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RESEARCH ARTICLE

Heat transfer in a porous medium having an ordered gyroid-based macrostructure

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Abstract
Objectives. Triply periodic minimal surfaces are non-intersecting surfaces with zero mean curvature, consisting of elements repeating in three directions of the Cartesian coordinate system. The use of structures based on minimal surfaces in heat engineering equipment is associated with their advantages over classical lattice and honeycomb structures, often used in practice. The aim of the work is to study heat transfer during filtration flow in a porous medium of an incompressible fluid having an ordered macrostructure based on gyroid triply periodic minimal surface.
Methods. In order to solve the problem of heat transfer in a porous medium, the finite difference method is used. As a means of implementing the finite difference method algorithm, the *Heat Transfer Solver* software was developed in the Python programming language.
Results. The described software program was used to obtain a numerical solution of the heat transfer problem in a porous medium with an ordered macrostructure using the finite difference method. The program functionality enables the investigation of the heat transfer process dynamics and the influence of various parameters on the temperature distribution. The program was used to study the heat transfer process in a porous medium based on gyroid triply periodic minimal surface. Graphical dependencies of the solid framework and fluid temperatures, as well as the heat flux on the coordinate at different time steps, were obtained. Characteristic time intervals with the highest absolute temperature gradient values were identified.
Conclusions. The results of the work, including both the developed software and the obtained temperature dependencies, can be used in a number of engineering problems where it is important to predict the temperature distribution in porous materials under various operating conditions.

Keywords: porous medium, fluid flow, heat transfer, triply periodic minimal surface, gyroid, finite difference method

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НАУЧНАЯ СТАТЬЯ

Исследование теплопереноса в пористой среде с упорядоченной макроструктурой на основе гироида

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Резюме

Цели. Трижды периодические минимальные поверхности – это непересекающиеся поверхности с нулевой средней кривизной, состоящие из повторяющихся в трех направлениях декартовой системы координат элементов. Использование конструкций, основанных на минимальных поверхностях, в теплотехническом оборудовании связано с их преимуществами перед классическими решетчатыми и сотовыми конструкциями, часто применяемыми на практике. Целью работы является исследование теплопереноса при фильтрационном течении несжимаемой жидкости в пористой среде с упорядоченной макроструктурой на основе трижды периодической минимальной поверхности (гироида).

Методы. Для решения задачи теплопереноса в пористой среде применяется метод конечных разностей. Для реализации алгоритма метода конечных разностей разработано программное обеспечение *Heat Transfer Solver* на языке программирования Python.

Результаты. В рамках настоящего исследования разработано программное обеспечение на языке программирования Python для численного решения методом конечных разностей задачи теплопереноса в пористой среде с упорядоченной макроструктурой. Функционал программы позволяет исследовать динамику процесса теплопереноса и влияние различных параметров на распределение температуры. При помощи данной программы изучен процесс теплопереноса в пористой среде на основе гироида. Получены графические зависимости температуры твердотельного каркаса и жидкости, а также теплового потока от координаты в различные моменты времени. Определены характерные временные интервалы, в которых наблюдается наибольшее абсолютное значение градиента температур.

Выводы. Результаты работы, включающие как разработанное программное обеспечение, так и зависимости температур, могут найти применение в ряде инженерных задач, где важным является прогнозирование температурного распределения в пористых материалах при различных условиях эксплуатации.

Ключевые слова: пористая среда, течение жидкости, теплообмен, трижды периодическая минимальная поверхность, гироид, метод конечных разностей

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INTRODUCTION

At present, an important task in theoretical and applied heat engineering is associated with the need to increase the energy efficiency of various heat engineering equipment [1] used in a wide range of industries. One of the methods for intensifying heat exchange consists in the use of porous materials in the design of heat exchange devices [2], catalysts [3], etc. Such porous materials may be used for increasing the heat exchange surface area and flow turbulization in the heat exchanger paths.

