Mathematical modeling

Математическое моделирование

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RESEARCH ARTICLE

Mathematical modeling of technological parameters of laser powder surfacing based on approximation of the deposition track profile

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Abstract

Objectives. Laser powder surfacing is a promising mechanical engineering technology used to effectively restore worn surfaces of parts and create special coatings with valuable properties. In the research and development of laser cladding technology, mathematical modeling methods are of crucial importance. The process of applying powder coating involves moving the spray head relative to the surface of the part to form a roller or spray path, whose sequential application results in the formation of coatings. The study sets out to evaluate methods of profile approximation and optimization of technological parameters in laser powder cladding processes.

Methods. In order to describe the dependencies of the profile parameters of the deposition paths during laser surfacing on the technological parameters of the process, mathematical modeling methods were used. The contours of the profiles of the surfacing section were obtained by analyzing images of microphotographs of thin sections of the cross sections of parts with applied surfacing. To approximate the curves of the section contours, methods of linear and nonlinear regression analysis were used. The dependence of the parameters of the profile contours of the surfacing section on the technological parameters of the spraying was represented by a two-factor parabolic regression equation. The search for optimal values of spraying technological parameters was carried out using the method of conditional optimization with linear approximation of the confidence region.

Results. A nonlinear two-parameter function was selected from three options for approximating functions of the section profile of a surfacing track. Technological surfacing parameters were mapped onto a set of parameters of the approximating contour line. Optimal values of the technological parameters of surfacing were obtained using regression models of these mappings to provide the maximum value of the area of the surfacing contour under restrictions on the proportion of the sub-melting area to the total cross-sectional area. The approximating function of the cross-sectional profile of the surfacing track was used to calculate the optimal pitch of the tracks that provides the most even surface.

Conclusions. The results of the study represent a technique for optimizing the technological parameters of laser surfacing with powder metals to ensure specified characteristics of the deposition track profile and select the track deposition step at which the most even deposition surface is achieved.

Keywords: mathematical modeling, laser cladding, section contour, approximation, regression analysis, optimization

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НАУЧНАЯ СТАТЬЯ

Математическое моделирование технологических параметров порошковой лазерной наплавки на основе аппроксимации профиля дорожки напыления

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Резюме

Цели. Лазерная порошковая наплавка – перспективная технология в машиностроении, позволяющая эффективно восстанавливать изношенные поверхности деталей и создавать специальные покрытия с ценными свойствами. Методы математического моделирования имеют решающее значение в исследовании и развитии технологии лазерной наплавки. Процесс нанесения порошкового покрытия предполагает перемещение распылительной головки относительно поверхности детали, образуя валик – дорожку напыления. Покрытия формируются путем последовательного нанесения этих дорожек. Целью исследования является изучение различных методов аппроксимации профиля и оптимизация технологических параметров в процессах порошковой лазерной наплавки.

Методы. Использованы методы математического моделирования для описания зависимостей параметров профиля дорожек напыления при лазерной наплавке от технологических параметров процесса. Получение контуров профилей сечения наплавки осуществлялось методами анализа изображений микрофотографий шлифов поперечных сечений деталей с наплавкой. Для аппроксимации кривых контуров сечений использовались методы линейного и нелинейного регрессионного анализа. Зависимость параметров контуров профилей сечения наплавки от технологических параметров напыления аппроксимировалась двухфакторным уравнением параболической регрессии. Поиск оптимальных значений технологических параметров напыления осуществляли методом условной оптимизации с линейной аппроксимацией доверительной области.

Результаты. Рассмотрены три варианта аппроксимирующих функций профиля сечения дорожки наплавки, из которых была выбрана нелинейная двухпараметрическая функция. Получены отображения множества технологических параметров наплавки во множество параметров аппроксимирующей линии контура. С использованием регрессионных моделей данных отображений найдены оптимальные значения технологических параметров наплавки, обеспечивающие максимальную величину площади контура наплавки при ограничениях на долю области подплавления к общей площади сечения. Аппроксимирующая функция профиля сечения дорожки наплавки использована для расчета оптимального шага нанесения дорожек, обеспечивающего наиболее ровную поверхность наплавки.

Выводы. Результаты проведенного исследования могут рассматриваться в качестве методики оптимизации технологических параметров лазерной наплавки порошковых металлов, позволяющей обеспечивать заданные характеристики профиля дорожки напыления и выбирать шаг нанесения дорожек, при котором достигается наиболее ровная поверхность наплавки.

