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RESEARCH ARTICLE

Development of a microwave low-pass filter based on a microstrip line projection model

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Abstract
Objectives. Sections of microstrip lines having finite length are widely used to develop integrated circuits and microwave devices for various purposes, such as power dividers, directional couplers, attenuators, and filters. In particular, low-pass filters in the microwave range are comprised of a cascade connection of regular sections of microstrip lines having various geometric parameters. However, modern approaches to calculating microwave filters using commercial software require large computational and time-consuming resources, especially when carrying out electrodynamic analysis of microstrip lines. The work set out to develop an algorithm and a method for calculating filters using a projection approach to the electrodynamic analysis of microstrip lines that reduces the time required to calculate characteristics of microwave filters while maintaining high accuracy of the obtained results.
Methods. The proposed projection approach to the electrodynamic analysis of a microstrip line can be used to rapidly and accurately calculate the main electrodynamic parameters of retardation coefficient and wave impedance across a wide range of changes in the geometrical parameters of the line, as well as its dielectric constant and frequency.
Results. Formulas obtained on the basis of analytical expressions for calculating the electrodynamic parameters of a microstrip line are used to describe the nature of changes in the elements of the scattering matrix of multistage low-pass filters in a given frequency band. A developed computer program was used to calculate the values of the elements of the low-pass filter scattering matrix across a wide range of substrate dielectric constant and frequency parameters. The obtained results were compared with the characteristics of filters calculated using commercial software.
Conclusions. The proposed approach to calculating the electrodynamic parameters of microstrip lines and consequent elements of the scattering matrix of multistage low-pass filters can significantly reduce the calculation time while achieving a sufficiently high accuracy of the obtained results to significantly reduce labor costs when calculating microwave filters in engineering practice.

Keywords: microstrip line, projection approach, low-pass filter, retardation coefficient, wave impedance, scattering matrix, reflection coefficient, transmission coefficient

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НАУЧНАЯ СТАТЬЯ

Разработка сверхвысокочастотного фильтра нижних частот на основе проекционной модели микрополосковой линии

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Резюме

Цели. Отрезки микрополосковых линий конечной длины применяются для разработки интегральных схем и устройств сверхвысоких частот (СВЧ) различного назначения, таких как делители мощности, направленные ответвители, аттенюаторы и фильтры, имеющих, в большинстве случаев, сложную топологическую структуру. В частности, фильтры нижних частот (ФНЧ) СВЧ-диапазона представляют собой ступенчатое соединение регулярных отрезков микрополосковых линий с различными геометрическими параметрами. Однако современные подходы к расчету фильтров СВЧ при помощи коммерческих программ требуют больших вычислительных и временных затрат, связанных, в частности, с предложенными подходами к электродинамическому анализу микрополосковых линий. Целью статьи является разработка алгоритма и методики расчета фильтров с использованием проекционного подхода к электродинамическому анализу микрополосковых линий, позволяющих сократить время расчета характеристик фильтров СВЧ при сохранении высокой точности полученных результатов.

Методы. Предложен проекционный подход к проведению электродинамического анализа микрополосковой линии, позволяющий быстро и с высокой точностью проводить расчет ее основных электродинамических параметров – коэффициента замедления и волнового сопротивления в широком диапазоне изменения геометрических параметров линии, ее диэлектрической проницаемости и частоты.

Результаты. На базе аналитических выражений для расчета электродинамических параметров микрополосковой линии получены формулы для описания характера изменений элементов матрицы рассеяния многокаскадных ФНЧ в заданной полосе частот. Разработана компьютерная программа, позволяющая рассчитывать значения элементов матрицы рассеяния ФНЧ в широком диапазоне диэлектрической проницаемости подложки и частоты. Проведено сравнение полученных результатов с характеристиками фильтров, рассчитанных при помощи коммерческих программ.

Выводы. Предложенный подход к расчету электродинамических параметров микрополосковых линий и, как следствие, элементов матрицы рассеяния многокаскадных ФНЧ позволяет значительно сократить время расчетов при достижении достаточно высокой точности полученных результатов, что значительно снижает трудозатраты при проектировании фильтров СВЧ в инженерной практике.

Ключевые слова: микрополосковая линия, проекционный подход, фильтр нижних частот, коэффициент замедления, волновое сопротивление, матрица рассеяния, коэффициент отражения, коэффициент передачи

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INTRODUCTION

Today, the vast majority of microwave devices and modules are structurally based on microstrip transmission lines (MTL). This is due to their small mass and size parameters, as well as the ease of transition to their topology from elements offering concentrated parameters during low-frequency prototyping of microwave devices [1]. For example, the simplest microwave low-pass filter (LPF) topology comprises a cascade of regular sections of finite-length MTLs having different strip conductor widths [2]. The geometric parameters of the MTL, as well as the dielectric constant and frequency of its substrate, determine the value of the main electrodynamic parameters of the line, namely the retardation coefficient and wave impedance, which play a key role in calculating the characteristics of microwave filters [3]. However, contemporary approaches to their calculation using a number of commercially available software products imply rather high computational and time costs both in the calculation of the basic electrodynamic parameters of MTLs [4] and in the design of microwave filters in general [5]. Considering this, the task of applying the MTL projection model detailed in [6] is relevant. In this model, an open MTL is simulated as a shield over a wide range of geometrical parameters, as well as substrate permittivity and frequency. Furthermore, [7] defines the minimum shield size that allows an open MTL to be modelled with a given accuracy, while [8] presents methods to improve the efficiency of the proposed MTL model. This paper describes the application of the MTL model proposed in [9] to the calculation of the LPF microwave scattering matrix. The presented model demonstrates high accuracy of results along with a significant reduction in the time requirement.

