}}-\sin\theta_0)\right]\right\}\sum_{n=1}^{N}\frac{A_n}{P_{0n}}\exp\left[j\frac{2\pi d_{\mathrm{r}}(n-1)}{\lambda}(\sin\theta_{\mathrm{TRG}}-\sin\theta_0)\right]\right|^2. \quad (16)$$

The processing can then be divided into two consecutive stages.

Stage 1.1

In Eq. (16), the expression within the curly brackets represents the summation of the results from the temporal accumulation of M signal components that compensates for their spatial phase in the expected direction θ_0. The second sum is the product of the N-element receiving AA with a d_r step, where the amplitude distribution, $\{A_n\}_{n=1}^{N}$, is normalized to the noise power in the radiator channels.

It should be noted that the sum over the index "m" corresponds formally to the multiplier of an M-element AA with a d_t step, where the maxima of MF responses to the signal components $A_m = U_{m\,\mathrm{mf}}(0)$ act as the amplitude distribution. The N weakly directional elements of the receiving AA are transformed into a VSA [5, 16]. The DP $F_\mathrm{v}(\theta_n, \theta_0)$ for this VSA is determined by the parameters of the transmitting AA. For $V_{m\,\mathrm{mf}}(0) = \mathrm{const}$, we obtain the following:

$$F_\mathrm{v}(\theta_\mathrm{TRG},\theta_0) = \mathrm{sinc}\left[\frac{\pi L_\mathrm{an.t}}{\lambda_0}(\sin\theta_\mathrm{TRG} - \sin\theta_0)\right], \tag{17}$$

where $\mathrm{sinc}\, x = \sin x/x$, $L_\mathrm{an.t} = (M-1)d_\mathrm{t}$ is the length of the transmitting AA.

If the transmitting AA step is chosen from the unambiguous scanning condition within the angular survey sector, $\Delta\theta_\mathrm{svy}$, then it is limited by the $d_\mathrm{t} \le \lambda_0[1 + |\sin\theta_0|]^{-1}$ ratio. As a result, diffraction lobes will not occur within the VSA DP $F_\mathrm{v}(\theta_n, \theta_0)$.

Stage 1.2

The spatial accumulation stage in the receiving AA virtually consists of N subarrays, each having a DP in the form of Eq. (17).

If the receiving AA step is the same length as the transmitting AA step, $d_\mathrm{r} = L_\mathrm{an.t}$, then its equivalent length can be assumed to be $L_\mathrm{eq.r} = (N-1)L_\mathrm{an.t} = (N-1)(M-1)d_\mathrm{t}$. In this case, the "equivalent" receiving AA is completely filled with $N \times M$ elements with a d_t step. Taking into account (17), the DP will take the following form:

$$\begin{aligned} F_\mathrm{r}(\theta_\mathrm{TRG},\theta_0) &= F_\mathrm{el}(\theta_\mathrm{TRG})F_\mathrm{v}(\theta_\mathrm{TRG},\theta_0)\sum_{n=1}^{N}\frac{1}{P_{0n}}\left[j2\pi\frac{d_\mathrm{r}}{\lambda_0}(n-1)(\sin\theta_\mathrm{TRG}-\sin\theta_0)\right] \approx \\ &\approx F_\mathrm{v}(\theta_\mathrm{TRG},\theta_0)\sum_{n=1}^{N}\frac{1}{P_{0n}}\left[j2\pi\frac{d_\mathrm{t}}{\lambda}(M-1)(n-1)(\sin\theta_\mathrm{TRG}-\sin\theta_0)\right]. \end{aligned} \tag{18}$$

Here no diffraction maxima are present. The second equality takes into account the isotropic nature of the elementary radiator DPs, $F_\mathrm{el}(\theta) = \mathrm{const}$.

For a MIMO radar in target search mode, it is essential to perform a parallel space survey, since the components of the ranging signal simultaneously "illuminate" the entire survey area. Here, due to the lack of a priori information about the expected target direction, multipath beamforming is required in order to prevent energy loss in the parallel survey.

To achieve this, the "equivalent" receiving AA should form $M \times N$ orthogonal beams (reception channels) in the following sequence. First, a fan of M beams in all N VSAs should be generated to create "secondary" reception channels ("primary" means N radiator channels here). Then, "tertiary" channels corresponding to a fan of N sharply directed beams should be generated within each of the M secondary beams.

