UDC 539.3, 621.762
https://doi.org/10.32362/2500-316X-2025-13-2-74-92
EDN KPQMIJ

RESEARCH ARTICLE

Mathematical modeling of hot isotatic pressing of tubes from powder materials

Vasiliy A. Goloveshkin [1, 2, @], Artem A. Nickolaenko [1], Victor N. Samarov [3], Gerard Raisson [4], Daria M. Fisunova [1]

[1] *MIREA – Russian Technological University, Moscow, 119454 Russia*
[2] *Institute of Applied Mechanics, Russian Academy of Sciences, Moscow, 125040 Russia*
[3] *Laboratory of new technologies, Moscow, 121352 Russia*
[4] *Clermond Ferrand, France*
[@] *Corresponding author, e-mail: vag-1953@yandex.ru*

Abstract
Objectives. The work set out to create a mathematical model to investigate the process of hot isostatic pressing (HIP) process of long tubes from powder materials in metal capsules. By analyzing the stress-strain state in the areas far from the top and bottom borders in the cylindrical system of coordinates, the axial strain rate at every moment of the process can be considered to be constant through the entire volume.
Methods. Mathematical modeling methods were used to describe mechanical properties in the process of HIP deformation by Green's model of porous compressible media. The HIP capsule material, which is considered to be non-compressible, is described by the ideal plasticity model. The temperature field is assumed to be uniform over the volume and constant during the time of deformation.
Results. The hypothesis of the uniform density over the cross section at each moment of the process was considered during analysis to the extent that the wall thickness of the tube is substantially less than its diameter. This hypothesis allowed us to reduce the task of determining the strain rates at every step of the process to a solution comprising two equations having two variables. When the strain rates are determined, the deformation field is built to obtain the final dimensions of the tube when the powder material is fully consolidated at the end of the HIP process.
Conclusions. The proposed model for describing the process hot isostatic pressing of long tubes from powder materials takes all the features of this process into account depending on the system parameters. The possibility of using tubular samples to determine the functions included in the Green's condition is demonstrated.

Keywords: mathematical modeling, plastically compressible media, Hot Isostatic Pressing, powder material, plastically irreversible compressible media, Green's plasticity criterion, ideal plasticity

• **Submitted:** 25.06.2024 • **Revised:** 29.08.2024 • **Accepted:** 06.02.2025

For citation: Goloveshkin V.A., Nickolaenko A.A., Samarov V.N., Raisson G., Fisunova D.M. Mathematical modeling of hot isotatic pressing of tubes from powder materials. *Russian Technological Journal.* 2025;13(2):74–92. https://doi.org/10.32362/2500-316X-2025-13-2-74-92, https://elibrary.ru/KPQMIJ

Financial disclosure: The authors have no financial or proprietary interest in any material or method mentioned.

The authors declare no conflicts of interest.

НАУЧНАЯ СТАТЬЯ

Математическое моделирование процесса горячего изостатического прессования труб из порошковых материалов

В.А. Головешкин [1, 2, @], А.А. Николаенко [1], В.Н. Самаров [3], Ж. Рейссон [4], Д.М. Фисунова [1]

[1] *МИРЭА – Российский технологический университет, Москва, 119454 Россия*
[2] *Институт прикладной механики, Российская академия наук, Москва, 125040 Россия*
[3] *ООО «Лаборатория новых технологий», Москва, 121352 Россия*
[4] *Клермон Ферран, Франция*
[@] *Автор для переписки, e-mail: vag-1953@yandex.ru*

Резюме

Цели. Цель работы – создание модели, которая позволяет с помощью математического моделирования исследовать процесс горячего изостатического прессования (ГИП) длинных труб из порошковых материалов. Напряженно-деформируемое состояние исследуется вдали от верхней и нижней границ капсулы в цилиндрической системе координат, поэтому осевая скорость деформации в каждый момент процесса предполагается постоянной по объему.

Методы. Используются методы математического моделирования. Порошковый материал моделируется как пластически сжимаемая сплошная среда. Для описания его механических свойств в процессе деформации используется модель Грина. Для анализа механического поведения материала капсулы применяется модель идеальной пластичности при условии несжимаемости. Температурное поле предполагается постоянным по объему и по времени в течение всего процесса.

Результаты. Поскольку, как правило, толщина стенок труб существенно меньше их радиуса, то в процессе исследования принималась гипотеза о постоянстве относительной плотности порошкового материала по объему в каждый момент процесса. Принятая гипотеза позволила свести задачу определения скоростей деформаций на каждом шаге процесса к решению некоторой системы двух уравнений с двумя неизвестными. По известным скоростям деформации определяются скорости перемещений, что позволяет получить конечные размеры трубы (при относительной плотности порошкового материала равной единице). Анализируются усадки всех размеров трубы (вертикального, внутреннего радиуса, наружного радиуса), как функции относительной плотности.

Выводы. Предложенная модель описания процесса ГИП длинных труб из порошковых материалов позволяет учитывать все особенности данного процесса в зависимости от параметров системы. Показана возможность использования трубчатых образцов для определения функций, входящих в условие Грина.

Ключевые слова: математическое моделирование, пластически сжимаемая среда, горячее изостатическое прессование, порошковый материал, условие Грина, идеальная пластичность

• **Поступила:** 25.06.2024 • **Доработана:** 29.08.2024 • **Принята к опубликованию:** 06.02.2025

Для цитирования: Головешкин В.А., Николаенко А.А., Самаров В.Н., Рейссон Ж., Фисунова Д.М. Математическое моделирование процесса горячего изостатического прессования труб из порошковых материалов. *Russian Technological Journal*. 2025;13(2):74–92. https://doi.org/10.32362/2500-316X-2025-13-2-74-92, https://elibrary.ru/KPQMIJ

Прозрачность финансовой деятельности: Авторы не имеют финансовой заинтересованности в представленных материалах или методах.

Авторы заявляют об отсутствии конфликта интересов.

INTRODUCTION

Hot isostatic pressing (HIP) is a process of high-temperature compaction (~1000°C) of powder materials under the action of external pressure (~1000 atm). Products manufactured using the HIP powder metallurgy process have high performance characteristics. However, it is precisely because of their high strength properties that their subsequent processing presents certain difficulties, requiring maximum precision at the manufacturing stage. As part of the HIP process, the powder material is placed in a metal container (capsule). This capsule is deformed together with the powder material until the latter is completely compacted, then removed chemically or mechanically. The function of the capsule is to induce the powder monolith to take the desired shape following the completion of the process. The mathematical modeling of the process sets out to design the capsule in such a way that the powder monolith takes the desired geometric shape after the end of the process.