Ключевые слова: математическое моделирование, лазерная наплавка, контур сечения, аппроксимация, регрессионный анализ, оптимизация

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INTRODUCTION

A promising technology in mechanical engineering, laser powder cladding is used to effectively restore the surfaces of worn parts, as well as to create special coatings for parts having valuable properties such as increased heat resistance, wear resistance, and chemical resistance [1–3]. Various methods of gas-thermal spraying of powder coatings [4–6] involve heating powdered materials to temperatures higher than their melting points and applying them to the surface of parts using high-speed gas flows. In the case of laser cladding, the source of particle heating is an infrared laser, whose beam is focused directly on or near the surface of the part. This allows more precise control of the temperature of the material to be clad and more accurate positioning of the cladding track. Due to these advantages, laser cladding forms the basis of state-of-the-art additive metal technologies [7].

Mathematical modeling methods are the most important research tool in the development of laser cladding technology [1, 8, 9]. Traditionally, the problems of heat flux distribution over the cross-section of the cladding material and the adjacent area of the workpiece are solved. Both analytical and numerical methods are used, among which finite element methods [10–12], including those using universal engineering analysis packages, have become very common in recent years. At the same time, in practical terms, a very important characteristic of the cladding process is the shape of the cross-sectional profile of the cladding track [13]. The laser cladding process, like the processes used in other powder spraying methods, consists in the movement of the atomizer head relative to the surface of the workpiece. As a result, a roller—a spraying (surfacing) track—is formed on the surface of the workpiece. By successive application of the tracks the coating is formed. The shape of the cross-sectional profile of the spraying track determines the thickness of the coating and the quality of its surface [14]. Due to the complexity of physical and chemical processes occurring during the formation of the sprayed track, modeling of the track cross-section profile on the basis of physical principles is difficult; therefore, in practice, track cross-section profiles are approximated by processing microphotographs of experimentally obtained cross-sectional slides of sprayed tracks [15]. As approximating functions of the cross-sectional profiles, rather simple mathematical functions such as parabola, circle arc or ellipse have been used in [16–18], although in practice the shape of the profile can be more complex [15, 19]. In this regard, the aim of the present work was to investigate by mathematical modeling methods various methods of approximation of cross-sectional profiles of spraying tracks with subsequent optimization of technological parameters of laser powder cladding.

APPROXIMATION AND OPTIMIZATION METHODS

The contours of the cross-sectional profiles of the sputtering tracks were obtained by processing micrographs of cross-sectional slides of the sputtering tracks with image analysis methods using the Python OpenCV library. For this purpose, auxiliary inscriptions (if any) were manually removed from the image and the color space was converted to grayscale. Next, an array of contours was extracted from the image using the algorithm [20], in which the contour with the maximum number of elements corresponded to the track contour. To perform the procedures of image file conversion and track contour extraction, we created a Python program module that enables batch processing of the scanned image array, returning a set of files in csv format containing an array of coordinates of the selected contour.

An approximation of the section profile contour and construction of mathematical models of dependencies of approximating function parameters on technological parameters of track spraying was carried out by methods of linear and nonlinear regression analysis [21, 22]. The mathematical formulation of these tasks was as follows.

Let there be a random function $y_j(x_j, \omega_j) \in \mathrm{Y}$ values for the fixed $x_j \in \mathrm{X} \subset \mathbb{R}$, $j = 1, \ldots, n$, where ω_j is a random event from Ω for a given sigma algebra A and probability measure P. The purpose of the approximation is to recover in X the function $Ey(x, \omega) = \eta(x)$, which is referred to as the regression function. In this work, we consider three variants of regression functions of the type $\eta_i(x, \theta_i)$, $i = 1, 2, 3$. Here η_i are known functions comprising regression models, whose specific type will be described in the main part of the article, while θ_i are parameters from the given parametric sets Θ_i as determined by the values of y_j.

Among the three regression models studied in this paper, two are parametrically linear, while one is parametrically nonlinear. The responses of the linear in parameters model y_j can be represented in the form of:

$$y_j = \eta_{\text{lin}}(x_j, \boldsymbol{\theta}) + \varepsilon_j = \boldsymbol{\theta}^{\mathrm{T}} f(x_j) + \varepsilon_j, \tag{1}$$

where ε_j are random variables with distribution assumed to be normal with zero expectation $E\varepsilon_j = 0$ and diagonal covariance matrix $E\varepsilon_j\varepsilon_k = \sigma^2\delta_{jk}$; $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_m)^{\mathrm{T}}$ is a vector of unknown parameters from $\mathbb{R}^m$; $\mathbf{f}(x) = (f_1(x), \ldots, f_m(x))^{\mathrm{T}}$ is a vector of given, linearly independent functions on the set X.

