

Mathematical modeling
Математическое моделирование

UDC 004.02

<https://doi.org/10.32362/2500-316X-2025-13-4-78-94>

EDN CVZOXD



REVIEW ARTICLE

Modern optimization methods and their application features

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• Submitted: 29.10.2024 • Revised: 13.01.2025 • Accepted: 13.05.2025

Abstract

Objectives. The authors conduct an analytical review of available optimization methods and simulation tools to identify their key features, effectiveness, and possible applications. The aim was to form an integrated picture of modern approaches, which may facilitate decision making when selecting the most appropriate method for a particular task. The key objective was to review and classify various optimization tools, which of theoretical and practical value for developers of new models.

Methods. Scientific publications and analytical materials were retrieved from specialized databases and technical documentation libraries.

Results. The analysis and classification of existing optimization methods allowed the authors to identify their advantages, disadvantages, and application features, as well as to determine the relationship between theoretical concepts and their practical implementation. During the analysis, various optimization approaches were considered, covering both classical and modern simulation methods.

Conclusions. The importance of informed selection of optimization methods, which raise the efficiency and accuracy of simulation procedures, is highlighted. The results obtained indicate the need for further study and comparative analysis of the methods used in practice in order to establish their efficiency and applicability in various scenarios. Future research directions include experimental testing of the effectiveness of various approaches based on several models in order to determine their advantages and disadvantages for a more informed selection of the method suitable for a particular task.

Keywords: optimization methods, application features, multi-criteria optimization methods, optimization algorithms, evolutionary algorithms, optimization of digital models, optimization problem, optimization software tools

For citation: Beketov S.M., Zubkova D.A., Gintciak A.M., Burlutskaya Zh.V., Redko S.G. Modern optimization methods and their application features. *Russian Technological Journal*. 2025;13(4):78–94. <https://doi.org/10.32362/2500-316X-2025-13-4-78-94>, <https://www.elibrary.ru/CVZOXD>

Financial disclosure: The authors have no financial or proprietary interest in any material or method mentioned.

The authors declare no conflicts of interest.

ОБЗОРНАЯ СТАТЬЯ

Современные методы оптимизации и особенности их применения

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• Поступила: 29.10.2024 • Доработана: 13.01.2025 • Принята к опубликованию: 13.05.2025

Резюме

Цели. Цель статьи – провести аналитический обзор методов и инструментов оптимизации, используемых в моделировании, для выявления их ключевых особенностей, эффективности и областей возможного применения. Исследование направлено на формирование целостной картины современных подходов, что позволит специалистам выбирать наиболее удобные методы для решения разнообразных задач. Ключевая задача – составить систематизированное представление об инструментах оптимизации, охватывающее различные методики и подходы, которые обеспечат как теоретическую, так и практическую ценность для разработки более эффективных моделей.

Методы. Для достижения поставленных целей исследование основывалось на обширной выборке научных публикаций и аналитических материалов, отобранных из специализированных баз данных и технической документации.

Результаты. Проведены анализ и классификация существующих методов оптимизации, что позволило выявить их сильные и слабые стороны, особенности применения, а также определить взаимосвязь между теоретическими концепциями и их практической реализацией. В ходе анализа рассмотрены различные подходы к оптимизации, охватывающие как классические, так и современные методы, что обеспечило всесторонний обзор применимых подходов в моделировании.

Выводы. Проведенное исследование подтверждает важность грамотного подбора методов оптимизации, что способствует более эффективному и точному моделированию. Полученные результаты подчеркивают необходимость дальнейшего изучения и сравнительного анализа методов на практике с целью более глубокого понимания их эффективности и применимости в различных условиях. Перспективы будущих исследований включают экспериментальное тестирование эффективности различных подходов на базе нескольких моделей, что позволит определить их преимущества и недостатки для более точного выбора метода в зависимости от специфики задач.

Ключевые слова: методы оптимизации, особенности применения, многокритериальные методы оптимизации, оптимизационные алгоритмы, эволюционные алгоритмы, оптимизация цифровых моделей, оптимизационная задача, программные инструменты оптимизации

Для цитирования: Бекетов С.М., Зубкова Д.А., Гинцяк А.М., Бурлуцкая Ж.В., Редько С.Г. Современные методы оптимизации и особенности их применения. *Russian Technological Journal*. 2025;13(4):78–94. <https://doi.org/10.32362/2500-316X-2025-13-4-78-94>, <https://www.elibrary.ru/CVZOXD>

Прозрачность финансовой деятельности: Авторы не имеют финансовой заинтересованности в представленных материалах или методах.

Авторы заявляют об отсутствии конфликта интересов.

INTRODUCTION

Optimization plays an important role in simulation modeling, allowing optimal results to be obtained, resources to be saved, and the quality of developed products to be improved. Optimization is becoming an essential element of modern engineering processes aimed at solving complex problems and achieving high technological standards [1]. However, to ensure the efficiency and accuracy of optimization, unnecessary repetition should be avoided while taking into account the diversity of methods and their complexity in software implementation. This is particularly important in such areas as design activity [2], energy [3], and healthcare [4], where even minor improvements may yield significant consequences.

