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RESEARCH ARTICLE

On the identification of decentralized systems

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Abstract

Objectives. The work set out to consider the problem of identification of decentralized systems (DS). Due to the increasing complexity of systems and *a priori* uncertainty, it becomes necessary to identify appropriate approaches and methods. In particular, this concerns the parametric identifiability (PI) of DSs. This condition can be explained in terms of the complexity of the DS and the presence of internal relationships that complicates the process of parametric estimation. Thus it becomes necessary to propose an approach to PI based on meeting the conditions of the excitation constancy that takes subsystem relationships into account. A class of nonlinear DS is considered whose nonlinearities satisfy the sectoral condition. By taking this condition into account a more rational approach can be taken to the analysis of the DS properties. The work additionally set out to: (1) develop an approach to the analysis of the properties of adaptive identification systems (AIS), taking into account the requirements for the quality of the processes and synthesis of adaptive parametric algorithms; (2) investigate the possibility of using signal adaptation algorithms in DS identification systems and searching for a class of Lyapunov functions for the analysis of AIS with such algorithms; (3) model the proposed methods and algorithms in order to confirm the results obtained.

Methods. The research is based on adaptive identification, implicit identification representation, S-synchronization of a nonlinear system, sector condition, and Lyapunov vector function methods.

Results. The conditions for the parametric identifiability of the DS at the output and in the state space are obtained. A criterion is proposed for estimating the stability of an AIS with signal adaptation. Algorithms for adjusting the parameters of an AIS are synthesized. The exponential dissipativity of the evaluation system is confirmed. The influence of interrelations in the subsystems on the properties of the obtained parameter estimates is considered. An adaptive algorithm can be described by a dynamic matrix system if a functional constraint is imposed on the AIS. The proposed methods and algorithms are modeled to confirm their validity.

Conclusions. Considering the problem of identifying nonlinear DS under uncertainty, estimates have been obtained for the nonlinear part of the system satisfying the quadratic condition. The parametric identifiability of nonlinear DS has been confirmed. Algorithms for parametric and signal adaptive identification have been synthesized. A class of Lyapunov functions is proposed for evaluating the properties of an adaptive system with signal adaptation. The exponential dissipativity of processes in an adaptive system is demonstrated.

Keywords: adaptive identification, identifiability, exponential stability, excitation constancy, Lyapunov vector function, signal adaptation, sector condition

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НАУЧНАЯ СТАТЬЯ

Об идентификации децентрализованных систем

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Резюме

Цели. Рассматривается задача идентификации децентрализованных систем (ДС). Усложнение систем и априорная неопределенность требуют разработки соответствующих подходов и методов. Это касается, прежде всего, параметрической идентифицируемости (ПИ) ДС. Такое состояние можно объяснить сложностью ДС, наличием внутренних взаимосвязей, которые усложняли процесс параметрического оценивания. Необходимо предложить подход к ПИ, основанный на выполнении условия постоянства возбуждения и учете взаимосвязей в подсистемах. Рассматривается класс нелинейных ДС, нелинейности в которых удовлетворяют секторному условию. Учет этого условия позволяет обоснованно подойти к анализу свойств ДС, что является одной из целей данного исследования. Кроме того, целями работы являются: 1) разработка подхода к анализу свойств адаптивных систем идентификации (АСИ) с учетом требований к качеству процессов и синтез адаптивных параметрических алгоритмов; 2) исследование возможности применения алгоритмов сигнальной адаптации в системах идентификации ДС и поиск класса функций Ляпунова для анализа АСИ с такими алгоритмами; 3) моделирование предлагаемых методов и алгоритмов с целью подтверждения полученных результатов.

Методы. Применяются метод адаптивной идентификации, неявное идентификационное представление, S-синхронизация нелинейной системы, секторное условие, метод векторных функций Ляпунова.

Результаты. Получены условия ПИ ДС по выходу и в пространстве состояния. Предложен критерий, позволяющий получить оценки устойчивости АСИ с сигнальной адаптацией. Синтезированы алгоритмы настройки параметров АСИ. Доказана экспоненциальная диссипативность системы оценивания. Рассмотрено влияние взаимосвязей в подсистемах на свойства получаемых оценок параметров. Показано, что адаптивный алгоритм можно описать динамической матричной системой, если на АСИ наложить функциональное ограничение. Выполнено моделирование предлагаемых методов и алгоритмов.

Выводы. Рассмотрена проблема идентификации нелинейных ДС в условиях неопределенности. Получены оценки для нелинейной части системы, удовлетворяющей квадратичному условию. Доказана ПИ нелинейных ДС. Синтезированы алгоритмы параметрической и сигнальной адаптивной идентификации. Предложен класс функций Ляпунова для оценки свойств адаптивной системы с сигнальной адаптацией. Доказана экспоненциальная диссипативность процессов в адаптивной системе.

Ключевые слова: адаптивная идентификация, идентифицируемость, экспоненциальная устойчивость, постоянство возбуждения, векторная функция Ляпунова, сигнальная адаптация, секторное условие

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INTRODUCTION

Decentralized control systems are widely used to solve various problems. In most cases, the focus is on ensuring the stability and quality of a system. Typically, decentralized systems (DS) operate under conditions of insufficient *a priori* information [1–7]. Various approaches and methods are used for the management of DS. Identification methods based on adaptive observers [2, 5], neural networks [6], frequency methods [7], model-based approaches [8], correlation analysis, and least squares methods [9] are used to recover the missing information. In [10], a two-step identification procedure is implemented. After applying adaptive control regularities, various procedures are used to estimate their parameters [11–13].

The analysis shows that different identification procedures and methods can be applied depending on the subject area for which a DS is being developed. Existing parametric uncertainties are typically compensated by adjusting the parameters of the adaptive control law. However, retrospective identification methods do not always take into account the current state of the system. In addition, the properties of the proposed algorithms and the identifiability of the system itself, as well as the influence of links in the system, are not always taken into account. This is compensated by the use of multistep identification procedures.

The present work considers the problem of adaptive identification of nonlinear DS (NDS) with nonlinearities for which the quadratic condition is satisfied (Section 1). Section 2 analyzes the important problem of parametric identifiability (PI) of an NDS in both the state variable space and the output space. The influence of the properties of the information space of the system on the PI is analyzed. An approach for the synthesis of adaptive algorithms (AA) for identification based on the second Lyapunov method is proposed. Two classes of algorithms, parametric and signaling, are considered. The properties of the identification system are studied. The exponential dissipativity of the adaptive identification system (AIS) is proved.

1. PROBLEM STATEMENT

The system consisting of m interconnected subsystems is considered:

$$S_i : \begin{cases} \dot{\mathbf{X}}_i = \mathbf{A}_i \mathbf{X}_i + \mathbf{B}_i u_i + \sum\limits_{j=1, j\neq i}^{m} \bar{\mathbf{A}}_{ij} \mathbf{X}_j + \mathbf{F}_i\left(\mathbf{X}_i\right), \\ \mathbf{Y}_i = \mathbf{C}_i \mathbf{X}_i, \end{cases} \quad (1)$$

where $\mathbf{X}_i \in \mathbb{R}^{n_i}$, $\mathbf{Y}_i \in \mathbb{R}^{q_i}$ are state and output vectors of S_i subsystem, $u_i \in \mathbb{R}$ is control, $i = \overline{1,m}$, $\sum\limits_{i=1}^{m} n_i = n.$

Elements of matrices $\mathbf{A}_i \in \mathbb{R}^{n_i \times n_i}, \mathbf{B}_i \in \mathbb{R}^{n_i}, \bar{\mathbf{A}}_{ij} \in \mathbb{R}^{n_i \times n_j}$ are unknown, $\mathbf{C}_i \in \mathbb{R}^{q_i \times n_i}$. The matrix $\bar{\mathbf{A}}_{ij}$ reflects the mutual influence of the S_j subsystem. $\mathbf{F}_i\left(\mathbf{X}_i\right) \in \mathbb{R}^{n_i}$ takes into account the nonlinear state of the S_i subsystem, and the matrix $\mathbf{A}_i \in \mathscr{H}$ is Hurwitz (stable).

Assumption 1. $\mathbf{F}_i(\mathbf{X}_i)$ belongs to class

$$\mathscr{N}_{\mathbf{F}}\left(\pi_1, \pi_2\right) = \\ = \left\{\mathbf{F}(\mathbf{X}) \in \mathbb{R}^n : \pi_1 \mathbf{X} \le \mathbf{F}(\mathbf{X}) \le \pi_2 \mathbf{X},\ \mathbf{F}(0) = 0\right\} \quad (2)$$

and satisfies the quadratic condition

$$\left(\pi_2 \mathbf{X} - \mathbf{F}\left(\mathbf{X}\right)\right)^{\mathrm{T}} \left(\mathbf{F}\left(\mathbf{X}\right) - \pi_1 \mathbf{X}\right) \ge 0, \quad (3)$$

where $\pi_1 > 0$, $\pi_2 > 0$ are given numbers.