Most porous materials used in practice have a stochastic structure, i.e., the pores have a random shape and size. However, there are whole classes of porous materials having an ordered structure, such as lattice and honeycomb structures [4, 5], as well as structures based on thrice periodic minimal surfaces (TPMS). Such surfaces have been the subject of much research [6–11] due to their excellent strength properties at high porosity values, which enables the design of lightweight yet strong structures. One of the main advantages of porous materials having a TPMS structure consists in the possibility of varying their properties (thermophysical, mechanical, etc.) by changing the characteristic geometrical parameters of the minimal surfaces [8].

Despite the above-mentioned advantages, a number of difficulties can be encountered when using porous TPMS-materials in the design of heat engineering equipment due to the absence of mathematical models for describing the transfer processes (heat, mass, momentum). Thus, due to the large expenditures of time and resources entailed in the manufacture of prototypes and full-scale experiments to obtain the desired information, the mathematical modeling of heat and mass transfer in TPMS-based porous media becomes an important task.

Both numerical and analytical methods can be used to solve heat and mass transfer problems [12–14]. Analytical methods, such as the method of separation of variables and the method of integral transformations, may be used to obtain exact solutions in particular cases. However, such methods are applicable only to solving a limited number of problems within certain constraints and conditions. When solving complex problems (with nonlinear properties or complex geometry), numerical methods (finite difference method, finite element method, etc.) are often used in practice [15, 16] to find approximate solutions with a given accuracy.

In the present work, a one-dimensional problem of heat transfer of an incompressible fluid flowing in a porous medium having a gyroid-type TPMS structure is solved using the finite difference method.

TASK STATEMENT

The transfer of heat in a porous medium having a gyroid-based structure at a low fluid velocity characteristic of filtration flows is as described in [17, 18]. The schematic of the problem is shown in Fig. 1. Figure 1a shows a unit cell of the gyroid whose characteristic geometric parameters are expressed in terms of the wall thickness δ and length of the cube edge a in which the cell is inscribed. When varying these parameters, the porosity of the material changes to affect both the hydrodynamic characteristics of the flow and the intensity of heat transfer.

The porous medium through which the fluid flows is shown in Fig. 1b. The TPMS lattice is formed by copying the unit cell (Fig. 1a) in orthogonal directions of the Cartesian coordinate system with a period equal to a.

To describe the heat transfer in fluid flow through a porous medium, we will use the two-temperature model proposed by Wakao and Kagei [19]. The use of separate energy equations to describe each component of the porous medium (solid and fluid) for each phase two-temperature model provides a basis for a more accurate description of the heat transfer process, especially in the absence of local equilibrium.

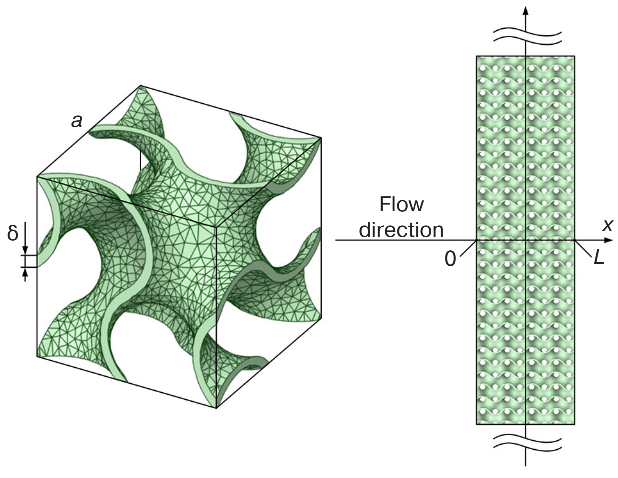

Fig. 1. Task diagram:
(a) gyroid unit cell; (b) porous TPMS medium.
x is a coordinate, L is a porous material thickness