A bibliometric analysis of optimization techniques conducted in [5, 6] described the evolution of the concept of “optimization” from trial and error to more formalized approaches. Due to the rapid progress in this direction, continued efforts are required to trace new methods for an approximate solution to optimization problems and review their computational aspects, applicability areas, and practical merits. The choice of an optimization method must be tailored to the specific features of the models. This requires consideration of the requirements for suitable methods and tools based on the nature of the models and the characteristics of the systems in order to create more flexible and adaptive conditions for the developer [7, 8].

The relevance of the analysis of optimization methods is explained by a number of factors, including the increasing complexity of sociotechnical and socioeconomic systems, the growth of computing power, and the emergence of more efficient methods [9, 10].

In this article, we carry out an analytical review of optimization methods with the purpose of their classification and assessment of their applicability in simulation modeling. Consideration of a large variety of optimization methods and their further analysis simplifies the research process and makes optimization approaches more accessible to a wider range of specialists.

RESEARCH METHODS

The aim of this literature review was to identify key optimization methods that are used in various research areas. The growing interest in model optimization problems has generated a significant amount of research over the past few years, which highlights the relevance and importance of developing efficient optimization methods.

A literature review was conducted using the databases of Scopus¹, Russian Science Citation Index², and the List of the Higher Attestation Commission for Academic Degrees and Titles³ (VAK) under the Ministry of Science and Higher Education of the Russian Federation. The literature search was conducted using the following keywords: optimization type, multicriteria optimization methods, computational complexity of optimization methods, parallel computing, optimization methods, decision making, optimization algorithms, evolutionary algorithms, statistical methods, mathematical methods, simulation modeling, model optimization, optimization problem, optimization problems. The total number of articles on these queries for the period of five years comprised 434.

The analysis of the retrieved articles was performed to identify the most common optimization methods used in different research areas. Figure 1 presents the results of the conducted analysis, i.e., the ratio of published articles on the topic to the total volume over the past five years.

Model optimization methods determine the conditions of existence of an object or process under which the highest value of some property of this object or process is achieved. They are used to search for optimal solutions to various problems where it is necessary to optimize certain parameters or criteria [11].

¹ <https://www.scopus.com/>. Accessed October 29, 2024.

² Russian Science Citation Index is a database that distinguishes the best Russian journals and places them on the Web of Science platform.

³ Higher Attestation Commission under the Ministry of Science and Higher Education of the Russian Federation. <https://vak.minobrnauki.gov.ru/> (in Russ.). Accessed October 29, 2024.

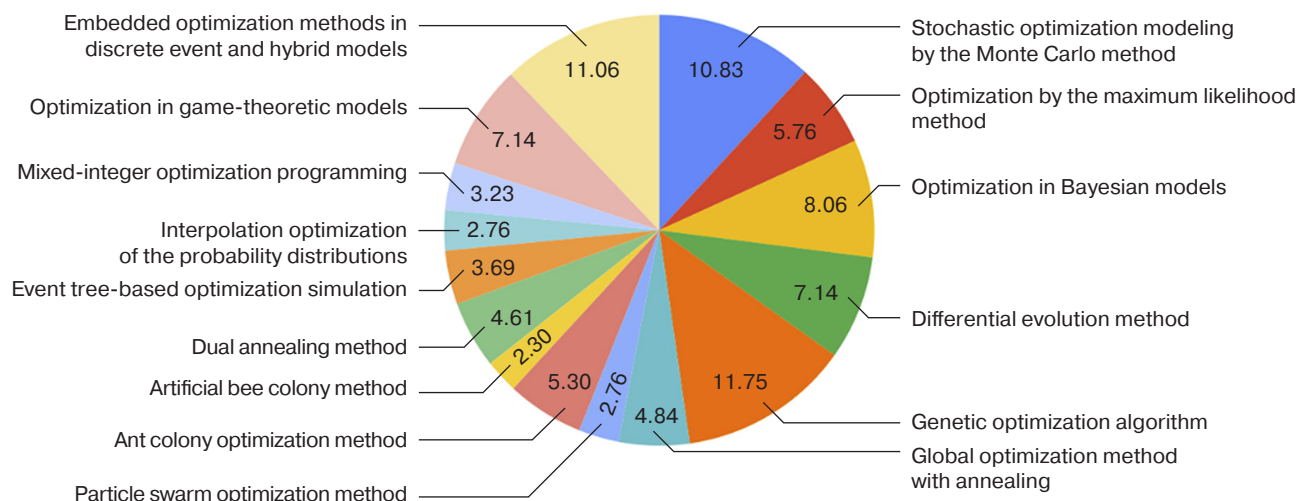


Fig. 1. Share of articles in different research areas over five years

The results offer an integrated picture of optimization methods and their classification, enabling researchers to select a specific model optimization method depending on the objectives, type of optimization, ability to account for complex target functions, and the advantages and disadvantages of the methods.