The mathematical model for system (1) is the following:

$$\dot{\hat{\mathbf{X}}}_i = \mathbf{K}_i\left(\hat{\mathbf{X}}_i - \mathbf{X}_i\right) + \hat{\mathbf{A}}_i \mathbf{X}_i + \hat{\mathbf{B}}_i u_i + \\ + \sum_{j=1, j\neq i}^{m} \hat{\bar{\mathbf{A}}}_{ij} \mathbf{X}_j + \hat{\mathbf{F}}_i\left(\mathbf{X}_i\right), \quad (4)$$

where $\mathbf{K}_i \in \mathscr{H}$ is the Hurwitz matrix with known parameters; $\hat{\mathbf{X}}_i$ is the model state vector; $\hat{\mathbf{A}}_i$, $\hat{\mathbf{B}}_i$, and $\hat{\bar{\mathbf{A}}}_{ij}$ are adjustable matrices of appropriate dimensions and $\hat{\mathbf{F}}_i$ is the *a priori* given nonlinear vector function with known structure. The matrix $\mathbf{K}_i$ is chosen according to control or identification system requirements.

It is necessary to find algorithms for estimating the parameters of model (4) based on the analysis of the set of measurement data $\mathbb{I}_{o,i} = \left\{\mathbf{X}_i(t), u_i(t), \mathbf{X}_j(t), t \in \mathbb{J} = \left[t_0, t_k\right]\right\}$ ($\mathbb{J}$ is the data acquisition interval; t is the time; and t_0, t_e are the beginning and end of the time interval) and satisfying assumption 1, such that

$$\lim_{t\to\infty} \left\|\hat{\mathbf{X}}_i(t) - \mathbf{X}_i(t)\right\| \le \delta_i,$$

where $\delta_i \ge 0$.

2. ON THE IDENTIFICATION OF THE S_i SUBSYSTEM

The possibility of parameter estimation depends on the identifiability of the S_i subsystems. It is known that in adaptive systems, the excitation constancy condition (marginal non-degeneracy) of elements $\mathbb{I}_{o,i}$ should be satisfied when implementing estimation algorithms.

Let the function $u_i(t)$ satisfy the excitation constancy condition:

$$\mathscr{EC}_{\underline{\alpha}_{u_i},\overline{\alpha}_{u_i}}: \underline{\alpha}_{u_i} \le u_i^2(t) \le \overline{\alpha}_{u_i} \quad \forall t \in \left[t_0, t_0+T\right], \tag{5}$$

where $\underline{\alpha}_{u_i}, \overline{\alpha}_{u_i}$ are positive numbers, $T > 0$. Further, the condition (5) is written as $u_i(t) \in \mathscr{EC}_{\underline{\alpha}_{u_i},\overline{\alpha}_{u_i}}$. If $u_i(t)$ does not have the excitation constancy property, then it is written as $u_i(t) \notin \mathscr{EC}_{\underline{\alpha}_{u_i},\overline{\alpha}_{u_i}}$ or $u_i(t) \notin \mathscr{EC}$.

Note 1. For the identification (identifiability) of nonlinear systems, condition (5) should be chosen taking into account the S-synchronizability of the nonlinear system [14]. In this case, property (5) is written in the following form:

$$\mathscr{EC}^{\mathrm{S}}_{\underline{\alpha}_{u_i},\overline{\alpha}_{u_i}}: \left(\underline{\alpha}_{u_i} \le u_i^2(t) \le \overline{\alpha}_{u_i}\right) \& \left(\Omega_{u_i}(\omega) \subseteq \Omega_{\mathrm{S}}(\omega)\right),$$

where $\Omega_{u_i}(\omega)$ is the set of frequencies u_i; $\Omega_{\mathrm{S}}(\omega)$ is the set of admissible frequencies of the input u_i, providing S-synchronizability of the system. In the following, the excitation constancy property is denoted by $\mathscr{EC}_{\underline{\alpha}_{u_i},\overline{\alpha}_{u_i}}$, noting that it guarantees $\mathscr{EC}^{\mathrm{S}}_{\underline{\alpha}_{u_i},\overline{\alpha}_{u_i}}$.

The model (4) is considered to obtain the identifiability conditions of S_i. For the error $\mathbf{E}_i = \hat{\mathbf{X}}_i - \mathbf{X}_i$, the following equation is derived:

$$\dot{\mathbf{E}}_i = \mathbf{K}_i\mathbf{E}_i + \Delta\mathbf{A}_i\mathbf{X}_i + \Delta\mathbf{B}_i u_i + \sum_{j=1, j\ne i}^{M} \Delta\overline{\mathbf{A}}_{ij}\mathbf{X}_j + \Delta\mathbf{F}_i\left(\mathbf{X}_i\right), \tag{6}$$

where $\Delta\mathbf{A}_i = \hat{\mathbf{A}}_i - \mathbf{A}_i$, $\Delta\mathbf{B}_i = \hat{\mathbf{B}}_i - \mathbf{B}_i$, $\Delta\overline{\mathbf{A}}_{ij} = \hat{\overline{\mathbf{A}}}_{ij} - \overline{\mathbf{A}}_{ij}$, $\Delta\mathbf{F}_i = \hat{\mathbf{F}}_i - \mathbf{F}_i$ are parametric discrepancies.

Lemma 1. If the nonlinearity $\mathbf{F}(\mathbf{X}) \in \mathbb{R}^n$ belongs to the class $\mathbf{F}(\mathbf{X}) \in \mathscr{N}_{\mathbf{F}}\left(\pi_1, \pi_2\right)$ and

$$\pi_1\mathbf{X} \le \mathbf{F}\left(\mathbf{X}\right) \le \pi_2\mathbf{X}, \tag{7}$$

then the following estimate holds for $\mathbf{F}(\mathbf{X})$:

$$\left\|\mathbf{F}\left(\mathbf{X}\right)\right\|^2 \le \eta\overline{\alpha}_{\mathbf{X}}, \tag{8}$$

where $\mathbf{X} \in \mathbb{R}^n$, $\pi_1 > 0$, $\pi_2 > 0$, $\eta = \eta\left(\pi_1, \pi_2\right) > 0$, $\overline{\alpha}_{\mathbf{X}} = \overline{\alpha}_{\mathbf{X}}\left(\mathbf{X}\right) > 0$.

Lemma 2. If the conditions of Lemma 1 are satisfied, then the following estimate holds for $\Delta\mathbf{F}(\mathbf{X})$:

$$\Delta\mathbf{F}^{\mathrm{T}}\Delta\mathbf{F} \le 2\eta\overline{\alpha}_{\mathbf{X}} + \delta_{\mathbf{F}},$$

where $\eta = 2\overline{\pi} + \pi^2$, $\overline{\pi} = \pi_1\pi_2$, $\pi = \pi_1 + \pi_2$, $\delta_{\mathbf{F}} > 0$.

Consider the system (6) and the Lyapunov function (LF) $V_i\left(\mathbf{E}_i\right) = 0.5\mathbf{E}_i^{\mathrm{T}}\mathbf{R}_i\mathbf{E}_i$, where $\mathbf{R}_i = \mathbf{R}_i^{\mathrm{T}} > 0$ is a positive definite symmetric matrix. We denote the matrix norm $\Delta\mathbf{A}_i$: $\left\|\Delta\mathbf{A}_i\right\| = \sqrt{\mathrm{Sp}(\Delta\mathbf{A}_i^{\mathrm{T}}\Delta\mathbf{A}_i)}$, $\left\|\Delta\overline{\mathbf{A}}_{ij}\right\| = \sqrt{\mathrm{Sp}(\Delta\mathbf{A}_{ij}^{\mathrm{T}}\Delta\mathbf{A}_{ij})}$, $\mathrm{Sp}(\cdot)$ is the trace of the matrix.