1. DESIGN METHODOLOGY FOR MICROWAVE LPFS

The basic methodology of microwave filter design is given in [10–12]. According to the classical approach, the first stage of microwave filter design is the calculation of its low-frequency prototype on concentrated elements. It starts with the determination of the number of sections (links) in the calculated filter. This number is determined based on the required type of filter approximation and the amount of barrier band attenuation. In this way, for the Butterworth filter, the number of sections can be determined by the following formula:

$$N = \frac{\lg\left(10^{\left[L(\omega)/10\right]} - 1\right)}{2\lg\left(\omega/\omega_{\mathrm{CO}}\right)}, \tag{1}$$

and for the Chebyshev filter:

$$N = \frac{\operatorname{arch}\left\{\left(10^{\left[L(\omega)/10\right]} - 1\right)\Big/\left(10^{\left[G_{\mathrm{T}}/10\right]} - 1\right)\right\}^{1/2}}{\operatorname{arch}\left(\omega/\omega_{\mathrm{CO}}\right)}, \tag{2}$$

where $L(\omega)$ is the value of attenuation at frequency ω in the cut-off band; ω_{CO} is the filter cut-off frequency; G_{T} is the pulse amplitude (Throb) in the passband (in decibels).

Having determined the number of links in the filter, its equivalent low-frequency circuit can be constructed in which each filter section is represented as a concentrated element (inductance or capacitance) according to Table 5.2 from [12]. The circuit shown in Fig. 1 is an example of such an equivalent circuit for a five-link LPF.

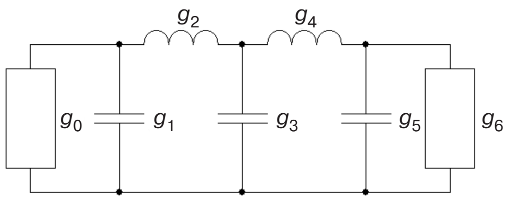

Fig. 1. Equivalent circuit of the low-pass prototype of the five-section LPF. g_i stands for normalized parameters of the equivalent circuit

The normalized parameters of the equivalent circuit (*g*-parameters) are determined by the type of filter approximation and the total number of sections *N*. For the Butterworth filter, the *g*-parameters are determined by the following formulae:

$$g_0 = g_{N+1} = 1,$$
$$g_k = 2\sin\left[\frac{(2k-1)\pi}{2N}\right],\ k = \overline{1, N}, \tag{3}$$

and for the Chebyshev filter by

$$\begin{gathered} g_0 = 1, \\ g_1 = \frac{2a_1}{\psi}, \\ g_k = \frac{4a_{k-1}a_k}{b_{k-1}g_{k-1}},\ k = \overline{2, N}, \\ g_{N+1} = \begin{cases} 1 \text{ at uneven } N, \\ \mathrm{cth}^2(\beta/4) \text{ at even } N, \end{cases} \\ \beta = \ln\left[\mathrm{cth}\left(G_\mathrm{T}/17.37\right)\right], \\ \psi = \mathrm{sh}\left(\frac{\beta}{2N}\right), \\ a_k = \sin\left[\frac{(2k-1)\pi}{2N}\right], \\ b_k = \psi^2 + \sin^2\left(\frac{\pi k}{N}\right). \end{gathered} \tag{4}$$

Once the g-parameters have been calculated, they should be denormalized to determine the absolute values of the capacitances C_k and inductances L_k in the equivalent circuit, as well as the generator and load resistances R_k, thus determining the wave impedance of the supply line. Denormalization is carried out according to the following rule:

$$R_k = R_\mathrm{LD} g_k,\ C_k = \frac{g_k}{R_\mathrm{LD}\omega_\mathrm{CO}},\ L_k = \frac{g_k R_\mathrm{LD}}{\omega_\mathrm{CO}}, \tag{5}$$

where R_LD is the load resistance equal to the wave impedance of the supply line.

Having calculated the parameters of the equivalent circuit, a transition to the topology of the filter on distributed elements should be carried out according to Table 5.3 from [10]. An example of the topology of a five-section LPF on MTL is shown in Fig. 2. As can be seen from the figure, this consists of a linear strip conductor with a varying strip width W along its length. The section l_1 of the MTL has a large wave impedance with respect to the wave impedance Z of the supply line, while the section l_2 has a smaller wave impedance. If $l_1 < \frac{\lambda_\mathrm{CO}}{8}$ and $l_2 < \frac{\lambda_\mathrm{CO}}{8}$ $\left(\lambda_\mathrm{CO} = \frac{6\pi \cdot 10^8}{\omega_\mathrm{CO}}\right)$, then section l_1 has an inductive impedance and l_2 has a capacitive impedance, where λ_CO is the wavelength corresponding to the cut-off frequency. Therefore, the above conditions should be checked when selecting the wave impedance and calculating the length of the sections.

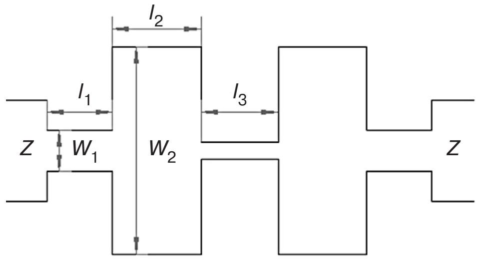

Fig. 2. Topology of a five-section microwave LPF on an MTL. W_1, W_2 are the widths of the strip conductors

In order to ensure a single-wave mode in the line (no transverse resonance), the width of the strip conductor should not exceed $\frac{\lambda_\mathrm{CO}}{4}$.