CLASSIFICATION OF OPTIMIZATION METHODS

Optimization methods can be divided into two categories: those based on the estimation of parameters of the *a posteriori* distribution, and those based on imprecise and approximation estimation. The *a posteriori* estimate is an estimate obtained empirically by performing an experiment, while the approximation estimate is an

estimate approximated to real values. Within the latter, two more subgroups can be distinguished: those based on evolutionary algorithms and those based on simulation modeling. The classification of optimization methods developed by the authors is shown in Fig. 2.

Stochastic optimization modeling by Monte Carlo method

The Monte Carlo method is widely used in stochastic modeling, especially in situations where probabilistic parameters need to be taken into account. The popularity of this method is explained by the possibility of predicting different outcomes based on probabilistic factors.

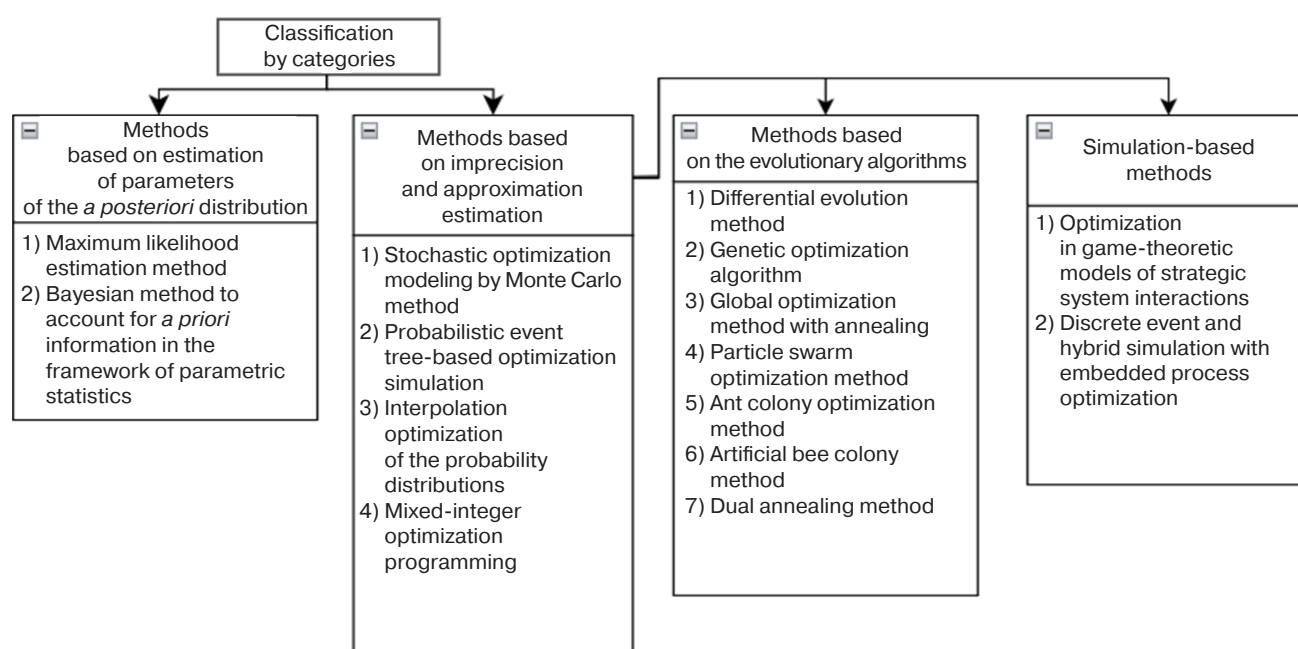


Fig. 2. Classification of optimization methods

The process of stochastic Monte Carlo modeling includes the following steps: generation of random input data, execution of simulation, generalization of results, and evaluation of phenomena, which can be generally described by the formula:

$$mc = \frac{1}{N} \sum_{i=1}^N f(i), \quad (1)$$

where mc is the estimation of the phenomenon by the Monte Carlo method, N is the number of the performed simulations, $f(i)$ is the result of the i th model run.

Unlike deterministic approaches that involve strict adherence to fixed algorithms, the Monte Carlo method is based on a multiple repetition of experiments (simulations) using random input data. Each run of the model yields one possible result, and the multitude of such results makes it possible to estimate the statistical distribution of outcomes and, thus, to predict probabilistic phenomena more accurately.

The process of stochastic Monte Carlo modeling includes such stages as generation of random input data, modeling, accumulation of results (the results of each individual modeling are saved), evaluation of phenomena, where statistical processing is performed on the basis of accumulated data and mean values, dispersions, probabilities of occurrence of events, and other characteristics are calculated.

The application of the Monte Carlo method is not limited to one specific area, finding application in various forecasting tasks. For example, the Monte Carlo method was used in combination with an error function in the form of a Gaussian function to predict the spread of the COVID-19 virus in Italy [12]. The authors conducted 150 Monte Carlo simulations to obtain an accurate prediction of the number of deaths in China and Italy.

The Monte Carlo method was also applied to compare the performance of multicriteria prioritization methods for selecting the sites for solar power plants in Iran [13]. The study used two reliability measures to evaluate different prioritization methods, and the Monte Carlo simulation evaluated the reliability values in each of these methods by conducting sensitivity analysis.