Theorem 1. Let 1) the matrix $\mathbf{A}_i \in \mathscr{H}$; 2) $\mathbf{X}_i(t) \in \mathscr{EC}_{\underline{\alpha}_{\mathbf{X}_i},\overline{\alpha}_{\mathbf{X}_i}}$, $\mathbf{X}_j(t) \in \mathscr{EC}_{\underline{\alpha}_{\mathbf{X}_j},\overline{\alpha}_{\mathbf{X}_j}}$ $u_i(t) \in \mathscr{EC}_{\underline{\alpha}_{u_i},\overline{\alpha}_{u_i}}$; and 3) the conditions of Lemmas 1, 2 are satisfied for $\mathbf{F}_i(\mathbf{X}_i)$. Then the subsystem (1) is identifiable on the set $\mathbb{I}_{\mathrm{o},i}$, provided that

$$2\left(\overline{\alpha}_{\mathbf{X}_i}\left\|\Delta\mathbf{A}_i\right\|^2 + \overline{\alpha}_{u_i}\left\|\Delta\mathbf{B}_i\right\|^2 + \sum_{j=1, j\ne i}^{m}\overline{\alpha}_{\mathbf{X}_j}\left\|\Delta\overline{\mathbf{A}}_{ij}\right\|^2 + 2\eta\overline{\alpha}_{\mathbf{X}_i} + \delta_{\mathbf{F}_i}\right) \le \overline{\lambda}_i V_i, \tag{9}$$

where $\overline{\lambda}_i = \lambda_i - k_i$, $\lambda_i > 0$ is the minimum eigenvalue of the matrix $\mathbf{Q}_i$, $k_i > 0$, $\mathbf{K}_i\mathbf{R}_i + \mathbf{K}_i^{\mathrm{T}}\mathbf{R}_i = -\mathbf{Q}_i$, $\mathbf{Q}_i$ is a positive symmetric matrix, $\eta = 2\overline{\pi} + \pi^2$, $\pi = \pi_1 + \pi_2$, $\overline{\pi} = \pi_1\pi_2$, $\delta_{\mathbf{F}_i} \ge 0$.

If the conditions of Theorem 1 are satisfied, then the S_i subsystem is called identifiable on the set $\mathbb{I}_{\mathrm{o}}$ or $\mathscr{PI}_{\mathbf{X}_i}$-identifiable.

We consider the identifiability of the subsystem S_i on the set "Input–Output":

$$\mathbb{I}_{\mathrm{o},\mathbf{Y}_i} = \left\{\mathbf{Y}_i(t),\, u_i(t),\, \mathbf{Y}_j(t),\, t \in \mathbb{J} = \left[t_0, t_k\right]\right\}.$$

For the S_i subsystem, the following representation is true:

$$\dot{\mathbf{Y}}_i = \mathbf{A}_{\#,i}\mathbf{Y}_i + \mathbf{B}_{\#,i}u_i + \sum_{j=1, j\ne i}^{m}\overline{\mathbf{A}}_{\#,ij}\mathbf{Y}_j + \tilde{\mathbf{C}}_i^{\#}\mathbf{F}_i\left(\tilde{\mathbf{C}}_i\mathbf{Y}_i\right), \tag{10}$$

where $\tilde{\mathbf{C}}_i = \left(\mathbf{C}_i^{\mathrm{T}}\mathbf{C}_i\right)^{\#}\mathbf{C}_i^{\mathrm{T}}$, $\mathbf{A}_{\#,i} = \tilde{\mathbf{C}}_i^{\#}\mathbf{A}_i\tilde{\mathbf{C}}_i$, $\mathbf{B}_{\#,i} = \tilde{\mathbf{C}}_i^{\#}\mathbf{B}_i$, $\overline{\mathbf{A}}_{\#,ij} = \tilde{\mathbf{C}}_i^{\#}\overline{\mathbf{A}}_{ij}\tilde{\mathbf{C}}_j$, # is the sign of the matrix pseudoinversion.

Taking into account the specificity of equation (10), the model for (10) has a similar structure to (4). We introduce the error $\mathbf{E}_{\#,i} = \hat{\mathbf{Y}}_i - \mathbf{Y}_i$ and LF $V_{\#,i}\left(\mathbf{E}_{\#,i}\right) = 0.5\mathbf{E}_{\#,i}^{\mathrm{T}}\mathbf{R}_i\mathbf{E}_{\#,i}$.

Theorem 2. Let: 1) the matrix $\mathbf{A}_{\#,i} \in \mathscr{H}$; 2) $\mathbf{Y}_i(t) \in \mathscr{EC}_{\underline{\alpha}_{\mathbf{Y}_i},\overline{\alpha}_{\mathbf{Y}_i}}$, $\mathbf{Y}_j(t) \in \mathscr{EC}_{\underline{\alpha}_{\mathbf{Y}_j},\overline{\alpha}_{\mathbf{Y}_j}}$ $u_i(t) \in \mathscr{EC}_{\underline{\alpha}_{u_i},\overline{\alpha}_{u_i}}$; 3) the conditions of Lemmas 1, 2 are satisfied for $\mathbf{F}_i(\mathbf{X}_i)$;

and 4) the S_i subsystem is observable. Then the subsystem (1) is identifiable on the set $\mathbb{I}_{o,\mathbf{Y}_i}$, provided that

$$2\left(\bar{\alpha}_{\mathbf{Y_i}}\left\|\Delta\mathbf{A}_{\#,i}\right\|^2+\bar{\alpha}_{u_i}\left\|\Delta\mathbf{B}_{\#.i}\right\|^2+\right.$$
$$\left.+\sum_{j=1,j\neq i}^{m}\bar{\alpha}_{\mathbf{Y}_j}\left\|\Delta\bar{\mathbf{A}}_{\#,ij}\right\|^2+2\eta\bar{\alpha}_{\mathbf{Y}_i}+\delta_{\mathbf{F}_i}\right)\leq\bar{\lambda}_{\#,i}V_{\#,i},$$

where $\Delta\mathbf{A}_{\#,i}=\hat{\mathbf{A}}_{\#,i}-\mathbf{A}_{\#,i}$, $\Delta\mathbf{B}_{\#,i}=\hat{\mathbf{B}}_{\#,i}-\mathbf{B}_{\#,i}$, $\Delta\bar{\mathbf{A}}_{\#,ij}=\hat{\bar{\mathbf{A}}}_{\#,ij}-\bar{\mathbf{A}}_{\#,ij}$, $\Delta\mathbf{F}_i=\hat{\mathbf{F}}_i-F_i$, $\bar{\lambda}_{\#,i}>0$.

The proof of Theorems 1 and 2 is reduced to ensuring the non-positivity of the LF derivatives.

3. SYNTHESIS OF ADAPTATION ALGORITHMS

We consider the LF $V_i(\mathbf{E}_i)=0.5\mathbf{E}_i^{\mathrm{T}}\mathbf{R}_i\mathbf{E}_i$ whose derivative on the motions (6) has the following form:

$$\dot{V}_i=-\mathbf{E}_i^{\mathrm{T}}\mathbf{Q}_i\mathbf{E}_i+\mathbf{E}_i^{\mathrm{T}}\mathbf{R}_i\times$$
$$\times\left(\Delta\mathbf{A}_i\mathbf{X}_i+\Delta\mathbf{B}_iu_i+\sum_{j=1,j\neq i}^{m}\Delta\bar{\mathbf{A}}_{ij}\mathbf{X}_j+\Delta\mathbf{F}_i\left(\mathbf{X}_i\right)\right).$$

We require that in the process of adaptation the following functional constraint is satisfied:

$$\dot{V}_i\leq-\chi_i\left(\Delta\mathbf{A}_i,\Delta\bar{\mathbf{A}}_i,\Delta\mathbf{B}_i,\Delta\mathbf{F}_i\right),$$

where

$$\chi\left(\Delta\mathbf{A}_i,\Delta\bar{\mathbf{A}}_i,\Delta\mathbf{B}_i,\Delta\mathbf{F}_i\right)=0.5\left(\varphi_{\mathbf{A}_i}(t)\left\|\Delta\mathbf{A}_i(t)\right\|^2+\right.$$
$$\left.+\varphi_{\bar{\mathbf{A}}_i}(t)\left\|\Delta\bar{\mathbf{A}}_i(t)\right\|^2+\varphi_{\mathbf{B}_i}(t)\left\|\Delta\mathbf{B}_i\right\|^2+\varphi_{\mathbf{F}_i}(t)\left\|\Delta\mathbf{F}_i(t)\right\|^2\right),$$

$\varphi_{\mathbf{A}_i}(t)$, $\varphi_{\bar{\mathbf{A}}_i}(t)$, $\varphi_{\mathbf{B}_i}(t)$, $\varphi_{\mathbf{F}_i}(t)$ are bounded non-negative functions. Then

$$\eta_i=\dot{V}_i+\chi_i=-\mathbf{E}_i^{\mathrm{T}}\mathbf{Q}_i\mathbf{E}_i+\chi\left(\Delta\mathbf{A}_i,\Delta\bar{\mathbf{A}}_i,\Delta\mathbf{F}_i\right)+$$
$$+\mathbf{E}_i^{\mathrm{T}}\mathbf{R}_i\left(\Delta\mathbf{A}_i\mathbf{X}_i+\Delta\mathbf{B}_iu_i+\sum_{j=1,j\neq i}^{m}\Delta\bar{\mathbf{A}}_{ij}\mathbf{X}_j+\Delta\mathbf{F}_i\left(\mathbf{X}_i\right)\right).\quad(11)$$