In matrix notation $\mathbf{Y} = (y_1, \ldots, y_n)^{\mathrm{T}}$, $\boldsymbol{\varepsilon} = (\varepsilon_1, \ldots, \varepsilon_n)^{\mathrm{T}}$, $\mathbf{F} = (f_1(x_j), \ldots, f_m(x_j))_{j=1}^{n}$ the system (1) is written in the form:

$$\mathbf{Y} = \mathbf{F}\boldsymbol{\theta} + \boldsymbol{\varepsilon}, \tag{2}$$

where $E\mathbf{Y} = \mathbf{F}\boldsymbol{\theta}$ and the covariance matrix $\mathbf{DY}$ is equal to $\sigma^2\mathbf{I}_n$, $\mathbf{I}_n$ is a unity matrix.

In the present work, the estimates $\hat{\boldsymbol{\theta}}$ of the unknown parameters $\boldsymbol{\theta}$ were computed using the least squares method (LSM):

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}\in\Theta} \sum_{j=1}^{n} \sigma_j^{-2}(y_j - \eta(x_j, \boldsymbol{\theta}))^2 \tag{3}$$

or in the matrix notations:

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}\in\Theta}(\mathbf{Y} - \mathbf{F}\Theta)^{\mathrm{T}}(\mathbf{Y} - \mathbf{F}\Theta). \tag{4}$$

The solution of problem (4) is reduced to the well-known formula of regression analysis

$$\hat{\boldsymbol{\theta}} = (\mathbf{F}^{\mathrm{T}}\mathbf{F})^{-1}\mathbf{F}^{\mathrm{T}}\mathbf{Y}. \tag{5}$$

The model adequacy dispersion s^2, which is an unbiased estimate of the variance σ^2, in this case is calculated by the formula $s^2 = SS_{\text{reg}}/(n - m)$, where

$$SS_{\text{reg}} = (\mathbf{Y} - \mathbf{F}\hat{\boldsymbol{\theta}})^{\mathrm{T}}(\mathbf{Y} - \mathbf{F}\hat{\theta}). \tag{6}$$

Since it was not possible to estimate the error variance from parallel experiments in the present work, the adequacy of the model could not be validated by comparing the adequacy and error variance. Therefore, the adequacy of the models was assessed qualitatively by the closeness to unity of the value of the coefficient of determination

$$R^2 = 1 - SS_{\text{reg}}/SS_{\text{tot}}, \tag{7}$$

where $SS_{\text{tot}} = (\mathbf{Y} - \bar{\mathbf{Y}})^{\mathrm{T}}(\mathbf{Y} - \bar{\mathbf{Y}})$, $\bar{\mathbf{Y}}$ is the average value of the responses.

For calculations according to formulas (5)–(7), a Python program module was created using the linear algebra package numpy.linalg, which is used to process csv files in batch mode with coordinates of track contours obtained as a result of image processing from microphotographs of cross-sectional profiles and plot points of the original contours and regression lines obtained as a result of calculating parameter estimates of regression equations.

For the regression model that is parametrically nonlinear, instead of representing the responses in the form (2), we used the representation of:

$$\mathbf{Y} = \mathbf{H}(\mathrm{X}, \boldsymbol{\theta}) + \boldsymbol{\varepsilon}, \tag{8}$$

where $\mathbf{H}(\mathrm{X}, \boldsymbol{\theta}) = (\eta(x_1, \boldsymbol{\theta}), \ldots, \eta(x_n, \boldsymbol{\theta}))^{\mathrm{T}}$ is the vector of values of the nonlinear function $\eta(x, \boldsymbol{\theta})$ at points x_j with parameters $\boldsymbol{\theta}$.

The formulation (4) of the LSM in this case takes the following form:

$$\hat{\boldsymbol{\theta}} = \arg\min_{\theta\in\Theta}(\mathbf{Y} - \mathbf{H}(\mathrm{X}, \Theta))^{\mathrm{T}}(\mathbf{Y} - \mathbf{H}(\mathrm{X}, \Theta)). \tag{9}$$

Since the system of normal equations of LSM becomes nonlinear, it is not possible to use the simple formula (5) to calculate the parameter estimates. Therefore, we used a numerical optimization method to solve this problem [23]. Formula (6) for the nonlinear model is as follows:

$$SS_{\text{reg}} = (\mathbf{Y} - \mathbf{H}(\mathrm{X}, \hat{\boldsymbol{\theta}}))^{\mathrm{T}}(\mathbf{Y} - \mathbf{H}(\mathrm{X}, \hat{\boldsymbol{\theta}})), \tag{10}$$

while the general form of formula (7) for calculating the coefficient of determination remains the same.