Optimization by the maximum likelihood estimation method

The method of maximum likelihood estimation (MLE) is a statistical approach widely used either alone or in combination with other methods to process data in research or problem solving. The method provides an estimation of unknown parameters by maximizing the likelihood function, being used for a variety of purposes [14].

The MLE method searches for the value of θ at which the likelihood function is maximized:

$$\theta_{MLE} = \arg \max_{\theta} L(\theta|X), \quad (2)$$

where X is the data sample, θ is the parametric model; $L(\theta|X)$ is the likelihood function that, for a given model, measures the probability of obtaining the observed data X given the value of the θ parameter.

This algorithm for implementing the method can be interpreted as follows: provided that the observed data can be described by a certain statistical model, MLE selects such model parameters that make the observed data the most probable ones.

The basic steps of the maximum likelihood method include defining the parametric model, formulating the likelihood function, and maximizing the likelihood function.

The maximum likelihood method is widely used in statistics to estimate parameters in various models, such as linear and generalized linear models, factor analysis, structural equation modeling, hypothesis testing, confidence interval formation, and discrete choice models. It is based on maximizing the likelihood function, which reflects the probability of selecting a particular parameter given known events. The MLE method is aimed at finding the parameters at which the results obtained by the model are most consistent with the actual data. The estimation of the parameter correctness is performed using the Akaike information criterion, which balances the model complexity and increases the validity at the optimal ratio with the maximum likelihood. The model with the minimum Akaike information criterion and the maximum likelihood value is considered the best. The following formula for the Akaike information criterion (AIC) is used:

$$AIC = -2\ln L + 2K, \quad (3)$$

where L is the value of the likelihood function of the model (conditional probability of obtaining observed data with given model parameters), K is the number of model parameters.

The AIC can be understood as a measure of the trade-off between model accuracy and its simplicity. In cases where the model is extremely complicated (e.g., contains a large number of parameters), it may produce a good fit of the data, but fail to generalize the data. The AIC criterion avoids this problem by adding a penalty for increasing the number of parameters.

Optimization in Bayesian models

Bayesian models are based on Bayes' theorem, which offers a way of updating probability estimates of model parameters based on new data. These models take

into account *a priori* information about the parameters and adjust this information to the observed data. Bayes' theorem is defined by the formula:

$$P(\theta|D) = \frac{P(D|\theta)P(\theta)}{P(D)}, \quad (4)$$

where θ is the vector of model parameters; D is the data set on which the model is trained; $P(\theta|D)$ is the *a posteriori* probability of the model parameters given the data; $P(D|\theta)$ is the probability of the data given the model parameters; $P(\theta)$ is the *a priori* probability of the model parameters; $P(D)$ is the marginal probability of the data.

The application of Bayesian models begins with the specification of an *a priori* distribution that reflects the initial assumptions about the model parameters. This distribution can be informative provided the presence of evidence from previous studies, or uninformative provided the minimum initial assumptions. The likelihood ratio is then calculated, which shows the likelihood of the observed data to be observed under different parameter values. In the next step of applying Bayes' theorem, the *a priori* distribution is combined with the likelihood to obtain an *a posteriori* distribution, which reflects the updated knowledge of the model parameters after accounting for the data. The resulting *a posteriori* distribution is interpreted to estimate parameters, determine confidence intervals, and make other statistical inferences. Thus, Bayesian models are a method that enables combining *a priori* knowledge and new data for a more accurate parameter estimation [15].

Such models are used in statistical analysis, machine learning, and other areas where uncertainty must be accounted for and probability estimates updated as new data become available.

For example, [16] proposed a mathematical model for detecting anomalous observations using neural network sensitivity analysis. The use of the Bayesian approach enabled the researchers to take into account the uncertainty parameter and accumulate knowledge about the data behavior. This makes the method effective for anomaly detection in complex multidimensional data spaces, such as medical monitoring or disease prediction systems.

In the economic domain, Bayesian models are helpful in assessing the impact of tax schemes on different social groups [17]. The use of theoretical foundations of planned economy in combination with Bayesian methods enables analysis of the probabilistic effects of changes in tax policy and developing optimal taxation schemes that take into account the uncertainty of initial data and forecasts. Bayesian methods are used in the development and analysis of machine learning algorithms, especially in optimization problems.

For example, the authors [18] proposed a machine learning model for organizing employee performance in remote and hybrid modes. The Bayesian approach is effective in estimating the uncertainty of the data related to employee performance, taking into account the probability distribution of various factors such as work schedule, team interaction, and workflow features. This makes the model more adaptive and flexible.

Interpolation optimization of the probability distributions

Interpolation optimization of the probability distributions is a method that combines interpolation of data with the construction of probability distributions to fill in intermediate values between known discrete points. Splines, i.e., smooth piecewise polynomial functions that can approximate complex relationships with a high degree of accuracy, are used for this purpose.