The secondary and tertiary beams can be generated using beamforming circuit in the form of analogue Butler matrix [17] or M- and N-point fast Fourier transform (FFT) according to the assumption that $M = 2^p$ and $N = 2^q$ ($p, q = 2, 3, 4, \ldots$) [18].

To generate the secondary channels at the VSA output for the ith DP, which are oriented in the direction $\theta_{0i} = \arcsin\left[\left(i-\frac{1}{2}\right)\frac{\lambda_0}{(M-1)d_\mathrm{t}}\right]$, the transformation matrix $\mathbf{W}(\theta_{0i}) = \left\{\exp\left(-j\frac{2\pi i m}{M}\right)\right\}_{i=-M/2}^{M/2}$ can be used. If vector $\mathbf{V}_{0\,\mathrm{mf}}(0)$ represents digitized samples at time $t = 0$, then the following response is obtained for an arbitrary direction towards the target in the ith secondary channel:

$$F_\mathrm{v}(0,\theta_\mathrm{TRG},\theta_{0i}) = \sum_{m=1}^{M/2} U_{m\,\mathrm{mf}}(0)\cos\left[\frac{2\pi d_\mathrm{t}}{\lambda_0}\left(m-\frac{1}{2}\right)\left(\sin\theta_\mathrm{TRG} - \frac{\left(i-\frac{1}{2}\right)\lambda_0}{L_\mathrm{an.t}}\right)\right]. \tag{19}$$

As a result, the same M-beam amplitude DPs are generated at the output of the N VSAs. However, the beams with the same index will differ in terms of phase progression, which is determined by the d_r step of the receiving AA.

In order to generate tertiary beams, the secondary beam signals having the same index (i) must be re-indexed according to the necessary compensation of phase progression. Since $L_{eq.r} = (N-1)L_{an.t}$, the width of the tertiary beams is N times narrower than that of secondary beams. The relationship between the primary, secondary, and tertiary beams is clearly illustrated in Fig. 2.

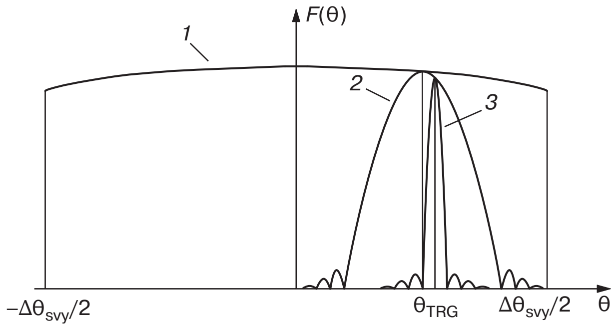

Fig. 2. DP of receiving channels:
(*1*) DP of the primary receiving radiator; (*2*) DP of the *i*th secondary channel at the VSA output; and (*3*) DP of the *s*th tertiary channel at the output of the *i*th secondary channel

The generation of tertiary beams is also facilitated by the FFT matrix $\mathbf{W}(\theta_{0s}) = \left\{\exp\left(-j\dfrac{2\pi sn}{N}\right)\right\}_{n=-N/2}^{N/2}$. Beams with orientation $\theta_{0s} = \arcsin\left[\left(s-\dfrac{1}{2}\right)\lambda_0 \Big/ (N-1)d_r\right]$ are implemented in each ith secondary reception channel $(i \in \overline{1,M})$. For this reason, it is convenient to assign a double indexing of θ_{0is} to the phasing angles of the tertiary beams, amounting to $M \times N$, i.e., the sth ray in the ith direction in the multipath VSA DP. Consequently, a fan of tertiary beams is generated from the instantaneous (at $t = 0$) values of the envelope signals sampled from the $N \times M$ outputs of the VSA.

