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RESEARCH ARTICLE

# Estimation of the Gaussian blur parameter by comparing histograms of gradients with a standard image

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**Abstract**
**Objectives.** The aim of this study is to develop a method for automatic quantitative estimation of the Gaussian blur parameter in digital images, which typically arises due to defocus of the optical system, various optical and camera-induced aberrations, as well as the influence of the propagation medium. This task is highly relevant for a wide range of applied fields, including remote sensing, forensic analysis, photogrammetry, medical imaging, automated inspection, and preprocessing of visual data prior to solving restoration, classification, or recognition problems.
**Methods.** The proposed method is based on comparing the two-dimensional histogram of gradients of the analyzed image with reference histograms precomputed for a high-sharpness image with similar texture and scale. The reference image is artificially blurred using convolution with a Gaussian kernel at various blur levels. For each level of blur, a two-dimensional gradient histogram is constructed, representing the distribution of directions and magnitudes of local intensity changes. The comparison with the corresponding histogram of the target image is performed after applying a logarithmic transformation and computing the Euclidean norm. This approach provides high sensitivity, interpretability, and numerical stability. The method does not require edge detection, neural network training, or labeled data, and can be implemented with minimal computational cost.
**Results.** Tests on synthetic data demonstrate that the proposed approach achieves high accuracy: the relative error in estimating the Gaussian blur parameter within the range of 0.7 to 2.0 pixels is less than 5%, and in most cases does not exceed 2–3%. The method is robust to noise, compression, local artifacts, and texture inhomogeneities.
**Conclusions.** The developed approach can be applied in automated image analysis systems as well as in blind deconvolution preprocessing tasks. It offers high accuracy, implementation simplicity, and reproducibility, providing reliable blur estimation under minimal data assumptions.

**Keywords:** image blur, Gaussian blur, blur parameter, gradient histogram, distribution comparison, distortion estimation, reference image, blind deconvolution, sharpness measurement, histogram distance metric

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НАУЧНАЯ СТАТЬЯ

# Оценка параметра гауссовского размытия методом сопоставления гистограмм градиентов с эталонным изображением

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**Резюме**
**Цели.** Целью настоящего исследования является разработка метода автоматической количественной оценки параметра гауссовского размытия цифрового изображения, возникающего, как правило, вследствие дефокусировки оптической системы, некоторых других погрешностей вносимых оптической системой и камерой, а также вследствие влияния среды распространения света. Данная задача актуальна для множества прикладных областей, включая дистанционное зондирование, техническую экспертизу, фотограмметрию, медицинскую визуализацию, автоматическую инспекцию и предварительную обработку изображений перед решением задач их восстановления, классификации или распознавания.
**Методы.** Предложенный метод основан на сравнении двумерной гистограммы градиентов анализируемого изображения с эталонными гистограммами, заранее вычисленными для изображения высокой четкости, обладающего сходной текстурой и масштабом. Эталонное изображение искусственно размывается с различными значениями параметра размытия путем вычисления свертки с гауссовским ядром. Для каждого уровня размытия строится двумерная гистограмма градиентов, отражающая распределение направлений и величин локальных изменений яркости. Сравнение с аналогичной гистограммой обрабатываемого изображения выполняется после логарифмирования по евклидовой норме. Это дает высокую чувствительность, интерпретируемость и численную устойчивость. Метод не требует выделения резких границ, обучения нейросетей или наличия размеченных данных и может быть реализован с минимальными вычислительными затратами.
**Результаты.** На синтетических данных показано, что предложенный подход обеспечивает высокую точность: относительная ошибка оценки параметра размытия в диапазоне его значений 0.7–2.0 пикселя составляет менее 5%, а в большинстве случаев не превышает 2–3%. Метод устойчив к шуму, сжатию, локальным артефактам и текстурным неоднородностям.
**Выводы.** Разработанный подход может применяться в системах автоматического анализа изображений, а также в качестве предварительного этапа в задачах слепой деконволюции. Он отличается высокой точностью, простотой реализации и воспроизводимостью, обеспечивая надежную оценку степени размытия при минимальных требованиях к исходным данным.