To ensure a jump in resistance at the transition from inductive to capacitive element and vice versa, the ratio of wave impedances for these elements should not be less than 3 times. The width of the strip conductors, the dielectric permittivity of the substrate, and its height are determined on the basis of the required wave impedances and taking into account the conditions described above.

Once the wave impedances and the widths of the strip conductors have been selected, the lengths of all the line segments in the filter should be determined. The length of the segment that implements the inductance is determined by the following formula:

$$l_L = \frac{3 \cdot 10^8}{\omega_\mathrm{CO}\sqrt{\varepsilon}} \arcsin\left(\frac{\omega_\mathrm{CO} L}{Z_L}\right), \tag{6}$$

while the length of the segment that implements the capacitance is given by

$$l_C = \frac{3 \cdot 10^8}{\omega_\mathrm{CO}\sqrt{\varepsilon}} \arcsin\left(\omega_\mathrm{CO} C Z_C\right), \tag{7}$$

where ε is the dielectric permittivity of the substrate; L and C are the inductance and capacitance values calculated at the low-frequency prototyping stage, respectively; and Z_L and Z_C are the wave resistances of the inductive and capacitive MTL segments, respectively.

The final stage in the design of the filter topology is the correction of the lengths of its capacitive and inductive line segments, taking into account the influence of terminal capacitances and inductances. Their values are subtracted from the initial values of inductance and capacitance for each section of the filter. Taking into account the obtained values, the corrected length of the line segments is calculated again using Eqs. (6) and (7).

The most labor-intensive part of the whole design process consists in the determination of the geometrical parameters of the strip conductor segments based on the required ratio of the line wave impedances, which can be determined using special graphs, for example, in [13]. However, it should be noted that all elements of the matrix are frequency-dependent functions when calculating the scattering matrix of the microwave filter. This requires the calculation of wave impedances and propagation constants of MTL segments as dispersion properties, which leads to significant computation time when modeling filters using commercial software.

2. PROJECTION APPROACH FOR THE CALCULATION OF MTL ELECTRODYNAMIC PARAMETERS

The projection approach considered in [14] for reducing the computational and time costs represents the surface current density on the strip conductor as a system of basic functions in the form of Chebyshev polynomials that take into account the field characteristics at the edges of the strip conductor. By decomposing the longitudinal component of the surface current density by the Chebyshev basis of only one basis function, the dispersion equation used to determine the MTL retardation coefficient n_0 can be obtained as follows:

$$\sum_{m=1}^{\infty}\left[\frac{1}{\chi_m^2}\left(n_0^2 G_m^{\mathrm{E}} + \alpha_m^2 G_m^{\mathrm{M}}\right)\right] J_0^2(m\alpha)\sin^2(m\beta) = 0, \quad (8)$$

where $G_m^{\mathrm{E}} = \left(\frac{\varepsilon}{\beta_{m1}}\mathrm{ctg}\left(k_0\beta_{m1}h\right) + \frac{1}{\beta_{m2}}\mathrm{ctg}\left[k_0\beta_{m2}(b-h)\right]\right)^{-1}$, $G_m^{\mathrm{M}} = \left(\beta_{m1}\mathrm{ctg}\left(k_0\beta_{m1}h\right) + \beta_{m2}\mathrm{ctg}\left[k_0\beta_{m2}(b-h)\right]\right)^{-1}$ are the functions obtained by solving the electric (E) and magnetic (M) eigenwave problems, respectively; $J_0(m\alpha)$ is the Bessel function; m is an integer that determines the field structure in MTL; h is the substrate height; $\beta_{m1} = \sqrt{\varepsilon - \chi_m^2}$; $\beta_{m2} = \sqrt{1 - \chi_m^2}$; $\chi_m^2 = \alpha_m^2 + \mathrm{G}^2$; $\alpha_m = \frac{\pi}{k_0 a}m$; $\alpha = \frac{\pi}{2}\cdot\frac{W}{a}$; $\beta = \alpha\left(1 + \frac{S}{W}\right)$; $k_0 = 2\pi f/c$ is the wave number; f is the frequency; c is the speed of light in a vacuum; W is the strip width; a, b are the shield dimensions; and S is the distance from the strip edge to the shield wall.

The wave impedance Z_0 is determined by the power carried through the line cross section and the current in the strip conductor, as follows:

$$Z_0 = \frac{240\pi}{k_0 a} n \times$$
$$\times \sum_{m=1}^{\infty}\left[-\left(n_0^2\left(G_m^{\mathrm{E}}\right)' + \alpha_m^2\left(G_m^{\mathrm{M}}\right)'\right) + \frac{\alpha_m^2}{\chi_m^2}\left(G_m^{\mathrm{M}} - G_m^{\mathrm{E}}\right)\right] \times \quad (9)$$
$$\times \frac{1}{\chi_m^2} J_0^2(m\alpha)\sin^2(m\beta),$$

where $\left(G_m^{\mathrm{E,M}}\right)'$ is the derivative of function $G_m^{\mathrm{E,M}}$ with respect to n_0^2.