One of the three regression models, which was selected according to the results of the adequacy analysis, was further used to build the dependencies of the shape of the cladding track cross-sectional profile on the technological parameters of the spraying process. For this purpose, the calculated parameter estimates of the selected model were used to construct mappings of the set of technological spraying parameters $u_k \in \mathrm{U} \subset \mathbb{R}^p$, $k = 1, \ldots, p$, into the set of profile line parameters $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_m)^{\mathrm{T}}$. Since the parameter estimates $\hat{\boldsymbol{\theta}}$ are random variables and the technological parameters u_k are set parameters, linear regression analysis was used to construct such mappings. The specific type of regression functions is described in the main part of the paper. These functions were further used to optimize the technological mode of sputtering using the algorithms described in [24].

CALCULATION RESULTS AND DISCUSSION

The cross-sectional profile of the clad track can be divided into two areas [13]: the part located above the workpiece surface comprising the cladding area, while the part located below the workpiece surface is referred to as the under-melting area, which is formed as a result of deepening of the molten metal bath into the volume of the workpiece. A comparison of methods of approximating the cross-sectional profiles of the cladding region by simple approximating functions—circle arc, elliptical arc, sinusoid, and parabola—is presented in [15]. According to the comparison of residual dispersions of the compared approximating functions, it was concluded that the parabola approximation is the best option among the studied ones. It should be noted, however, that the profiles of the roll sections studied in this paper were quite regular in contrast to the profiles presented in other works, including [13]. In addition, the above approximating functions do not describe the sub-melting region, whose profile turns out to be more complex. In this connection, in the present work, first of all, polynomials of higher order compared to the parabola were studied as approximating functions.

Figure 1 shows the profile contour points obtained by image processing in [13] and their approximations by two types of polynomials. The first type of regression model represented a seventh degree polynomial of the following form:

$$y = (1 - x^2)\sum_{i=0}^{5}\theta_i^1 x^i, \tag{11}$$

where θ_i^1 are the regression parameters, $x \in [-1; 1] \subset \mathbb{R}$ are the normalized values of the argument.

Hereinafter, the upper index y of the parameter θ denotes the number of the approximating function, while the upper index y of the argument x denotes the degree.

The second type of polynomial approximating function for the profile of the roll section, proposed in [25] where it is called a biquadratic approximator, takes the form:

$$y = (1 - x^2)(\theta_0^2 + \theta_1^2 x^2 + \theta_2^2 x^4). \tag{12}$$

In contrast to model (11), only even degrees of the argument are retained here, thus achieving symmetry of the profile with respect to the OY axis. Theoretically, this symmetry, which should be fulfilled in coaxial surfacing, was also taken into account in [15] when choosing simple symmetric approximating functions. In both models, the common term $(1 - x^2)$ is removed to zero the functions at points $x = \pm 1$. This may be explained in terms of the practical convenience of representing the argument of regression functions in a dimensionless normalized form:

$$x^{\mathrm{cond}} = \frac{x^{\mathrm{nat}} - x_0}{\Delta x}, \tag{13}$$

where x^{nat} is the value of the argument in natural units;
x^{cond} is the value in dimensionless scale;
$x_0 = (\max(x^{\mathrm{nat}}) + \min(x^{\mathrm{nat}}))/2$ is the mean level;
$\Delta x = (\max(x^{\mathrm{nat}}) - \min(x^{\mathrm{nat}}))/2$ is the step of variation.

Since both models (11) and (12) are parametrically linear, their estimates are easily calculated using formula (5). As can be seen from Fig. 1, both functions adequately approximate the cladding region and the sub-melting region. At the same time, the coefficient of determination of function *1* was $R^2 = [0.995; 0.996]$ for approximating functions of the cladding and sub-melting regions, respectively, while that of function *2* was slightly lower: $R^2 = [0.985; 0.976]$. Based on this, it would be possible to adopt, for example, function (11) as the main tool for approximating the track cross-section profiles. However, when processing micrographs of less regular profiles, the disadvantage of polynomial approximation when processing complex or irregular profiles became clear due to the appearance of additional extrema on the approximating curve, which should not exist in the physical sense. To address this issue, we propose a nonlinear approximating function of the following form:

$$y = A\cos^B\left(\frac{\pi}{2}x\right), \tag{14}$$

where the approximation parameters, which were calculated by the nonlinear estimation method, are denoted as A, B.