**Ключевые слова:** размытие изображения, гауссовское размытие, параметр размытия, гистограмма градиентов, сравнение распределений, оценка искажений, эталонное изображение, слепая деконволюция, измерение резкости, метрика расстояния между гистограммами

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## INTRODUCTION

Estimating image blur parameters is an important task in image processing and analysis. It plays a key role in improving sharpness, diagnosing data quality, and preprocessing images in satellite monitoring, medicine, industrial control, and other fields. The task of estimating the blur parameter σ in the case of Gaussian blur is particularly relevant. It is widely used as a model of spatial image degradation caused by defocusing or other forms of optical degradation. A highly accurate method for estimating the image blur parameter is important for the successful solution of the blind deconvolution problem.

Existing methods for evaluating blur parameters can be divided into several classes. The first class includes methods based on sharp edge analysis. They require accurate extraction and approximation of the transition profile, making them sensitive to noise, compression, and complex scene structure. The second class consists of frequency-based approaches, including methods based on the evaluation of the Modulation Transfer Function (MTF). These methods are also susceptible to global texture and can produce systematically overestimated values in heterogeneous image regions. The third class consists of methods based on machine learning, in particular convolutional neural networks [1, 2]. They demonstrate a high level of accuracy but require a large amount of labeled data for training and may not transfer well to new domains different from the training sample [1–7]. There are also individual studies in the literature devoted to the direct identification of the Gaussian blur parameter [8].

The method proposed in this paper aims to overcome these limitations. It does not require the localization of sharp boundaries and is resistant to noise and compression artifacts. It is based on comparing the two-dimensional histogram of gradients of the analyzed image with similar histograms obtained in advance from a reference image of a similar texture, artificially blurred with different values of σ. This allows for the degree of blurring to be estimated using the nearest neighbor principle in the histogram space. The method is easy to implement, requires no training, and demonstrates a high level of accuracy (the relative error, depending on the blur parameter value, ranges from 2% to 5%). In order to implement it, only one sufficiently arbitrary high-quality image is required as a reference.

The objective of this article is to describe formally the proposed method, experimentally evaluate its accuracy on synthetically blurred images, and compare its effectiveness with existing methods for estimating blur parameters.

## 1. IMAGE BLURRING MODEL

Let us consider the Cartesian coordinate system *Oxy* associated with the matrix of light-sensitive image elements. Let $q[x, y]$ be the discrete image of the scene—the reflected light signal coming from the object being photographed before passing through the propagation medium and the optical system. Let us assume that the camera pixel size Δ meets the condition $\Delta < 1/F_{max}$, where $F_{max}$ is the highest significant spatial frequency contained in the frequency spectrum of the image $q[x, y]$, which in this case, according to the sampling theorem, is represented as:

$$q[x,y] = \sum_{x',y'\in\mathbb{Z}} q[x',y']\mathrm{sinc}(x-x')\mathrm{sinc}(y-y'), \quad (1)$$

wherein, for convenience, $\Delta = 1$ is assumed.

Taking into account distortions when the signal passes through the optical channel and the camera's optical system, a Gaussian blurred image is projected onto the camera's touch panel:

$$q'(x,y) = \int_{\mathbb{R}^2} q(x',y')g_\sigma(x-x',y-y')dx'dy', \quad (2)$$

wherein

$$g_\sigma[x,y] = g_\sigma(x)g_\sigma(y), \quad g_\sigma(x) = \frac{1}{\sqrt{2\pi}\sigma}\mathrm{e}^{-\frac{x^2}{2\sigma^2}}.$$

Here, it is assumed that the Gaussian blur parameter σ, which models the properties of the optical channel and certain design and manufacturing errors in the optical system, is sufficiently large (σ > 0.7 pixels) and therefore dominates over the effect caused by diffraction.

During the exposure time τ a charge equal to $p[x, y] = \tau\Delta^2 q'(x, y) = q'[x, y]$ accumulates on the sensor with the indices *x*, *y*, where τ = 1 and Δ = 1 are assumed.

From (2), taking into account (1), it follows that

$$p[x, y] = \\ = \sum_{x', y' \in \mathbb{Z}} q[x', y'](\text{sinc} * h_\sigma)(x - x')(\text{sinc} * h_\sigma)(y - y'), \quad (3)$$

wherein $(\text{sinc} * h_\sigma)(x) = \int_{\mathbb{R}} \text{sinc}(x') g_\sigma(x - x') dx$, '*' is the convolution operation, defined by Eq. (2).