Two main problems of mathematical modeling of the HIP process can be distinguished. Firstly, the HIP process is characterized by large deformations (initial powder density is about 65% of the monolith density). In mathematical terms, this means that the defining relations will be nonlinear with the boundary conditions set on a time-variable boundary. The second more fundamental problem consists in the difficulty of constructing the constitutive relations (under constitutive relations, we understand the relations defining the relationship between the stress tensor in the medium and the parameters characterizing the state of the medium). This problem is characteristic of all problems of mechanics of deformable solids that investigate their behavior beyond the elasticity limit. Since any defining relations will be approximate, any calculation will be approximate even if mathematical problems are excluded. Therefore, the actual powder manufacturing process must be an iterative process, a schema of which is outlined in [1]. The essence is as follows: a mathematical model is built, a capsule is designed on the basis of this model, and the product is manufactured. The product parameters are then compared with the required to provide a basis for refining the mathematical model. This method is some analog of the complex loading computer method proposed by Ilyushin in [2, 3]. A model satisfying the following requirements is considered to be an acceptable mathematical model of the HIP process:

1) it provides a close first approximation;
2) it correctly considers the influence of parameters;
3) it enables changes to be made to the model parameters based on the experimental results and, if necessary, additional parameters to be introduced.

Typically, 2–3 experimental iterations are required to put a product into production. At the present stage, the task of mathematical modeling is to consistently obtain the desired geometry at the second experimental iteration. For this purpose, experience shows that it is necessary to have an error of about 10% at the first iteration.

There are various approaches to describing the behavior of powder medium, some of which, for example [4], consider the medium as discrete. Within such approaches, when considering the interaction of individual particles, it is necessary to take into account the effects arising on the surface of their interaction [5, 6]. The use of such an approach requires the application of statistical methods [7]. More often, powder material is considered as a single continuum; thus, since we are interested in the kinematic aspects of behavior in the process of HIP as shown in [8–10], the kinematic aspects of the behavior of powder materials do not differ significantly from the behavior of continuous media.

The defining relations for powder materials have one essential difference from those used in classical theories of plasticity [11–14] due to the fact that these works proceed, as a rule, from small volume deformations or their equality to zero. For powder materials, the volume strain (or equivalent parameters: relative density, porosity) is an important parameter characterizing the state of the medium. It should be noted that it is the shear part of the strain tensor that is of real interest in describing the HIP process. Since the purpose of ISU is to obtain a monolithic product and the initial density can be determined with a high degree of accuracy, the volume component of the strain tensor can be considered as known. The time of the compaction process can be determined quite accurately (at known temperature and pressure) by compaction diagrams (Ashby diagram) [15–18]. Various models describing the behavior of plastically compressible media are presented in the works of Druyanov [19], Green [20], Shtern [21], Skorokhod [22].

The presence of the capsule causes its walls to “shield” the external pressure differently in different directions. This is particularly evident in the fabrication of pipes made of powder materials. In this process, there are actually three external boundaries: an outer and inner radial boundary, and another boundary at the ends. As shown in [23], under certain conditions this can lead to a looping motion of the inner wall. The peculiarities of the behavior of powder tubes at the initial stage of the process are investigated in [24]. The purpose of this work is to develop a model that for qualitatively assessing the change of the main parameters to determine the pipe geometry at the entire stage of the HIP process, as well as to establish the possibility of using pipe pressing experiments to determine the parameters characterizing the mechanical behavior of powder material.

1. MATHEMATICAL TASK STATEMENT

Let there be the following system in the cylindrical coordinate system (r, z): at $R_1 < r < R_2$ and $0 < z < H$, there is a capsule with perfectly plastic material with yield strength T_1; at $R_3 < r < R_4$ and $0 < z < H$, there is a capsule with perfectly plastic material with yield strength T_2; at $R_2 < r < R_3$, and $0 < z < H$, there is a powder material, whose behavior is described by the Green's model, with a monolithic yield strength Y. Here R_1, R_2, R_3, R_4, and H are the current geometrical dimensions of the pipe.

The elliptic Green's yield condition is used to describe the mechanical properties of powder material [20, 22]:

$$\frac{\sigma^2}{f_2^2} + \frac{s^2}{f_1^2} = Y^2, \tag{1.1}$$

where $\sigma = \sigma_{ij}/3$ is the first invariant of the stress tensor (σ_{ij} are the components of the stress tensor); s is the intensity of the deviator of the stress tensor, $s^2 = (3/2)s_{ij}s_{ij}$, $s_{ij} = \sigma_{ij} - \sigma\delta_{ij}$; indices i, j take integer values 1, 2, 3, with the r axis corresponding to index 1, φ axis—to index 2, z axis—to index 3; δ_{ij} is the Kronecker symbol ($\delta_{ij} = 0$ at $i \neq j$, $\delta_{ij} = 1$ at $i = j$); f_1 and f_2 are the relative density ρ functions known from experiment.

According to the flow law

$$\varepsilon_{ij} = \omega \frac{\partial \Phi}{\partial \sigma_{ij}}, \tag{1.2}$$

where ε_{ij} are the components of the strain rate tensor; $\Phi(\sigma_{ij}) = 0$ is the yield surface equation (1.1); ω is the proportionality factor determined at each point of space during the solution process.