In [9], the problem of calculating the retardation coefficient and the wave impedance of the shielded MTL using Eqs. (8) and (9) is considered. The slow convergence of the series included in these expressions results in a considerable time needed to calculate the parameters to ensure the convergence of the series. Thus, simple formulae for calculating the main electrodynamic parameters of the line in the quasi-static approximation are proposed along with the limits of their applicability. After determining the dependence of retardation coefficient and wave impedance on frequency based on the obtained expressions, the following simple formulas can be used for calculating dispersion characteristics of shielded MTL:

$$n(f) \approx \xi_n n_0(f),\ Z(f) \approx \xi_Z Z_0(f), \quad (10)$$

where $Z_0(f)$ is the wave impedance calculated in the “first approximation” at a given frequency f; ξ_n, ξ_Z are coefficients that depend on the width of the strip conductor and are determined by the following approximation formulae:

$$\xi_n \approx \begin{cases} 1, \text{ at } W/h < 2, \\ 1 + 3.6\cdot 10^{-3}\left(W/h - 2\right), \text{ at } 2 \le W/h \le 10, \end{cases}$$
$$\xi_Z \approx \begin{cases} 1, \text{ at } W/h < 2, \\ 1 - 8.725\cdot 10^{-3}\left(W/h - 2\right), \text{ at } 2 \le W/h \le 10. \end{cases} \quad (11)$$

Given the proposed formulae, the table shows the values of the propagation constant and wave impedance of MTL over a wide range of variations of the strip conductor width, the dielectric constant of the substrate, and the frequency. The first lines of each cell show the values of the propagation constant G = nk_0 and the wave impedance Z for a line with a quartz substrate (ε = 3.8). The second lines show the values for a line with a polycore substrate (ε = 9.6), while the third lines show the values for a line with an arsenide-gallium substrate (ε = 13.3). Here, the width of the strip conductor and the frequency are normalized to the substrate height h to unify the values of the electrodynamic parameters of the line. These tables allow the design engineer to determine the material and substrate height, as well as the geometric parameters of the microwave filter topology, on the basis of the specified cut-off frequency. The numerical values of the propagation constant and the retardation coefficient given for the selected geometrical and physical parameters of the filter are also necessary for the calculation of the scattering matrix elements.

Table. MTL propagation constant and wave impedance

fh, GHz · mm		0.1	1	3	5	7	10	15
$W/h = 0.1$	G	0.19 0.29 0.34	1.93 2.90 3.37	5.79 8.72 10.16	9.67 14.60 17.04	13.57 20.55 24.03	19.45 29.63 34.72	29.38 45.15 53.13
	Z	163.70 109.01 93.66	163.71 108.99 93.63	163.85 109.05 93.68	164.16 109.34 94.01	164.67 109.92 94.70	165.81 111.45 96.56	168.81 116.12 102.46
$W/h = 0.5$	G	0.20 0.30 0.35	1.97 2.98 3.48	5.92 8.99 10.50	9.90 15.10 17.67	13.90 21.31 25.01	19.97 30.85 36.31	30.26 47.28 55.92
	Z	101.64 67.23 57.69	101.65 67.21 57.66	101.75 67.23 57.67	102.00 67.44 57.91	102.40 67.90 58.44	103.29 69.06 59.83	105.57 72.36 63.82
$W/h = 1$	G	0.20 0.30 0.36	2.01 3.05 3.56	6.03 9.22 10.79	10.09 15.51 18.20	14.18 21.94 25.81	20.40 31.83 37.58	30.97 48.90 57.96
	Z	75.74 49.82 42.70	75.74 49.80 42.67	75.82 49.80 42.66	76.03 49.97 42.86	76.36 50.35 43.31	77.09 51.31 44.42	78.91 53.82 47.31
$W/h = 1.5$	G	0.20 0.31 0.36	2.03 3.11 3.63	6.12 9.41 11.02	10.24 15.85 18.63	14.41 22.44 26.44	20.74 32.59 38.53	31.51 50.09 59.40
	Z	61.45 40.24 34.46	61.45 40.21 34.42	61.52 40.21 34.41	61.69 40.37 34.60	61.98 40.70 34.99	62.61 41.51 35.93	64.13 43.51 38.16
$W/h = 2$	G	0.21 0.32 0.37	2.06 3.16 3.69	6.19 9.56 11.22	10.37 16.12 18.98	14.59 22.85 26.96	21.02 33.20 39.28	31.95 50.98 60.46
	Z	52.01 33.93 29.03	52.01 33.91 29.00	52.06 33.91 28.99	52.22 34.07 29.17	52.48 34.40 29.54	53.05 35.15 30.36	54.36 36.89 32.16
$W/h = 3$	G	0.21 0.32 0.38	2.09 3.23 3.79	6.31 9.80 11.52	10.57 16.55 19.51	14.88 23.47 27.73	21.45 34.08 40.35	32.61 52.22 61.89
	Z	40.06 25.99 22.21	40.05 25.97 22.18	40.10 25.98 22.19	40.23 26.15 22.37	40.46 26.46 22.69	40.93 27.09 23.34	41.97 28.41 24.60
$W/h = 4$	G	0.21 0.33 0.38	2.12 3.29 3.85	6.39 9.99 11.75	10.72 16.87 19.90	15.10 23.91 28.26	21.76 34.69 41.06	33.06 53.01 62.78
	Z	32.71 21.14 18.05	32.71 21.12 18.03	32.75 21.15 18.05	32.87 21.32 18.22	33.08 21.60 18.51	33.49 22.14 19.04	34.35 23.17 19.99
$W/h = 5$	G	0.21 0.33 0.39	2.14 3.33 3.91	6.46 10.13 11.92	10.83 17.11 20.19	15.26 24.24 28.65	22.00 35.12 41.56	33.40 53.55 63.36
	Z	27.70 17.85 15.24	27.70 17.84 15.21	27.74 17.88 15.24	27.86 18.05 15.42	28.04 18.30 15.67	28.41 18.75 16.12	29.13 19.56 16.87