In this approach, the key problem is to construct a function that not only passes through given points of probability distributions, but also preserves their important characteristics, such as the mathematical expectation. To construct splines, we define the following function [19]:

$$S_i = a_i + b_i(x - x_i) + \frac{c_i}{2}(x - x_i)^2 + \frac{d_i}{6}(x - x_i)^3, \quad (5)$$

where a_i, b_i, c_i, d_i are the coefficients to be determined at each of the segments; $i = 1, n, x_{i-1} \leq x \leq x_i$.

The method is applied in cases where it is necessary to predict the probability distribution values at intermediate points on the basis of discrete data. For example, in models, splines can be used for interpolation of the probability distributions in time or by the values of the parameters.

Special attention is paid to preserving the mean characteristics of the distribution. To that end, splines are used, which provide coincidence of the mathematical expectation of the approximated function with the mathematical expectation of the original distribution. This approach makes it possible to preserve statistical properties of the data, which is especially important in tasks where correct representation of probabilistic characteristics is required.

One example demonstrating the application of this approach is the study of failures of complex systems under combined loads. The work [20] described an interpolation method that enables the probability of system failure to be estimated from a limited amount of test data under different types of loads. The use of splines to interpolate failure probabilities makes it possible to recover distributions for intermediate values of load parameters while preserving the original statistical characteristics.

Another example is related to solar radiation modeling, where splines with preservation of the mean value are used for interpolation of solar activity data [21]. The author proposed an interpolation method that can correctly reconstruct the distribution of solar radiation at intermediate points of the time series, ensuring the coincidence of the mathematical expectation of the spline function with the initial average value.

Mixed-integer optimization programming

Mixed-integer programming is the following optimization task [22]:

$$\max\{\mathbf{c}^T \mathbf{x} : \mathbf{b}^1 \leq \mathbf{A}\mathbf{x} \leq \mathbf{b}^2, \mathbf{d}^1 \leq \mathbf{x} \leq \mathbf{d}^2, x_i \in S\}, \quad (6)$$

where $\mathbf{b}^1, \mathbf{b}^2, \mathbf{c}, \mathbf{d}^1, \mathbf{d}^2 \in \mathbb{R}^m$; $\mathbf{A}$ is a real $m \times n$ -matrix; $\mathbf{x}$ is a n -vector of variables (unknowns); $S \subseteq \{1, \dots, n\}$ is a set of integer variables, and x_i is a subset of S bounded by integers.

Several approaches are used in practice for solving mixed-integer programming problems.

The branch-and-bound method [23] is based on splitting the original problem into subproblems, each of which is solved separately. The main idea is to build a decision tree, where the search space is partitioned at each level, and binding is used to exclude irrelevant variants based on the solutions of previous nodes of the tree. This method is used in logistics problems where it is necessary to consider both integer and continuous variables, e.g., when optimizing delivery routes.

The method of cutting planes [24] is aimed at improving the approximate solution by adding new constraints that exclude non-integer solutions from the domain of admissible solutions. Such constraints, referred to as “cuts”, gradually refine the search domain and improve the quality of the solution. The cut method is frequently used in inventory management and resource allocation problems.

Decomposition methods [25], such as Benders’ generalized decomposition, involve breaking down a complex problem into simpler subtasks. Each of the subtasks is solved separately followed by combining the results to obtain a solution to the original problem. This approach is effective for large-scale problems, e.g., in network planning or power system modeling problems.

The study presented in [26] carried out a detailed analysis of these methods and their application in linear and nonlinear optimization problems. The specific features of each approach and their effectiveness depending on the specifics of a particular problem were the research focus. The examples include both scheduling and model optimization problems where mixed constraints need to be considered.

Event tree-based optimization simulation

A decision tree is a method for modeling, classification, and regression [27]. It is a hierarchical structure in which each node represents a test of some attribute, each branch represents the result of the test, and the leaves contain the values of the target variable or the decision. Decision trees enable the construction of models capable of making decisions based on the analysis of multiple input attributes, which makes them useful in the tasks of data analysis and forecasting.

Problems that require consideration of the time factor and probabilities of the sequence of events are solved using continuous event trees [28]. Such structures further develop the idea of decision trees and enable modeling of the temporal dependencies and probabilistic nature of events. In such trees, events are described by statistical probabilities or exponentially distributed event rates.

Events in the event tree are associated with statistical probabilities or Poisson-exponentially distributed constant rates. For example, component failures typically occur at some constant failure rate λ (constant hazard function). In the simplest case, the failure probability depends on the rate λ and the exposure time t :

$$P = 1 - e^{-\lambda t}, \quad (7)$$

where $P \approx \lambda t$, if $\lambda t < 0.001$.

This formula describes the exponential distribution of events characteristic of processes that occur randomly over time, such as failures of equipment or other systems.

Continuous event trees are useful for risk assessment in projects where time-dependent chains of events are possible. For example, in project management, an event tree can model the risks associated with equipment failures or delays in the process, taking into account the probability of problems at each stage [29]. This approach provides an opportunity to predict the probable scenarios of event development and develop strategies for risk minimization.

Optimization in game-theoretic models

Game-theoretic modeling is a method of analyzing strategic interactions between participants in order to optimize their strategies [30]. This method is widely used to study situations of conflict or cooperation, where each participant seeks to maximize their gains or minimize losses depending on the decisions of other participants.