The process is investigated away from the pipe ends. This allows us to assume that the strain rate ε_z is constant throughout the entire volume of the system. Then, by virtue of the assumption made and axial symmetry, $\varepsilon_{rz} = \varepsilon_{r\varphi} = \varepsilon_{z\varphi} = 0$, $\sigma_{rz} = \sigma_{r\varphi} = \sigma_{z\varphi} = 0$. Using (1.1), (1.2), taking into account the last remarks, we obtain:

$$\begin{aligned}
\varepsilon_r &= \omega/9\left[\left(2/f_2^2 + 18/f_1^2\right)\sigma_r + \left(2/f_2^2 - 9/f_1^2\right)\sigma_\varphi + \left(2/f_2^2 - 9/f_1^2\right)\sigma_z\right],\\
\varepsilon_\varphi &= \omega/9\left[\left(2/f_2^2 - 9/f_1^2\right)\sigma_r + \left(2/f_2^2 + 18/f_1^2\right)\sigma_\varphi + \left(2/f_2^2 - 9/f_1^2\right)\sigma_z\right],\\
\varepsilon_z &= \omega/9\left[\left(2/f_2^2 - 9/f_1^2\right)\sigma_r + \left(2/f_2^2 - 9/f_1^2\right)\sigma_\varphi + \left(2/f_2^2 + 18/f_1^2\right)\sigma_z\right].
\end{aligned} \tag{1.3}$$

Considering (1.3) as a system of equations with respect to stresses, we obtain

$$\begin{aligned}
\sigma_r &= 1/(18\omega)\left[\left(9f_2^2 + 4f_1^2\right)\varepsilon_r + \left(9f_2^2 - 2f_1^2\right)\varepsilon_\varphi + \left(9f_2^2 - 2f_1^2\right)\varepsilon_z\right],\\
\sigma_\varphi &= 1/(18\omega)\left[\left(9f_2^2 - 2f_1^2\right)\varepsilon_r + \left(9f_2^2 + 4f_1^2\right)\varepsilon_\varphi + \left(9f_2^2 - 2f_1^2\right)\varepsilon_z\right],\\
\sigma_z &= 1/(18\omega)\left[\left(9f_2^2 - 2f_1^2\right)\varepsilon_r + \left(9f_2^2 - 2f_1^2\right)\varepsilon_\varphi + \left(9f_2^2 + 4f_1^2\right)\varepsilon_z\right].
\end{aligned} \tag{1.4}$$

Substituting (1.4) into (1.1), we obtain:

$$\frac{1}{\omega} = \frac{6Y}{\sqrt{\left(9f_2^2 - 2f_1^2\right)\left(\varepsilon_r + \varepsilon_\varphi + \varepsilon_z\right)^2 + 6f_1^2\left(\varepsilon_r^2 + \varepsilon_\varphi^2 + \varepsilon_z^2\right)}}. \tag{1.5}$$

The power of internal forces in a unit volume w is determined by the relation

$$w = \sigma_{ij}\,\varepsilon_{ij}. \tag{1.6}$$

According to (1.4)–(1.6)

$$w = \frac{Y}{3}\sqrt{\left(9f_2^2 - 2f_1^2\right)\left(\varepsilon_r + \varepsilon_\varphi + \varepsilon_z\right)^2 + 6f_1^2\left(\varepsilon_r^2 + \varepsilon_\varphi^2 + \varepsilon_z^2\right)}. \tag{1.7}$$

The capsule material is assumed to be incompressible. Its behavior is described by the law of perfect plasticity.

$$s^2 = T^2, \tag{1.8}$$

where T is the yield strength.

According to the flow law (1.2) and the yield surface Eq. (1.8),

$$\varepsilon_r = \omega\left[2\sigma_r - \sigma_\varphi - \sigma_z\right],\ \varepsilon_\varphi = \omega\left[2\sigma_\varphi - \sigma_r - \sigma_z\right],\ \varepsilon_z = \omega\left[2\sigma_z - \sigma_\varphi - \sigma_r\right]. \tag{1.9}$$

We suppose

$$\sigma_r + \sigma_\varphi + \sigma_z = -3p. \tag{1.10}$$

Since due to the incompressibility condition

$$\varepsilon_r + \varepsilon_\varphi + \varepsilon_z = 0, \tag{1.11}$$

then from (1.9)–(1.11) we have:

$$\sigma_r = -p + \varepsilon_r / 3\omega,\ \sigma_\varphi = -p + \varepsilon_\varphi / 3\omega,\ \sigma_z = -p + \varepsilon_z / 3\omega. \tag{1.12}$$

According to (1.8), (1.12),

$$\frac{1}{\omega} = \frac{T\sqrt{6}}{\sqrt{\left(\varepsilon_r^2 + \varepsilon_\varphi^2 + \varepsilon_z^2\right)}}. \tag{1.13}$$

Power of internal forces per unit volume:

$$w = T\sqrt{\frac{2}{3}}\sqrt{\left(\varepsilon_r^2 + \varepsilon_\varphi^2 + \varepsilon_z^2\right)}. \tag{1.14}$$

Taking into account the axial symmetry, the radial equation of equilibrium in the quasi-static approximation has the form:

$$\frac{\partial\sigma_r}{\partial r} + \frac{\left(\sigma_r - \sigma_\varphi\right)}{r} = 0. \tag{1.15}$$

The equilibrium equation along the z axis is satisfied integrally, i.e., the total stress forces σ_z in each section z = const are balanced by the external pressure on the capsule face:

$$2\pi\int_{R_1}^{R_4} \sigma_z r dr = -\pi P\left(R_4^2 - R_1^2\right), \tag{1.16}$$

where P is the external pressure.

At the boundaries $r = R_2$, $r = R_3$ we assume the condition of equality of radial velocities U_r. At the boundaries $r = R_1$, $r = R_4$ we assume

$$\sigma_r = -P. \tag{1.17}$$

By assumption of the constancy of the strain rate ε_z over the volume, since the problem statement allows the system to move as a rigid body along the z axis, we can put the following expression for the axial velocity U_z:

$$U_z = \left(V / H\right)z, \tag{1.18}$$

where V is the value of velocity U_z at $z = H$, determined during the solution process.

The radial velocity at each moment is a function of radius only $U_r = U_r(r)$. For strain rates we have the following relations:

$$\varepsilon_r = \frac{\partial U_r}{\partial r},\ \varepsilon_\varphi = \frac{U_r}{r},\ \varepsilon_z = \frac{\partial U_z}{\partial z} = V / H. \tag{1.19}$$

Relations (1.1)–(1.13), (1.15), (1.19) with additional conditions (1.16)–(1.18) mathematically define the problem of finding the velocity field at each moment of time at known distribution of relative density of powder material.

The law of change of powder material density is determined by the continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho U_r r)}{\partial r} = 0, \tag{1.20}$$

where t is time.

2. FIELD OF VELOCITIES

As shown in [24], a nonuniform density distribution along the radius occurs when the tube is pressed. However, as a rule, the wall thickness of the tube is significantly less than its radius. Under these conditions, it can be approximated that the relative density of the powder material is constant along the radius at each moment of the process. Consequently, for the radial velocity of displacements U_r in the powder material we have an approximate representation:

$$U_r = Ar + B/r, \tag{2.1}$$

where A, B are coefficients determined in the solution process.