From the condition $\eta\leq0$, the AA is derived, as follows:

$$\Delta\dot{\mathbf{A}}_i=-\mathbf{\Gamma}_{\mathbf{A}_i}\left(\varphi_{\mathbf{A}_i}\Delta\mathbf{A}_i+\mathbf{E}_i\mathbf{R}_i\mathbf{X}_i^{\mathrm{T}}\right),$$
$$\Delta\dot{\bar{\mathbf{A}}}_{ij}=-\mathbf{\Gamma}_{\bar{\mathbf{A}}_{ij}}\left(\varphi_{\bar{\mathbf{A}}_{ij}}\Delta\bar{\mathbf{A}}_{ij}+\mathbf{E}_i\mathbf{R}_i\mathbf{X}_j^{\mathrm{T}}\right),\quad(12)$$
$$\Delta\dot{\mathbf{B}}_i=-\mathbf{\Gamma}_{\mathbf{B}_i}\left(\varphi_{B_{ij}}\Delta\mathbf{B}_i+\mathbf{R}_i\mathbf{E}_iu_i\right),$$

where $\mathbf{\Gamma}_{\mathbf{A}_i}$, $\mathbf{\Gamma}_{\bar{\mathbf{A}}_{ij}}$, and $\mathbf{\Gamma}_{\mathbf{B}_i}$ are diagonal matrices of corresponding dimensions with positive diagonal elements. This guarantees the stability of the adaptation processes.

3.1. Parametric algorithm for $\mathbf{F}_i$

Parametric and signaling algorithms can be used to estimate $\mathbf{F}_i$. We consider the parametric approach [15].

Assumption 2. The function $\mathbf{F}_i(\mathbf{X}_i)$ is defined on the following set:

$$\mathbf{F}_i\in\mathbb{F}_{\mathbf{F}_i}=\left\{\mathbf{F}_i\in\mathscr{N}_{\mathbf{F}}\left(\pi_1,\pi_2\right):\mathbf{F}_i(\mathbf{X}_i)=\right.$$
$$=\tilde{\mathbf{F}}_i^{\mathrm{T}}\left(\mathbf{X}_i,\mathbf{N}_{i,1}\right)\mathbf{N}_{i,2},\quad(13)$$
$$\left.\mathbf{N}_i=\left[\mathbf{N}_{i,1}^{\mathrm{T}},\mathbf{N}_{i,2}^{\mathrm{T}}\right]^{\mathrm{T}},\ \mathbf{N}_i\in\mathbb{N}_{i,\mathrm{a}}\right\},$$

where $\mathbb{N}_{i,\mathrm{a}}=\left\{\mathbf{N_i}\in\mathbb{R}^n:\underline{\mathbf{N}}_i\leq\mathbf{N}_i\leq\bar{\mathbf{N}}_i\right\}$ is the *a posteriori* formed parametric domain for $\mathbf{F}_i$; $\underline{\mathbf{N}},\bar{\mathbf{N}}$ are the vector bounds for $\mathbf{N}$, understood as $\underline{n}_i\in\underline{\mathbf{N}}_i$, $\bar{n}_i\in\bar{\mathbf{N}}_i$; $\mathbf{N}_{i,\,1}$ is the *a priori* known vector of nonlinearity parameters, $\mathbf{N}_{i,\,2}$ is the *a priori* unknown set of parameters, which is further considered as a vector to be estimated. Some elements of $\underline{\mathbf{N}}_i$, $\bar{\mathbf{N}}_i$ may be unknown. The structure $\tilde{\mathbf{F}}_i\left(\mathbf{X}_i,\mathbf{N}_{i,1}\right)$ is formed *a priori* taking into account the known vector $\mathbf{N}_{i,\,1}$.

It follows from (13) that the estimate of the vector function $\mathbf{F}_i(\mathbf{X}_i)$ in the identification step is sought in the following form:

$$\hat{\mathbf{F}}_i(X_i)=\tilde{\mathbf{F}}_i^{\mathrm{T}}\left(\mathbf{X}_i,\hat{\mathbf{N}}_{i,1}\right)\hat{\mathbf{N}}_{i,2},\quad(14)$$

where $\hat{\mathbf{N}}_{i,1}\in\mathbb{R}^{n_{i,1}}$ is the *a priori* estimate of known parameters, $\hat{\mathbf{N}}_{i,2}\in\mathbb{R}^{n_{i,2}}$ is the vector of adjustable parameters.

We assume $\mathbb{N}_i=\mathbb{N}_{i,1}\cup\mathbb{N}_{i,2}$. The set $\mathbb{N}_{i,1}\subset\mathbb{R}^{n_{i,1}}$ $\left(\mathbf{N}_{i,1}\in\mathbb{N}_{i,1}\right)$ contains elements that are not available for tuning with AA. The elements $\mathbf{N}_{i,2}\in\mathbb{N}_{i,2}\subset\mathbb{R}^{n_{i,2}}$ are estimated in the identification step. The matrix $\tilde{\mathbf{F}}_i\left(y,\hat{\mathbf{N}}_{i,1}\right)$ is formed in the step of structural synthesis (analysis) of the system. The representation (14) is a consequence of the proposed parametric concept for $\mathbf{F}_i(\mathbf{X}_i)$.

Note 2. The vector $\hat{\mathbf{N}}_{i,1}$ can be tuned iteratively based on the forcing algorithm [15].

Since $\Delta\mathbf{F}=\tilde{\mathbf{F}}_i^{\mathrm{T}}\left(y,\hat{\mathbf{N}}_{i,1}\right)\hat{\mathbf{N}}_{i,2}-\mathbf{F}_i(\mathbf{X}_i)$, then from the condition $\dot{V}_i\leq0$ we obtain AA for $\hat{\mathbf{N}}_{i,2}$, as follows:

$$\dot{\hat{\mathbf{N}}}_{i,2} = -\mathbf{\Gamma}_{\mathbf{F}_i}\left(\varphi_{F_i}\dot{\hat{\mathbf{N}}}_{i,2} + \tilde{\mathbf{F}}_i^{\mathrm{T}}\mathbf{R}_i\mathbf{E}_i\left(\mathbf{X}_i, \hat{\mathbf{N}}_{i,1}\right)\right), \quad (15)$$

where $\mathbf{\Gamma}_{\mathbf{F}_i}$ is a diagonal matrix with positive diagonal elements.

In the following, the system (6), (11), (15) will be denoted as AS_{AF_i} to simplify the references.

3.2. Signaling algorithm

We consider the following model:

$$\dot{\hat{\mathbf{X}}}_i = \mathbf{K}_i\left(\hat{\mathbf{X}}_i - \mathbf{X}_i\right) + \hat{\mathbf{A}}_i\mathbf{X}_i + \hat{\mathbf{B}}_i u_i + \sum_{j=1, j\neq i}^{m} \hat{\bar{\mathbf{A}}}_{ij}\mathbf{X}_j + \mathbf{U}_i \quad (16)$$

and the error equation:

$$\dot{\mathbf{E}}_i = \mathbf{K}_i\mathbf{E}_i + \Delta\mathbf{A}_i\mathbf{X}_i + \Delta\mathbf{B}_i u_i + \sum_{j=1, j\neq i}^{m} \Delta\bar{\mathbf{A}}_{ij}\mathbf{X}_j + \mathbf{U}_i\left(\mathbf{X}_i\right) - \mathbf{F}_i\left(\mathbf{X}_i\right). \quad (17)$$

The LF derivative has the following form:

$$\dot{V}_i = -\mathbf{E}_i^{\mathrm{T}}\mathbf{Q}_i\mathbf{E}_i + \mathbf{E}_i^{\mathrm{T}}\mathbf{R}_i \times \left(\Delta\mathbf{A}_i\mathbf{X}_i + \Delta\mathbf{B}_i u_i + \sum_{j=1, j\neq i}^{m} \Delta\bar{\mathbf{A}}_{ij}\mathbf{X}_j + \mathbf{U}_i - \mathbf{F}_i\right), \quad (18)$$

and the algorithm for $\mathbf{U}_i$ is as follows:

$$\mathbf{U}_i = -\mathbf{D}_i\mathbf{R}_i\mathbf{E}_i, \quad (19)$$

where $\mathbf{D}_i \in \mathbb{R}^{n_i \times n_i}$ is the diagonal matrix with positive elements.

Since $\mathbf{F}_i\left(\mathbf{X}_i\right) \in \mathscr{N}_{\mathbf{F}}\left(\pi_1, \pi_2\right)$, Lemma 1 holds for $\mathbf{F}_i(\mathbf{X}_i)$.

