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Математическое моделирование

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RESEARCH ARTICLE

Dynamic model of BSF portfolio management

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Abstract

Objectives. The work compares studies on BSF portfolios consisting of a risk-free Bond (B) asset, a Stock (S), and a cash Flow (F) that represents risky asset prices in the form of a tree structure. On the basis of existing models for managing dynamic investment portfolios, the work develops a dynamic model for managing a BSF portfolio that combines risk-free and risky assets with a deposit. Random changes in the prices of a risky asset are reflected in the developed model according to a tree structure. Two approaches to portfolio formation are proposed for the study: (1) initial capital is invested in a risk-free asset, while management is conducted at the expense of a risky asset; (2) the initial capital is invested in a risky asset, but management is carried out at the expense of a risk-free asset.

Methods. A binomial model was used to predict the prices of risky assets. Changes in risky asset prices in the model are dynamically managed via a branching tree structure. A comparative analysis of modeling results reveals the optimal control method.

Results. A dynamic model for unrestricted management of a BSF portfolio has been developed. By presenting risky asset prices according to a tree structure, the model can be used to increase the accuracy of evaluating investments by from 2.4 to 2.7 times for the first approach and from 1.7 to 2.7 times for the second. The increased accuracy of evaluating investments as compared with previously proposed models is achieved by averaging prices at various vertices of the tree.

Conclusions. The results of the research suggest that the use of a dynamic management model based on a tree-like price structure can significantly increase the accuracy of evaluating investments in an investment portfolio.

Keywords: optimal control, dynamic system with random parameters, dynamic programming, investment portfolio, tracking a reference portfolio, binomial price structure of a risky asset

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НАУЧНАЯ СТАТЬЯ

Динамическая модель управления BSF-портфелем без ограничений

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Резюме
Цели. Рассматриваются модели управления инвестиционными портфелями, носящими динамический характер, проводится сравнение исследований, посвященных BSF-портфелям (состоящим из безрискового актива (bond), акции (stock) и потока платежей (cash flow)) с древовидной структурой цен рискового актива. Целью работы является разработка динамической модели управления BSF-портфелем, включающим безрисковый, рисковый активы и депозит. В отличие от проведенных ранее исследований, в разрабатываемой модели цены рискового актива изменяются случайным образом, следуя древовидной структуре. К исследованию предлагается два подхода формирования портфеля: 1) начальный капитал вкладывается в безрисковый актив, управление происходит за счет рискового актива; 2) начальный капитал вкладывается в рисковый актив, управление происходит за счет безрискового актива.
Методы. Использована биномиальная модель для моделирования цен рискового актива. Динамическая модель управления на основе древовидной структуры цен рискового актива позволяет учитывать изменения в ценах активов. Сравнительный анализ результатов моделирования выявляет оптимальный способ управления.
Результаты. Разработана динамическая модель управления BSF-портфелем без ограничений. Показано, что динамическая модель управления на основе древовидной структуры цен рискового актива позволяет повысить точность оценки объема вложений от 2.4 до 2.7 раз для первого подхода и от 1.7 до 2.7 раз – для второго. Повышение точности оценки объемов вложений по сравнению с ранее предложенными моделями достигается путем усреднения цен по различным вершинам дерева.
Выводы. Проведенное исследование позволяет говорить о том, что применение динамической модели управления, основанной на древовидной структуре цен, позволяет значительно повысить точность оценки объема вложений в инвестиционный портфель.

Ключевые слова: оптимальное управление, динамическая система со случайными параметрами, динамическое программирование, инвестиционный портфель, слежение за эталонным портфелем, биномиальная структура цен рискового актива

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INTRODUCTION

The management of investment portfolios (IP) can be analyzed in terms of multi-period dynamic decision-making problems pertaining to transactions that occur at discrete points in time. An evaluation carried out by the investor concerning possible future changes in interest rates, prices, or cash flows from securities forms the basis for further decisions to buy or sell, open deposits or lend, i.e., decisions to reshape the IP. The present work sets out to develop and verify a dynamic model for the management of a portfolio that combines a risky asset (RA), a riskless asset (RLA), and a deposit.

Dynamic models of the IP management have been studied in detail in a number of works [1–12]. The study carried out by V.V. Dombrovsky discusses problems involved in controlling discrete stochastic systems and applying the quadratic criterion in this area. The author considers systems characterized by their functional dependency on states and control actions whose various random parameters include additive and multiplicative sources of noise. As well as deriving equations for optimal linear static and dynamic output regulators, the study applies the obtained conclusions to solve the dynamic IP optimization problem for a portfolio whose financial assets having variable price volatility are analyzed in discrete time. The practical significance of the work lies in the possibility of developing effective IP management strategies [1]. In the study [3], the problems of synthesizing control strategies in discrete systems using a predictive model are considered. These systems also include random parameters comprising additive and multiplicative noise phenomena that depend on the states and controls. The work develops control strategies using prediction for closed-loop and open-loop systems taking into account random factors and restrictions. The results are applied to solve the dynamic optimization problem of IP taking into account restrictions on trading operations. Another study [8] considers the problem of managing an IP consisting of RA and RLA taking into account dynamic tracking of the benchmark portfolio. Price changes on RA are described by stochastic equations with Gaussian and impulsive Poisson perturbations. The method for determining an optimal control strategy using feedback based on the application of a quadratic criterion can be used to evaluate the quality of control and select the best strategy for minimizing uncertainty and achieving the best results. The main scientific contribution of the study to the field of IP control consists in its innovative use of stochastic analysis and feedback techniques. The study by D.V. Dombrovsky and E.A. Lyashenko [9] analyzes the dynamics of the IP control model taking into account restrictions on the trading operations. The model includes stochastic difference equations with random volatility to describe the dynamics of prices of risky financial assets within the given IP. An important problem of IP management arises when trying to ensure effective investment management under conditions of restrictions on trading operations. In order to minimize risks and achieve the best results under financial market conditions where asset prices are subject to random fluctuations, volatility is considered as a random variable. The dissertation by T.Yu. Pashinskaya[1] synthesizes the results of research devoted to the control of nonlinear discrete systems with random parameters under constraints. The author develops a methodology for tracking a hypothetical benchmark portfolio with a predetermined growth trajectory in the field of IP management. The results of the study are used to derive equations for determining optimal strategies of IP management with feedback in the presence of constraints. In [13], a dynamic model of IP management using a linear quality criterion is developed.

Studies based on BSF-portfolios comprising RLA Bond (B), Stock (S), and cash Flow (F) with a tree-like RA price structure have also been conducted [14–21]. These studies analyze market structures including such assets as stocks, RLA bonds, and cash flow. The essence of the model is revealed under certain conditions for completeness and absence of arbitrage in the market. A numerical approach to the development of a self-financing strategy provides a payment function superior to the one established in the terminal vertices of the price tree given an initial portfolio of minimum value. The works [17–21] analyze the properties of the (B, S)-market when market completeness and arbitrage-free conditions are violated. Particular attention is paid to the problems related to the inadequacy of the model representation of the RA price evolution in the process of exchange trading using the binomial pricing mechanism in an incomplete (B, S)-market. The described methods take the impact of market trends on the process of RA price evolution into account.

The present study proposes a new dynamic model of BSF portfolio management including RLA, RA, and deposits. Unlike those described in works [1–12], the presented model considers random RA price changes according to a tree structure. The novelty of the model consists in the increased accuracy of investment evaluation as compared to the model described in the work of T.Yu. Pashinskaya. This effect is achieved by averaging prices across different vertices of the tree.

[1] Pashinskaya T.Yu. *Control with prediction of nonlinear discrete systems with random parameters under constraints*: Cand. Sci. Thesis (Phys.-Math.). Tomsk: Tomsk State University; 2021. http://vital.lib.tsu.ru/vital/access/manager/Repository/koha:000702951 (in Russ.). Accessed February 26, 2024.

MODEL CONSTRUCTION

Let us consider a portfolio consisting of RLA, RA, and deposits at discrete moments of time 0, 1, 2, …, n. We will denote the RLA return rate as $r_1(t)$. The price of RLA is known at each moment of time. The randomly changing price of RLA can take one of two possible values at any one time, i.e., possible prices of the stock have the structure of a binary tree (Fig. 1) with terminal vertices (Fig. 2). Let us denote the probability of the RA price increasing by a random value η as p, while the probability that the asset price will decrease by a random value η will be denoted as $q = 1 - p$.