A key concept in game theory is the Nash equilibrium. A Nash equilibrium occurs when no participant has motivation to change their strategy given the selected strategies of other participants. This means that given each player’s strategy, no one has the desire to change their actions, since any deviation from the current strategy will not improve their winnings.

Mathematical formulation: (S, H) is a noncooperative game of n persons in normal form, where S is the set of pure strategies, and H is the set of wins. When each player $i \in \{1, \dots, n\}$ chooses a strategy $x\{i\}$ belonging to S in the strategy profile $(x\{1\}, \dots, (x\{n\})$, player i receives his win $H\{i\}(x)$. A strategy profile $x^*\{i\} \in S$ is a Nash equilibrium if changing one's strategy from $x^*\{i\}$ to $x\{i\}$ is not profitable for any player i , i.e., for any player i : $H_i(x^*) \geq H_i(x_i, x_{-i}^*)$.

The work [31] was dedicated to solving optimization problems under uncertainty using game theory, in particular, a move-by-nature model. This model is applied when a participant (player) interacts with an external environment whose behavior cannot be controlled and which is represented as "nature" or an external uncertainty factor. Three classical criteria were used to select the most optimal strategy, including Wald, Bayes, and Hurwitz criteria.

Embedded optimization methods in discrete-event and hybrid models

Discrete-event and hybrid modeling with embedded process optimization represent methods for optimizing system processes in the context of scientific research. Discrete-event modeling enables the representation of a system as a set of entities and resources, emphasizing the changes associated with discrete events. Conversely, hybrid modeling combines discrete and continuous aspects, thus providing a more accurate reflection of the dynamics of systems [32].

Process optimization embedded in the models is an integrated mechanism aimed at automatic parameter adjustment, efficient resource management, and dynamic process adaptation during the simulation process.

This optimization approach is effective for simple variations in the parameters being varied (e.g., different discrete values of resource capacity); however, it becomes significantly more complex with the advent of differently structured process implementations. In addition, some of the existing simulation environments equipped with a built-in process optimization subsystem assume manual configuration of replications. This causes difficulties due to the large number of replications required for the task and the probability of human error; thus, the user may accidentally omit the optimal variant from the list of runs.

Hybrid modeling is being increasingly used to describe complex socioeconomic and sociotechnical systems. This approach includes combinations of basic paradigms of simulation modeling. Hybrid modeling allows the system to be considered from different perspectives, thereby providing a more complete and accurate picture of its behavior [33].

Differential evolution method

The differential evolution method is one of the evolutionary modeling methods designed to solve multidimensional optimization problems. According to the classification of optimization methods, it belongs to the class of stochastic methods due to the use of a random number generator in the process of finding a solution. It also uses some ideas of genetic algorithms; however, unlike such algorithms, it does not require working with variables in binary code.

The algorithm behind this method starts with initialization of the initial population of points (candidates), which is randomly generated in the search space. This population represents potential solutions for the optimization task [34]. The next step involves selecting three random points (vectors) from the current population. These three points will be used to create a new vector that represents a descendant in the mutation process.

Difference vector refinement is the creation of a new vector (descendant) by the difference of two random vectors multiplied by a scaling factor. This step simulates mutation, introducing diversity into the population and providing new candidates for optimization.

The next stage is crossing (crossover), where the elements of the new vector are compared with the elements of the base vector. If an element of the new vector is better (smaller) than the corresponding element of the base vector, it replaces the corresponding element of the base vector. This mechanism ensures preservation of the best characteristics of vectors in the population. Adaptability is assessed by using a target (cost) function. The new vector is evaluated, and if being better (has a lower cost) than the old vector, the new vector is accepted into the population.

The steps are repeated until a stopping criterion is reached, such as the maximum number of iterations or reaching the desired level of fitness. Thus, the differential evolution method effectively and iteratively improves the candidate population in search of an optimal solution to the optimization task.

Genetic optimization algorithm

Genetic algorithms are an evolutionary optimization method based on the mechanisms of natural selection and genetic evolution [35]. This method is aimed at finding an optimal solution to a problem by simulating the evolutionary process. The algorithm starts with initialization, which creates an initial population consisting of a set of possible solutions to the problem represented as chromosomes, where each chromosome encodes a particular solution in the search space. At each step of the algorithm, the fitness of each individual is

evaluated using a target function whose value reflects the quality of the corresponding solution. Individuals with the highest adaptability gain an advantage in the selection process for further stages of evolution.

The evolutionary process is based on the operations of selection, interbreeding, and mutation. Selection is carried out to select the best adapted individuals that form the basis of a new population. Inbreeding combines the genetic data of two parents, producing offspring with combined characteristics, which facilitates the study of new combinations of traits. Mutation introduces random changes to the genotype of individual chromosomes, enabling the exploration of new regions of the solution space and preventing the algorithm from converging prematurely into local extrema. Once these operations are applied, a new population is formed to replace the previous one. The process is repeated until a given number of iterations is reached or a solution with an acceptable level of quality is obtained.