Table. Continued

fh, GHz · mm		0.1	1	3	5	7	10	15
$W/h = 6$	G	0.22 0.34 0.39	2.16 3.37 3.95	6.51 10.24 12.06	10.93 17.30 20.42	15.39 24.49 28.95	22.18 35.44 41.92	33.64 53.94 63.78
	Z	24.06 15.47 13.19	24.05 15.45 13.17	24.09 15.50 13.21	24.21 15.67 13.38	24.38 15.89 13.61	24.71 16.27 13.99	25.33 16.92 14.61
$W/h = 7$	G	0.22 0.34 0.40	2.17 3.40 3.99	6.56 10.33 12.17	11.00 17.45 20.60	15.50 24.69 29.17	22.32 35.69 42.19	33.84 54.23 64.09
	Z	21.27 13.65 11.64	21.27 13.64 11.62	21.31 13.70 11.67	21.42 13.85 11.83	21.58 14.05 12.04	21.88 14.36 12.37	22.42 14.90 12.91
$W/h = 8$	G	0.22 0.34 0.40	2.19 3.42 4.02	6.60 10.41 12.27	11.06 17.57 20.74	15.59 24.84 29.35	22.44 35.88 42.40	33.99 54.45 64.32
	Z	19.08 12.23 10.42	19.08 12.23 10.41	19.12 12.28 10.46	19.23 12.42 10.61	19.38 12.59 10.80	19.65 12.86 11.09	20.13 13.31 11.57
$W/h = 10$	G	0.22 0.34 0.40	2.20 3.46 4.07	6.65 10.53 12.41	11.16 17.75 20.96	15.72 25.07 29.62	22.62 36.16 42.71	34.21 54.76 64.65
	Z	15.84 10.12 8.63	15.83 10.12 8.61	15.88 10.18 8.68	15.98 10.30 8.81	16.11 10.43 8.97	16.33 10.63 9.21	16.72 10.98 9.60

A linear function can be used to approximate the numerical values of the propagation constant and wave impedance within two adjacent frequencies. This allows sufficient accuracy in calculating the elements of the scattering matrix.

3. CALCULATION OF SCATTERING MATRIX ELEMENTS FOR MULTI-CASCADE LPF

According to [12], the most convenient approach to calculating the scattering matrix of multi-cascade microwave filters is the transition to the transmission matrices calculated separately for each irregularity in the filter, followed by their multiplication and the reverse transition from the final transmission matrix to the final scattering matrix. In more detail, we consider the calculation of the scattering and transmission matrix for the ith-step transition shown in Fig. 3.

It is assumed that the source resistance $Z_{src}(f) = Z_1(f)$ and the load resistance $Z_{LD}(f) = Z_2(f)$ when the source is connected to the left arm and the load to the right arm. In this case, the coefficient of reflection from the first arm, S_{11}, and the coefficient of transmission from the first arm to the second arm, S_{21}, are functions of the frequency which are determined as follows:

$$S_{11}(f) = \frac{Z_2(f) - Z_1(f)}{Z_2(f) + Z_1(f)} e^{-i\mathrm{G}_1(f)2l_1}, \quad (12)$$

$$S_{21}(f) = \sqrt{1 - |S_{11}(f)|^2} e^{-i(\mathrm{G}_1(f)l_1 + \mathrm{G}_2(f)l_2)} = \frac{2\sqrt{Z_1(f)Z_2(f)}}{Z_2(f) + Z_1(f)} e^{-i(\mathrm{G}_1(f)l_1 + \mathrm{G}_2(f)l_2)}. \quad (13)$$

Similar expressions can be obtained for the reflection coefficient from the second arm, S_{22}, and the transmission coefficient from the second arm to the first arm, S_{12}, by substituting 1 for 2 and 2 for 1 in expressions (12) and (13):

$$S_{22}(f) = \frac{Z_1(f) - Z_2(f)}{Z_1(f) + Z_2(f)} e^{-i\mathrm{G}_2(f)2l_2}, \quad (14)$$

$$S_{12}(f)=\sqrt{1-\left|S_{22}(f)\right|^2}\,\mathrm{e}^{-i\left(\mathrm{G}_2(f)l_2+\mathrm{G}_1(f)l_1\right)}=$$
$$=\frac{2\sqrt{Z_2(f)Z_1(f)}}{Z_1(f)+Z_2(f)}\mathrm{e}^{-i\left(\mathrm{G}_2(f)l_2+\mathrm{G}_1(f)l_1\right)}. \quad (15)$$

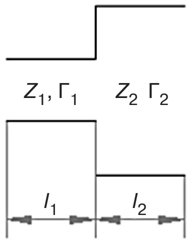

Fig. 3. The MTL step transition topology.
Z_1, Z_2 are the wave impedance of the MTL segment;
G_1, G_2 are the propagation constants of this segment