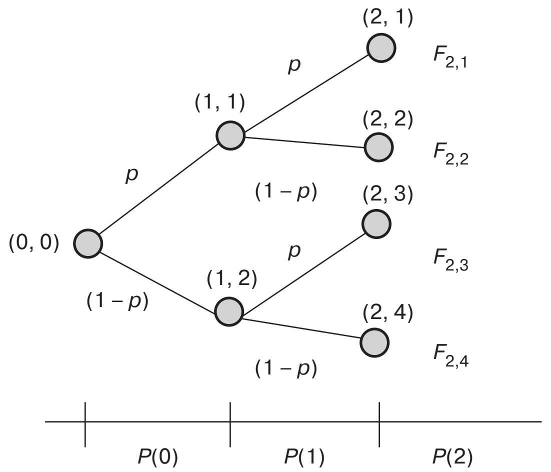


Fig. 1. Two-period RA tree. $F_{t,i}$ is the payment function for the point with number (t, i); $P(t)$ is the payment for time step t

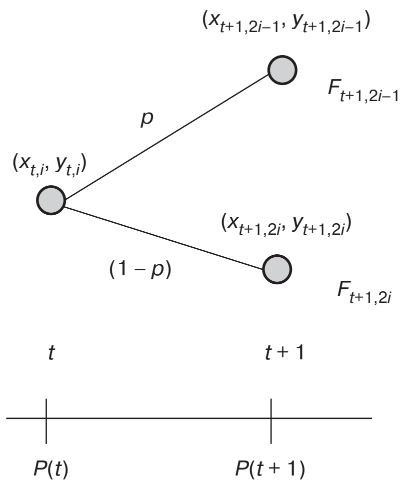


Fig. 2. Terminal vertices of the RA tree

Let us denote the share of RLA as x, and the share of RA as y. The successors of the (t, i)th vertex are the vertices with the numbers $(t + 1, 2i - 1)$ and $(t + 1, 2i)$. The price of RLA in the vertex with number (t, i) corresponding to the moment of time t is equal to $C(t)$, while the price of RLA in the vertices with numbers $(t + 1, 2i - 1)$ and $(t + 1, 2i)$ corresponding to the moment of time $t + 1$ is equal to $C'(t + 1)$. The price of RA in the vertex with number (t, i) corresponding to the moment of time t is equal to $C''_{t,i}$, while the prices of RA in the vertices with numbers $(t + 1, 2i - 1)$ and $(t + 1, 2i)$ corresponding to the moment of time $t + 1$ are equal to $C''_{t+1,2i-1} = C''_{t,i}(1+\eta_i)$ and $C''_{t+1,2i} = C''_{t,i}(1-\eta_i)$ respectively. For each time step t, the payments $P(t)$ are set.

We will suppose that at the initial stage all available funds were invested in RLA and no borrowed funds were used. It is important to note that both RLA and RA can be acquired or alienated at any time, which implies their high availability and readiness for trading [15, 16]. One of the key features of payment flow is its limited liquidity according to which payments are constrained. Since each path from the initial vertex of the price tree structure to the terminal vertex represents a particular scenario, it can be randomized.

The method of asset portfolio management consists in determining at each point of the price tree the RLA x_i and RA y_i under the following conditions [16, 17]:

a) for each endpoint of the tree, a payment function $F_{t,i} \geq 0$, $i = 1, 2, \ldots, 2^t$, is defined, representing the amount that the investor expects to receive when asset prices reach the corresponding tree vertex, after selling assets and making payments or receipts of funds along the payment stream;

b) there is a fee for borrowing assets. For example, if x units of RLA are borrowed, at the next moment of time λx units (RLA) should be returned, and μ is the RA loan fee;

c) the market is self-financing, i.e., the investor can buy and sell assets, providing payments and receipts on the payment flow so that the portfolio value at each moment of time does not change, but at the same time the vertex-average value of the portfolio with time changes according to the given law in accordance with the law of change of the vertex-average payment function. Specific vertices can be mathematically written as follows:

$$C'(t+1)\lambda x_{t,i} + C''_{t+1,2i-1}\,\mu\, y_{t,i} + P(t+1) = \\ = C'(t+1)x_{t+1,2i-1} + C''_{t+1,2i-1}y_{t+1,2i-1}, \quad (1)$$

$$C'(t+1)\lambda x_{t,i} + C''_{t+1,2i}\,\mu\, y_{t,i} + P(t+1) = \\ = C'(t+1)x_{t+1,2i} + C''_{t+1,2i}y_{t+1,2i} \quad (2)$$

at $i = \overline{1, 2^t}$.

In the terminal vertices the following inequalities must be met:

$$C'(t+1)\lambda x_{t,i} + C''_{t+1,2i-1}\,\mu\, y_{t,i} + P(t+1) \geq F_{t+1,2i-1}, \quad (3)$$

$$C'(t+1)\lambda x_{t,i} + C''_{t+1,2i}\,\mu\, y_{t,i} + P(t+1) \geq F_{t+1,2i}. \quad (4)$$

Constructing a dynamic model of a BSF portfolio with one RA and one RLA

Let us analyze the IP, where the components are RA with variable returns and risk-free deposits with constant return. At the moment of time t the funds invested in RA are equal to $V''(t)$, and the funds invested in RLA are equal to $V'(t)$. Then the total amount of investments at the moment of time t will be equal, taking into account the deposit

$$V(t) = V'(t) + V''(t) - P(t). \quad (5)$$

Using formulas (1), (2) for the moment of time $t = 1$ (vertices (1, 1) and (1, 2)), we obtain

$$C'(1)x_{1,1} + C''_{1,1}y_{1,1} - P(1) = C'(1)\lambda x_{0,0} + \mu C''_{1,1}y_{0,0},$$
$$C'(1)x_{1,2} + C''_{1,2}y_{1,2} - P(1) = C'(1)\lambda x_{0,0} + \mu C''_{1,2}y_{0,0}.$$

Given the fact that RA prices in vertices (1, 1) and (1, 2) are random, accepting values $C''_{1,1}$ and $C''_{1,2}$ with probabilities p and $q = 1 - p$ respectively, the value of the portfolio at the moment of time $t = 1$ will be equal to

$$V(1) = \lambda C'(1)x_{0,0} + \mu C''(1)y_{0,0}. \quad (6)$$

Here $C'(1)x_{0,0}$ is the RLA cost at the moment of time $t = 1$; $C''(1)y_{0,0}$ is the RA cost at the moment of time $t = 1$;

$$C''(1) = pC''_{1,1} + qC''_{1,2}. \quad (7)$$

$C''(1)$ is the average value of RA price at the moment of time $t = 1$.

For the moment of time $t = 2$ (vertices (2, 1), (2, 2), (2, 3) and (2, 4)) we obtain:

$$V(2) = \lambda C'(2)x_1 + \mu C''(2)y_1. \quad (8)$$

Here

$$C''(2) = p\left(pC''_{2,1} + qC''_{2,2}\right) + q\left(pC''_{2,3} + qC''_{2,4}\right). \quad (9)$$

$C''(2)$ is the vertex-averaged value of RA price at the moment of time $t = 2$; x_1 is the average value of RLA share at the moment of time $t = 1$; y_1 is the average value of RA share at the moment of time $t = 1$.

For the moment of time $t = 3$ (vertices (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (3, 7), (3, 8)) we obtain:

$$V(3) = \lambda C'(3)x_2 + \mu C''(3)y_2, \quad (10)$$

where x_2 is the average value of the RLA share at the moment of time $t = 2$; y_2 is the average value of the RA share at the moment of time $t = 2$;

$$C''(3) = p\left(p\left(pC''_{3,1} + qC''_{3,2}\right) + q\left(pC''_{3,3} + qC''_{3,4}\right)\right) + q\left(p\left(pC''_{3,5} + qC''_{3,6}\right) + q\left(pC''_{3,7} + qC''_{3,8}\right)\right). \quad (11)$$

$C''(3)$ is the average value of RA price at the moment of time $t = 3$.

Continuing this process, we obtain:

$$V(t) = \lambda C'(t)x_{t-1} + \mu C''(t)y_{t-1},\ t = 1, 2, 3, \ldots, \quad (12)$$

where $x_0 = x_{0,0}$, $y_0 = y_{0,0}$. Here x_{t-1} is the vertex average of the RLA share at the moment of time $t - 1$; y_{t-1} is the average value of the RA share at the moment of time $t - 1$; $C''(t)$ is the average value of the RA price at the moment of time t; y_t is the average value of the RA share at the moment of time t.

Let us introduce the values

$$m_{t,i} = pC''_{t,2i-1} + qC''_{t,2i},\ i = \overline{1, 2^{t-1}}. \quad (13)$$

Then the average price of RA at the moments of time 1, 2, 3, ... can be represented as

$$C''(1) = m_{1,1}, \quad (14)$$

$$C''(2) = pm_{1,1} + qm_{2,2}, \quad (15)$$

$$C''(3) = p(pm_{3,1} + qm_{3,2}) + q(pm_{3,3} + qm_{3,4}). \quad (16)$$

It is easy to show that for any moment of time t the sum of probabilities is equal to 1. Indeed,

$$\sum_{k=0}^{n} C_n^k p^k q^{n-k} = (p+q)^n = 1, \quad (17)$$

where $C_n^k = \dfrac{n!}{k!(n-k)!}$ are the binomial coefficients.

In the terminal vertices the following inequalities must be met:

for the moment of time $t = 1$

$$\lambda C'(1)x_{0,0} + \mu C''(1)y_{0,0} + P(1) \geq F(1); \quad (18)$$

for the moment of time $t = 2$

$$\lambda C'(2)x_1 + \mu C''(2)y_1 + P(2) \geq F(2); \quad (19)$$

for the moment of time t

$$\lambda C'(t)x_{t-1} + \mu C''(t)y_{t-1} + P(t) \geq F(t),\ t = \overline{1, n}. \quad (20)$$

Here $F(t)$ is the average value of the payment function at the moment of time t. In our case it is a deterministic a priori known value.