The efficiency of genetic algorithms is largely determined by tuning the key parameters such as population size, mutation and crossing probability, and the number of iterations. These parameters regulate the balance between exploring the solution space and refining the current optimal values, which makes this approach particularly versatile.

For example, the researchers [36] considered the application of a genetic algorithm based on the theory of natural selection to solve optimization problems. The authors demonstrated the effectiveness of the proposed approach using the example of optimizing the load distribution function in resource-constrained systems. A genetic algorithm was used to find the optimal distribution of tasks among the available computational nodes in such a way as to minimize the total execution time and simultaneously balance the load between the nodes.

Global optimization method with annealing

The global optimization method with simulated annealing is a heuristic algorithm that found its inspiration in the physical process of metal annealing [37]. The method has become part of the toolbox for solving complex global optimization problems, particularly in situations where the search space may be complicated, containing many local minima or having uncertain characteristics.

The basic concept of the method simulates the process of material cooling after heating. The initial “temperature” of the algorithm determines the degree of aggressiveness of its steps in the solution space. At the initial stages of the process, the algorithm is more tolerant to random changes, which enables it to avoid getting stuck in local minima. Over time, the temperature

gradually decreases, which reduces the probability of making the worst decisions, making the algorithm more focused on finding global optima.

The mechanism of operation of the global optimization method with annealing enables the algorithm to easily overcome local minima, due to random natural wandering through the solution space. This provides a wider coverage of the search for optimal solutions in complex and multidimensional spaces.

Particle swarm optimization method

The method of particle swarm optimization (PSO) is one of the heuristic algorithms inspired by the collective behavior of natural systems, such as flocks of birds or schools of fish [38]. The algorithm is used to solve optimization problems, particularly when the search space is characterized by high dimensionality, nonlinearity, and the presence of multiple local optima.

PSO is based on the idea of simulating the motion of particles, each of which represents a potential solution to the problem. Each particle is characterized by its position and velocity, which are updated at each step of the algorithm. The update is based on two key factors: the personal experience of a particle (the best position it has previously discovered) and the collective experience of the group (the best solution found among all particles). This enables particles to simultaneously explore the search space and focus on the most promising areas [39].

The PSO algorithm has several parameters, such as inertia coefficient, cognitive and social influence parameters, which determine the dynamics of particle movement and their ability to explore and search. Proper tuning of these parameters helps to balance between global exploration of the solution space and its local exploration.

Applications of the PSO method cover a wide range of optimization problems including machine learning and neural network design, finding application in various engineering and scientific fields.

Ant colony optimization method

Ant colony optimization (ACO) is a heuristic method developed based on observations of the collective behavior of ants in nature [40]. It is based on the concept of using pheromones and stigmergy to solve optimization problems. The principle of ACO is based on modeling the behavior of ants that leave a chemical trace—pheromones—when searching for food. These traces serve as a guide for other ants, increasing the probability for them to follow the same route.

One specific feature of this algorithm consists in dynamic updating of pheromone traces. In this case,

shorter and more efficient routes are reinforced by more pheromones left by ants. This mechanism enables the algorithm to gradually concentrate on the most optimal solutions, eliminating less promising routes [41]. The ACO method is effective in problems where it is necessary to explore a large space of possible solutions and select the best of them.

The application of ACO covers a wide range of optimization problems. These include, e.g., solving routing problems and finding optimal paths in networks, such as transportation systems or computer networks. Parallelization of the algorithm, which consists in dividing the work into several independent agents, makes it possible to significantly reduce computation time and make the algorithm applicable to problems with a large number of variables.

Artificial bee colony method

The artificial bee colony (ABC) method is an optimization algorithm that simulates the strategies of bees searching for food sources in nature [42]. The algorithm distinguishes three types of bees: workers, observers, and scouts.

At the initial stage, there is preliminary information about the location of food sources representing admissible solutions to the optimization task. The worker bees head to these sources, search in their vicinity, and memorize new solutions that improve the parameters of the task.

After the search phase is completed, worker bees return to the hive and relay information to observer bees about more attractive sources. The observer bees probabilistically select a source to start their search, conducting it in the same way as the worker bees. New solutions are retained only when the quality improves on the given parameters. The search process continues until a given number of iterations is reached [43].

Dual annealing method

Dual annealing is a stochastic global optimization method that is an extension of the simulated annealing algorithm [44]. The method is designed to improve the efficiency of finding the global optimum by using a local search algorithm in addition to the simulated annealing procedure. This approach improves the accuracy of finding the optimal solution, particularly in problems with a large number of local extrema.

The basic principle of dual annealing implies the initial application of the stochastic global search process based on the principles of thermodynamic annealing. Simulated annealing successfully determines the region of the solution space in which the global optimum is located, although not guaranteeing the exact solution inside this region. To overcome this limitation, the dual annealing algorithm applies a local optimization method to the solution found by simulated annealing in the final step. The local search enables the refinement of the solution found, minimizing the risk of missing the global optimum due to insufficient exploration of a limited region of the solution space.