The mathematical formulation of the problem, including basic differential equations, as well as initial and boundary conditions, is as follows:

$$(1-\phi)(\rho c_{\mathrm{p}})_{\mathrm{s}} \frac{\partial T_s(x,t)}{\partial t} = \\ = \lambda_{\mathrm{eff.s}} \frac{\partial^2 T_{\mathrm{s}}(x,t)}{\partial x^2} + \alpha_{\mathrm{s.f}}(T_{\mathrm{f}}(x,t) - T_{\mathrm{s}}(x,t)), \quad (1) \\ (0 < x < L, t > 0);$$

$$\phi(\rho c_{\text{p}})_{\text{f}}\left[\frac{\partial T_{\text{f}}(x,t)}{\partial t}+u\frac{\partial T_{\text{f}}(x,t)}{\partial x}\right]=$$

$$=\lambda_{\text{eff.f}}\frac{\partial^2 T_{\text{f}}(x,t)}{\partial x^2}+\alpha_{\text{s.f}}(T_{\text{s}}(x,t)-T_{\text{f}}(x,t)),\quad (0<x<L,\ t>0); \qquad (2)$$

$$T_{\text{s}}(x,0)=T_{\text{f}}(x,0)=T_0; \qquad (3)$$

$$-\lambda_{\text{s}}\left.\frac{\partial T_{\text{s}}(x,t)}{\partial x}\right|_{x=0}=\alpha_1(T_1-T_{\text{s}}(0,t)); \qquad (4)$$

$$\lambda_{\text{s}}\left.\frac{\partial T_{\text{s}}(x,t)}{\partial x}\right|_{x=L}=\alpha_2(T_2-T_{\text{s}}(L,t)); \qquad (5)$$

$$T_{\text{f}}(0,t)=T_1; \qquad (6)$$

$$\left.\frac{\partial T_{\text{f}}(x,t)}{\partial x}\right|_{x=L}=0, \qquad (7)$$

where t is the time; $T_{\text{s}}(x, t)$ and $T_{\text{f}}(x, t)$ are temperature functions of the solid frame and fluid, respectively; ϕ is the porosity; u is the fluid flow velocity; $(\rho c_{\text{p}})_{\text{s}}$ and $(\rho c_{\text{p}})_{\text{f}}$ are density and heat capacity of solid material and fluid, respectively; $\lambda_{\text{eff.s}}$ and $\lambda_{\text{eff.f}}$ are effective thermal conductivity of solid frame and fluid; T_0 is the initial temperature; T_1 and T_2 are temperatures of surrounding media before and after porous zone; $\alpha_{\text{s.f}}$ is the heat transfer coefficient between the fluid and the solid frame; α_1 is the heat transfer coefficient at the boundary between the solid frame and the environment at the point $x = 0$; α_2 is the heat transfer coefficient at the boundary between the solid frame and the environment at the point $x = L$.

Since the heat transfer problem is considered separately from the problem of mass transfer in a porous medium (in some particular cases, the flow dynamics may have little influence on the thermal effects), the equations of conservation of mass and momentum are not included in the system of equations. In addition, the properties of fluid and solid will change insignificantly in the range of considered temperatures. For this reason, the functions to describe the temperature dependence of the properties of the corresponding phases are also not included in the problem formulation.

The same temperature T_0 is taken as the initial condition for both solid and fluid. The boundary condition of the 3rd kind is set at the boundaries of the solid frame (at the points $x = 0$ and $x = L$). The fluid temperature at the point $x = 0$ has a constant value that corresponds to the ambient temperature at the entrance to the porous zone. Since there is no explicit data on the fluid temperature at the point $x = L$, we assume that the heat flux at the boundary is equal to zero.