In accordance with the approach outlined in the dissertation of T.Yu. Pashinskaya, let us introduce the RA return rate for the period of time $[t, t+1]$:

$$v(t+1) = \frac{C''(t+1) - C''(t)}{C''(t)}. \quad (21)$$

Earlier we have introduced the value $r_1(t)$—the RLA return rate. Let us introduce the value $r_2(t)$ the rate on RLA loan (deposit rate).

The dynamics of RLA price and risk-free borrowing are defined by the expressions:

$$C'(t+1) = C'(t)(1 + r_1(t+1)), \quad (22)$$

$$P(t+1) = P(t)(1 + r_2(t+1)). \quad (23)$$

Then from the formulas (20), (22), and (23) for the moment of time $t + 1$ it follows:

$$\lambda C'(t)(1 + r_1(t+1))x_t + \mu C''(t)(1 + v(t+1))y_t + P(t)(1 + r_2(t+1)) \geq F(t+1)$$

or

$$\lambda V'(t)(1 + r_1(t+1)) + \mu V''(t)(1 + v(t+1)) + P(t)(1 + r_2(t+1)) \geq F(t+1). \quad (24)$$

Model 1

Let us consider the change in IP capital in discrete time. Such a change can be written using the equation taking into account (24) and (5):

$$\begin{gathered} V(t+1) = \lambda(1 + r_1(t+1))V(t) + \\ + V''(t)[\mu(1 + v(t+1)) - \lambda(1 + r_1(t+1))] + \\ + [\lambda(1 + r_1(t+1)) - (1 + r_2(t+1))]P(t), \\ t = 0, 2, \ldots, n-1, \end{gathered} \quad (25)$$

where n is the depth of the tree.

The capital placed in RLA is equal to

$$V'(t) = V(t) - V''(t) + P(t). \quad (26)$$

Note that expression (25) coincides with a similar formula for the dynamics of capital at $\lambda = 1$, $\mu = 1$, obtained in the works of V.V. Dombrovsky [10] and T.Yu. Pashinskaya for the RA random rate.

Let us define the equation of the benchmark portfolio by an expression for the payment function:

$$F(t+1) = [1 + \mu_0(t)]F(t), \quad (27)$$

where $\mu_0(t)$ is a given benchmark portfolio rate. This indicator characterizes the investor's risk aptitude: the larger it is, the higher the risk aptitude. $F(0) = V(0)$ (at the initial moment of time the capital of the reference portfolio coincides with the capital of real IP).

Let us introduce the notations $u1_1(t) = V''(t)$, $u1_2(t) = P(t)$,

$$\mathbf{A1}(t) = \begin{pmatrix} \lambda\left(1 + r_1(t+1)\right) & 0 \\ 0 & (1 + \mu_0(t+1)) \end{pmatrix}, \quad (28)$$

$$\mathbf{B1}(t) = \begin{pmatrix} \mu(1 + v(t+1)) - \lambda(1 + r_1(t+1)) & \lambda(1 + r_1(t+1)) - (1 + r_2(t+1)) \\ 0 & 0 \end{pmatrix}, \quad (29)$$

$$\mathbf{z}(t) = (V(t)\ F(t))^{\mathrm{T}}. \quad (30)$$

Taking into account (29), (30), and (31), the expression (27) will take the form:

$$\mathbf{z}(t+1) = \mathbf{A1}(t)\mathbf{z}(t) + \mathbf{B1}(t)\mathbf{ul}(t), \quad (31)$$

where

$$\mathbf{u1}(t) = (V''(t)\ P(t))^{\mathrm{T}}. \quad (32)$$

The control variables here are the values

$$u1_1(t) = V''(t),\ u1_2(t) = P(t).$$

The cost of the riskless part of the portfolio in this case is equal to

$$V'(t) = V(t) - V''(t) + P(t) = V(t) - u1_1(t) + u1_2(t). \quad (33)$$

Model 2

Let us describe the IP capital dynamics in discrete time by the equation taking into account (24) and (5):

$$\begin{gathered} V(t+1) = \mu(1 + v(t+1))V(t) + \\ + V'(t)[\lambda(1 + r_1(t+1)) - \mu(1 + v(t+1))] + \\ + [\mu(1 + v(t+1)) - (1 + r_2(t+1))]P(t), \\ t = 0, 2, \ldots, n-1, \end{gathered} \quad (34)$$

where n is the depth of the tree.

Then the matrices for Model 2 will have the following form:

$$\mathbf{A2}(t) = \begin{pmatrix} \mu\left(1 + v(t)\right) & 0 \\ 0 & (1 + \mu_0(t)) \end{pmatrix}, \quad (35)$$

$$\mathbf{B2}(t) = \begin{pmatrix} \lambda(1 + r_1(t)) - \mu(1 + v(t)) & \mu(1 + v(t)) - (1 + r_2(t)) \\ 0 & 0 \end{pmatrix}. \quad (36)$$

The control vector will now be

$$\mathbf{u2}(t) = (V'(t)\ P(t))^{\mathrm{T}}. \quad (37)$$

The value of the risk part of the portfolio in this case is equal to

$$V''(t) = V(t) - V'(t) + P(t) = V(t) - u2_1(t) + u2_2(t). \quad (38)$$

Tracking task

As an optimality criterion we choose a quadratic functional

$$J = \sum_{t=0}^{n-1}\left[[V(t)-F(t)]^2 + (\mathbf{u}(t))^{\mathrm{T}}\mathbf{R}(t)\mathbf{u}(t) + [V(n)-F(n)]^2\right] \to$$

$$\to \min_u \frac{-b \pm \sqrt{b^2-4ac}}{2a}, \qquad (39)$$

$\mathbf{R}(t)$ is a diagonal matrix of weight coefficients of dimension 2 × 2. Here $\mathbf{u}(t)$ means either $\mathbf{u1}(t)$ or $\mathbf{u2}(t)$.

The second summand in the functional (39) imposes restrictions on the size of monetary amounts that are used to buy/sell securities.

Let us write the functional (39) as follows:

$$J = \sum_{t=0}^{n-1}\left[\mathbf{z}^{\mathrm{T}}(t)\mathbf{h}^{\mathrm{T}}hz(t) + (\mathbf{u}(t))^{\mathrm{T}}\mathbf{R}(t)\mathbf{u}(t) + \mathbf{z}^{\mathrm{T}}(n)\mathbf{h}^{\mathrm{T}}\mathbf{h}z(n)\right], \qquad (40)$$

where $h = [1, -1]$.

In order to determine the optimal control strategy with quadratic criterion feedback, a linear control law of the following form is used

$$\mathbf{u}(t) = K_1(t)V(t) + K_2(t)F(t) = \mathbf{K}(t)\mathbf{z}(t), \qquad (41)$$

where $\mathbf{K}(t) = [K_1(t), K_2(t)]$—the matrix of feedback coefficients—is chosen from the condition of the minimum of the functional (40).

The functional (40) can be rewritten in the form

$$J = tr\left\{\sum_{t=0}^{n-1}\left[\mathbf{h}^{\mathrm{T}}\mathbf{h}\mathbf{S}(t) + \mathbf{K}^{\mathrm{T}}(t)\mathbf{R}(t)\mathbf{K}(t)\mathbf{S}(t)\right] + \mathbf{h}^{\mathrm{T}}\mathbf{h}\mathbf{S}(n)\right\}, \qquad (42)$$

where $tr\{\cdot\}$ is the trace of the matrix, and the matrix is $\mathbf{S}(t) = \mathbf{z}(t)\mathbf{z}^{\mathrm{T}}(t) = \begin{pmatrix} (V(t))^2 & V(t)\cdot F(t) \\ V(t)\cdot F(t) & (F(t))^2 \end{pmatrix}$.

Equation of state

Based on (31) and (41), the dynamics of the matrix $\mathbf{S}(t) = \mathbf{z}(t)\mathbf{z}^{\mathrm{T}}(t)$ is determined by the expression:

$$\mathbf{S}(t+1) = \left[\mathbf{A}(t) + \mathbf{B}(t)\mathbf{K}(t)\right]\mathbf{S}(t)\left[\mathbf{A}(t) + \mathbf{B}(t)\mathbf{K}(t)\right]^{\mathrm{T}}. \qquad (43)$$

Here, either $\mathbf{A1}(t)$, $\mathbf{B1}(t)$, or $\mathbf{A2}(t)$, $\mathbf{B2}(t)$ is taken as $\mathbf{A}(t)$ and $\mathbf{B}(t)$.

The optimal control strategy is determined by solving the system optimization problem [22, 23]. In this problem, the equation of state dynamics (43) is considered, where the matrix $\mathbf{K}(t)$ represents the control action and the functional (44) serves as a quality criterion.

In the context of this task it is required to minimize the criterion (42) under dynamic constraints, which are described by the difference matrix equation (43). To solve this problem, the maximum principle in the matrix formulation, which was developed earlier in [3, 4], is applied.