Consequently, the receiving AA has an $N \times M$ beam DP for the receiving AA consisting of N VSAs with M-ray partial DPs (19). The dependence of the response of the tertiary channel "is" for a signal from an arbitrary direction θ_{TRG} takes the following form:

$$F_r(0,\theta_{TRG},\theta_{0is}) = F_{el}(\theta_{TRG})F_v(0,\theta_{TRG},\theta_{0is})\sum_{n=1}^{N/2}\frac{A_n}{P_{0n}}\cos\left\{\frac{\pi(M-1)d_t}{\lambda_0}\left(n-\frac{1}{2}\right)\left(\sin\theta_{TRG}-\frac{\left(s-\frac{1}{2}\right)\lambda_0}{L_{an.r}}\right)\right\}. \quad (20)$$

This formula can be interpreted as the tertiary DP of the spatial channel "is" or as the cross-section of the ambiguity function at time $t = 0$ in the "zero" speed channel.

Variant 2 (spatial + temporal accumulation)

In this STP variant, the coherent summation in Eq. (14) is first implemented by the "n" index and then subsequently by the "m" index.

We rewrite Eq. (14) in a different order using the $p_{0n}^{(m)} = p_0^{(m)}$ condition:

$$\xi = \left|\left\{\sum_{n=1}^{N} A_n \exp\left[j\frac{2\pi d_r(n-1)}{\lambda}(\sin\theta_{TRG}-\sin\theta_0)\right]\right\}\sum_{m=1}^{M}\frac{1}{P_0^{(m)}}U_{m\,\mathrm{mf}}\exp\left[j\frac{2\pi d_t(m-1)}{\lambda}(\sin\theta_{TRG}-\sin\theta_0)\right]\right|. \quad (21)$$

As in the first case, we split the processing into two stages.

Stage 2.1

For this variant, since θ_{TRG} in (21) is unknown a priori, spatial coherent integration must first be performed for the N expected directions of signal arrival. Using the FFT algorithm with the transformation matrix

$\mathbf{W}(\theta_{0s}) = \left\{ \exp\left(-j\frac{2\pi sn}{N} \right) \right\}_{n=-N/2}^{N/2}$, the first sum in (21) for an arbitrary sth phasing angle can be expressed as follows:

$$F_{\mathrm{r}}(0,\theta_{\mathrm{TRG}},\theta_{0s}) = \sum_{n=0}^{N/2} A_n \cos\left[\frac{\pi d_{\mathrm{r}}}{\lambda_0}\left(n-\frac{1}{2}\right)\left(\sin\theta_{\mathrm{TRG}} \frac{\left(s-\frac{1}{2}\right)\lambda_0}{(N-1)d_{\mathrm{r}}} \right) \right]. \tag{22}$$

For the maximum angle of phasing $\sin\theta_{0\,N/2}$, it is important that the condition $d_{\mathrm{r}} \le \lambda_0(1+|\sin\theta_{0\,m/2}|)^{-1}$ be imposed on a step of the reception array in order to prevent diffraction maxima in DP. Then, at the first stage, an N-beam DP can be generated in the receiving AA that overlaps a given survey sector. This corresponds to the implementation of secondary receiving channels.

Unlike the previous variant, it should be noted that the DP in (22) does not have the index "v," since the channels of the receiving AA physically exist. In particular, during digital charting, analogue-to-digital converters (ADCs) must be installed in each of the N primary channels. The DP $F_{\mathrm{r}}(0, \theta_{\mathrm{TRG}}, \theta_{0\,s/2})$ can be interpreted as "virtual" if, in formula (21), the summation over the "m" index is considered an array multiplier and the summation over the "n" index is considered a partial DP of the subarray.

Within each sth secondary channel, M tertiary channels should now be generated. As in the first variant, $N \times M$ receiving channels will also be implemented in the second variant.

Stage 2.2

The use of MF in each sth secondary channel enables the temporary accumulation of signal components and compensates for their spatial phase progression. Following the temporary accumulation at time $t = 0$, the second sum in (21) can be interpreted as an M element AA multiplier, where the maximum signal component values at the SF output act as amplitude distribution.

In this variant, the equivalent MIMO radar receiving AA is considered as an array consisting of M subarrays, each having N elements and a single d_{r} step. For the equivalent $N \times M$-element AA to be completely filled with radiators of the same step size and of the same length as in variant 1, the transmitting AA can be sparse with an equidistant step size equal to $d_{\mathrm{t}} = (N-1)d_{\mathrm{r}}$.