The convenience of the function (14) is its symmetry due to having only two parameters: similar to functions (11), (12), it has zeros at $x = \pm 1$. At the same time, it always has only one extremum in the interval $x \in [-1; 1]$.

Figure 2 shows the points of the profile of the cladding track cross-section according to the processing of microphotography [13] for cladding parameters: laser power is 310 W, while powder feed is 29 g/m and approximating curves are obtained by the three regression functions considered above. As can be seen, the polynomial functions adequately approximate the upper part of this profile, but are not workable for the lower part (the sub-melting region), whereas function (14) adequately describes the entire profile, including both the cladding region and the sub-melting region.

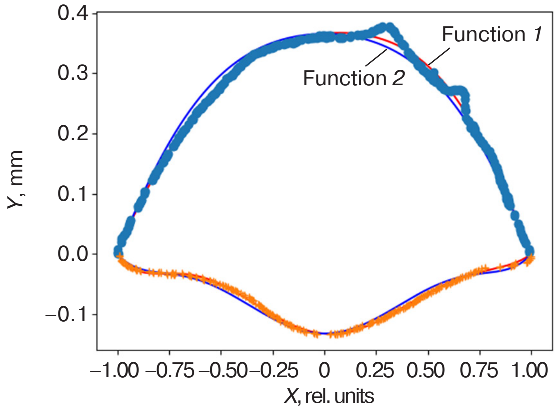

Fig. 1. Points of the cladding track cross-section profile according to microphotography processing [13] (positive *Y* values—cladding area, negative *Y* values—sub-melting area) and approximating curves: by model (11)—function *1*, by model (12)—function *2*

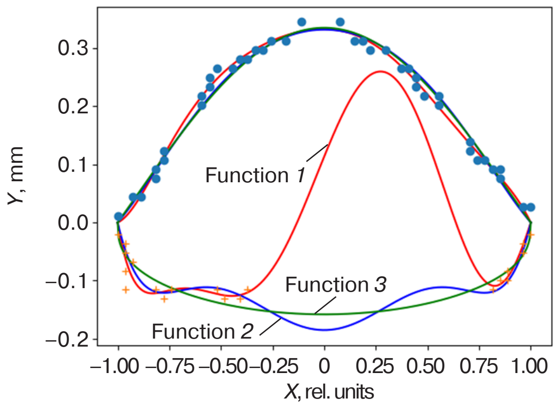

Fig. 2. Points of the cladding track cross-section profile by microphotography processing [13] and approximating curves: by model (11)—function *1*, by model (12)—function *2*, by model (14)—function *3*

Tables 1 and 2 show the parameters A, B of the approximating functions of the model (14) and the values of Δx for various technological parameters of sputtering NiCr16 nickel-chromium alloy on a steel billet obtained from the results of processing the images of profiles given in [13]. These data can be used for validation of mathematical models of cladding based on numerical solution of the equations of hydrodynamics and heat transfer using finite element methods [11, 12, 18]. By considering a wide enough range of variation of cladding parameters, it is possible to use the approximation results to construct mappings of a set of technological parameters of cladding into a set of profile line parameters and thus solve the problem of optimization of technological parameters. To construct such mappings, the method of regression analysis with approximation of profile parameter dependencies on technological parameters by two-factor parabolic regression equations of the following form was used in the present work:

$$\theta_i = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_1^2 + b_4 x_2^2 + b_5 x_1 x_2, \quad (15)$$

where θ_i are the profile parameters; x_1 and x_2 are the laser power and powder feed in normalized units (13); b_j, $j = 0, \ldots, 5$ are the regression coefficients calculated with the help of LSM.

Table 1. Parameters of approximation of cladding path profiles at different spraying process parameters (upper part of the contour)

Laser power, W	Powder feed, g/m	Δx, mm	A_0, mm	B_0
310	12	0.333	0.145	0.661
310	29	0.429	0.335	1.022
310	45	0.500	0.606	0.689
570	12	0.476	0.228	1.035
570	29	0.524	0.495	0.558
570	45	0.587	0.521	0.664
570	85	0.802	1.084	0.541
720	12	0.508	0.233	0.973
720	29	0.516	0.481	0.597
720	45	0.603	0.598	0.678
720	63	0.746	0.815	0.769
1150	12	0.619	0.248	0.893
1150	29	0.643	0.572	0.904
1150	45	0.778	0.721	0.654
1150	63	0.873	0.959	0.463
1150	85	1.095	0.988	0.506
1150	100	1.159	0.919	0.513