Algorithm for finding a solution

1. We find $\mathbf{Q}(t)$, $t = n, n-1, \ldots, 1, 0$ from the equation

$$\mathbf{Q}(t) = \mathbf{A}(t)\mathbf{Q}(t+1)\mathbf{A}(t) + \mathbf{A}(t)\mathbf{Q}(t+1)\mathbf{B}(t)\times$$
$$\times\left(\mathbf{R}(t) - \mathbf{B}^{\mathrm{T}}(t)\mathbf{Q}(t+1)\mathbf{B}(t)\right)^{-1}\left(\mathbf{B}^{\mathrm{T}}(t)\mathbf{Q}(t+1)\mathbf{A}(t)\right) - \mathbf{h}^{\mathrm{T}}\mathbf{h}.$$

2. Then, we calculate $\mathbf{K}(t)$, $t = 0, 1, \ldots, n-1$ in accordance with the formula

$$\mathbf{K}(t) = \left(\mathbf{R}(t) - \mathbf{B}^{\mathrm{T}}(t)\mathbf{Q}(t+1)\mathbf{B}(t)\right)^{-1}\left(\mathbf{B}^{\mathrm{T}}(t)\mathbf{Q}(t+1)\mathbf{A}(t)\right).$$

3. By found $\mathbf{K}(t)$, we calculate $\mathbf{S}(t)$, $t = 1, 2, \ldots, n$, where $\mathbf{S}(t) = \begin{pmatrix} (V(t))^2 & V(t)F(t) \\ V(t)F(t) & (F(t))^2 \end{pmatrix}$.

The elements of the matrix $\mathbf{S}(t)$ and the matrix $\mathbf{K}(t)$ are the desired solution to the benchmark portfolio tracking problem.

Knowing the matrix $\mathbf{S}(t)$, we have:

$$F(t) = \sqrt{S_{22}(t)};\ V(t) = S_{12}(t)/F(t),$$

where $V(t)$ is the investments in the real portfolio.

The portfolio management is calculated by the formula

$$\mathbf{u}(t) = K_1(t)V(t) + K_2(t)F(t).$$

4. In order to calculate the amount of investment in the portfolio, it is necessary to solve the system of relations:

$$\begin{aligned} &\mathbf{u}(t) = K_1(t)V(t) + K_2(t)F(t) = \mathbf{K}(t)\mathbf{z}(t),\ t = \overline{0, n-1}; \\ &\mathbf{z}(t+1) = \mathbf{A}(t)\mathbf{z}(t) + \mathbf{B}(t)\mathbf{u}(t). \end{aligned} \qquad (44)$$

Here for Model 1: $\mathbf{A}(t) = \mathbf{A1}(t)$, $\mathbf{B}(t) = \mathbf{B1}(t)$, $\mathbf{z}(t) = \begin{pmatrix} V(t) \\ F(t) \end{pmatrix}$, $\mathbf{u}(t) = \begin{pmatrix} V''(t) \\ P(t) \end{pmatrix}$; for Model 2: $\mathbf{A}(t) = \mathbf{A2}(t)$, $\mathbf{B}(t) = \mathbf{B2}(t)$, $\mathbf{z}(t) = \begin{pmatrix} V(t) \\ F(t) \end{pmatrix}$, $\mathbf{u}(t) = \begin{pmatrix} V'(t) \\ P(t) \end{pmatrix}$.

5. The RLA investments is calculated by (33) (for Model 1)

$$V'(t) = V(t) - V''(t) + P(t)$$

or the RA investments in is calculated by (38) (for Model 2)

$$V''(t) = V(t) - V'(t) + P(t).$$

6. Let us calculate the RA and RLA shares in the portfolio

$$x_t = \begin{cases} [V'(t)/C'(t)], & \text{if } V'(t) \ge 0, \\ [|V'(t)|/(\lambda C'(t))], & \text{if } V'(t) < 0, \end{cases} \quad t = \overline{0,n}, \quad (45)$$

$$y_t = \begin{cases} [V''(t)/C''(t)], & \text{if } V''(t) \ge 0, \\ [|V''(t)|/(\mu C''(t))], & \text{if } V''(t) < 0, \end{cases} \quad t = \overline{0,n}. \quad (46)$$

Here $[\cdot]$ is the integer part of the number.

NUMERICAL MODELING RESULTS

RA prices were modeled on the basis of a mixture of two normal distributions with parameters:

$$m1_j = C01 + h1 \cdot j, \ j = \overline{0,500}, \ \ \sigma 1;$$

$$m2_j = C02 + h2 \cdot j, \ j = \overline{0,500}, \ \ \sigma 2,$$

where $m1_j$, $m2_j$ are the price distributions; C01, C02 is the initial price for the corresponding distribution; $h1$, $h2$ are possible price fluctuations at a given sample size; σ1 and σ2 are standard deviations.

The parameter values were as follows: C01 = 100, C02 = 90, $h1$ = 0.04, $h2$ = 0,02, σ1 = 10, σ2 = 15.

Figure 3 shows the calculated RA prices. Hereinafter monetary values are shown in conventional units.

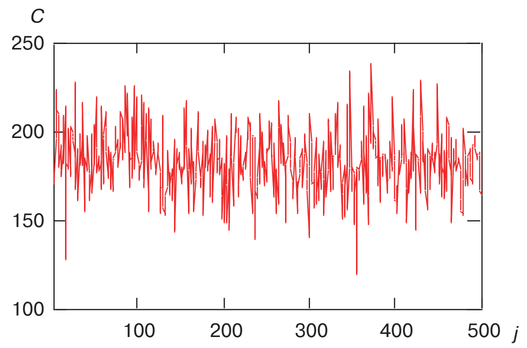


Fig. 3. RA prices. *C* is the RA price, units of money; *j* is the number of realizations

Figure 4 shows the obtained probability distribution of RA prices.

This distribution is treated as an analog of the empirical distribution. Then RA prices at the tree nodes are modeled based on this distribution. The probability of price growth was estimated based on the constructed distribution for the price difference $C_i'' - C_1''$, $i = \overline{1,500}$. The probability of price growth (the probability that $C_i'' - C_1'' > 0$) was $p = 0.495$.

The values of the other parameters were as follows: $C''(0) = 150$, $C'(0) = 10$, $\lambda = 1.02$, $\mu = 1.02$, $F(0) = 10000$, $V(0)$ = 10000, $P(0)$ = 0, $r_1(t)$ = 0.02, $r_2(t)$ = 0.015, $\mu_0(t) = 0.02$, tree depth $n = 5$.

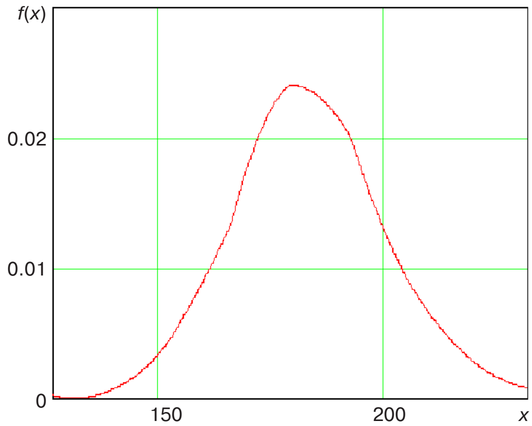


Fig. 4. Probability distribution of RA prices. *f*(*x*) is the probability distribution density; *x* is the RA price, unit of money

The values of investments were taken as follows. For Model 1, $V'(0)$ = 10000, $V''(0)$ = 0, i.e., at the initial moment of time all funds were invested in RLA. For Model 2, $V'(0)$ = 0, $V''(0)$ = 10000, i.e., all funds were invested in RA.

Modeling results are given in Tables 1–5.

Figure 5 shows graphs of tracking the desired portfolio value. Here and further in the graphs time t is given in relative units.

Figure 6 shows the necessary changes in the RA investments to achieve the portfolio value not less than the desired one.

Figure 7 shows the necessary changes in the RLA investments to achieve the portfolio value not less than the desired one.

Figure 8 shows the tracking for the payment function (desired portfolio value).

Figure 9 shows the necessary changes in the RA investments to achieve the portfolio value not less than the desired one.

Figure 10 shows the necessary changes in the RLA investments to achieve the portfolio value not less than the desired one.

Figure 11 shows the tracking for the payment function (desired portfolio value).

Figure 12 shows the necessary changes in RA investments to achieve the portfolio value not less than the desired one.

Figure 13 shows the necessary changes in the RLA investments in to achieve the portfolio value not less than the desired one.

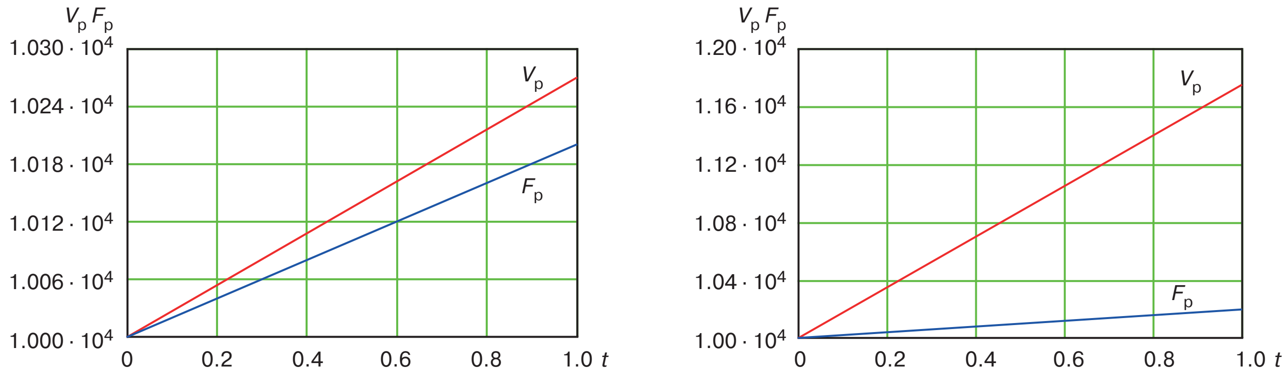
Figure 14 shows the tracking for the payment function (desired portfolio value).