The dual annealing algorithm is successfully applied in problems where the search space is complex and multidimensional, containing many local extrema. Dual annealing demonstrates high efficiency in engineering when solving design optimization problems, in development of complex technological systems, or in tuning hyperparameters in machine learning models. Its ability to combine the advantages of global and local search makes this method versatile and solvable.

Nevertheless, a detailed study of evolutionary optimization methods is a separate area of scientific research.

A comparative analysis of optimization methods is presented in the table.

Table. Comparison of optimization methods

Method	Type	Field of application	Advantages	Disadvantages
Stochastic optimization modeling by Monte Carlo method	Stochastic	Simulation of random processes, probability estimation, optimization of complex systems	Simple, versatile, high accuracy	Requires a large number of calculations, can be computationally expensive
Optimization by the maximum likelihood method	Deterministic	Estimation of the model parameters based on data	Accuracy, efficiency	Requires knowledge of the target function form, can be sensitive to the choice of initial values
Optimization in Bayesian models	Deterministic	Probability analysis, consideration of <i>a priori</i> data, forecasting	Accuracy, flexibility, consideration of <i>a priori</i> data	Requires knowledge of the target function form, can be computationally expensive

Table. Continued

Method	Type	Field of application	Advantages	Disadvantages
Differential evolution method	Evolutionary	Global optimization of the multivariate functions	Efficiency, resistance to local minima	Requires knowledge of the target function form, can be sensitive to parameter selection
Genetic optimization algorithm	Evolutionary	Finding optimal solutions in complex problems	Stability to local minima, solving multicriteria tasks	Requires a large number of calculations, can be computationally expensive
Global optimization method with annealing	Evolutionary	Simulation of metal annealing process to find the global minimum	Efficiency, resistance to local minima	Requires a large number of calculations, can be computationally expensive
Particle swarm optimization method	Evolutionary	Collective search for optimal solutions, simulation of the bee swarm behavior	Efficiency, resistance to local minima	Requires a large number of calculations, can be computationally expensive
Ant colony optimization method	Evolutionary	Shortest paths search, route optimization	Efficiency, resistance to local minima	Requires a large number of calculations, can be computationally expensive
Artificial bee colony method	Evolutionary	Simulating the behavior of scout bees and forager bees to find solutions	Efficiency, resistance to local minima	Requires a large number of calculations, can be computationally expensive
Dual annealing method	Evolutionary	Use of two temperature modes	Efficiency, resistance to local minima	Requires a large number of calculations, can be computationally expensive
Event tree-based optimization simulation	Stochastic	Risk analysis, assessment of event occurrence probability	Accuracy, flexibility	Requires knowledge of the target function form, can be computationally expensive
Interpolation optimization of the probability distributions	Stochastic	Modeling of complex probability distributions	Accuracy, flexibility	Requires knowledge of the target function form, can be computationally expensive
Mixed-integer optimization programming	Deterministic	Solving optimization tasks with discrete variables	Accuracy, efficiency	Requires knowledge of the target function form, can be computationally expensive
Optimization in game-theoretic models	Deterministic	Analysis and optimization of strategies in the competitive environment	Accuracy, flexibility	Requires knowledge of the target function form, can be computationally expensive
Discrete event and hybrid models	Stochastic	Optimization of dynamic systems with discrete events	Accuracy, flexibility	Requires knowledge of the target function form, can be computationally expensive

Stochastic methods are used in various fields due to their capacity to estimate model parameters, account for various probabilistic factors, and for model fitting.

In deterministic optimization methods, the process is completely defined and predictable. Deterministic optimization algorithms seek to find a solution by following a strictly defined set of rules or procedure.

Evolutionary algorithms work by creating candidate populations, subjecting them to evolutionary operators such as crossbreeding, mutation, and selection to gradually improve the population toward the optimization of a given function. The methods can also be used for multicriteria optimization when multiple target functions need to be considered.

On the basis of the conducted review, the following algorithm for selecting the necessary optimization method can be proposed:

- first, determine the quality of the input data, completeness of the sample, and the number of attributes to be considered;
- then, select a group of methods based on *a posteriori* or approximation estimation;
- consider the application area, estimate the complexity of the system, and select the optimization object;
- finally, select the method depending on the task and expected results.

CONCLUSIONS

In this article, we carry out an analytical review of optimization methods and provide their classification. The methods under consideration are distinguished into two main groups: those based on the estimation of *a posteriori* distribution parameters, and those based

on an inaccurate and approximation estimation. The advantages and disadvantages of each method in terms of its applicability in simulation modeling are discussed.

The results of the study can be used when conducting scientific research in the field of model optimization problems and for the practical tasks of developing models and decision-support systems on their basis.

Future research should be aimed at a detailed study of evolutionary optimization methods and a comparison of the performance and applicability of optimization methods based on various models.

ACKNOWLEDGMENTS

The research was supported by the Ministry of Science and Higher Education of the Russian Federation (State Assignment No. 075-03-2025-256 dated January 16, 2025).

Authors' contribution

All authors equally contributed to the research work.

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Translated from Russian into English by L. Bychkova

Edited for English language and spelling by Thomas A. Beavitt