The value of the velocity U_z is determined by Eq. (1.18).

From the incompressibility condition (1.11) in the capsule material we have: $\frac{dU_r}{dr} + \frac{U_r}{r} + \varepsilon_z = 0.$ Consequently:

$$U_r = -1/2\varepsilon_z r + C / r, \tag{2.2}$$

where C is the coefficient determined in the solution process.

We suppose that

$$U_r = U_1 \text{ at } r = R_2;\ U_r = U_2 \text{ at } r = R_3. \tag{2.3}$$

Finally, taking into account (2.3) and the condition of continuity of velocities at $r = R_2$, $r = R_3$, the velocities in the total area are equal to:

at $R_1 < r < R_2$, $0 < z < H$ (capsule)

$$U_z = V / H,\quad U_r = -(1/2)\varepsilon_z r + C_1 / r,\quad C_1 = (1/2)\varepsilon_z R_2^2 + U_1 R_2; \tag{2.4}$$

at $R_3 < r < R_4$, $0 < z < H$ (capsule)

$$U_z = V / H,\quad U_r = -(1/2)\varepsilon_z r + C_3 / r,\quad C_3 = (1/2)\varepsilon_z R_3^2 + U_2 R_3; \tag{2.5}$$

at $R_2 < r < R_3$, $0 < z < H$ (powder)

$$U_z = (V / H)z,\quad U_r = Ar + B_1 R_3^2 / r,\quad A = (U_2 R_3 - U_1 R_2) / (R_3^2 - R_2^2),\ B_1 = (U_1 R_2 - U_2 R_2^2 / R_3) / ({R_3}^2 - R_2^2). \tag{2.6}$$

According to (2.4)–(2.6), (1.19) strain rates in the entire domain:
at $R_1 < r < R_2$, $0 < z < H$ (capsule)

$$\varepsilon_z = V / H,\ \ \varepsilon_r = -(1/2)\varepsilon_z - C_1 / r^2,\ \ \varepsilon_\varphi = -(1/2)\varepsilon_z + C_1 / r^2; \tag{2.7}$$

at $R_3 < r < R_4$, $0 < z < H$ (capsule)

$$\varepsilon_z = V / H,\ \ \varepsilon_r = -(1/2)\varepsilon_z - C_3 / r^2,\ \varepsilon_\varphi = -(1/2)\varepsilon_z + C_3 / r^2; \tag{2.8}$$

at $R_2 < r < R_3$, $0 < z < H$ (powder)

$$\varepsilon_z = V / H,\ \varepsilon_r = A - B_1 R_3^2 / r^2,\ \varepsilon_\varphi = A + B_1 R_3^2 / r^2. \tag{2.9}$$

The total internal force power W consists of three components: $W = W_1 + W_2 + W_3$, where W_1 is the internal force power in the inner capsule at $R_1 < r < R_2$, $0 < z < H$; W_3 is the internal force power in the outer capsule at $R_3 < r < R_4$, $0 < z < H$; W_2 is the internal force power in the powder at $R_2 < r < R_3$, $0 < z < H$.

At $R_1 < r < R_2$, $0 < z < H$ the power of internal forces per unit volume, according to (1.14), (2.7), is equal to $w_1 = T_1\sqrt{2/3}\sqrt{(3/2)\varepsilon_z^2 + 2C_1^2 / r^4}$, where T_1 is the yield strength. Then the total power in the internal capsule:

$$W_1 = 2\pi H T_1 \sqrt{2/3}\int_{R_1}^{R_2} \sqrt{(3/2)\varepsilon_z^2 + 2C_1^2 / r^4}\, r dr.$$

After the corresponding calculations:

$$W_1 = \pi H T_1 \sqrt{\frac{1}{3}}\left\{\left[\sqrt{3\varepsilon_z^2 R_2^4 + 4C_1^2} - \sqrt{3\varepsilon_z^2 R_1^4 + 4C_1^2}\right] + 2C_1 \ln\left(\frac{R_2^2}{R_1^2}\cdot\frac{2C_1 + \sqrt{3\varepsilon_z^2 R_1^4 + 4C_1^2}}{2C_1 + \sqrt{3\varepsilon_z^2 R_1^4 + 4C_1^2}}\right)\right\}. \tag{2.10}$$

At $R_3 < r < R_4$, $0 < z < H$ the power of internal forces per unit volume, according to (1.14), (2.8) is equal to $w_3 = T_2\sqrt{2/3}\sqrt{(3/2)\varepsilon_z^2 + 2C_3^2 / r^4}$, where T_2 is the yield strength. Total power:

$$W_3 = 2\pi H T_2 \sqrt{2/3}\int_{R_3}^{R_4} \sqrt{(3/2)\varepsilon_z^2 + 2C_3^2 / r^4}\, r dr.$$

By calculating, we get:

$$W_3 = \pi H T_2 \sqrt{\frac{1}{3}}\left\{\left[\sqrt{3\varepsilon_z^2 R_4^4 + 4C_3^2} - \sqrt{3\varepsilon_z^2 R_3^4 + 4C_3^2}\right] + 2C_3 \ln\left(\frac{R_4^2}{R_3^2}\cdot\frac{2C_3 + \sqrt{3\varepsilon_z^2 R_3^4 + 4C_3^2}}{2C_3 + \sqrt{3\varepsilon_z^2 R_4^4 + 4C_3^2}}\right)\right\}. \tag{2.11}$$

At $R_2 < r < R_3$, $0 < z < H$ ((in powder), the power of internal forces in a unit volume, according to (1.7), (2.9), is equal to $w_2 = \frac{Y}{3}\sqrt{\left[\left(9f_2^2 - 2f_1^2\right)\left(2A + \varepsilon_z\right)^2 + 6f_1^2\left(\varepsilon_z{}^2 + 2A^2\right)\right] + \frac{12 f_1^2 R_3^4 B_1^2}{r^4}}$. Total power:

$$W_2 = (2/3)\pi H Y \int_{R_2}^{R_3} \sqrt{\left[\left(9f_2^2 - 2f_1^2\right)\left(2A + \varepsilon_z\right)^2 + 6f_1^2\left(\varepsilon_z^2 + 2A^2\right)\right] + 12 f_1^2 R_3^4 B_1^2 / r^4}\, r dr.$$