In [9], it is shown that for values of the blur parameter σ > 0.7 pixels can be considered with high accuracy as $(\text{sinc} * h_\sigma)(x) = g_\sigma(x)$. Taking this into account, Eq. (3) can be written as:

$$p[x, y] = \sum_{x', y' \in \mathbb{Z}} q[x', y'] h_\sigma[x - x', y - y'],$$

wherein $h_\sigma[x, y] = g_\sigma(x) g_\sigma(y)$ is the two-dimensional isotropic Gaussian model of the discrete point blur function (PBF) with the blur parameter σ > 0 that we accept.

The accepted model is typical of satellite imaging of the Earth in cases where directional distortions (related to camera movement, object movement, or platform instability) are compensated. This work assumes that such directional blurring has been preliminarily eliminated by hardware or software and therefore, in a first approximation, the image can be considered isotropically blurred, without any predominant direction in the blurring.

## 2. THEORETICAL ASSUMPTIONS OF THE METHOD

The proposed method for estimating linear blur parameters is based on the assumption that, given the known nature of the image texture and a specified shooting scale (the ratio of the linear size of the terrain to the pixel size), the two-dimensional histogram of image gradients contains sufficient information to restore the blur parameters. In other words, it is assumed that the distribution of pixel brightness gradients, considered as a realization of a two-dimensional random variable, depends primarily on the properties of the scene texture, image scale, and blur parameters, while not depending on the content of the scene as a whole. This means that images with similar textures and the same scale, but different degrees of blur, will have different gradient statistics. This dependence can be used to estimate the distortion parameters.

It is assumed that there are broad classes of images for which the distribution of gradients can be described by a parametric family depending only on blur parameters. Such assumptions are typical for statistical models of natural scenes (natural scene statistics [1, 3, 4]) used in a number of works on distortion estimation [10–12].

It ca be presumed that such classes are determined primarily by texture characteristics: large or small details; contrasting or homogeneous structures; and linear scale. As will be shown in Section 3, this assumption is confirmed in practice: histograms of gradients of images with similar textures but different blur parameters demonstrate a stable dependence on distortion parameters.

The main idea of the method is to use a reference image belonging to the same class (i.e., with similar texture and scale) as the one being analyzed. The reference is synthetically subjected to Gaussian blurring with a blurring parameter which varies across a regular grid. For each blurred image obtained in this way, a two-dimensional gradient histogram (reference histogram) is calculated. The gradient histogram of the image being analyzed is then compared with the pre-calculated reference histograms. The following measures were considered:

- $l_p$-norms ($p = 1, 2, \infty$) from the difference of logarithms of histograms;
- Kullback–Leibler and Jensen–Shannon divergences.

This corresponds to standard practice in image quality assessment tasks [13]. However, significantly better results were obtained using logarithmic transformation and the $l_2$-norms.

Thus, the task is narrowed down to finding the nearest neighbor in the space of reference histograms. Each node of the parameter grid corresponds to one reference histogram and, therefore, to a uniquely defined vector of the blur parameters.

### 2.1. Plotting a two-dimensional histogram of image gradients

For each pixel of the input image *p*[*y*, *x*], the discrete gradient is calculated in the following way:

$$\nabla p[y, x] = (p_x[y, x],\ p_y[y, x]),$$

wherein the gradient components are defined as the convolution of the image with the Sobel operator *D*:

$$p_x[y, x] = (p * D_x)[y, x], \quad p_y[y, x] = (p * D_y)[y, x],$$

and the operators themselves have the form

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}, \quad D_y = D_x^{\mathrm{T}}.$$

Each gradient vector $\nabla p[y, x]$ is further interpreted as the realization of a two-dimensional random variable observed in independent "tests" across the entire image area. Based on the set of these values, a two-dimensional histogram of gradient distribution is constructed:

$$H_{\nabla p}[i, j], \; i, j = \overline{1, N},$$

wherein $N$ is the number of histogram cells for each measurement ($p_x$ and $p_y$ axes).

Let us assume that the gradient values fall within a fixed limited window (e.g., $[-G_{\max}, G_{\max}] \times [-G_{\max}, G_{\max}]$, which is evenly divided into $N \times N$ cells.

Choosing the $N$ parameter requires a compromise between smoothing and discretization. If $N$ is too small, the histogram becomes excessively smooth and loses important features of the distribution. If $N$ is too large, the number of observations in each cell drops sharply, and the histogram becomes noisy. Thus, choosing the number of cells is part of the method configuration and should correspond to the sampling density and the nature of the gradients.