Table 2. Parameters of approximation of sub-melting area profiles at different technological spraying parameters (lower part of the contour)

Laser power, W	Powder feed, g/m	Δx, mm	A_1, mm	B_1
310	12	0.333	−0.054	0.550
310	29	0.429	−0.158	0.367
310	45	0.500	−0.106	0.264
570	12	0.476	−0.245	2.518
570	29	0.524	−0.139	1.191
570	45	0.587	−0.323	0.365
570	85	0.802	−0.357	0.320
720	12	0.508	−0.213	1.088
720	29	0.516	−0.208	0.769
720	45	0.603	−0.317	0.431
720	63	0.746	−0.388	0.473
1150	12	0.619	−0.603	2.246
1150	29	0.643	−0.445	2.384
1150	45	0.778	−0.523	1.553
1150	63	0.873	−0.675	1.715
1150	85	1.095	−0.676	0.792
1150	100	1.159	−0.577	0.414

The values of the parameters Δx, A_0, A_1, B_0, B_1 given in Tables 1 and 2, as well as the areas under the profile curve S_0, S_1 calculated on their basis, were used as profile parameters. Indices 0 and 1 of the parameters correspond to the upper and lower parts of the profile, respectively. The areas were calculated by numerical integration of the profile functions (14) for the corresponding technological parameters.

Table 3 shows the calculated values of estimates of regression coefficients (15) for the studied parameters and the coefficients of determination of the models. In practice, the most interesting parameters are Δx, A_0, A_1, of which the first two characterize the half-width and height of the cladding roll, respectively, while the third characterizes the depth of the underfusion area into the part volume. Regression models for these parameters, as can be seen from Table 3, are characterized by R^2 values close to unity, which indicates their adequacy.

The areas under the profile curve are also important. They are used to calculate the relative share of the under-melting area (under-melting coefficient):

$$D = \frac{S_1}{S_0 + S_1}. \qquad (16)$$

For high-quality cladding, the value of the D parameter should be optimal: small values of D provide an insufficiently strong bond between the cladding and the substrate, while excessively large values worsen the properties of the base material of the part.

The parameters of the mathematical models were derived on the basis of the calculated values of the regression equation coefficients: coordinates of critical points and eigenvalues of Hesse matrices. The graphs of regression surfaces were also plotted. According to the analysis of the obtained regression models, the dependencies of profile parameters Δx, A_0, A_1, S_0, S_1 on technological parameters x_1, x_2 are monotonic in the studied area of technological parameter changes, while the dependence $D(x_1, x_2)$ has the character of a hyperbolic paraboloid. As an example, Figs. 3–5 are plots of surfaces $A_0(x_1, x_2)$, $S_0(x_1, x_2)$, $D(x_1, x_2)$.

Table 3. Coefficients of regression equations of profile parameter dependencies on technological cladding parameters

Parameter	b_0	b_1	b_2	b_3	b_4	b_5	R^2
Δx	0.6932	0.1514	0.2539	−0.0083	0.0543	0.0356	0.9877
a_0	0.7772	0.0339	0.4114	−0.0055	−0.1445	−0.0717	0.9543
a_1	−0.3444	−0.1908	−0.1000	−0.0445	−0.0048	0.0406	0.9052
B_0	0.6222	−0.0459	−0.1436	0.0225	0.1185	−0.0918	0.5243
B_1	0.6396	0.6522	−0.7602	0.0881	0.2115	−0.2000	0.7039
S_0	0.9716	0.2314	0.8311	0.0162	0.0200	0.0824	0.9704
S_1	0.4409	0.3013	0.3442	0.0466	0.0876	0.1216	0.9594

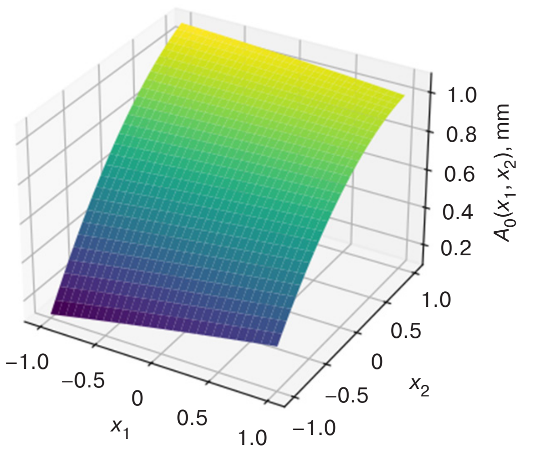

Fig. 3. Dependence of the cladding roll height on normalized values of laser power and powder feed rate