Integrating, we come to the expression:

$$W_2 = \pi HYR_3^2 / 3\left\{\sqrt{\left[\left(9f_2^2 - 2f_1^2\right)\left(2A+\varepsilon_z\right)^2 + 6f_1^2\left(\varepsilon_z^2 + 2A^2\right)\right] + 12f_1^2B_1^2} - \right.$$
$$- \sqrt{\left[\left(9f_2^2 - 2f_1^2\right)\left(2A+\varepsilon_z\right)^2 + 6f_1^2\left(\varepsilon_z^2 + 2A^2\right)\right]R_2^4 / R_3^4 + 12f_1^2B_1^2} + \tag{2.12}$$
$$\left. + 2\sqrt{3}f_1B_1 \ln\left(\frac{R_3^2}{R_2^2}\frac{\sqrt{12}f_1B_1 + \sqrt{\left[\left(9f_2^2 - 2f_1^2\right)\left(2A+\varepsilon_z\right)^2 + 6f_1^2\left(\varepsilon_z^2 + 2A^2\right)\right]R_2^4 / R_3^4 + 12f_1^2B_1^2}}{\sqrt{12}f_1B_1 + \sqrt{\left[\left(9f_2^2 - 2f_1^2\right)\left(2A+\varepsilon_z\right)^2 + 6f_1^2\left(\varepsilon_z^2 + 2A^2\right)\right] + 12f_1^2B_1^2}}\right)\right\}.$$

Let P be the external pressure. Let us determine the power of external forces. The total power of external forces contains three components: N_1 is the power of external forces at the boundary $z = H$; N_2 is the power of external forces at the boundary $r = R_1$; N_3 is the power of external forces at the boundary $r = R_4$. After the corresponding calculations, we obtain:

$$N_1 = -\pi PV\left(R_4^2 - R_1^2\right),$$
$$N_2 = 2\pi PR_1H\left(-1/2\varepsilon_z R_1 + 1/2\varepsilon_z R_2^2 / R_1 + U_1R_2 / R_1\right),$$
$$N_3 = -2\pi PR_4H\left(-1/2\varepsilon_z R_4 + 1/2\varepsilon_z R_3^2 / R_4 + U_2R_3 / R_4\right).$$

Total power $N = N_1 + N_2 + N_3$.
Considering that $\varepsilon_z = V/H$:

$$N = -PH\left(R_3^2 - R_2^2\right)\pi\left[V/H + 2\left(U_2R_3 - U_1R_2\right)/\left(R_3^2 - R_2^2\right)\right]. \tag{2.13}$$

From the condition $N = W_1 + W_2 + W_3$ we obtain:

$$P = -\frac{W_1 + W_2 + W_3}{H\left(R_3^2 - R_2^2\right)\pi\left[\frac{V}{H} + 2\left(U_2\frac{R_3}{R_3^2 - R_2^2} - U_1\frac{R_2}{R_3^2 - R_2^2}\right)\right]}. \tag{2.14}$$

Since the velocities are determined with accuracy up to a constant multiplier and there is no characteristic time in the problem formulation, we can put:

$$\left[\frac{V}{H} + 2\left(U_2\frac{R_3}{R_3^2 - R_2^2} - U_1\frac{R_2}{R_3^2 - R_2^2}\right)\right] = -1. \tag{2.15}$$

Let us represent the external pressure P in the form:

$$P = T_1M_1 + YM_2 + T_2M_3. \tag{2.16}$$

Let us denote:

$$\beta_1 = T_1 / Y,\ \beta_2 = T_2 / Y,$$
$$\alpha_1 = R_2 / R_1,\ \alpha_2 = R_3 / R_2,\ \alpha_3 = R_4 / R_3, \tag{2.17}$$
$$\mu_1 = U_1 / R_2,\ \mu_2 = U_2 / R_3.$$

Relations (2.15), (2.16) will take the form:

$$P/Y = \beta_1M_1 + M_2 + \beta_2M_3, \tag{2.18}$$

$$\varepsilon_z + 2\left(\alpha_2^2\mu_2 - \mu_1\right)/\left(\alpha_2^2 - 1\right) = -1. \tag{2.19}$$

Then, according to (2.10)–(2.14), (2.16)–(2.19):

$$\begin{aligned} M_1 = M_1\left(\mu_1, \varepsilon_z\right) = \frac{1}{\sqrt{3}\left(\alpha_2^2 - 1\right)} \Bigg\{ \Bigg[\sqrt{3\varepsilon_z^2 + \left(2\mu_1 + \varepsilon_z\right)^2} - \sqrt{\frac{3\varepsilon_z^2}{\alpha_1^4} + \left(2\mu_1 + \varepsilon_z\right)^2} \Bigg] \\ + \left(2\mu_1 + \varepsilon_z\right) \ln\left(\frac{\left(2\mu_1 + \varepsilon_z\right)\alpha_1^2 + \sqrt{3\varepsilon_z^2 + \alpha_1^4\left(2\mu_1 + \varepsilon_z\right)^2}}{\left(2\mu_1 + \varepsilon_z\right) + \sqrt{3\varepsilon_z^2 + \left(2\mu_1 + \varepsilon_z\right)^2}} \right) \Bigg\}, \end{aligned} \tag{2.20}$$

$$\begin{aligned} M_3 = M_3\left(\mu_2, \varepsilon_z\right) = \frac{\alpha_2^2}{\sqrt{3}\left(\alpha_2^2 - 1\right)} \Bigg\{ \Bigg[\sqrt{3\varepsilon_z^2\alpha_3{}^4 + \left(2\mu_2 + \varepsilon_z\right)^2} - \sqrt{3\varepsilon_z^2 + \left(2\mu_2 + \varepsilon_z\right)^2} \Bigg] + \\ + \left(2\mu_2 + \varepsilon_z\right) \ln\left(\alpha_3^2 \frac{\left(2\mu_2 + \varepsilon_z\right) + \sqrt{3\varepsilon_z^2 + \left(2\mu_2 + \varepsilon_z\right)^2}}{\left(2\mu_2 + \varepsilon_z\right) + \sqrt{3\varepsilon_z^2\alpha_3^4 + \left(2\mu_2 + \varepsilon_z\right)^2}} \right) \Bigg\}, \end{aligned} \tag{2.21}$$