Figure 15 shows the necessary changes in RA investments to achieve the portfolio value not less than the desired one.

Figure 16 shows the necessary changes in the RLA investments to achieve the portfolio value not less than the desired one.

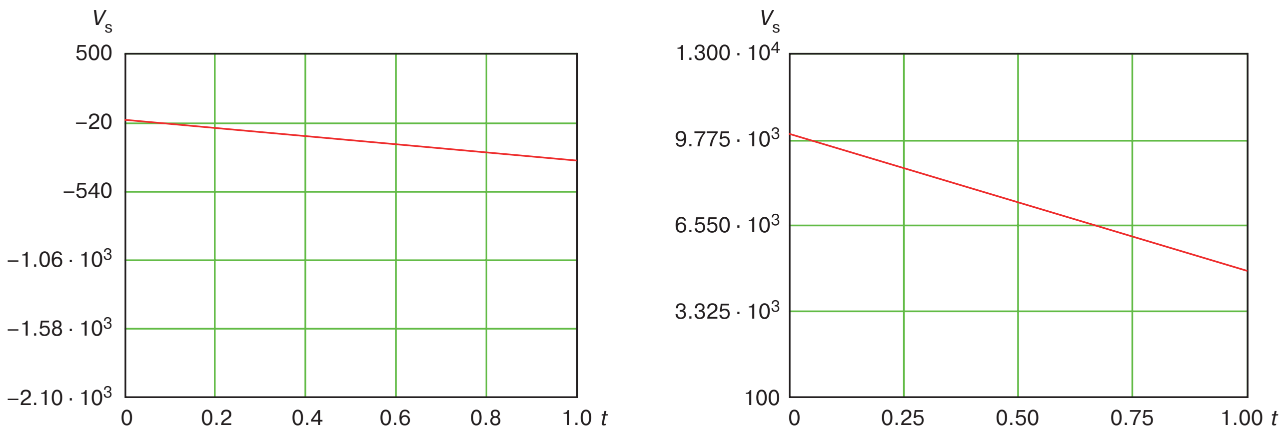
Table 1. Tree depth up to $n = 1$

Tree depth	Model 1				Model 2			
	Investments in the portfolio	RLA share	RA share	Deposit	Investments in the portfolio	RLA share	RA share	Deposit
0	10000	1	0	0	10000	0	1	0
1	10340	1.028	−0.028	−0.368	11750	0.725	0.272	−35.890



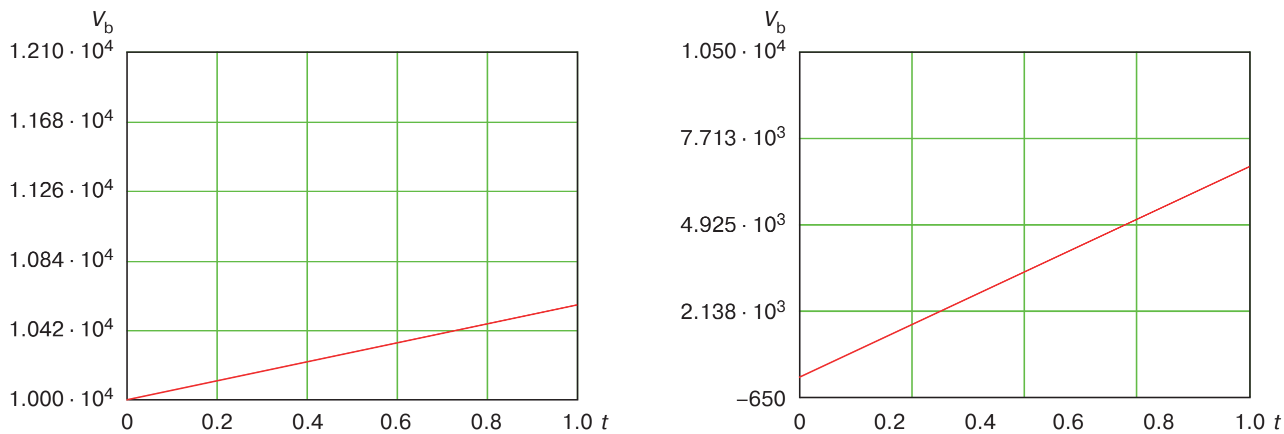
(a) (b)

Fig. 5. Tracking the desired portfolio value: (a) Model 1, (b) Model 2.
V_p is the investments or portfolio capital, units of money; F_p is the payment function, units of money; t is time, arb. units



(a) (b)

Fig. 6. Necessary changes in the RA investments: (a) Model 1, (b) Model 2.
V_s is the RA investments, units of money; t is time, arb. units



(a) (b)

Fig. 7. Necessary changes in the RLA investments: (a) Model 1, (b) Model 2.
V_b is the investments in RLA, units of money; t is time, arb. units

Table 2. Tree depth up to $n = 2$

Tree depth	Model 1				Model 2			
	Investments in the portfolio	RLA share	RA share	Deposit	Investments in the portfolio	RLA share	RA share	Deposit
0	10000	1	0	0	10000	0	1	0
1	10260	1.067	−0.067	−0.874	11400	0.427	0.568	−54.988
2	10690	1.079	−0.079	−1.089	11820	0.698	0.294	−93.140

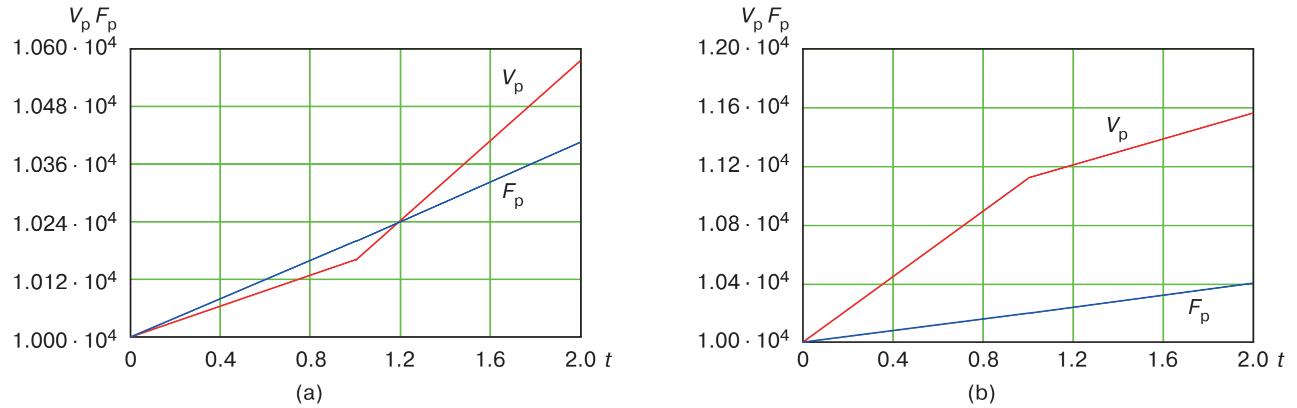


Fig. 8. Tracking for the payment function: (a) Model 1, (b) Model 2.
V_p is the investments or portfolio capital, units of money; F_p is the payment function, units of money; t is time, arb. units

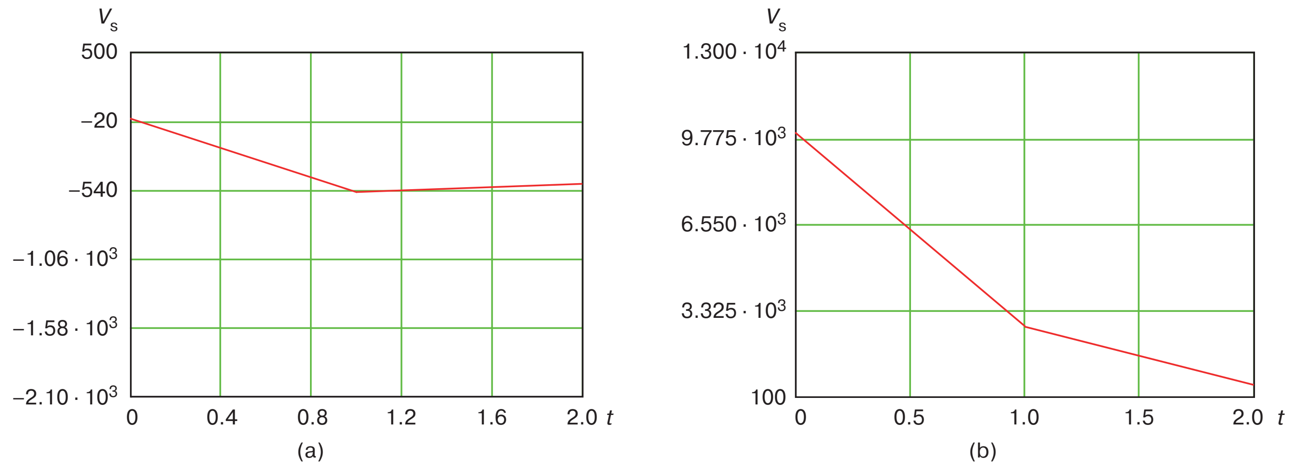


Fig. 9. Necessary changes in the RA investments: (a) Model 1, (b) Model 2.
V_s is the RA investments, units of money; t is time, arb. units