NUMERICAL SOLUTION

The solution of the problem (1)–(7) is carried out by the finite difference method. For this purpose, let us introduce a spatiotemporal grid, where N is the number of steps in coordinate, and M is the number of steps in time:

$$x^i=i\Delta x,\quad i\in[0,N],$$
$$t^n=n\Delta t,\quad n\in[0,M],$$

where Δx, Δt are coordinate and time steps, respectively.

The finite-difference approximation of differential equations (1), (2) and boundary conditions (3)–(7) is as follows:

$$(1-\phi)(\rho c_{\text{p}})_{\text{s}}\left[\frac{T_{\text{s}}^{n+1,i}-T_{\text{s}}^{n,i}}{\Delta t}\right]= =\lambda_{\text{eff.s}}\left[\frac{T_{\text{s}}^{n,i-1}-2T_{\text{s}}^{n,i}+T_{\text{s}}^{n,i+1}}{\Delta x^2}\right]+\alpha_{\text{s.f}}(T_{\text{f}}^{n,i}-T_{\text{s}}^{n,i}); \qquad (8)$$

$$\phi(\rho c_{\text{p}})_{\text{f}}\left[\frac{T_{\text{f}}^{n+1,i}-T_{\text{f}}^{n,i}}{\Delta t}+u\frac{T_{\text{f}}^{n,i+1}-T_{\text{f}}^{n,i}}{\Delta x}\right]= =\lambda_{\text{eff.f}}\left[\frac{T_{\text{f}}^{n,i-1}-2T_{\text{f}}^{n,i}+T_{\text{f}}^{n,i+1}}{\Delta x^2}\right]+\alpha_{\text{s.f}}(T_{\text{s}}^{n,i}-T_{\text{f}}^{n,i}); \qquad (9)$$

$$T_{\text{s}}^{0,i}=T_{\text{f}}^{0,i}=T_0; \qquad (10)$$

$$-\lambda_{\text{s}}\frac{T_{\text{s}}^{n,1}-T_{\text{s}}^{n,0}}{\Delta x}=\alpha_1(T_1-T_{\text{s}}^{n,0}); \qquad (11)$$

$$\lambda_{\text{s}}\frac{T_{\text{s}}^{n,N}-T_{\text{s}}^{n,N-1}}{\Delta x}=\alpha_2(T_2-T_{\text{s}}^{n,N}); \qquad (12)$$

$$T_{\text{f}}^{0,i}=T_1; \qquad (13)$$

$$\frac{T_{\text{f}}^{n,N}-T_{\text{f}}^{n,N-1}}{\Delta x}=0. \qquad (14)$$

The task is solved according to an explicit scheme to obtain the values of temperatures T_{f} and T_{s} at each grid node at each time step. The explicit scheme was chosen due to its simple implementation and sufficient accuracy for the problem under consideration. At the same time, the stability of the solution is ensured by selecting the time step Δt depending on Δx in accordance with the Courant criterion.

To implement the algorithm for solving the problem by the finite difference method, a software program has been developed in the Python programming language. The graphical interface of the program, which is based on the Tkinter library is shown in Fig. 2.