$$\begin{aligned} M_2 = M_2\left(A, B_1\right) = \frac{\alpha_2^2}{3\left(\alpha_2^2 - 1\right)} \Bigg\{ \sqrt{\left[9f_2^2 + 4f_1^2\left(1 + 3A\right)^2\right] + 12f_1^2B_1^2} - \\ - \sqrt{\frac{1}{\alpha_2^4}\left[9f_2^2 + 4f_1^2\left(1 + 3A\right)^2\right] + 12f_1^2B_1^2} + \\ + 2\sqrt{3}f_1B_1 \ln\frac{\sqrt{12}\alpha_2^2f_1B_1 + \sqrt{\left[9f_2^2 + 4f_1^2\left(1 + 3A\right)^2\right] + 12\alpha_2^4f_1^2B_1^2}}{\sqrt{12}f_1B_1 + \sqrt{\left[9f_2^2 + 4f_1^2\left(1 + 3A\right)^2\right] + 12f_1^2B_1^2}} \Bigg\}, \end{aligned} \tag{2.22}$$

where

$$A = \left(\mu_2\alpha_2^2 - \mu_1\right)/\left(\alpha_2^2 - 1\right),\ B_1 = \left(\mu_1 - \mu_2\right)/\left(\alpha_2^2 - 1\right). \tag{2.23}$$

3. SYSTEM OF EQUATIONS FOR DETERMINING VARIABLES UNKNOWN

At each step of the HIP process it is necessary to determine the unknown values μ_1, μ_2, ε_z, and P. At known values of μ_1 and μ_2, the values ε_z and P are determined by Eqs. (2.18) and (2.19). The values μ_1, μ_2 are determined from the condition of minimum P in the area bounded by the lines $\mu_1 \geq \mu_2$; $\mu_2 \leq 0$, $\varepsilon_z \leq 0$ on the plane of parameters μ_1, μ_2 (see Figure).

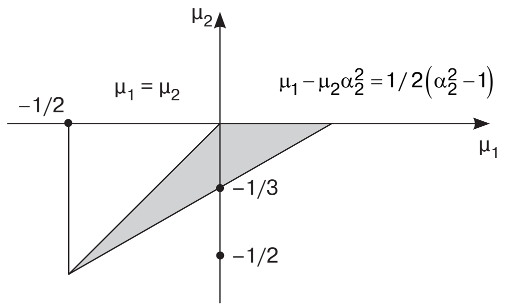

Figure. Area of finding the minimum

According to (2.18) the system of equations for determining μ_1, μ_2 is obtained from the conditions:

$$\frac{\partial}{\partial\mu_1}\left(\beta_1 M_1 + M_2 + \beta_2 M_3\right) = 0, \tag{3.1}$$

$$\frac{\partial}{\partial\mu_2}\left(\beta_1 M_1 + M_2 + \beta_2 M_3\right) = 0. \tag{3.2}$$

So we have:

$$\frac{\partial\varepsilon_z}{\partial\mu_1} = 2/\left(\alpha_2^2 - 1\right), \quad \frac{\partial\varepsilon_z}{\partial\mu_2} = -2\alpha_2^2/\left(\alpha_2^2 - 1\right);$$
$$\frac{\partial A}{\partial\mu_1} = -1/\left(\alpha_2^2 - 1\right), \quad \frac{\partial A}{\partial\mu_2} = \alpha_2^2/\left(\alpha_2^2 - 1\right);$$
$$\frac{\partial B_1}{\partial\mu_1} = 1/\left(\alpha_2^2 - 1\right), \quad \frac{\partial B_1}{\partial\mu_2} = -1/\left(\alpha_2^2 - 1\right).$$

Consequently, Eqs. (3.1), (3.2) will take the form:

$$\beta_1\frac{\partial M_1}{\partial\mu_1} + \frac{\partial M_1}{\partial\varepsilon_z}\frac{2\beta_1}{\left(\alpha_2^2 - 1\right)} - \frac{\partial M_2}{\partial A}\frac{1}{\left(\alpha_2^2 - 1\right)} + \frac{\partial M_2}{\partial B_1}\frac{1}{\left(\alpha_2^2 - 1\right)} + \frac{\partial M_3}{\partial\varepsilon_z}\frac{2\beta_2}{\left(\alpha_2^2 - 1\right)} = 0, \tag{3.3}$$

$$-\frac{\partial M_1}{\partial\varepsilon_z}\frac{2\beta_1\alpha_2^2}{\left(\alpha_2^2 - 1\right)} + \frac{\partial M_2}{\partial A}\frac{\alpha_2^2}{\left(\alpha_2^2 - 1\right)} - \frac{\partial M_2}{\partial B_1}\frac{1}{\left(\alpha_2^2 - 1\right)} + \beta_2\frac{\partial M_3}{\partial\mu_2} - \frac{\partial M_3}{\partial\varepsilon_z}\frac{2\beta_2\alpha_2^2}{\left(\alpha_2^2 - 1\right)} = 0. \tag{3.4}$$

For the above derivatives the corresponding relations are as follows:

$$\frac{\partial M_1}{\partial\mu_1} = \frac{1}{\left(\alpha_2^2 - 1\right)}\frac{2}{\sqrt{3}}\ln\left(\frac{\left(2\mu_1 + \varepsilon_z\right)\alpha_1^2 + \sqrt{3\varepsilon_z^2 + \alpha_1^4\left(2\mu_1 + \varepsilon_z\right)^2}}{\left(2\mu_1 + \varepsilon_z\right) + \sqrt{3\varepsilon_z^2 + \left(2\mu_1 + \varepsilon_z\right)^2}}\right), \tag{3.5}$$

$$\frac{\partial M_1}{\partial\varepsilon_z} =$$
$$= \frac{1}{\left(\alpha_2^2 - 1\right)}\sqrt{\frac{1}{3}}\left\{\ln\left(\frac{\left(2\mu_1 + \varepsilon_z\right)\alpha_1^2 + \sqrt{3\varepsilon_z^2 + \alpha_1^4\left(2\mu_1 + \varepsilon_z\right)^2}}{\left(2\mu_1 + \varepsilon_z\right) + \sqrt{3\varepsilon_z^2 + \left(2\mu_1 + \varepsilon_z\right)^2}}\right) + \frac{1}{\alpha_1^4}\frac{3\varepsilon_z\left(\alpha_1^4 - 1\right)}{\sqrt{3\varepsilon_z^2 + \left(2\mu_1 + \varepsilon_z\right)^2} + \sqrt{3\varepsilon_z^2/\alpha_1^4 + \left(2\mu_1 + \varepsilon_z\right)^2}}\right\}, \tag{3.6}$$