Next, a multipath diagram is implemented in the equivalent receiving AA to ensure that the sharply directed tertiary beam falls within the same ith direction as the sth secondary beam. Therefore, in this variant, the beams are double-indexed "si", rather than "is," as in variant 1. We apply the FFT algorithm to the samples of the output signals of the secondary channels using the transformation matrix $\mathbf{W}(\theta_{0i}) = \left\{ \exp\left(-j\frac{2\pi im}{M} \right) \right\}_{i=1}^{M}$. Thus, the DP of the si-tertiary channel, taking into account Eq. (22), acquires the following form:

$$F_{\mathrm{r}}(0,\theta_{\mathrm{TRG}},\theta_{0si}) = F_{\mathrm{el}}(\theta_{\mathrm{TRG}})F_{\mathrm{v}}(0,\theta_{\mathrm{TRG}},\theta_{0s})\sum_{n=0}^{M/2} \frac{A_n}{P_0^{(m)}}\cos\left[\frac{\pi d_{\mathrm{t}}}{\lambda_0}\left(m-\frac{1}{2}\right)\left(\sin\theta_{\mathrm{TRG}} \frac{\left(i-\frac{1}{2}\right)\lambda_0}{(N-1)d_{\mathrm{r}}} \right) \right], \tag{23}$$

where $F_{\mathrm{v}}(0, \theta_{\mathrm{TRG}}, \theta_{0\,s})$ is the DP defined by Eq. (22).

Thus, sufficient statistics for ξ (16) are maximized in the phasing directions.

COMPARATIVE ANALYSIS OF STP VARIANTS

In the synthesized STP variants, the sufficient pre-threshold statistics ξ, which is the same for all variants, is equal to the squared modulus of the correlation integral (14). This corresponds to the classical detector of a quasi-deterministic signal having a random uniform phase and random Rayleigh amplitude against the WGN background [9, 12]. Therefore, the same well-known formulas and curves can be used for detection characteristics that depend only on the power SNR (q^2) in both cases. According to the Neumann–Pearson criterion, the detection quality indicators D and F, as well as the detection threshold η, are related by the following approximate ratios:

$$D = F^{\frac{1}{1+q^2}},\ \eta = \sqrt{2\ln\frac{1}{F}}, \tag{24}$$

where D and F are the conditional probabilities of correct detection and false alarm, respectively.

It should also be noted that any parallel survey radar requires a device for selecting the spatial channel with the maximum SNR. Since the detection threshold may be exceeded in multiple reception channels simultaneously, the direction to the target is determined by the index of this channel.

The STP variants under comparison differ in the sequence of temporal and spatial signal accumulation. While this sequence does not affect the final result, it can significantly impact the practical implementation of MIMO radar. In particular, it affects the construction of the transmitting and receiving ARs.

The sparse array used for reception in variant 1 has a step size of $d_r = (M-1)d_t$, which is equal to the length of the transmitting AA. In variant 2, the transmitting AA with a step $d_t = (N-1)d_r$ is considered as sparse. Furthermore, the d_t step in variant 1 and the d_r step in variant 2 can be considered to be equivalent since they are both selected based on the absence of diffraction maxima in the angular survey areas for all scanning angles.

As shown in Fig. 3, the block diagram of the transmitting M-element AA is the same for both STP variants.

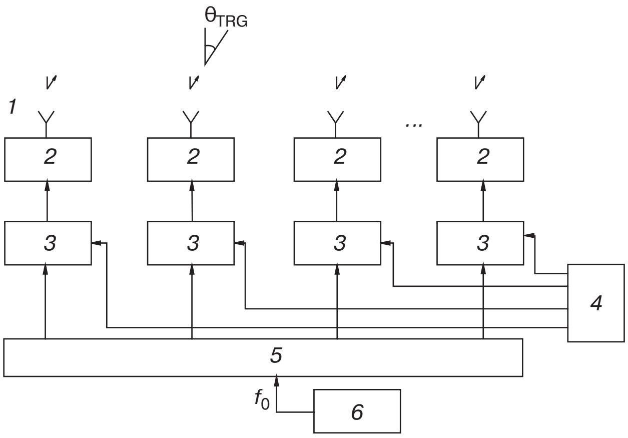

Fig. 3. Block diagram of the transmitting AA: (*1*) radiators, (*2*) power amplifier, (*3*) modulators (mixers), (*4*) complex signal generator, (*5*) divider 1 : *M*, and (*6*) driving oscillator