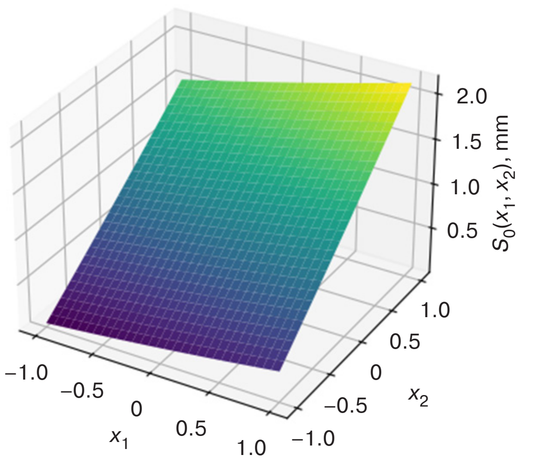

Fig. 4. Dependence of the cross-sectional area of the cladding roll on the normalized values of laser power and powder feed rate

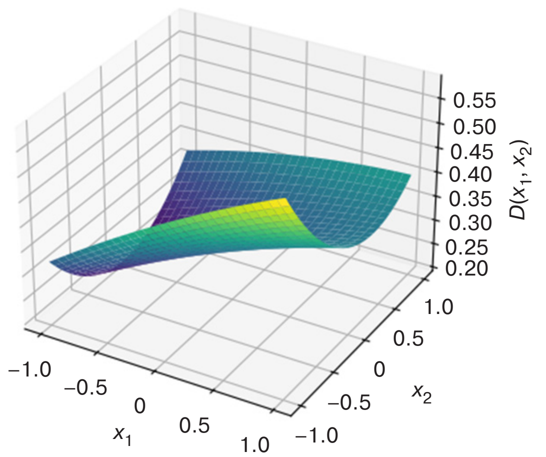

Fig. 5. Dependence of the relative share of the sub-melting area on normalized values of laser power and powder feed rate

The choice of values for the cladding profile parameters depends on the specific requirements of the product. If there is no optimization problem and it is only necessary to provide some specified profile characteristics, the construction of contour curves of regression functions can be used. As an example, Fig. 6 shows contour curves for functions $S_0(x_1, x_2)$ and $D(x_1, x_2)$. Having selected the required values of each function, the values of technological parameters x_1 and x_2 necessary for their achievement can be obtained as coordinates of the intersection points of the corresponding isolines. Thus, in particular, the values of coordinates $x_1 = -0.753$, $x_2 = -0.283$ meet the requirement $S_0 = 0.6$, $D = 0.25$. The coordinates are calculated from the solution of the nonlinear system of equations:

$$\begin{cases} S_0(x_1, x_2) - 0.6 = 0, \\ D(x_1, x_2) - 0.25 = 0. \end{cases}$$

An alternative option formulates the problem of finding the conditional extremum of one of the indicators in terms of constraints on the values of the others. For example, the coordinates of the conditional maximum of the function $S_0(x_1, x_2)$ under the constraint $D(x_1, x_2) = 0.35$, which is calculated by the conditional optimization method with a linear approximation of the confidence region [23], were the values $x_{\text{opt}} = (0.350, 0.748)$.

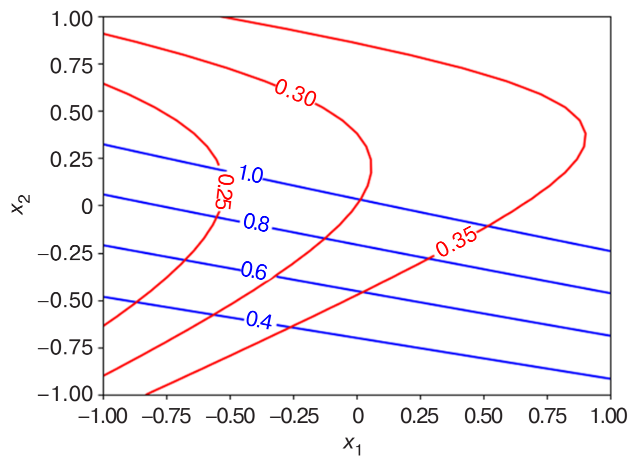

Fig. 6. Contour curves of regression functions $S_0(x_1, x_2)$ for levels {0.4, 0.6, 0.8, 1.0} and $D(x_1, x_2)$—for levels {0.25, 0.30, 0.35}