$$\frac{\partial M_3}{\partial\mu_2} = \frac{\alpha_2^2}{\left(\alpha_2^2 - 1\right)}\frac{2}{\sqrt{3}}\ln\left(\alpha_3^2\frac{\left(2\mu_2 + \varepsilon_z\right) + \sqrt{3\varepsilon_z^2 + \left(2\mu_2 + \varepsilon_z\right)^2}}{\left(2\mu_2 + \varepsilon_z\right) + \sqrt{3\varepsilon_z^2\alpha_3^4 + \left(2\mu_2 + \varepsilon_z\right)^2}}\right), \tag{3.7}$$

$$\frac{\partial M_3}{\partial\varepsilon_z} =$$
$$= \frac{\alpha_2^2}{\left(\alpha_2^2 - 1\right)}\sqrt{\frac{1}{3}}\left\{\ln\left(\alpha_3^2\frac{\left(2\mu_2 + \varepsilon_z\right) + \sqrt{3\varepsilon_z^2 + \left(2\mu_2 + \varepsilon_z\right)^2}}{\left(2\mu_2 + \varepsilon_z\right) + \sqrt{3\varepsilon_z^2\alpha_3^4 + \left(2\mu_2 + \varepsilon_z\right)^2}}\right) + \frac{3\varepsilon_z\left(\alpha_3^4 - 1\right)}{\sqrt{3\varepsilon_z^2\alpha_3^4 + \left(2\mu_2 + \varepsilon_z\right)^2} + \sqrt{3\varepsilon_z^2 + \left(2\mu_2 + \varepsilon_z\right)^2}}\right\}, \tag{3.8}$$

$$\frac{\partial M_2}{\partial A} =$$
$$= \frac{4f_1^2\left(1 + 3A\right)\left(\alpha_2^2 + 1\right)}{\alpha_2^2}\left\{\frac{1}{\sqrt{\left[9f_2^2 + 4f_1^2\left(1 + 3A\right)^2\right] + 12f_1^2B_1^2} + \sqrt{\left[9f_2^2 + 4f_1^2\left(1 + 3A\right)^2\right]\frac{1}{\alpha_2^4} + 12f_1^2B_1^2}}\right\}, \tag{3.9}$$

$$\frac{\partial M_2}{\partial B_1}=\frac{\alpha_2^2}{\left(\alpha_2^2-1\right)}\frac{2}{\sqrt{3}}f_1\left[\ln\left(\alpha_2^2\frac{\sqrt{12}f_1B_1+\sqrt{\left[9f_2^2+4f_1^2\left(1+3A\right)^2\right]\frac{1}{\alpha_2^4}+12f_1^2B_1^2}}{\sqrt{12}f_1B_1+\sqrt{\left[9f_2^2+4f_1^2\left(1+3A\right)^2\right]+12f_1^2B_1^2}}\right)\right]. \quad (3.10)$$

Equation (3.3) implicitly determines μ_1 at a given value of μ_2. Under certain conditions, the equation has no solutions for the parameters under study, and then the value of μ_1 is at the boundary of the area. That is, it can be argued, taking into account the above remark, that Eq. (3.3) implicitly defines μ_1 as a function of μ_2, i.e., $-\mu_1=\mu_1(\mu_2)$. Then, using Eq. (3.4), given that $\mu_1=\mu_1(\mu_2)$, we can determine μ_2. Again, the equation has no solutions, and the point of minimum is on the boundary of the domain. Knowing the parameters μ_1 and μ_2 allows us to determine all other parameters of the process. In order to determine the nature of change of the pipe parameters, it remains to find out what was meant by the concept of time when the condition (2.20) was accepted. From the law of conservation of mass of powder material it follows:

$$\rho\pi H\left(R_3^2-R_2^2\right)=\rho_0\pi H_0\left(R_{30}^2-R_{20}^2\right),$$

where ρ_0 is the initial relative density; R_{20} and R_{30} are the initial pipe dimensions.

Differentiating this relation, after some simplifications we obtain:

$$d\rho H\left(R_3^2-R_2^2\right)+\rho V\left(R_3^2-R_2^2\right)dt+2\rho H\left(R_3U_2-R_2U_1\right)dt=0. \quad (3.11)$$

Let us convert it to the form

$$d\rho/\rho+\left[\varepsilon_z+2\left(\alpha_2^2\mu_2-\mu_1\right)/\left(\alpha_2^2-1\right)\right]dt=0. \quad (3.12)$$

Then, considering (2.19), we have:

$$dt=d\rho/\rho. \quad (3.13)$$

Consequently, the relative density of the powder material ρ can be taken as a process parameter instead of time t. Then the laws of change of values of H, R_2 and R_3 are determined by the relations:

$$dH=\varepsilon_z Hdt\Rightarrow dH=\varepsilon_z Hd\rho/\rho\Rightarrow dH/d\rho=\varepsilon_z H/\rho, \quad (3.14)$$

$$dR_2=U_1dt\Rightarrow dR_2=\mu_1R_2d\rho/\rho\Rightarrow dR_2/d\rho=\mu_1R_2/\rho, \quad (3.15)$$

$$dR_3=U_2dt\Rightarrow dR_3=\mu_2R_3d\rho/\rho\Rightarrow dR_3/d\rho=\mu_2R_3/\rho. \quad (3.16)$$

When H, R_2, R_3 are known, the values of R_1, R_4 are determined from the condition of incompressibility of the capsule material.

$$\pi\left(R_2^2-R_1^2\right)H=\pi\left(R_{20}^2-R_{10}^2\right)H_0, \quad (3.17)$$

$$\pi\left(R_4^2-R_3^2\right)H=\pi\left(R_{40}^2-R_{30}^2\right)H_0, \quad (3.18)$$

where R_{10} and R_{40} are the initial dimensions of the pipe.

Table 1 shows the results of calculation of the process of HIP of the pipe with initial parameters—initial dimensions in millimeters: $R_{10}=18$, $R_{20}=20$, $R_{30}=30$, $R_{40}=32$, $H_0=100$, initial relative density $\rho_0=0.6$.