In order to consider the practicality and suitability of their implementation, we will now move on to a qualitative analysis of STP variants at the receiving AA output. The block diagram of variant 1 with initial temporary accumulation is shown in Fig. 4. This assumes $M > N$, which simplifies the construction of the receiving AA. Multibeam VSA DP is provided by an M-point FFT based on the signals of its primary channels. The signals in all secondary channels are processed by an N-point FFT to generate $M \times N$ tertiary channels with sharply directional DPs. These channels correspond to the equivalent length of the receiving AA, which is equal to $L_{\text{an.eq}} = (M-1)(N-1)d_t$.

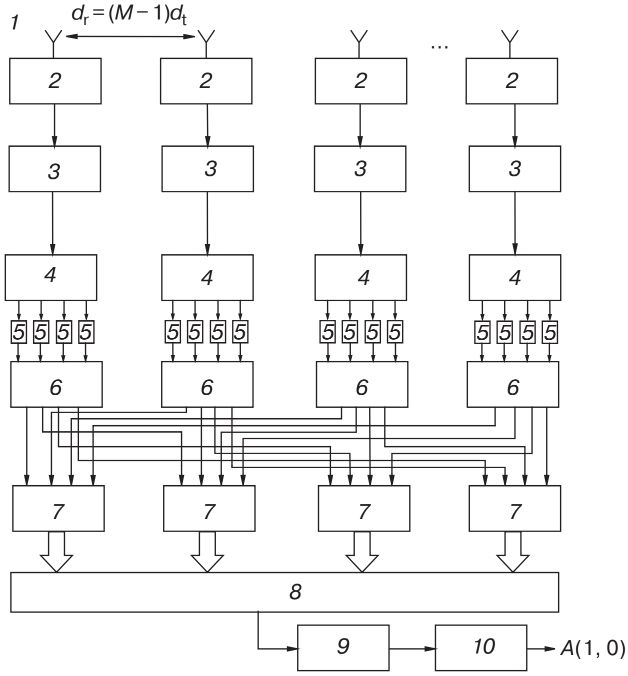

Fig. 4. Block diagram of the first STP variant (wide arrows represent *M*-dimensional vectors): (*1*) primary emitters; (*2*) amplifier-converter unit; (*3*) ADC; (4) divider 1 : *M*; (*5*) MF component of a complex signal; (*6*) FFT-temporary accumulation (generation of secondary channels); (*7*) FFT-spatial accumulation (generation of tertiary channels); (*8*) formation of pre-threshold statistics; (*9*) threshold device (decision-making); (*10*) maximum selection

Figure 5 shows the STP block diagram for variant 2 with initial spatial accumulation. The assumption of $M < N$ in this variant simplifies the construction of the transmitting AA. Spatial processing is provided here by an N-point FFT of samples in the primary channels, which is reduced to generating N secondary channels (beams). Following the temporary accumulation of secondary channel samples in the MF due to the M-point FFT, as in variant 1, $M \times N$ tertiary channels are generated, whose DP is determined by an AA length that is equal to $L_{\text{an.eq}} = (M-1)(N-1)d_r$.

Assuming a fixed SNR (since the d_t step in variant 1 is equal to the d_r step in variant 2 for a given scan sector), it can be inferred that the MIMO radar's angular resolution is the same for the compared STP variants, as determined by $L_{\text{an.eq}}$. Here, the blocks for formation of pre-threshold statistics (block *8*) involve selection of a tertiary channel having a maximum value of $\xi = \max|Z|^2$ (block *10*), while the threshold device (block *9*) in the compared STP variants are identical to those in conventional radars and have no specific features.