We use the approach described in [16]. Choose the elements of the matrix $\mathbf{D}_i$ from the condition $\left\|\mathbf{D}_i\right\| \geq d_i \geq \eta\bar{\alpha}_{X_i}$. Then we obtain the following for (18):

$$\dot{V}_i = -\mathbf{E}_i^{\mathrm{T}}\mathbf{Q}_i\mathbf{E}_i + \mathbf{E}_i^{\mathrm{T}}\mathbf{R}_i\left(\Delta\mathbf{A}_i\mathbf{X}_i + \Delta\mathbf{B}_i u_i + \sum_{j=1, j\neq i}^{m} \Delta\bar{\mathbf{A}}_{ij}\mathbf{X}_j - \mathbf{D}_i\mathbf{R}_i\mathbf{E}_i - \mathbf{F}_i\right). \quad (20)$$

Since $\mathbf{E}_i^{\mathrm{T}}\mathbf{R}_i\mathbf{D}_i\mathbf{R}_i\mathbf{E}_i \geq 2\eta\underline{\alpha}_{\mathbf{X}_i}\underline{\lambda}_{\mathbf{R}_i}V_i$, where $\underline{\lambda}_{\mathbf{R}_i}$ is the smallest eigenvalue of the matrix $\mathbf{R}_i$, then

$$\dot{V}_i \leq -\sigma V_i + \mathbf{E}_i^{\mathrm{T}}\mathbf{R}_i \times \left(\Delta\mathbf{A}_i\mathbf{X}_i + \Delta\mathbf{B}_i u_i + \sum_{j=1, j\neq i}^{m} \Delta\tilde{\mathbf{A}}_{ij}\mathbf{X}_j\right), \quad (21)$$

where $\sigma = \lambda_{\mathbf{Q}_i} + 2\eta\underline{\alpha}_{\mathbf{X}_i}\underline{\lambda}_{\mathbf{R}_i}$.

After simple transformations we obtain the following:

$$\dot{V}_i \leq -\sigma V_i + 2\left(\bar{\alpha}_{X_i}\left\|\Delta\mathbf{A}_i\right\|^2 + \bar{\alpha}_{u_i}\left\|\Delta\mathbf{B}_i\right\|^2 + \sum_{j=1, j\neq i}^{m} \bar{\alpha}_{X_j}\left\|\Delta\bar{\mathbf{A}}_{ij}\right\|^2\right).$$

If the condition

$$2\left(\bar{\alpha}_{\mathbf{X}_i}\left\|\Delta\mathbf{A}_i\right\|^2 + \bar{\alpha}_{u_i}\left\|\Delta\mathbf{B}_i\right\|^2 + \sum_{j=1, j\neq i}^{m} \bar{\alpha}_{\mathbf{X}_j}\left\|\Delta\tilde{\mathbf{A}}_{ij}\right\|^2 + \eta\bar{\alpha}_{i,\mathbf{X}_i}\right) \leq \sigma V_i$$

is satisfied, then the system (16) at $\left(u_i(t), \mathbf{X}_i(t), \mathbf{X}_j(t)\right) \in \mathscr{E}\mathscr{C}$ is parametrically identifiable on the set $\left\{u_i(t), \mathbf{X}_i(t), \mathbf{X}_j(t)\right\}$ on the class of algorithms (12), (19).

For ease of references, the system (6), (12), (19) is denoted below as AS_{AS_i}.

4. ADAPTIVE ALGORITHM AS A DYNAMIC SYSTEM

The considered approach to AA synthesis is based on meeting stability properties and is typical of AIS. Attempts to obtain more complex AAs involve imposing constraints on the system. This approach requires certain knowledge and does not always lead to workable algorithms. Below, a method is proposed that takes into account a number of requirements for AIS.

Consider the synthesis of AA using the matrix $\mathbf{A}_i$ as an example. We apply LF, as follows:

$$V_i^{\Delta}\left(\mathbf{E}_i, \Delta\mathbf{A}_i, \Delta\dot{\mathbf{A}}_i\right) = 0.5\mathbf{E}_i^{\mathrm{T}}\mathbf{R}_i\mathbf{E}_i + 0.5\mathrm{Sp}\left(\Delta\mathbf{A}_i^{\mathrm{T}}\Delta\dot{\mathbf{A}}_i\right).$$

We require that the functional constraint is satisfied:

$$\dot{V}_i^{\Delta} \leq -\chi_{\Delta} = -\alpha_{\Delta}\mathrm{Sp}\left(\Delta\mathbf{A}_i^{\mathrm{T}}\Delta\mathbf{A}_i\right).$$

Then for $\eta_{\Delta} = \dot{V}_i^{\Delta} + \alpha_{\Delta}\mathrm{Sp}\left(\Delta A_i^{\mathrm{T}}\Delta A_i\right)$ on the motions (6) we obtain the following:

$$\eta_\Delta = -\mathbf{E}_i^{\mathrm{T}}\mathbf{Q}_i\mathbf{E}_i + \mathbf{E}_i^{\mathrm{T}}\mathbf{R}_i\Delta\mathbf{A}_i\mathbf{X}_i + \mathrm{Sp}\left(\Delta\dot{\mathbf{A}}_i^{\mathrm{T}}\Delta\dot{\mathbf{A}}_i\right) + $$
$$+ \mathrm{Sp}\left(\Delta\mathbf{A}_i^{\mathrm{T}}\Delta\ddot{\mathbf{A}}_i\right) + \alpha_\Delta\mathrm{Sp}\left(\Delta\mathbf{A}_i^{\mathrm{T}}\Delta\mathbf{A}_i\right).$$

Hence AA follows for $\Delta\mathbf{A}_i$:

$$\Delta\ddot{\mathbf{A}}_i = -\Delta\dot{\mathbf{A}}_i - \alpha_\Delta\Delta\mathbf{A}_i - \boldsymbol{\Gamma}_{\mathbf{A}_i}\mathbf{E}_i\mathbf{R}_i\mathbf{X}_i^{\mathrm{T}}.$$

Let $\Delta\mathbf{A}_i = \mathbf{Z}_1$. Then

$$S_{\mathrm{AA}}: \begin{cases} \dot{\mathbf{Z}}_1 = \mathbf{Z}_2, \\ \dot{\mathbf{Z}}_2 = -\alpha_\Delta\mathbf{Z}_1 - \mathbf{Z}_2 - \boldsymbol{\Gamma}_{\mathbf{A}_i}\mathbf{E}_i\mathbf{R}_i\mathbf{X}_i^{\mathrm{T}}. \end{cases}$$

Thus, we obtain the AA whose state is described by the S_{AA} system, provided that the functional constraint $\chi_\Delta \geq 0$ is imposed on the AIS.

Note 3. Theoretically, different classes of algorithms can be obtained from this approach. Their implementation requires further structuring, taking into account the operating conditions of the loop system.

5. ADAPTIVE SYSTEM PROPERTIES

5.1. The AS_{AF_i} system

We consider the AS_{AF_i} and AS_{AF_j} systems along with LF $V_i\left(\mathbf{E}_i\right) = 0.5\mathbf{E}_i^{\mathrm{T}}\mathbf{R}_i\mathbf{E}_i$,

$$V_{\Delta,i} = 0.5\cdot\mathrm{Sp}\left(\Delta\mathbf{A}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{-1}\Delta\mathbf{A}_i\right) + $$
$$+ 0.5\sum_{j=1}^{m}\mathrm{Sp}\left(\Delta\bar{\mathbf{A}}_{ij}^{\mathrm{T}}\boldsymbol{\Gamma}_{ij}^{-1}\Delta\bar{\mathbf{A}}_{ij}\right) + \quad (22)$$
$$+ 0.5\Delta\mathbf{B}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{-1}\Delta\mathbf{B}_i + 0.5\Delta\mathbf{N}_{i,2}^{\mathrm{T}}\boldsymbol{\Gamma}_{\mathbf{N}_{i,2}}^{-1}\Delta\mathbf{N}_{\mathbf{N}_{i,2}}.$$