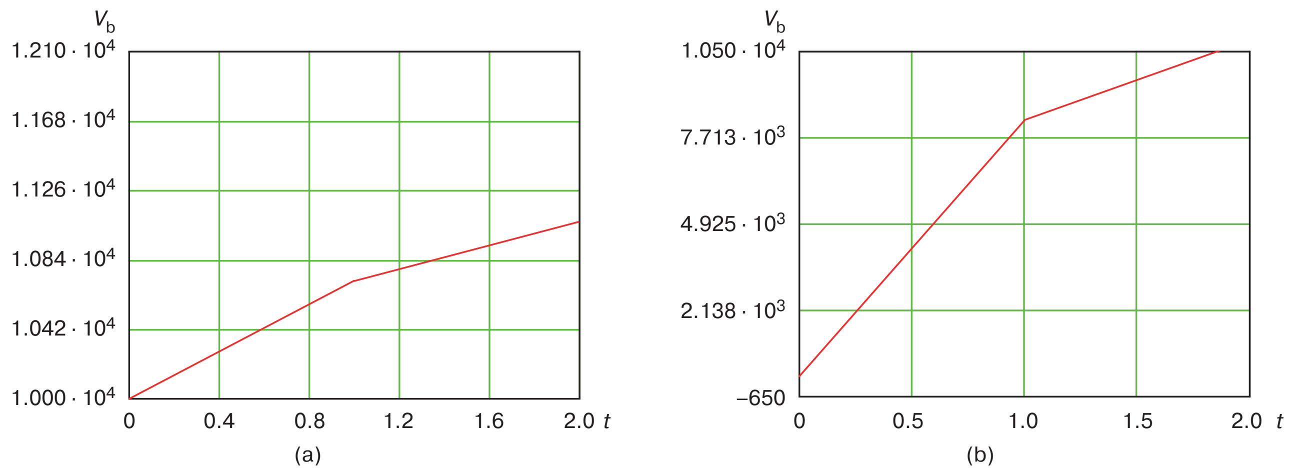


Fig. 10. Necessary changes in the RLA investments:
(a) Model 1, (b) Model 2. V_b is the RLA investments, units of money; t is time, arb. units

Table 3. Tree depth up to $n = 3$

Tree depth	Model 1				Model 2			
	Investments in the portfolio	RLA share	RA share	Deposit	Investments in the portfolio	RLA share	RA share	Deposit
0	10000	1	0	0	10000	0	1	0
1	10180	1.110	−0.111	−1.484	11190	0.538	0.456	−67.938
2	10610	1.100	−0.103	−1.450	11610	0.814	0.177	−106.597
3	11030	0.975	0.018	−2.705	11850	−0.024	1.024	−3.375

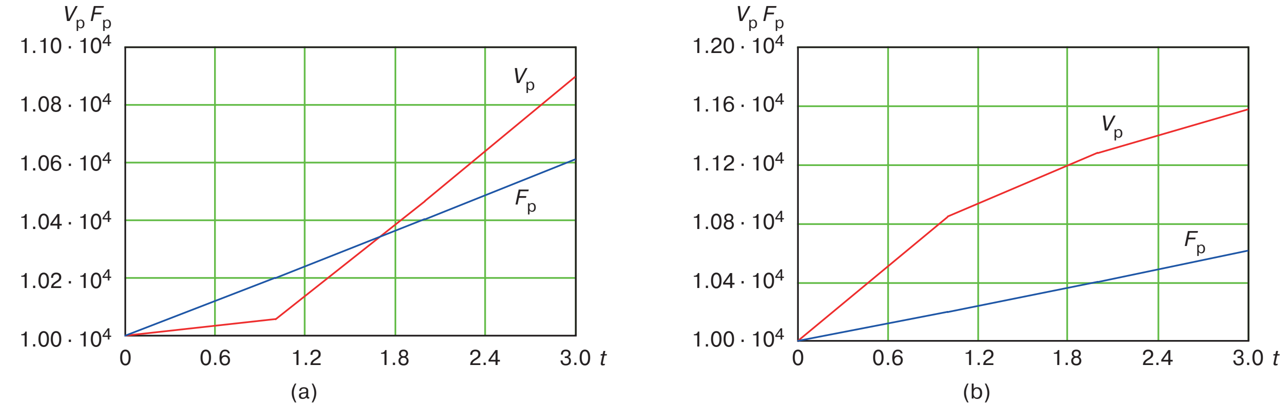


Fig. 11. Tracking for the payment function: (a) Model 1, (b) Model 2.
V_p is the investments or portfolio capital, units of money; F_p is the payment function, units of money; t is time, arb. units

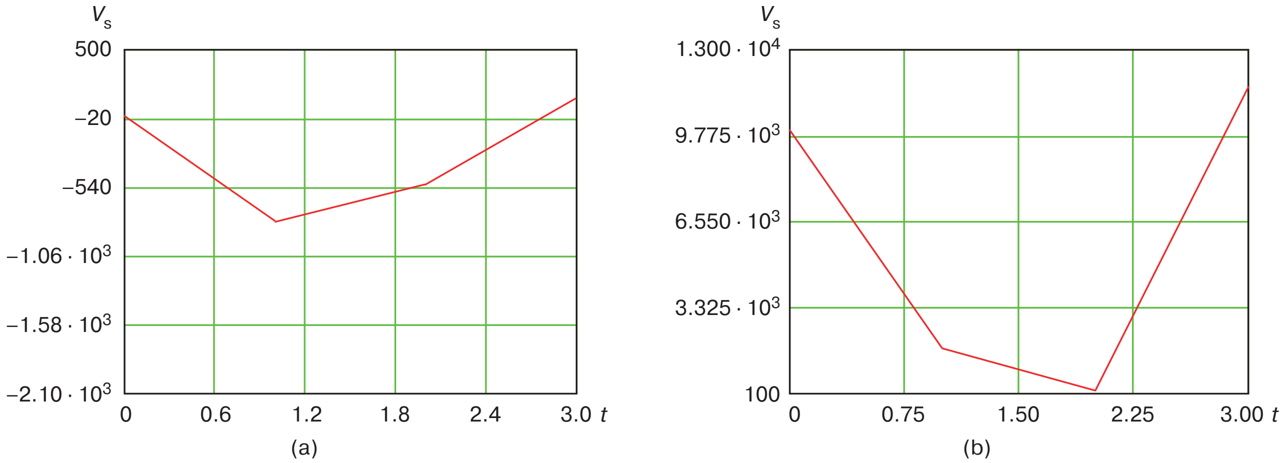


Fig. 12. Necessary changes in RA investments: (a) Model 1, (b) Model 2.
V_s is the RA investments, units of money; t is time, arb. units

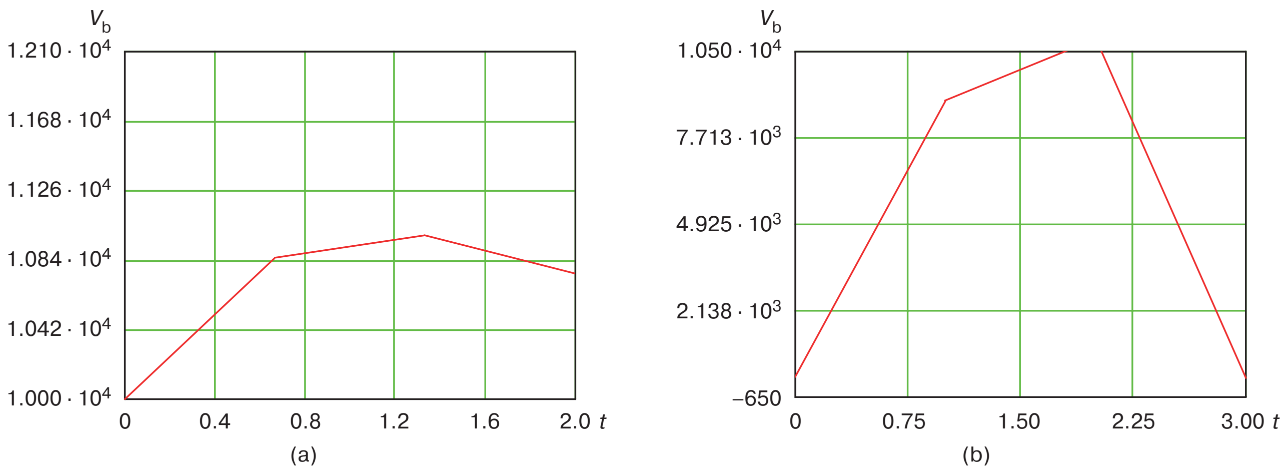


Fig. 13. Necessary changes in the RLA investments: (a) Model 1, (b) Model 2.
V_b is the RLA investments, units of money; t is time, arb. units