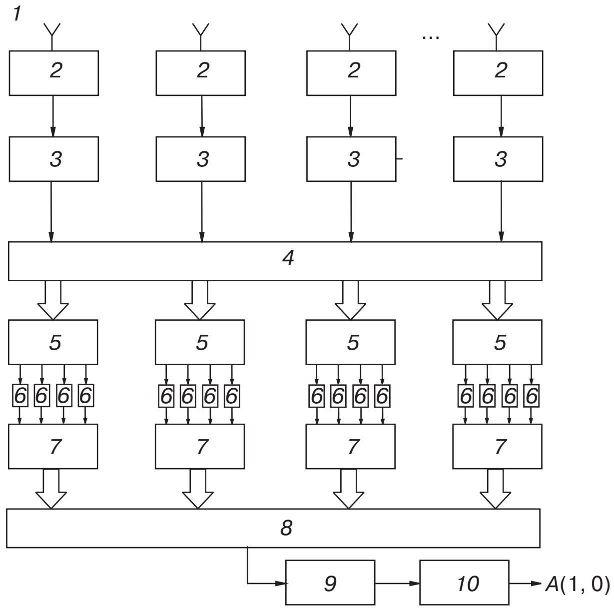

Fig. 5. Block diagram of the second STP variant (wide arrows represent *N*- and *M*-dimensional vectors): (*1*) primary radiators; (*2*) an amplifier-converter unit; (*3*) an ADC; (*4*) FFT spatial accumulation (generation of secondary channels); (*5*) divider 1 : *M*; (*6*) MF components of a complex signal; (*7*) FFT temporal accumulation (generation of tertiary channels); (*8*) formation of pre-threshold statistics; (*9*) a threshold device (decision-making); (*10*) maximum selection

An analysis of the STP schemes in Figs. 4 and 5 shows that variant 1 may be preferable when implementing a compact, multielement transmitting AA based on relatively low-power amplifier modules. A compact AA allows for easier synchronization of the modulators' operation to achieve the desired coherence of the orthogonal signal components. For a sparse receiving AA, this variant reduces the number of amplifier-converter units and ADCs. However, implementing their synchronous operation may present issues due to the spatial separation of the ADCs.

It should be noted that several sources regard STP variant 1 as the only viable ground-based radar, for instance [19, 20]. For medium- and long-range radars, recall that the transmitting AA of the MIMO radar serves as the VSA of the "equivalent" receiving AA.

The second STP variant is preferable when implementing a compact multibeam receiving AA with either analogue (Butler matrix) or digital (FFT) beamforming. In this variant, the transmitting AA is sparse; however, due to the increased power of the channel amplifiers, it should provide the same energy potential as variant 1, albeit having fewer *M* channels. Due to the spatial separation of the channels, maintaining signal component coherence may be challenging. Variant 2 has recently been used in airborne MIMO aircraft radars [21].

The qualitative comparative analysis reveals that the viability of implementing one variant over another depends significantly on several subjective technical factors. These include the frequency band, the ultrahigh frequency and digital element bases used, and the permissible design dimensions of the MIMO radar.

In conclusion, we explain the benefits of angular resolution provided by MIMO radar with any STP variant as compared to the fixed SNR of a conventional radar design. In radar theory, it is assumed that the angular resolution of the radar at the operating wavelength λ_0 has antenna aperture length L equal to $\Delta\theta_0 = \dfrac{\lambda_0}{L\cos\theta}$, where θ is the beam deviation angle from the normal. Often, the resolution is determined by the equivalent Rayleigh criterion through the antenna beam width at half power $\Delta\theta_{0.5}$.

The above relation for $\Delta\theta_0$ is derived from the following conditional criterion. Consider two radiation sources in space with amplitudes A_1 and A_2, separated by an angle $\delta\theta$. The total area at the opening of a linear antenna aligned along the z-axis is then given by:

$$E(x) = A_1 \exp(j2\pi f_0 t) \times \\ \times \exp\left(j\frac{2\pi}{\lambda_0} z\sin\theta\right)\left[1 + \frac{A_2}{A_1}\exp\left(j\frac{2\pi}{\lambda_0} z\delta\theta\cos\theta\right)\right]. \quad (25)$$

In Eq. (25), the multiplier in front of the square brackets corresponds to the linear phase progression along the aperture (along the z-axis) from the A_1 first source. The presence of a second source may be determined by the modulating multiplier inside the square brackets, which varies slowly depending on z. Here it is assumed that this modulation has been established if at least one complete period of it is observed. This implies a lower limit on the aperture length of $2\pi L\delta\theta\cos\theta/\lambda \geq 2\pi$. The condition of equality implies the expression for $\Delta\theta_0 = \delta\theta_{min}$.