### 2.2. Reference histograms generation

A set of synthetically blurred images $\{p_{\sigma_k}\}$ is generated from the selected reference image $p^{\text{etalon}}[y, x]$ using a fairly fine two-dimensional grid of the blur parameter $\sigma_k$. For each grid value $\sigma_k$, the reference image is blurred:

$$p_{\sigma_k}^{\text{etalon}} = \left(p^{\text{etalon}} * h_{\sigma_k}\right)[y, x],$$

wherein the blur kernel $h_{\sigma_k}[y, x]$ is calculated via formula (3).

Then, for each variant of the reference image blur $p_{\sigma_k}^{\text{etalon}}[y, x]$ a separate histogram of gradients $H_{\nabla p_{\sigma_k}^{\text{etalon}}}$ как is plotted, as indicated in section 2.1.

### 2.3. Procedure for evaluating the blur parameter based on comparison with reference values

An appropriate metric should be used to evaluate the distances between the gradient histogram of the analyzed image and variants of similar reference histograms. Experiments have shown that the best results are obtained using the following metric:

$$\operatorname{dist}_{\log}(H_{\nabla p}, H_{\nabla p_{\sigma_k}^{\text{etalon}}}) = \sum_{i,j} \left| \log H_{\nabla p}[i, j] - \log H_{\nabla p_{\sigma_k}^{\text{etalon}}}[i, j] \right|^2.$$

In order to improve numerical stability under the logarithm sign, a regularization procedure should be used which consists of replacing the values of zero cells with the smallest of all obtained cell values before logarithmization. In such a situation, the expression of the type $\log(x)$ is often replaced by $\log(x + \varepsilon)$, where, for example, $\varepsilon = 10^{-6}$. However, as the experiment has shown, such a solution, compared to the one proposed above, often leads to a multiple increase in the relative error of the blur parameter estimation.

Thus, the evaluation of the blur parameters present in the analyzed image is carried out according to the following rule:

$$\hat{\sigma} = \arg\min_k \operatorname{dist}_{\log}(H_{\nabla p}, H_{\nabla p_{\sigma_k}^{\text{etalon}}}).$$

## 3. MODELING RESULTS

Figure 1 shows two high-quality images. The first, blurred to different degrees, is considered as the processed image, the degree of blurring of which needs to be evaluated (at different blurring values). The second is used as a reference (standard) image.

The modeling results are presented in Table 1.

Table 2 presents the results of modeling when changing the roles of the images under consideration—the image that was previously evaluated became the reference image, and *vice versa*.

As can be seen from the tables above, when the evaluated image is artificially blurred by the values $\sigma \in [0.7, 2.0]$, the relative accuracy of the estimates in most cases does not exceed 5%. In fact, in most cases it is significantly lower. Individual outliers in the estimate values are associated with the high sensitivity of the method to the choice of histogram cell boundaries. The estimates given in Tables 1 and 2 were obtained with histogram cell boundaries taken in the range from −2.1 to 2.1 with a step of 0.02126.

Figure 2 shows a typical one-dimensional logarithmic histogram (natural logarithm of the histogram). The corresponding one-dimensional histograms are obtained by averaging the two-dimensional histogram in each of the two directions.

Figure 3 shows typical graphs of the dependence of the $\operatorname{dist}_{\log}$ metric value on the $\sigma_k$ parameter (at a fixed $\sigma$). The minima on these curves correspond to the estimation of the blur parameters. The graphs compare the results for two values of the grid step along $\sigma$: 0.01 and 0.001 pixels. As can be seen, the step of 0.01 is already small enough, and further reduction does not lead to a significant increase in accuracy.

(a) (b)