Table 4. Tree depth up to $n = 4$

Tree depth	Model 1				Model 2			
	Investments in the portfolio	RLA share	RA share	Deposit	Investments in the portfolio	RLA share	RA share	Deposit
0	10000	1	0	0	$1 \cdot 10^4$	0	1	0
1	10080	1.159	−0.159	−2.078	$1.057 \cdot 10^4$	0.913	0.078	−100.38
2	10510	1.119	−0.119	−1.653	$1.1 \cdot 10^4$	0.977	0.013	−111.746
3	10930	0.970	0.030	−4.141	$1.124 \cdot 10^4$	−0.027	1.026	−1.617
4	11370	0.977	0.023	−3.411	$1.146 \cdot 10^4$	$3.502 \cdot 10^{-3}$	0.996	−3.804

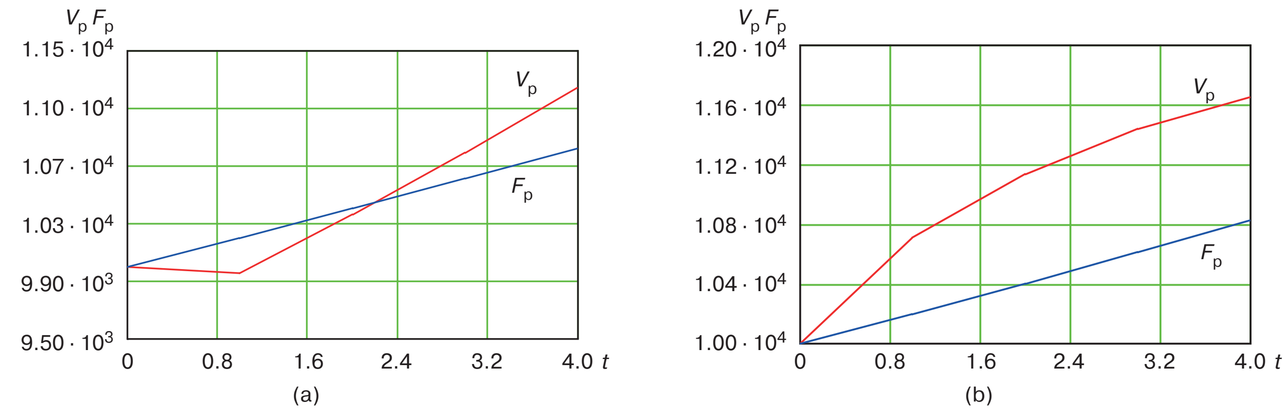


Fig. 14. Tracking for the payment function: (a) Model 1, (b) Model 2.
V_p is the investments or portfolio capital, units of money; F_p is the payment function, units of money; *t* is time, arb. units

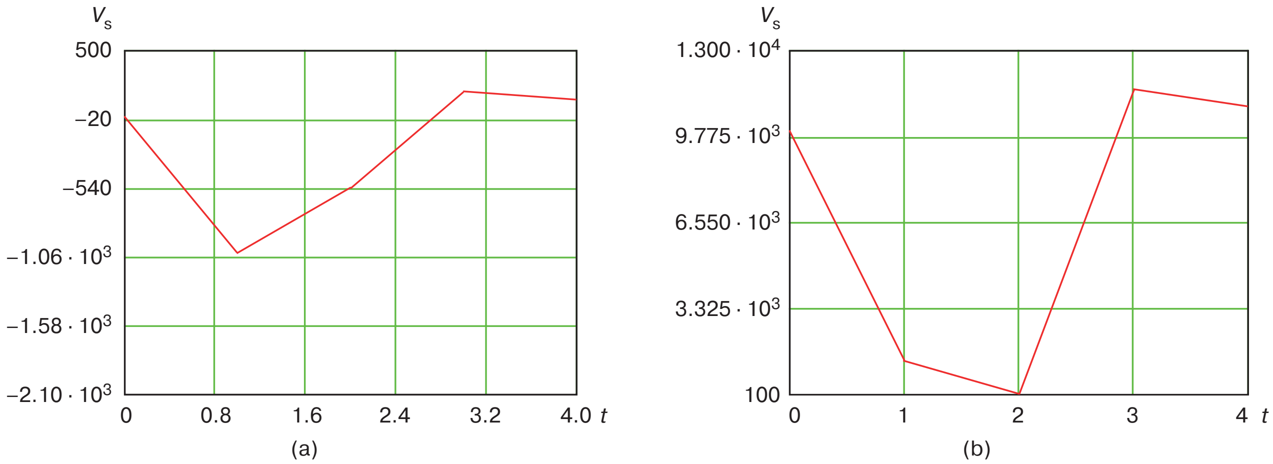


Fig. 15. Necessary changes in the RA investments: (a) Model 1, (b) Model 2.
V_s is the RA investments, units of money; *t* is time, arb. units

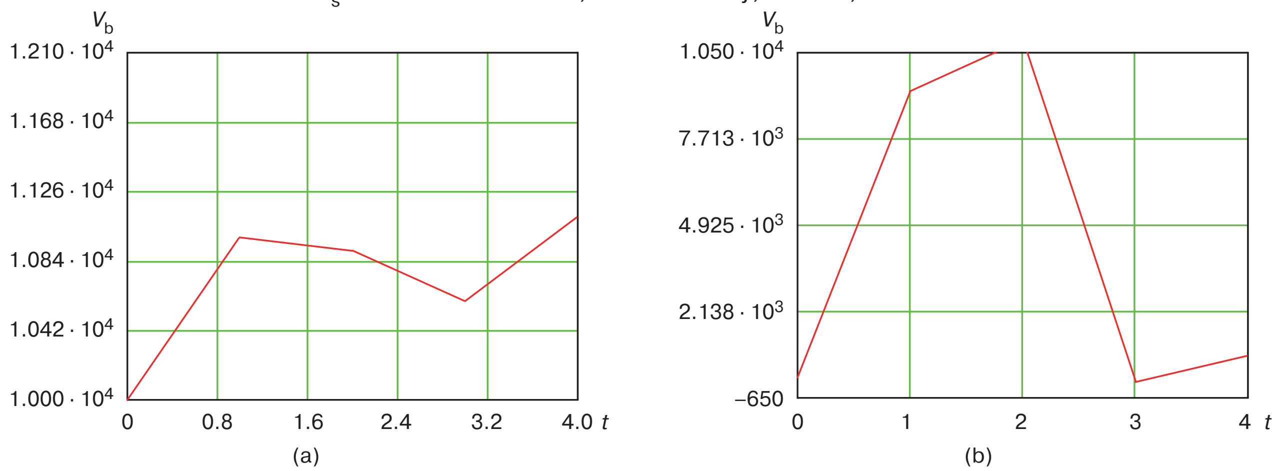


Fig. 16. Necessary changes in the RLA investments: (a) Model 1, (b) Model 2.
V_b is the RLA investments, units of money; *t* is time, arb. units

Table 5. Tree depth up to $n = 5$

Tree depth	Model 1				Model 2			
	Investments in the portfolio	RLA share	RA share	Deposit	Investments in the portfolio	RLA share	RA share	Deposit
0	10000	1	0	0	10000	0	1	0
1	9815	1.095	−0.095	−0.379	10480	0.934	0.056	−101.88
2	10220	1.034	−0.034	−0.140	10910	0.979	0.011	−111.062
3	10630	0.971	0.028	−3.587	11130	−0.025	1.025	−2.593
4	11060	0.976	0.023	−3.602	11350	0.003	0.996	−4.519
5	11500	0.980	0.019	−2.986	11580	−0.01	1.01	−1.555

Figure 17 shows the tracking for the payment function (desired portfolio value).

Figure 18 shows the necessary changes in RA investments to achieve the portfolio value not less than the desired one.

Figure 19 shows the necessary changes in the RLA investments to achieve the portfolio value not less than the desired one.

Negative deposit shares observed in Tables 1–5 can be interpreted as "short sales." Negative deposit fractions present in Tables 1–5 mean borrowing of funds. Such results within the framework of this dynamic model are explained by the fact that no restrictions were imposed on the investments and deposits.

It can be seen that portfolio reforming according to the dynamic model allows us to provide a given level of the payment function.

It is of interest to compare the error of the dynamic model based on the tree structure of RA price changes with the general model of RA price changes [13]. Tables 6 and 7 summarize the errors of investment estimation. Here σV is the error of portfolio value; σV_s is the error of RA investment; σV_b is the error of RLA investment; σx is the error of RA quantity; σy is the error of RLA quantity.