For STP variants 1 and 2 considered in Eqs. (16) and (21), the sufficient statistics involving double summations by indices $n \in (1 - N)$ and $m \in (1 - M)$ correspond to obtaining DP of equivalent receiving ARs consisting of $M \times N$ elements, with unambiguous steps for scanning d_t and d_r, respectively. Since this determines the elongation of the aperture resolution in MIMO radars, a proportional improvement in angular resolution is achieved as compared to N-element AA in conventional radars.

For example, in variant 1, the physical step of the receiving AA is equal to the length of the transmitting AA. This is due to the temporary processing at the radiators' outputs; these can be interpreted as the VSAs with a length equal to $L_{an.t}$. In this scenario, the equivalent length of the receiving AA aperture is $(M - 1)$ times greater than that of a conventional N-element array with a step size of d_t. Therefore, the modulating phase multiplier in Eq. (25) achieves a value of 2π with a separation of the sources $\delta\theta$ that is $(M-1)$ times lower.

CONCLUSIONS

The results of the study on STP synthesis against the WGN background in MIMO radar with transmitting M-element and receiving N-element AAs support the following conclusions:

1. According to the Neumann–Pearson criterion, the optimal STP against the WGN background is a combination of the temporal and spatial accumulation of M orthogonal signal components ranged by the transmitting AA. The accumulation sequence depends on the method used to represent the interference CM in Eqs. (10) and (11), resulting in two variants for constructing a consistent STP (Figs. 4 and 5). In both variants, the length of the equivalent (virtual) aperture $L_{\text{an.eq}}$ of the receiving AA is proportional to the product $M \times N$, whose single-element step is determined by the scanning sector.
2. The first variant of the agreed STP consistently implements the following:
   - Temporary accumulation of M signal components in the MF from the N outputs of the primary radiators of the receiving AA.
   - Spatial accumulation during M-beam diagramming in N VSAs due to the M-point FFT (generation of secondary receiving channels).
   - Spatial accumulation due to the N-point FFT at the output of all secondary channels, resulting in the generation of $M \times N$ orthogonal beams of equivalent AA (generation of tertiary receiving channels).
3. The second variant of the agreed STP consistently implements the following:
   - Spatial accumulation of the signal due to the N-point FFT at the outputs of the primary radiators of the receiving AA (generation of secondary reception channels.
   - Temporary accumulation of M signal components in the MF from the N outputs of the secondary channels.
   - Spatial accumulation due to the M-point FFT at the MF output of all secondary channels, resulting in the generation of $M \times N$ orthogonal beams of equivalent AA (generation of tertiary receiving channels).
4. The same detection quality indicators can be achieved in both STP variants if they provide the same radiated power and product $M \times N$, as well as having an unambiguous scanning step. Therefore, the processing steps relating to the formation of the pre-threshold statistics and the detection threshold for decision-making in Figs. 4 and 5 are identical.
5. The size of the equivalent aperture of receiving AA is the same for the variants under comparison. Consequently, they implement the same angular resolution, which is inversely proportional to $L_{\text{an.eq}}$. In the first variant, the equivalent AA is interpreted as an $M \times N$-element array consisting of $N \times M$-element VSAs ($M > N$). In the second variant, it is interpreted as an $M \times N$-element array consisting of $M \times N$-element VSAs ($M < N$).

Abbreviations

AA—antenna array;
ADC—analog-to-digital converter;
CM—correlation matrix;
DP—directional pattern;
FFT—Fast Fourier transform;
MF—matched filter AA;
MIMO—Multiple-Input, Multiple-Output;
SNR—signal-to-noise ratio;
STP—space-time processing;
VSA—virtual subarray;
WDA—weakly directional antenna;
WGN—white Gaussian noise.

Authors’ contributions

B.M. Vovshin—general statement of the problem, development of principles for constructing two options of STP, development of methods for their comparison.

A.A. Pushkov—development of structural design schemes, derivation of analytical ratios, and analysis of the results of comparison two options of STP.

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