Once the optimum values of the technological parameters of surfacing have been selected, the optimum step of track application can be calculated. The technological process of surfacing consists in the sequential application of powdered material on the surface in the form of tracks with a certain step, which we will further denote w. If the step of application is smaller than the width of the track profile, the profiles overlap, and the material of the second track fills the space between these profiles [15]. In this case, the problem arises of choosing the optimal value of w at which the excess material formed during the overlapping of profiles completely fills the free space between them. In [26], the optimal values of w for the tracks of some simple profiles are calculated. Let us calculate the optimal value of w at approximation of a profile by function (14), which parameters A_0, B_0 are calculated by regression equation (15).

Figure 7 shows the scheme for calculating the optimal value of w: curve *1* corresponds to the profile of the first track, while curve *2* corresponds to the profile of the second track overlapped with the first one. The excess material formed due to overlapping profiles theoretically forms curve *3*, which is obtained by summing up the profiles of tracks of curves *1* and *2* in the area of their overlapping. The optimum step of overlapping of tracks will be such that the area of the area ABC of intersection of profiles of tracks will be equal to the area of the area BDE located above the overlap zone. In this case, the molten metal from the area under curve *3* will evenly fill the free space between the tracks and the surface of the coating will be optimally smooth.

Taking into account the symmetry of the figures with respect to the vertical line FG, passing through the point of intersection of profiles B with dimensionless coordinate ζ, the optimal superposition of tracks assumes the equality of areas of areas ABG and BDF, which can be expressed as follows:

$$A_0\zeta - \int_0^{\zeta} A_0 \cos^{B_0}\left(\frac{\pi}{2}x\right)dx = \int_{\zeta}^{1} A_0 \cos^{B_0}\left(\frac{\pi}{2}x\right)dx. \quad (17)$$

Hence, it follows that

$$\zeta = \int_0^1 \cos^{B_0}\left(\frac{\pi}{2}x\right)dx. \quad (18)$$

It is clear that the optimal value of the track spacing in dimensionless units can be expressed in terms of ζ by means of the formula:

$$w_{\text{opt}} = 2 - 2(1 - \zeta) = 2\zeta. \quad (19)$$

In particular, for $B_0(0.35,\ 0.75) = 0.544$, the optimal step value is $w_{\text{opt}} = 1.498$, and for $B_0(-0.753, -0.283) = 0.700$ is equal to $w_{\text{opt}} = 1.409$.

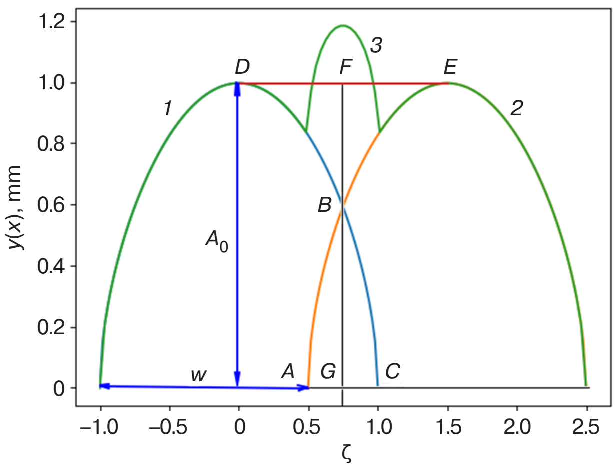

Fig. 7. Scheme for calculating the optimal value of the track overlap step

CONCLUSIONS

A method of approximating the profile of the spraying track cross-section during laser cladding of powder metals has been proposed for optimizing the cladding process parameters on its basis.

A variant of nonlinear dependence that includes two approximation parameters has been selected from three profile approximating function variants. The coefficients of the regression dependence equations of the approximating function parameters were calculated along with the contour cross-sectional area and the underfusion coefficient on the laser power and powder feed technological cladding parameters.

As a result of mathematical modeling of the dependence of the parameters of the spraying track cross-sectional profile function, the contour cross-sectional area is shown to increase monotonically with increasing laser power and powder feed, while the dependence of the sub-melting factor on the above parameters has the form of a surface with a saddle point. For these contour characteristics, the problem of conditional optimization of the contour cross-sectional area with a restriction on the value of the sub-melting coefficient is solved.

The approximating function of the cladding track cross-section profile proposed in the present work can be used to solve the problem arising in practice of calculating the optimal step of track application to ensure the achievement of an even cladding surface.

Authors' contribution. All authors equally contributed to the research work.

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