Thus, the scattering matrix $\mathbf{S}_i$ for the step transition can be represented by:

$$\mathbf{S}_i=\begin{bmatrix}\dfrac{Z_2(f)-Z_1(f)}{Z_2(f)+Z_1(f)}\mathrm{e}^{-i\mathrm{G}_1(f)2l_1} & \dfrac{2\sqrt{Z_2(f)Z_1(f)}}{Z_1(f)+Z_2(f)}\mathrm{e}^{-i\left(\mathrm{G}_2(f)l_2+\mathrm{G}_1(f)l_1\right)}\\ \dfrac{2\sqrt{Z_1(f)Z_2(f)}}{Z_2(f)+Z_1(f)}\mathrm{e}^{-i\left(\mathrm{G}_1(f)l_1+\mathrm{G}_2(f)l_2\right)} & \dfrac{Z_1(f)-Z_2(f)}{Z_1(f)+Z_2(f)}\mathrm{e}^{-i\mathrm{G}_2(f)2l_2}\end{bmatrix}. \quad (16)$$

The transition from the scattering matrix $\mathbf{S}_i$ to the transmission matrix $\mathbf{T}_i$ is performed according to [12] by the following rule:

$$\mathbf{T}_i=\begin{bmatrix}T_{11}(f) & T_{12}(f)\\ T_{21}(f) & T_{22}(f)\end{bmatrix}=\begin{bmatrix}\dfrac{1}{S_{21}(f)} & -\dfrac{S_{22}(f)}{S_{21}(f)}\\ \dfrac{S_{11}(f)}{S_{21}(f)} & S_{12}(f)-\dfrac{S_{11}(f)S_{22}(f)}{S_{21}(f)}\end{bmatrix}. \quad (17)$$

After calculating the transmission matrix $\mathbf{T}_i$ for each ith irregularity in the filter topology, the final LPF transmission matrix $\mathbf{T}$ can be obtained by multiplying the transmission matrices of all its irregularities:

$$\mathbf{T}=\prod_{i=1}^{N+1}\mathbf{T}_i. \quad (18)$$

The transition from the transmission matrix $\mathbf{T}$ to the LPF scattering matrix $\mathbf{S}$ is performed according to [12] by the following rule:

$$\mathbf{S}=\begin{bmatrix}\dfrac{T_{21}(f)}{T_{11}(f)} & T_{22}(f)-\dfrac{T_{21}(f)T_{12}(f)}{T_{11}(f)}\\ \dfrac{1}{T_{11}(f)} & -\dfrac{T_{12}(f)}{T_{11}(f)}\end{bmatrix}. \quad (19)$$

4. RESULTS OF NUMERICAL ANALYSIS

Based on the above algorithm, a software is developed using the *GNU Octave*[1] programming language to calculate the LPF characteristics in a wide range of varying strip conductor width ($0.1 \le W/h \le 10$), substrate permittivity ($2 \le \varepsilon \le 20$) and frequency (0.1 GHz $\le f \le$ 15 GHz). The structure of the software includes the main body, in which mathematical calculations are performed according to the algorithm given in this paper, as well as a subroutine for approximating the data from the table by frequency and dielectric permittivity. Based on the results obtained using the software, the LPF is calculated at different cut-off frequencies (1, 5, and 10 GHz) and its characteristics are compared with those obtained using commercial software. The amplitude-frequency characteristics of the corresponding filters (solid line is the filter

1 https://octave.org/. Accessed March 20, 2025.

calculation using the proposed approach; dashed line is the filter calculation using commercial software) and the absolute gain calculation error values (right) are shown in Fig. 4. The time taken to calculate the transmittance using the proposed approach is only a few seconds, while calculating the transmittance using commercial software takes several minutes.

By analyzing the obtained graphs, it can be concluded that the absolute value of the transmission coefficient error does not exceed 0.08 in the whole investigated range. Simultaneously, for the LPF with 1 GHz cut-off frequency, the maximum value of the absolute transmission coefficient error is observed in the barrier band at a frequency of 7 GHz and is 0.072. For the LPF having a cut-off frequency of 5 GHz, while the maximum value of the absolute error of the transmission coefficient at a frequency of 12 GHz is 0.04. For the LPF with a cut-off frequency of 10 GHz, the maximum value of the absolute error of the transmission coefficient is 0.04.