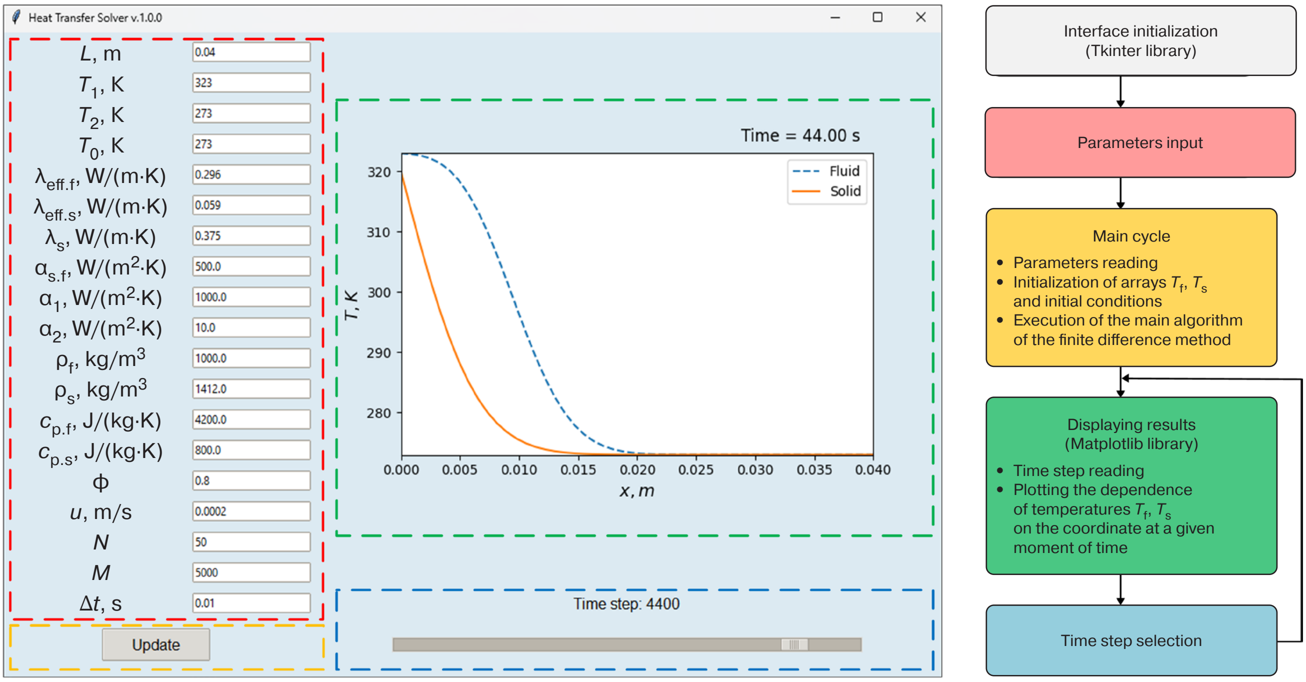

Fig. 2. Interface and algorithm of the developed program

```
for n in range(1, M):
    for i in range(1, N - 1):
        T_s[n, i] = ((1 / ((1 - phi) * p_s * co_s)) *
                    (((T_s[n - 1, i - 1] - 2 * T_s[n - 1, i] + T_s[n - 1, i + 1]) / dx ** 2) * k_es +
                    h_sf * (T_f[n - 1, i] - T_s[n - 1, i])) * dt + T_s[n - 1, i])

        T_f[n, i] = (((k_ef * ((T_f[n - 1, i - 1] - 2 * T_f[n - 1, i] + T_f[n - 1, i + 1]) / dx ** 2) +
                    h_sf * (T_s[n - 1, i] - T_f[n - 1, i])) / (phi * p_f * co_f)) - u * (T_f[n - 1, i] -
                    T_f[n - 1, i - 1]) / dx) * dt + T_f[n - 1, i]

    T_s[n, 0] = (h_ext1 * dx * T_ext1 + k_s * T_s[n, 1]) / (k_s + h_ext1 * dx)

    T_s[n, -1] = (k_s * T_s[n, -2] + h_ext2 * dx * T_ext2) / (k_s + h_ext2 * dx)

    T_f[n, -1] = T_f[n, -2]
```

Fig. 3. Finite-difference diagram in the Python programming language

The following stages can be distinguished in the main cycle of the program:

1. Input of the main parameters of the problem through special text fields. The program functionality allows both the thermophysical properties of the studied materials and the size of the spatiotemporal grid to be set manually.
2. Reading of parameters, initialization of arrays for T_f, T_s and execution of the main cycle of the finite difference method. The finite-difference scheme for the basic differential equations (8), (9) and boundary conditions (11), (12), (14) in the Python programming language is presented in Fig. 3.
3. Visualization of the results is provided by plotting the diagram of temperature change of the solid frame and fluid within the considered area. The Matplotlib library was used to display an interactive diagram of temperature change in a porous medium. The graph display can be controlled by means of a slider to change the time step and update the temperature field plots. This provides a means of studying the dynamics of the heat transfer process and the influence of various parameters on the temperature distribution.