Theorem 3. Let: 1) there exist LFs $V_i(t)$ and $V_{\Delta,i}(t)$ admitting an infinitesimal upper limit; 2) $\mathbf{A}_i \in \mathscr{H}$; 3) $\bar{\mathbf{A}}_{ij}$ guarantee the stability of the S_i subsystem; 4) $\mathbf{X}_i(t) \in \mathscr{EC}_{\underline{\alpha}_{\mathbf{X}_i},\bar{\alpha}_{\mathbf{X}_i}}$, $\mathbf{X}_j(t) \in \mathscr{EC}_{\underline{\alpha}_{\mathbf{X}_j},\bar{\alpha}_{\mathbf{X}_j}}$, $u_i(t) \in \mathscr{EC}_{\underline{\alpha}_{u_i},\bar{\alpha}_{u_i}}$; 5) $\mathbf{F}_i \in \mathbb{F}_{\mathbf{F}_i}$, where $\mathbb{F}_{\mathbf{F}_i}$ is the set of functions belonging to $\mathscr{N}_{\mathbf{F}}$; 6) the system of inequalities is valid for the vector LF $\mathbf{W}_i = \left[V_i, V_{\Delta,i}\right]^{\mathrm{T}}$:

$$\begin{bmatrix} \dot{V}_i \\ \dot{V}_{\Delta,i} \end{bmatrix} \leq \underbrace{\begin{bmatrix} -\mu_i & \frac{2}{\mu_i}\kappa_i \\ \vartheta_{\chi\alpha,i}\rho_i & -\beta_{\lambda\chi,i} \end{bmatrix}}_{\mathbf{A}_{\mathbf{W}_i}} \begin{bmatrix} V \\ V_{\Delta,i} \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{2}{\mu_i}\upsilon_{\tilde{\mathbf{F}}_i} \\ 0.5\delta_{\mathbf{N},i} \end{bmatrix}}_{\mathbf{L}_i}, \quad (23)$$

where $\mu_i, \kappa_i, \vartheta_{\chi\alpha,i}, \rho_i, \beta_{\lambda\chi,i}, \upsilon_{\tilde{F}_i}, \delta_{N,i}$ are positive numbers depending on the parameters of the S_i subsystem and the characteristics of the information set; and 7) the upper solution for $\mathbf{W}_i$ satisfies the system of equations $\dot{\mathbf{S}}_{\mathbf{W}_i} = A_{\mathbf{W}_i}\mathbf{S}_{\mathbf{W}_i} + \mathbf{L}_i$, provided that

$$w_\rho(t) \leq s_\rho(t) \quad \forall\left(t \geq t_0\right) \& \left(w_\rho\left(t_0\right) \leq s_\rho\left(t_0\right)\right),$$

$\rho = e, i; \Delta, i$ for elements $\mathbf{W}_i$, $w_\rho \in \mathbf{W}_i$, $s_\rho \in \mathbf{S}_{\mathbf{W}_i}$. Then the system is exponentially dissipative with the following estimate:

$$\mathbf{W}_i(t) \leq \mathrm{e}^{\mathbf{A}_{\mathbf{W}_i}(t-t_0)}\mathbf{S}_{\mathbf{W}_i}(t_0) + \int_{t_0}^{t}\mathrm{e}^{\mathbf{A}_{\mathbf{W}_i}(t-\tau)}\mathbf{L}_i d\tau, \quad (24)$$

if

$$\mu_i^2\beta_{\lambda\chi,i} \geq 2\kappa_i\vartheta_{\chi\alpha,i}\rho_i. \quad (25)$$

The limiting properties of $AS_{F,i}$ are determined by the elements of the vector $\mathbf{L}_i$. If the structure and the parameters of the vector $\hat{\mathbf{N}}_{i,1}$ are known, then the subsystem is exponentially stable, provided that $\tilde{\mathbf{F}}_i \in \mathscr{EC}$.

We consider the $AS_{F,i,j}$ system consisting of the $AS_{F,i}$ и $AS_{F,j}$ subsystems. For $AS_{F,i,j}$, the following system of inequalities applies:

$$\begin{bmatrix} \dot{\mathbf{W}}_i \\ \dot{\mathbf{W}}_j \end{bmatrix} \leq \begin{bmatrix} \mathbf{A}_{\mathbf{W}_i} & 0 \\ 0 & \mathbf{A}_{\mathbf{W}_j} \end{bmatrix} \begin{bmatrix} \mathbf{W}_i \\ \mathbf{W}_j \end{bmatrix} + \begin{bmatrix} \mathbf{L}_i \\ \mathbf{L}_j \end{bmatrix}, \quad (26)$$

where $\mathbf{A}_{\mathbf{W}_i}$ and $\mathbf{L}_i$ are of the form (23).

The conditions for exponential stability are as follows:

$$\mu_i^2\beta_{\lambda\chi,i} \geq 2\kappa_i\vartheta_{\chi\alpha,i}\rho_i, \; \mu_i^2\beta_{\lambda\chi,j} \geq 2\kappa_j\vartheta_{\chi\alpha,j}\rho_j.$$

5.2. The AS_{AS_i} system

We consider the LFs $V_i(t)$ and

$$V_{\Delta_S,i} = 0.5\cdot\mathrm{Sp}\left(\Delta\mathbf{A}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{-1}\Delta\mathbf{A}_i\right) + $$
$$+ 0.5\sum_{j=1}^{m}\mathrm{Sp}\left(\Delta\bar{\mathbf{A}}_{ij}^{\mathrm{T}}\boldsymbol{\Gamma}_{ij}^{-1}\Delta\bar{\mathbf{A}}_{ij}\right) + 0.5\Delta\mathbf{B}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{-1}\Delta\mathbf{B}_i + $$
$$+ 0.5\int_{t_0}^{t}\Delta_{\mathbf{U}_i}\mathbf{F}_i^{\mathrm{T}}(\tau)\Delta_{\mathbf{U}_i}\mathbf{F}_i(\tau)d\tau,$$

where $\Delta_{\mathbf{U}_i}\mathbf{F}_i = \mathbf{U}_i - \mathbf{F}_i$.

Theorem 4. Let the following conditions be satisfied: 1) there exist LFs $V_i(t)$ and $V_{\Delta_S,i}(t)$ admitting an infinitesimal upper limit; 2) $\mathbf{A}_i \in \mathscr{H}$; 3) $\bar{\mathbf{A}}_{ij}$ guarantee the stability of the S_i subsystem; 4) $\mathbf{F}_i(\mathbf{X}_i) \in \mathscr{N}_{\mathbf{F}}(\pi_1, \pi_2)$; 5) $u_i(t) \in \mathscr{E}\mathscr{C}_{\underline{\alpha}_{u_i}, \bar{\alpha}_{u_i}}$, $\mathbf{X}_i(t) \in \mathscr{E}\mathscr{C}_{\underline{\alpha}_{\mathbf{X}_i}, \bar{\alpha}_{\mathbf{X}_i}}$, $\mathbf{X}_j(t) \in \mathscr{E}\mathscr{C}_{\underline{\alpha}_{\mathbf{X}_j}, \bar{\alpha}_{\mathbf{X}_j}}$; 6) in some origin region there exist such $v_i > 0$ that $\mathbf{F}_i^{\mathrm{T}} \Delta_{\mathbf{U}_i} \mathbf{F}_i = \nu_i \left(\|\mathbf{F}_i\|^2 + \|\Delta_{\mathbf{U}_i} \mathbf{F}_i\|^2 \right)$ is valid at $t >> t_0$; 7) the system of inequalities for the vector LF $\mathbf{W}_{S,i} = \left[V_i, V_{\Delta_S, i} \right]^{\mathrm{T}}$ is satisfied:

$$\underbrace{\begin{bmatrix} \dot{V}_i \\ \dot{V}_{\Delta_{S,i}} \end{bmatrix}}_{\dot{\mathbf{W}}_{S_i}} \leq \underbrace{\begin{bmatrix} -\mu_i & \dfrac{2}{\mu_i} \kappa_{\Delta_{S,i}} \\ \vartheta_{\Delta_{S,i}} \rho_i & -\beta_{\Delta_{S,i}} \end{bmatrix}}_{\mathbf{A}_{\mathbf{W}_{S_i}}} \underbrace{\begin{bmatrix} V_i \\ V_{\Delta_{S,i}} \end{bmatrix}}_{\mathbf{W}_{S_i}} + \underbrace{\begin{bmatrix} 0 \\ \sqrt{0.125 v_i \eta_i \bar{\alpha}_{\mathbf{X}_i}} \end{bmatrix}}_{\mathbf{L}_{S,i}}, \tag{27}$$

where $\mu_i, \kappa_{\Delta_S, i}, \vartheta_{\Delta, i}, \rho_i, \beta_{\Delta, i}, v_i, \eta_i$ are positive numbers depending on the parameters of the AS_{AS_i} subsystem and properties of the information set $\mathbb{I}_{\mathrm{o},i}$; and 8) for the upper solution $\mathbf{W}_{S,i}$, the system of equation $\dot{\mathbf{S}}_{\mathbf{W}_{Si}} = \mathbf{A}_{\mathbf{W}_{S,i}} \mathbf{S}_{\mathbf{W}_{S,i}} + \mathbf{L}_{S,i}$ is valid, provided that

$$w_\rho(t) \leq s_\rho(t) \quad \forall (t \geq t_0) \,\&\, \left(w_\rho(t_0) \leq s_\rho(t_0) \right),$$

$\rho = e, i$; $\Delta_{S,i}$ for elements $\mathbf{W}_{S_i}$, $w_\rho \in \mathbf{W}_{S_i}$, $s_\rho \in \mathbf{S}_{\mathbf{W}_{S,i}}$. Then the AS_{AS_i} system is exponentially dissipative with the following estimate:

$$\mathbf{W}_{S_i}(t) \leq \mathrm{e}^{\mathbf{A}_{\mathbf{W}_{S_i}}(t - t_0)} \mathbf{S}_{\mathbf{W}_{S_i}}(t_0) + \int_{t_0}^{t} \mathrm{e}^{\mathbf{A}_{\mathbf{W}_{S_i}}(t - \sigma)} \mathbf{L}_{S,i} d\sigma, \tag{28}$$

if $\mu_i^2 \beta_{\Delta, i} \geq 2 \vartheta_{\Delta, i} \rho_i \kappa_{\Delta_S, i}$.