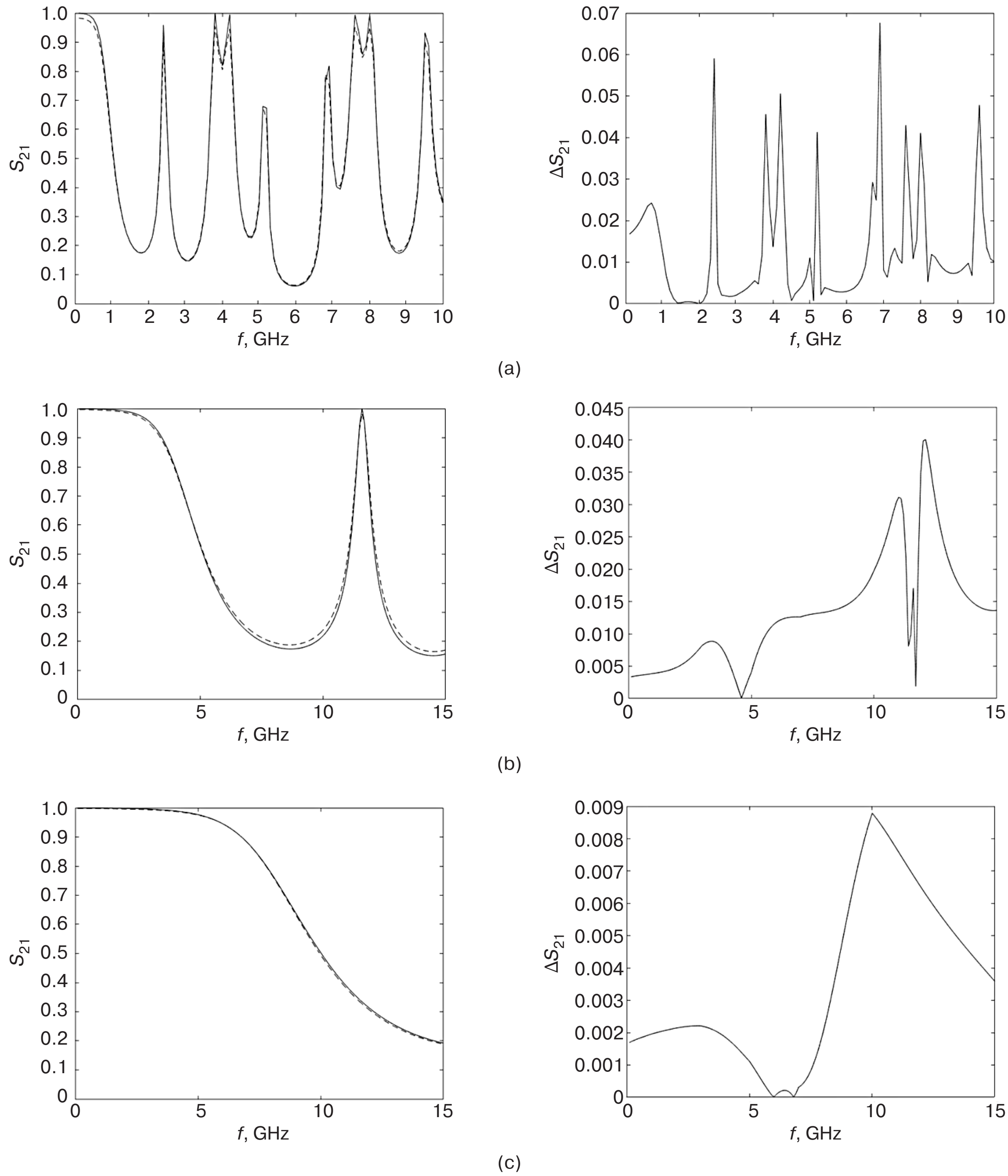

Fig. 4. LPF amplitude frequency response at the cut-off frequency:
(a) 1 GHz; (b) 5 GHz; (c) 10 GHz

CONCLUSIONS

The present work presents a microstrip LPF designed based on the MTL projection model along with a calculation of its scattering matrix. The accuracy of the obtained results is verified by comparing the transmission coefficient of the developed filter with a model constructed using modern computer-aided design systems. The absolute error of the transmission coefficient in a wide frequency band up to 15 GHz calculated based on the comparison does not exceed 0.08 for different filter cut-off frequencies. The use of the projection approach allows a significant (tenfold) reduction of the calculation time of the reflection and transmission coefficients for each pair of microwave multipole arms, together with a sufficiently high accuracy of the obtained results.

Authors' contribution. All authors equally contributed to the research work.

REFERENCES

1. Petrov I.A. Microwave filters with use broadband matching structures. *Fizika volnovykh protsessov i radiotekhnicheskie sistemy = Physics of Wave Processes and Radio Systems.* 2011;14(1):51–56 (in Russ.).
2. Lamanov Yu.A., Kudryavtseva T.O., Drobotun N.B. Design and Research Process of Microstrip Low-Pass Filters with High Slope Steepness. *Doklady Tomskogo gosudarstvennogo universiteta sistem upravleniya i radioelektroniki* (*Doklady TUSUR*) = *Proceedings of TUSUR University.* 2021;24(2):7–13 (in Russ.).
3. Fel'dshtein A.L. (Ed.). *Spravochnik po elementam poloskovoi tekhniki* (*Handbook of Elements of Strip Technology*). Moscow: Svjaz'; 1979. 336 p. (in Russ.).
4. Gupta K.C., Garg R., Chadha R. *Mashinnoe proektirovanie SVCh ustroistv* (*Computer-aided Design of Microwave Circuits*): transl. from Engl. Moscow: Radio i svjaz'; 1987. 432 p. (in Russ.).
[Gupta K.C., Garg R., Chadha R. *Computer-aided Design of Microwave Circuits*. Dedham, Mass.: Artech House Inc.; 1981. 636 p.]
5. Lopatin V.V., Khvorenko V.V. Modeling and implementation of a microstrip filter. In: *Information Technologies in Science, Industry and Education: collection of papers of the Scientific and Technical Conference*. Izhevsk; 2021. P. 166–176 (in Russ.).
6. Kovalenko A.N. Projection method for constructing full-wave models of striplines. *J. Commun. Technol. Electron*. 2019;64(2):93–99. https://doi.org/10.1134/S1064226919020128
[Original Russian Text: Kovalenko A.N. Projection method for constructing full-wave models of striplines. *Radiotekhnika i elektronika*. 2019;64(2):108–115 (in Russ.). https://doi.org/10.1134/S0033849419020128]
7. Kovalenko A.N., Yarlykov A.D. Numerical analysis of a shielded microstrip line. In: *Actual Problems and Prospects for the Development of Radio Engineering and Infocommunication Systems* (*Radioinfocom-2021*): *Proceedings of the 5th International Scientific and Practical Conference.* Moscow: RTU MIREA; 2021. P. 331–334 (in Russ.).
8. Kovalenko A.N., Yarlykov A.D. Increasing the efficiency of projection models of strip lines. *J. Commun. Technol. Electron*. 2021;66(9):997–1003. https://doi.org/10.31857/S0033849421090084
[Original Russian Text: Kovalenko A.N., Yarlykov A.D. Increasing the efficiency of projection models of strip lines. *Radiotekhnika i elektronika*. 2021;66(9):837–844 (in Russ.). https://doi.org/10.31857/S0033849421090084]
9. Kovalenko A.N., Yarlykov A.D. Analytical expressions for electrodynamic parameters of the shielded microstrip line. *Russian Technological Journal.* 2021;9(4):68–76 (in Russ.). https://doi.org/10.32362/2500-316X-2021-9-4-68-76
10. Fusco V. *SVCh tsepi. Analiz i avtomatizirovannoe proektirovanie* (*Microwave Circuits. Analysis and Computer-aided Design*): transl. from Engl. Moscow: Radio i svyaz'; 1990. 288 p. (in Russ.).
[Fusco V.F. *Microwave Circuits. Analysis and Computer-aided Design.* Prentice Hall; 1986. 358 p.]
11. Matthaei G.L., Yong L., Jones E.M.T. *Fil'try SVCh, soglasuyushchie tsepi i tsepi svyazi* (*Microwave Filters, Impedance-Matching Networks, and Coupling Structures*): transl. from Engl. Moscow: Svyaz'; 1972. V. 2. 496 p. (in Russ.).
[Matthaei G.L., Yong L., Jones E.M.T. *Microwave Filters, Impedance-Matching Networks, and Coupling Structures.* McGraw-Hill; 1964. 1096 p.]
12. Kostin M.S., Yarlykov A.D. *Ustroistva i moduli sverkhvysokikh chastot* (*Devices and Modules of Ultra-High Frequencies*). Moscow, Vologda: Infra-Inzheneriya; 2022. 400 p. (in Russ.).
13. Maloratskii L.G., Yavich L.R. *Proektirovanie i raschet SVCh-elementov na poloskovykh liniyakh* (*Design and Calculation of Microwave Elements on Strip Lines*). Moscow: Sovetskoe radio; 1972. 232 p. (in Russ.).
14. Kovalenko A.N. Natural modes of a microstrip line. *Radiophys. Quantum Electron.* 1978;21(2):128–133. https://doi.org/10.1007/BF01078702
[Original Russian Text: Kovalenko A.N. Natural modes of a microstrip line. *Izvestiya vysshikh uchebnykh zavedenii. Radiofizika*. 1978;21(2):188–194 (in Russ.).]