The program was tested for correctness by comparing numerical solutions with known analytical solutions for particular cases of the problem. The source code as well as the executable file of the program are presented on the Mendeley Data[1] repository [20].

[1] https://data.mendeley.com/datasets/kcn33tr7sb/2. Accessed July 23, 2025.

RESULTS

Using the developed program, the problem of calculating heat transfer in a gyroid-based porous medium has been solved. Graphical dependences of the temperature change of the solid frame and the fluid were obtained. In particular, a graph of the dependence of the temperature of the fluid and the solid frame on the coordinate at different moments of time is presented (Fig. 4). This graph was obtained with the following constants and values of thermophysical properties of the solid frame and fluid, as well as boundary and initial conditions: $c_{p.s}$ = 800 J/(kg·K), ρ_s = 1412 kg/m^3, λ_s = 0.375 W/(m·K), $c_{p.f}$ = 4200 J/(kg·K), ρ_f = 1000 kg/m^3, λ_f = 0.6 W/(m·K), $\lambda_{eff.f}$ = 0.296 W/(m·K), $\lambda_{eff.s}$ = 0.059 W/(m·K), $\phi = 0.8$, $L = 0.04$ m, $T_0 = 273$ K, $T_1 = 323$ K, $T_2 = 273$ K, α_1 = 1000 W/(m^2·K), α_2 = 10 W/(m^2·K), $\alpha_{s.f}$ = 500 W/(m^2·K), $u = 0.0002$ m/s. The coefficients of effective thermal conductivity of solid and fluid phases in porous material with gyroid-based structure were determined according to the methodology presented in [21].

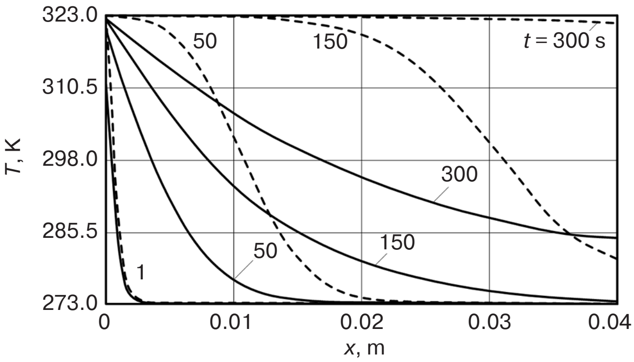

Fig. 4. Diagram of temperature distribution in a porous medium:
–––––– solid framework; – – – – fluid

From the diagram in Fig. 4 it can be seen that the temperatures of the fluid and the solid frame differ significantly over the entire range of the spatial coordinate under the given conditions. This emphasizes the importance of using a two-temperature model to accurately describe the temperature distribution in porous media.

The diagram in Fig. 5 demonstrates the change in temperature at different points of the porous material, namely, at the beginning, in the middle and at the end of the region under consideration. From the analysis of this graph, it can be concluded that under the given heat exchange conditions, the fluid temperature reaches the maximum value specified by the boundary condition (T_1 = 323 K). At $t > 300$ s, the temperature curves of the fluid and the solid framework begin to converge, which indicates a decrease in the absolute value of the temperature gradient and an approach to thermodynamic equilibrium in the system.