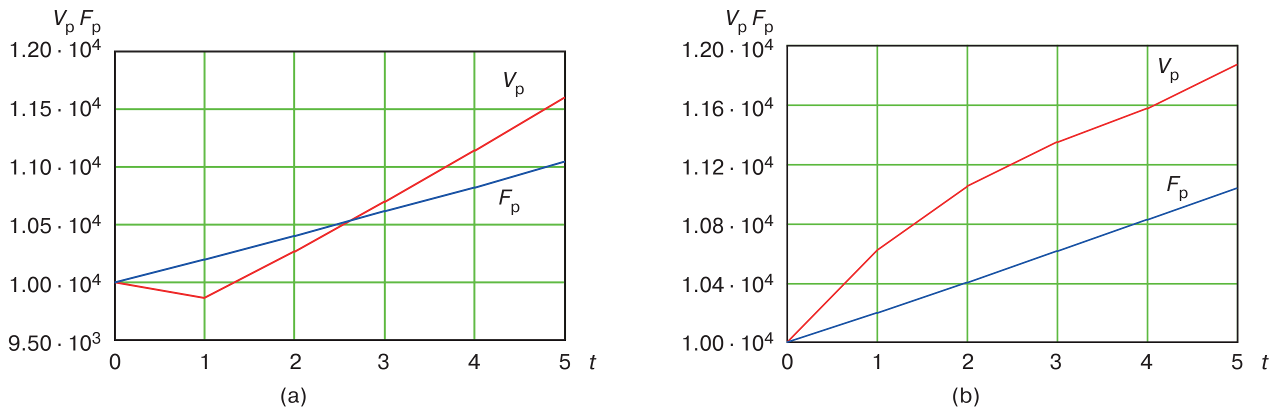


Fig. 17. Tracking for the payment function: (a) Model 1, (b) Model 2.
V_p is the investments or portfolio capital, units of money; F_p is the payment function, units of money; t is time, arb. units

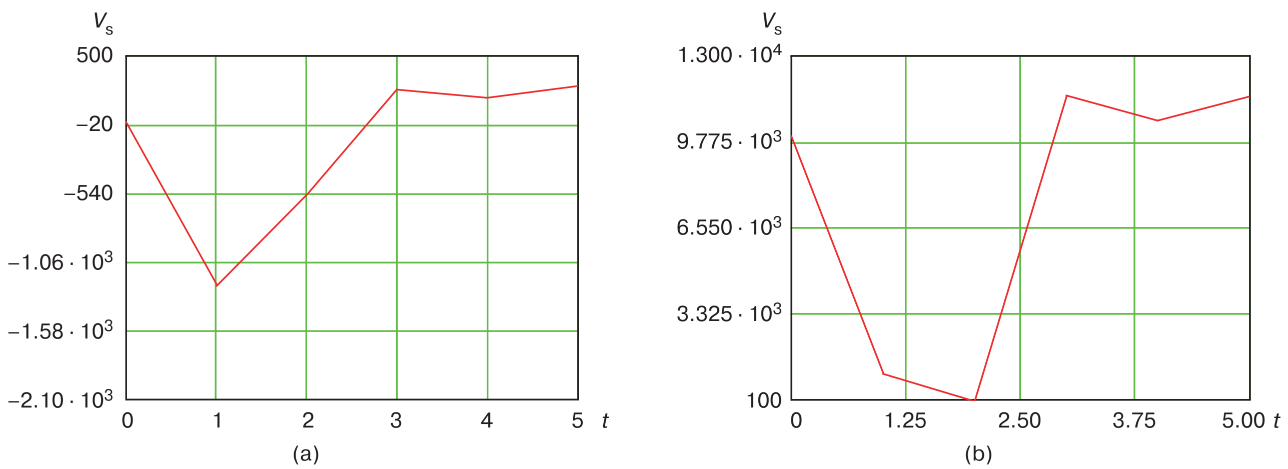


Fig. 18. Necessary changes in RA investments:
(a) Model 1, (b) Model 2. V_s is the RA investments, units of money; t is time, arb. units

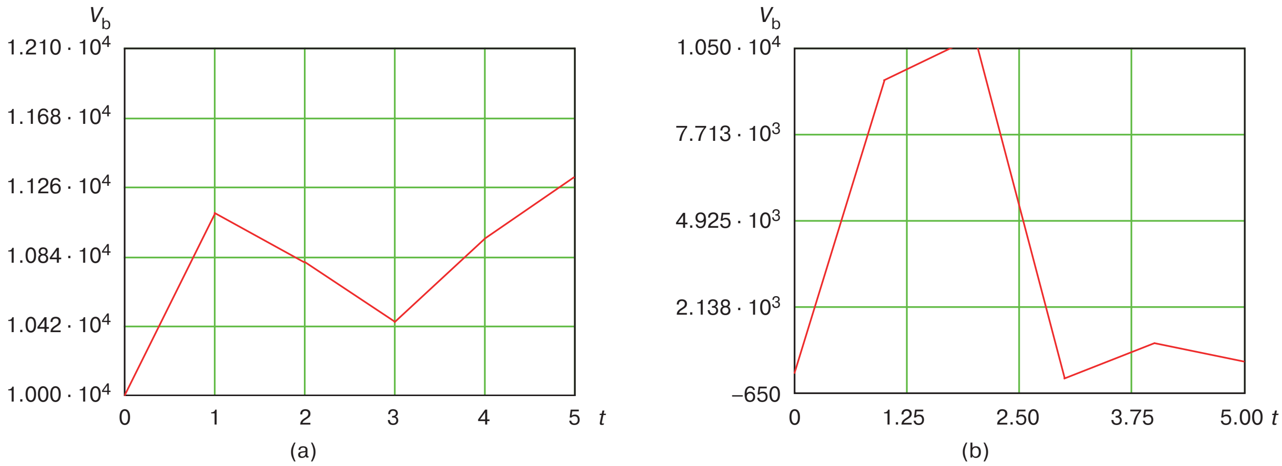


Fig. 19. Necessary changes in the RLA investments. (a) Model 1, (b) Model 2. V_b is the RLA investments, units of money; t is time, arb. units

Table 6. Estimation of the Model 1 error

Tree depth / investment horizon n	Tree structure of RA prices			Natural change in RA prices			Gain in model accuracy		
	σV	σV_s	σV_b	$\sigma V1$	$\sigma V_s 1$	$\sigma V_b 1$	$\frac{\sigma V1}{\sigma V}$	$\frac{\sigma V_s 1}{\sigma V_s}$	$\frac{\sigma V_b 1}{\sigma V_b}$
1	6.44	61.5	61.7	15.4	84.9	83.3	2.4	1.4	1.3
2	12.3	26.3	21.9	21.25	37.0	34.0	1.7	1.4	1.6
3	18.64	417	410	39.0	581	575	2.1	1.4	1.4
4	25.1	340	354	61.49	625	654	2.4	1.8	1.8
5	26.9	234	238	73.7	552	556	2.7	2.4	2.3

Table 7. Estimation of the Model 2 error

Tree depth / investment horizon n	Tree structure of RA prices			Natural change in RA prices			Gain in model accuracy		
	σV	σV_s	σV_b	$\sigma V2$	$\sigma V_s 2$	$\sigma V_b 2$	$\frac{\sigma V2}{\sigma V}$	$\frac{\sigma V_s 2}{\sigma V_s}$	$\frac{\sigma V_b 2}{\sigma V_b}$
1	194.4	5003	302	339.7	875	531	1.7	1.7	1.7
2	210.3	52.8	157	366.2	98.2	269	1.7	1.9	1.7
3	370.3	981	637	743.6	1726	1083	2.0	1.8	1.7
4	367.8	560	493	781.0	1900	1959	2.1	3.4	4.0
5	370.0	559	280	843.2	1685	1288	2.3	3.0	4.6

As follows from Table 6, the error of the dynamic model based on the tree structure of RA price changes is smaller than for the conventional model, and the gain in accuracy of the model increases with increasing investment horizon. Thus, for the portfolio value the gain in model accuracy varies from 2.4 (n = 1) to 2.7 (n = 5); for the RA investments, the gain in accuracy varies from 1.4 (n = 1) to 2.4 (n = 5); for the RLA investments, the gain in accuracy varies from 1.3 (n = 1) to 2.3 (n = 5).

Accuracy gains are also observed for Model 2. For example, the gain in accuracy of portfolio value estimation ranges from 1.7 (n = 1) to 2.3 (n = 5).

CONCLUSIONS

The study analyzes a securities portfolio comprising assets having different risk levels, as well as risk-free assets and deposits. A binomial model was used to model the RA price structure. The main objective of the study was to develop a management model for tracking the benchmark portfolio. For this purpose, a quality criterion in the form of a quadratic function served as the basis for the construction of the management model. The developed model belongs to the class of dynamic programming models for determining the optimal management strategy using feedback.

The study considers two approaches to portfolio formation. The first approach involves initial investment of capital in RLA and subsequent management carried out through RA. The second approach, conversely, includes initial capital investment in RA, with management is carried out through RLA. In order to optimize management for achieving the desired objective, the study applies a linear control law to determine the optimal values of the control parameters based on the current state of the system and the target value of the benchmark portfolio.

By using the described dynamic management model based on the tree structure of RA prices the accuracy of evaluating investments in the portfolio can be significantly increased. For the first approach (Model 1), there is an increase in evaluation accuracy from 2.4 to 2.7 times, while the second approach (Model 2) increases evaluation accuracy from 1.7 to 2.7 times.

Thus, the developed model can become a useful tool for financial analysts and investors by allowing them to make better informed decisions when forming and managing a securities portfolio. The model can be used to carry out a more accurate determination of the optimal amount of investments, leading to higher investment efficiency and better results for investors.

Authors' contributions

A.A. Mitsel—concept, structure, and scientific leadership of the study.

E.V. Viktorenko—analysis and interpretation of data, writing and editing the text of the manuscript.

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