**Fig. 1.** Two high-quality images:
(a) the image being processed; (b) the image used as a reference

**Table 1.** Modeling results

| True value σ | Estimation σ | Error value | Relative error, % |
|---|---|---|---|
| 0.7 | 0.69 | +0.01 | 4.3 |
| 0.8 | 0.79 | +0.01 | 1.3 |
| 0.9 | 0.88 | +0.02 | 1.1 |
| 1.0 | 0.99 | +0.01 | 1.0 |
| 1.1 | 1.07 | +0.03 | 1.8 |
| 1.2 | 1.24 | −0.04 | 2.5 |
| 1.3 | 1.35 | −0.05 | 2.3 |
| 1.4 | 1.42 | −0.02 | 3.6 |
| 1.5 | 1.54 | −0.04 | 7.3 |
| 1.6 | 1.61 | −0.01 | 3.1 |
| 1.7 | 1.74 | −0.04 | 5.9 |
| 1.8 | 1.82 | −0.02 | 1.7 |
| 1.9 | 2.00 | −0.10 | 7.9 |
| 2.0 | 2.10 | −0.10 | 6.5 |
| 2.1 | 2.20 | −0.10 | 6.7 |
| 2.2 | 2.30 | −0.10 | 6.8 |
| 2.3 | 2.40 | −0.10 | 7.8 |
| 2.4 | 2.50 | −0.10 | 3.8 |
| 2.5 | 2.60 | −0.10 | 10.0 |

**Table 2.** Modeling results when changing the roles of the images under consideration

| True value σ | Estimation σ | Error value | Relative error, % |
|---|---|---|---|
| 0.7 | 0.69 | +0.01 | 4.3 |
| 0.8 | 0.80 | 0.00 | 5.0 |
| 0.9 | 0.93 | −0.03 | 2.2 |
| 1.0 | 1.02 | −0.02 | 1.0 |
| 1.1 | 1.12 | −0.02 | 0.9 |
| 1.2 | 1.13 | +0.07 | 2.5 |
| 1.3 | 1.24 | +0.06 | 3.1 |
| 1.4 | 1.34 | +0.06 | 3.6 |
| 1.5 | 1.44 | +0.06 | 0.7 |
| 1.6 | 1.56 | +0.04 | 1.3 |
| 1.7 | 1.64 | +0.06 | 1.8 |
| 1.8 | 1.77 | +0.03 | 3.9 |
| 1.9 | 1.80 | +0.10 | 3.2 |
| 2.0 | 1.90 | +0.10 | 6.0 |
| 2.1 | 2.00 | +0.10 | 8.1 |
| 2.2 | 2.11 | +0.09 | 1.4 |
| 2.3 | 2.20 | +0.10 | 5.7 |
| 2.4 | 2.30 | +0.10 | 5.4 |
| 2.5 | 2.40 | +0.10 | 9.2 |