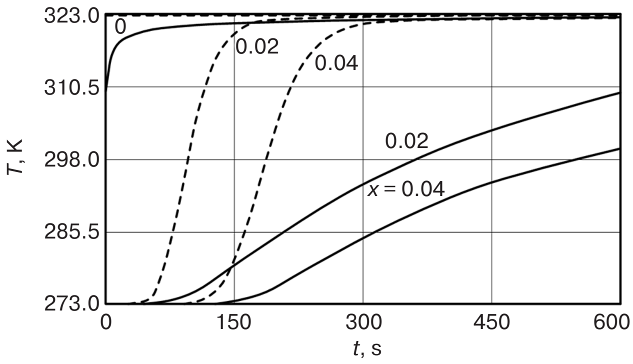

Fig. 5. Diagram of temperature change in a porous medium over time:
–––––– solid framework; – – – – fluid

Figure 6 shows a diagram of the heat flux distribution determined in accordance with the Newton–Richmann law ($|q| = \alpha_{s.f}(T_f - T_s)$) in a porous material along the coordinate. The pronounced peak in the heat flux distribution near the left boundary of the region at the early stages of the process ($t \approx 10$ s) is associated with a large absolute value of the temperature gradient caused by the adoption of the first-order boundary condition at the point $x = 0$. As time increases (10 s < t < 200 s), an increase in the heat flux is observed with a shift in the peak value to the right boundary of the region. The highest efficiency of the process observed at time $t \approx 200$ s is followed by a gradual decrease in the heat exchange intensity; at this point, the fluid temperature has the maximum possible value in the entire range of x (Fig. 3).

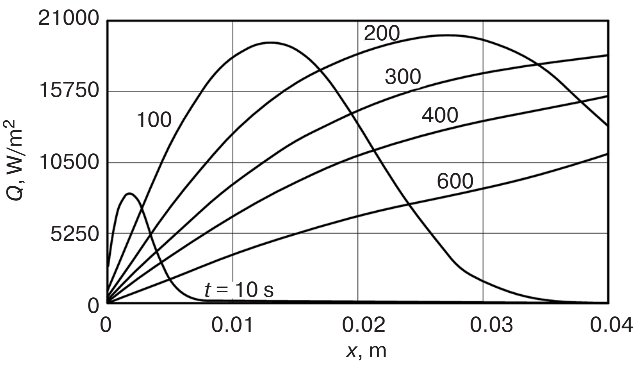

Fig. 6. Heat flow distribution diagram

CONCLUSIONS

We have presented a study of heat transfer in a porous medium with an ordered macrostructure based on a gyroid during filtration flow. For mathematical modeling of the process under study, a Wakao and Kagei two-temperature model was used.

In order to obtain the numerical solution of the heat transfer problem in a porous medium (1)–(7) using the finite difference method, a program in the Python programming language was developed. The functionality of the developed program provides for the construction of a graph of the temperature of a solid frame and fluid. The visualization of changes in temperature curves enabled by the built-in time step slider is important for studying the dynamics of the heat transfer process. As compared with direct modeling methods (for example, the finite element method in *ANSYS*[2]), the program represents a valuable means for quickly obtaining a solution to the problem, which is achieved due to the lack of need to construct computational geometry and a computational grid.

Porous materials having a TPMS-based (e.g. gyroid) structure can be used in the design of heat exchangers, catalysts and other devices where it is important to minimize the mass of the structure without compromising its strength and efficiency. The developed program can be used to model heat transfer in porous TPMS as a means for quickly predicting their behavior under real operating conditions. This has potential utility in solving a number of engineering problems in the energy, chemical industry, mechanical engineering, and other industries.

The results of this work can become the basis for further scientific research in the field of modeling heat and mass transfer in porous media. In the future, it is possible to expand the model and the program by taking into account the equation of fluid motion in a porous medium to determine both the velocity profile and pressure losses. An additional promising direction may consist in the development of similar models for other types of minimal surfaces to select structures having the desired heat-transfer-, strength-, and mass characteristics depending on the engineering tasks set.

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Authors' contributions

A.I. Popov—researching, writing the manuscript, developing of the program, visualization, and editing the article.

A.V. Eremin—task statement, funding acquisition, and editing the article.

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