It follows from the results that using the AS_{AS_i} system gives biased estimates of the S_i subsystem parameters.

The scheme used to prove Theorems 3, 4 is given in [17].

Note 4. It should be noted that signaling algorithms are widely used in adaptive control systems. Their use is justified on the basis of ensuring the non-positivity of the LF derivative. This is due to the use of quadratic LFs, which incompletely reflect the specificity of the processes in AS_{AS_i} systems. The LF $V_{\Delta_{S,i}}$ proposed in the paper makes it possible to substantiate the properties of the AS_{AS_i} system and to obtain estimates of the quality of its functioning.

Note 5. Algorithm (19) is a compensation control. Therefore, the term “signal adaptation” (SA) only refers to the choice of the gain matrix in (19). In identification problems, the SA use depends largely on the quality requirements of the identification system.

Note 6. The results obtained in [15] can be used to analyze the properties of the algorithm (12) with $\varphi_i = 0$.

6. EXAMPLE

We consider the following system:

$$S_1: \begin{cases} \begin{bmatrix} \dot{x}_{11} \\ \dot{x}_{12} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_{21} & -a_{22} \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + \begin{bmatrix} 0 \\ \bar{a}_1 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ b_1 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ c_1 \end{bmatrix} f_1(x_{11}), \\ y_1 = x_{11}, \end{cases}$$
$$S_2: \begin{cases} \dot{x}_2 = -a_2 x_2 + \bar{a}_2 x_{11} + b_2 u_2 + c_2 f_1(x_{11}), \\ y_2 = x_2, \end{cases} \tag{29}$$

where $\mathbf{X}_1 = \left[x_{11} \; x_{12} \right]^{\mathrm{T}}$; y_1 is the state vector and the output of the S_1 subsystem; u_1 is the input (control); $f_1(x_{11})$ = sat(x_{11}) is the saturation function; $f_2(x_2)$ = sign(x_2) is the sign function; and y_2 is the output of the S_2 subsystem. The parameters of the system (29) are: $b_1 = 1$, $a_{21} = 2$, $a_{22} = 3$, $\bar{a}_1 = 1.5$, $c_1 = 1$, $a_2 = 1.25$, $\bar{a}_2 = 0.2$, $b_2 = 1$, and $c_2 = 0.25$. The inputs $u_i(t)$ are sinusoidal.

Since the variable x_{12} is not measured, the S_1 subsystem is transformed to the form where only the observable variables are used. Applying the approach from [15], the representation in the “Input–Output” space for S_1 is obtained:

$$\dot{y}_1 = -\alpha_1 y_1 + \alpha_2 p_{y_1} + \beta_{12} p_{x_2} + b_1 p_{u_1} + c_1 p_{f_1}, \tag{30}$$

where α_1, α_2, β_{12}, b_1, c_1 are the coefficients to be estimated; $\mu > 0$,

$$\begin{gathered} \dot{p}_{y_1} = -\mu p_{y_1} + y_1, \; \dot{p}_{x_2} = -\mu p_{x_2} + x_2, \\ \dot{p}_{u_1} = -\mu p_{u_1} + u_1, \; \dot{p}_{f_1} = -\mu p_{f_1} + f_1. \end{gathered} \tag{31}$$

The phase portrait for S_1 is shown in Fig. 1. The processes in the subsystem are nonlinear. In addition, there is a relationship $y_2 = y_2(y_1)$ between y_1 and y_2 (coefficient of determination is 75%), which is reflected in the properties of the S_1 subsystem (see Fig. 1). In particular, y_2 affects the S-synchronizability of S_1 and the parameter estimation. Applying the approach from [15], it can be concluded that the subsystem S_1 is structurally identifiable.

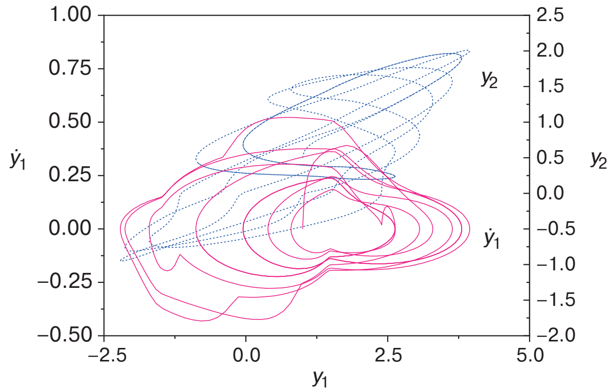

Fig. 1. Phase portrait of the S_1 subsystem

The models for the S_1 and S_2 subsystems are as follows

$$\dot{\hat{y}}_1 = -k_1 e_1 + \hat{a}_{11} y_1 + \hat{a}_{12} p_{y_1} + \\ + \hat{\beta}_{12} p_{x_2} + \hat{b}_1 p_{u_1} + \hat{c}_1 p_{f_1}, \quad (32)$$

$$\dot{\hat{y}}_2 = -k_2 e_2 + \hat{a}_2 y_2 + \hat{\bar{a}}_2 y_1 + \hat{b}_2 u_2 + \hat{c}_1 f_2, \quad (33)$$

where k_1, k_2 are *a priori* positive numbers, $e_1 = \hat{y}_1 - y_1$, $e_2 = \hat{y}_2 - y_2$ are identification errors, and $\hat{a}_i, \hat{\bar{a}}_i, \hat{b}_i, \hat{c}_i$ are adjustable parameters.

The algorithms (12) with $\varphi_i = 0$ are used to adjust the parameters of the models, as follows:

$$\dot{\hat{a}}_{11} = -\gamma_{a_{11}} e_1 y_1, \ \dot{\hat{a}}_{12} = -\gamma_{a_{12}} e_1 p_{y_1}, \\ \dot{\hat{\beta}}_{12} = -\gamma_{\beta_{12}} e_1 p_{x_2}, \\ \hat{b}_1 = -\gamma_{b_1} e_1 p_{u_1}, \ \hat{c}_1 = -\gamma_{c_1} e_1 p_{f_1}, \quad (34)$$

$$\dot{\hat{a}}_2 = -\gamma_{a_2} e_2 y_2, \dot{\hat{\bar{a}}}_2 = -\gamma_{\bar{a}_2} e_2 y_1, \\ \hat{b}_2 = -\gamma_{b_2} e_2 u_1, \ \hat{c}_2 = -\gamma_{c_2} e_2 f_2, \quad (35)$$

where $\gamma_{a_{ij}} > 0, \gamma_{\bar{a}_{ij}} > 0, \gamma_{\beta_{ij}} > 0, \gamma_{b_i} > 0, \gamma_{c_i} > 0$ are the gain coefficients of the adaptation loop.

The process of adjusting the model (32) parameters for S_1 is shown in Fig. 2.

The results of evaluating the adequacy of models (32) and (33) in the output space are shown in Fig. 3.

In Fig. 4, the dynamics of the adjustment processes of the model (33) parameters depending on e_2 is shown. It can be seen that the processes are nonlinear in the AIS for the S_2 subsystem, while the adjustment process is more regular in the AIS for the S_1 subsystem. The parameter adjustment processes of models (32) and (33) are multitemporal (Fig. 5).

We consider the AIS with SA for S_2. We apply the model:

$$\dot{\hat{y}}_2 = -k_2 e_2 + \hat{a}_2 y_2 + \hat{\bar{a}}_2 y_1 + \hat{b}_2 u_2 + u_{s,2}, \quad (32a)$$

where $u_{s,2} = -d_2 e_2 x_2$.