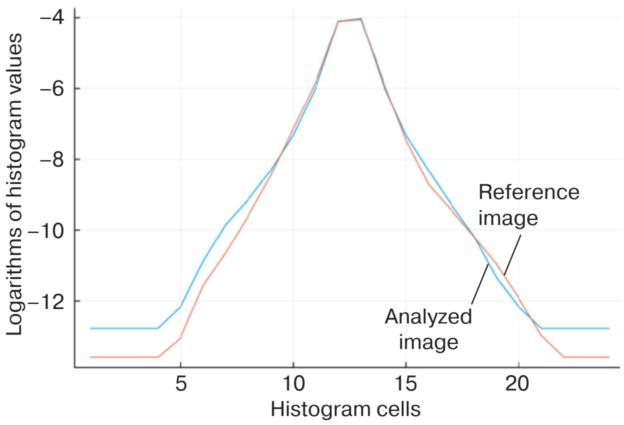


**Fig. 2.** A typical view of one-dimensional logarithmic histograms

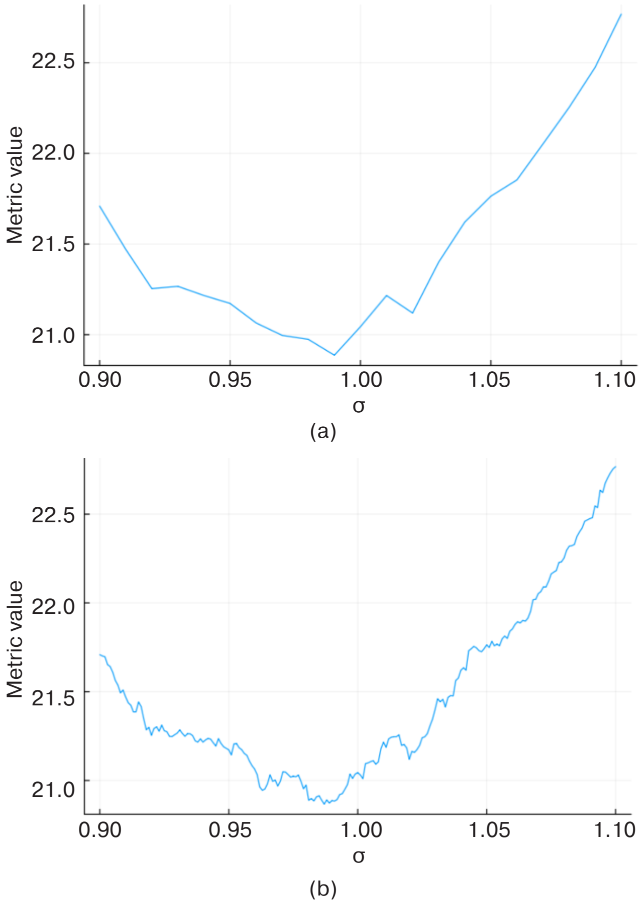


**Fig. 3.** A typical view of the dependence of the metric determining the similarity of the compared images on the value of the estimated blur parameter σ at grid step: (a) 0.01; (b) 0.001

## 4. COMPARISON WITH OTHER METHODS

This work proposes a method for estimating the Gaussian blur parameter of an image based on comparing a two-dimensional gradient histogram with pre-calculated reference histograms obtained from an image of similar texture and scale which has been artificially blurred. The method does not require boundary localization and works well even in the presence of compression and noise.

The simulation showed that the method demonstrates high accuracy in the range of blur parameter values $\sigma \in [0.7, 2.0]$, achieving a relative error that in the vast majority of cases does not exceed 5% and is often lower. At the same time, one of the key factors affecting the accuracy of the estimate is the choice of cell boundaries when constructing the gradient histogram.

The minimum grid step for the blur parameter $\sigma_k$ sufficient for high accuracy is 0.01 pixels. Further reduction does not provide any significant gain which makes the method efficient in terms of computational costs.

Thus, the proposed approach is simple to implement, noise-resistant, and easily adaptable to different blur ranges. It has the potential to be applied in automatic image analysis tasks in technical, medical, and remote applications, as well as used as a preliminary stage for subsequent image restoration.

Table 3 presents a brief comparison of the proposed method with a number of known approaches to estimating Gaussian or linear blur parameters.

Thus, comparison with existing methods shows that the approach proposed achieves accuracy comparable to the best modern methods based on trainable neural network models (2–5%), while requiring no prior training, labeled data, or complex infrastructure.

It should be noted that the proposed method essentially implements the simplest version of regression in feature space, similar to how convolutional neural networks work. Two-dimensional gradient histograms serve as features, and a set of reference images artificially blurred with known values of the parameter σ serves as the training sample. Instead of training the model parameters, an explicit comparison is made using a metric, making the method interpretable and robust. Thus, the approach proposed can be considered an effective and interpretable alternative to neural network methods for blur estimation [4, 5–7, 14, 15].

**Table 3.** Comparative table of different methods for evaluating image blur parameters

| Method | Requires a reference? | Requires a boundary? | Noise resistance | Automation | Accuracy at $\sigma \in [1.0, 2.0]$ | Comments |
|---|---|---|---|---|---|---|
| Proposed method | Yes (similar structure) | No | High | Full | 2–5% (in most cases) | High accuracy, highly scalable |
| Sharp boundary method | No | Yes (locally) | Low | Limited | 5–15% | Requires a clear sharp boundary and may be sensitive to noise and edge line direction |
| Frequency method (MTF-fit)[1] | No | No | Average | Yes | 5–10% | Requires correct window selection, depends on texture |
| Gradient statistics without a reference | No | No | Average | Yes | 10–20% | Easy to implement, low accuracy |
| Methods based on convolutional neural networks | Not always | No | High | Requires training | 2–5% | Good results on the trained sample, but difficult to apply |

[1] Modulation Transfer Function is an experimental fitting of the modulation transfer function.

## CONCLUSIONS

This work used a single reference image with texture characteristics similar to those of the image being analyzed. However, in order to increase the stability and versatility of the method in practice, it is reasonable to use not just one reference, but a representative set of references. This approach involves the preliminary formation of a basic set of high-quality images, their clustering by characteristics (for example, by gradient histograms at $\sigma = 0$), and the subsequent selection of the closest reference for each image under analysis. This can enable the method to be adapted to a variety of scene structures and textures, increasing the accuracy and expanding the applicability of the approach.

The method could be further developed in the direction of automatically selecting image areas with high textural informativeness, potentially reducing the requirements for selecting a reference image.

**Authors' contribution**

All authors contributed equally to the research work.

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