СПИСОК ЛИТЕРАТУРЫ

1. Петров И.А. Фильтры СВЧ с использованием широкополосных согласующих структур. *Физика волновых процессов и радиотехнические системы*. 2011;14(1):51–56.

2. Ламанов Ю.А., Кудрявцева Т.О., Дроботун Н.Б. Разработка и исследование микрополоскового фильтра низких частот с высокой крутизной спада АЧХ. *Доклады Томского государственного университета систем управления и радиоэлектроники* (*Доклады ТУСУР*). 2021;24(2):7–13. https://doi.org/10.21293/1818-0442-2021-24-2-7-13
3. *Справочник по элементам полосковой техники;* под ред. А.Л. Фельдштейна. М.: Связь; 1979. 336 с.
4. Гупта К., Гардж Р., Чадха Р. *Машинное проектирование СВЧ устройств*: пер. с англ. М.: Радио и связь; 1987. 432 с.
5. Лопатин В.В., Хворенко В.В. Моделирование и реализация микрополоскового фильтра. В сб.: *Информационные технологии в науке, промышленности и образовании: сборник трудов научно-технической конференции*. Ижевск; 2021. С. 166–176.
6. Коваленко А.Н. Проекционный метод построения электродинамических моделей полосковых линий. *Радиотехника и электроника*. 2019;64(2):108–115. https://doi.org/10.1134/S0033849419020128
7. Коваленко А.Н., Ярлыков А.Д. Численный анализ экранированной микрополосковой линии. В сб.: *Актуальные проблемы и перспективы развития радиотехнических и инфокоммуникационных систем* (*Радиоинфоком-2021*): *Сборник научных статей V Международной научно-практической конференции.* М.: РТУ МИРЭА; 2021. С. 331–334.
8. Коваленко А.Н., Ярлыков А.Д. Повышение эффективности проекционных моделей полосковых линий. *Радиотехника и электроника*. 2021;66(9):837–844. https://doi.org/10.31857/S0033849421090084
9. Коваленко А.Н., Ярлыков А.Д. Аналитические выражения для электродинамических параметров экранированной микрополосковой линии. *Russian Technological Journal.* 2021;9(4):68–76. https://doi.org/10.32362/2500-316X-2021-9-4-68-76
10. Фуско В. *СВЧ цепи. Анализ и автоматизированное проектирование*: пер. с англ. М.: Радио и связь; 1990. 288 с.
11. Маттей Д.Л., Янг Л., Джонс Е.М.Т. *Фильтры СВЧ, согласующие цепи и цепи связи*: пер. с англ. М.: Связь; 1972. Т. 1. 496 с.
12. Костин М.С., Ярлыков А.Д. *Устройства и модули сверхвысоких частот*. М., Вологда: Инфра-Инженерия; 2022. 400 с.
13. Малорацкий Л.Г., Явич Л.Р. *Проектирование и расчет СВЧ-элементов на полосковых линиях.* М.: Советское радио; 1972. 232 с.
14. Коваленко А.Н. Собственные волны микрополосковой линии. *Изв. вузов. Радиофизика.* 1978;21(2):188–194.

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