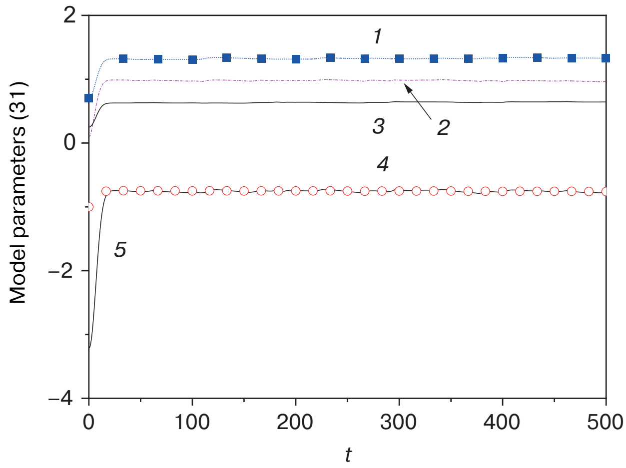

Fig. 2. Tuning of model (32) parameters: (*1*) $\hat{\beta}_{11}$, (*2*) $\hat{c}_{11}$, (*3*) $\hat{a}_{11}$, (*4*) $\hat{a}_{12}$, and (*5*) $\hat{b}_1$

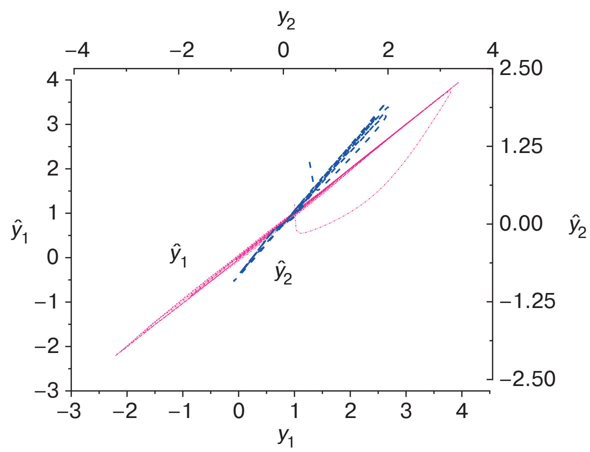

Fig. 3. Adequacy of models (32) and (33)

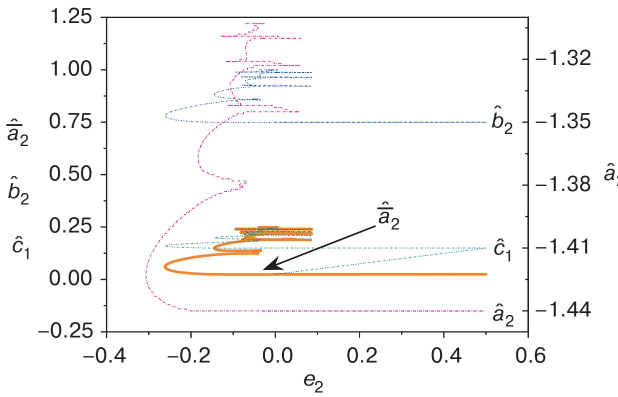

Fig. 4. Tuning of model (33) parameters

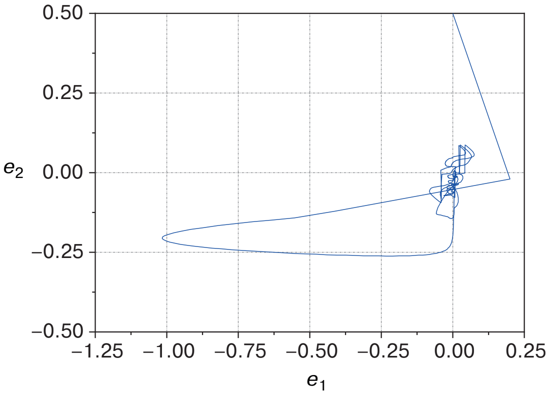

Fig. 5. AIS phase portrait in error space

The results of the AIS with SA are shown in Figs. 6–9. In Figs. 6 and 7, the adjustment processes of the model (32) parameters and the adequacy of the models in the output space are shown. The dynamics of the processes in AIS and the SA signal variation as a function of the identification error for the S_2 subsystem are shown in Fig. 8.

The results of AIS modeling with SA show that the output of the S_2 subsystem influences the AIS parameter adjustment of the S_1 subsystem. Therefore, provided that the requirements for the quality of the adaptation process are met, identification systems with SA should be used. Nevertheless, despite the compensating properties, SA may lead to complication of processes in AIS.

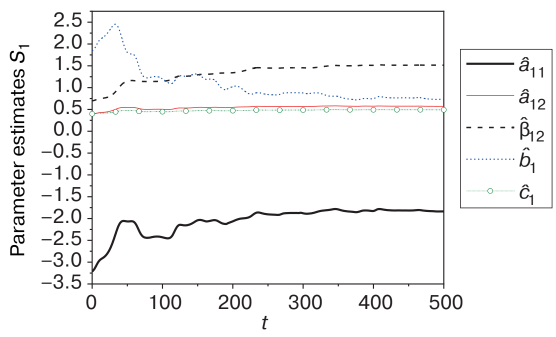

Fig. 6. Tuning of model (32) parameters

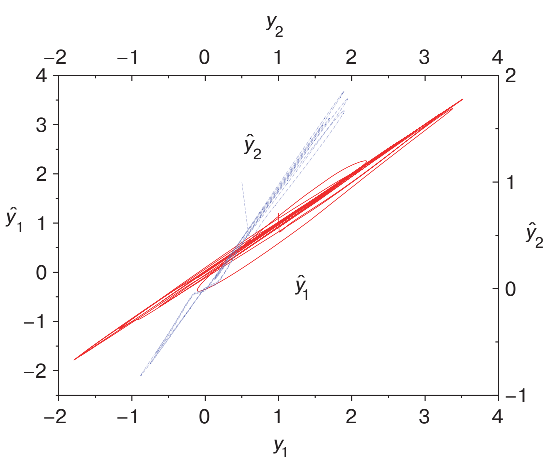

Fig. 7. Adequacy of models (32) and (32a)

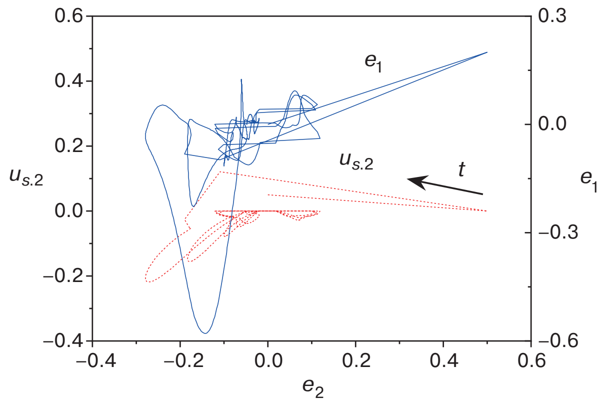

Fig. 8. AIS phase portraits and SA output in error space

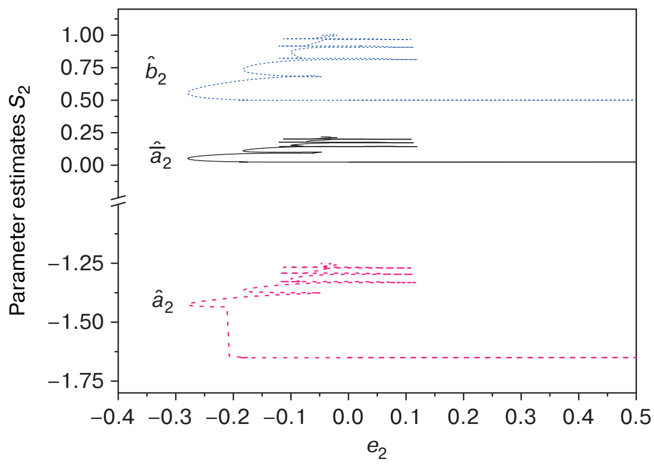

Fig. 9. Tuning of model (32a) parameters

CONCLUSIONS

An approach for the identification of NDS under uncertainty is proposed. The class of NDS is considered where the nonlinearity satisfies the sector condition (quadratic coupling). Estimates of the nonlinearity are obtained. The presence of internal correlations significantly complicates the problem of parametric estimation. Here, the problem of NDS identifiability plays an important role. The PI conditions of the DS are obtained both for the output and in the state space. These are based on the verification of the excitation constancy condition. The PI is shown to be influenced by the interconnections between the subsystems. Algorithms for parametric and signal adaptive identification of DSs are obtained along with the properties of signal-adaptive DS. For the first time, a justification of the stability of AIS with the above class of algorithms is carried out based on a special class of Lyapunov functions used for this purpose. The proposed approach to AA synthesis takes into account the requirements of the identification system.

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