The following function values were taken in the calculation:

$$f_1(\rho)=\sqrt{\left(\rho-\rho_0\right)/\left(1-\rho_0\right)},\ \ f_2(\rho)=\sqrt{\left(\rho-\rho_0\right)/\left(1-\rho\right)}.$$

Table 1. Results of parameter calculation as a function of relative density

ρ	0.659	0.719	0.778	0.838	0.897	0.937	0.977
P/Y	0.447	0.682	0.932	1.249	1.742	2.357	4.083
H/H_0	0.977	0.954	0.933	0.915	0.898	0.889	0.882
R_1/R_{10}	0.976	0.949	0.923	0.901	0.881	0.870	0.863
R_2/R_{20}	0.983	0.963	0.946	0.930	0.916	0.909	0.905
R_3/R_{30}	0.973	0.948	0.925	0.905	0.887	0.876	0.867
R_4/R_{40}	0.978	0.957	0.939	0.923	0.909	0.900	0.893

Here is another example of calculation of initial and final dimension ratios under other conditions. $\beta_1 = \beta_2 = 2/9$, $R_{10} = 15$, $R_{20} = 20$, $R_{30} = 30$, $R_{40} = 45$, $H_0 = 100$, $\rho_0 = 0.6$, $\rho = 0.978$, $P/Y = 4.309$, $H/H_0 = 1$, $R_1/R_{10} = 1$, $R_2/R_{20} = 1$, $R_3/R_{30} = 0.886$, $R_4/R_{40} = 0.918$.

In the latter case, the shrinkage for the powder was directed towards the radius reduction. Vertical shrinkage was not observed. The inner capsule remained nondeformable. The minimum was reached at the boundary of the area.

4. POSSIBILITY OF EXPERIMENTAL DETERMINATION OF FUNCTIONS

Traditionally, the functions $f_1(\rho)$, $f_2(\rho)$ are determined on the basis of two experiments [25, 26]. The first experiment involves the process of HIP of a cylindrical sample in a thin-walled capsule up to a certain relative density ρ, while the second investigates the free deposition of the obtained sample after removing the capsule. It should be assumed that the first experiment (considered to be the main one for determining the function $f_2(\rho)$) does not provide uniform all-round compression due to the influence of the capsule. The second precipitation experiment (considered basic for determining the function $f_1(\rho)$) is not easy to perform, especially for small values of relative density ρ. The second drawback of the free settlement experiment for a cylindrical specimen is as follows. In the real HIP process, all deformations are overwhelmingly compressive in nature, while in the settlement experiment, two out of three main deformations are tensile in nature. As shown in [27], the real vector of principal strain rates in the HIP process makes a significantly smaller angle with the vector of uniform compression as compared to that obtained in free settlement. While the ideal experiment in this respect involves one-dimensional pressing of a powder layer, it is difficult to realize due to technical issues. In [27–29] it is shown that both functions can in principle be determined in one experiment if the ratio of strain rates is known at each moment of the experiment. In [28], the possibility of determining the desired functions based on experiments with the same cylindrical specimens interrupted at different values of relative density is demonstrated. The main disadvantage of these experiments is that their results lie rather close to the hydrostatic axis (uniform all-round compression). However, the use of tubular specimens allows us to eliminate this disadvantage to a certain extent. The purpose of this section is to determine the feasibility in principle of determining the functions $f_1(\rho)$, $f_2(\rho)$.

Adding (3.3) to (3.4), we obtain:

$$\beta_1 \partial M_1 / \partial \mu_1 - 2\beta_1 \partial M_1 / \partial \varepsilon_z + \partial M_2 / \partial A + \beta_2 \partial M_3 / \partial \mu_2 - 2\beta_2 \partial M_3 / \partial \varepsilon_z = 0. \tag{4.1}$$

Using (3.9), we rewrite Eq. (4.1) in the form:

$$\frac{4 f_1^2 (1+3A)\left(\alpha_2^2+1\right)}{\alpha_2^2 \left\{ \sqrt{\left[9 f_2^2 + 4 f_1^2 (1+3A)^2\right] + 12 f_1^2 B_1^2} + \sqrt{\left[9 f_2^2 + 4 f_1^2 (1+3A)^2\right] \frac{1}{\alpha_2^4} + 12 f_1^2 B_1^2} \right\}} = -\beta_1 \partial M_1 / \partial \mu_1 + 2\beta_1 \partial M_1 / \partial \varepsilon_z - \beta_2 \partial M_3 / \partial \mu_2 + \beta_2 \partial M_3 / \partial \varepsilon_z. \tag{4.2}$$

Let us denote:

$$\Psi_1 = \varepsilon_z / \mu_2, \; \Psi_2 = \mu_1 / \mu_2. \tag{4.3}$$

Then

$$(3A+1)/B_1=\Delta_1=\left(\alpha_2^2-\Psi_2\right)/\left(\Psi_2-1\right)-\Psi_1\left(\alpha_2^2-1\right)/\left(\Psi_2-1\right), \tag{4.4}$$

$$1/B_1=\Delta_2=-\Psi_1\left(\alpha_2^2-1\right)/\left(\Psi_2-1\right)-2\left(\alpha_2^2-\Psi_2\right)/\left(\Psi_2-1\right). \tag{4.5}$$

According to (3.5)–(3.10) and (4.3)–(4.5), we obtain

$$\frac{\partial M_1}{\partial \mu_1}=\frac{1}{\left(\alpha_2^2-1\right)}\frac{2}{\sqrt{3}}\ln\left(\frac{-\left(2\Psi_2+\Psi_1\right)\alpha_1^2+\sqrt{3\Psi_1^2+\alpha_1^4\left(2\Psi_2+\Psi_1\right)^2}}{-\left(2\Psi_2+\Psi_1\right)+\sqrt{3\Psi_1^2+\left(2\Psi_2+\Psi_1\right)^2}}\right),$$

$$\frac{\partial M_3}{\partial \mu_2}=\frac{\alpha_2^2}{\left(\alpha_2^2-1\right)}\frac{2}{\sqrt{3}}\ln\left(\alpha_3^2\frac{-\left(2+\Psi_1\right)+\sqrt{3\Psi_1^2+\left(2+\Psi_1\right)^2}}{-\left(2+\Psi_1\right)+\sqrt{3\Psi_1^2\alpha_3^4+\left(2+\Psi_1\right)^2}}\right),$$

$$\frac{\partial M_1}{\partial \varepsilon_z}=$$

$$=\frac{1}{\left(\alpha_2^2-1\right)}\sqrt{\frac{1}{3}}\left\{\ln\left(\alpha_1^2\frac{-\left(2\Psi_2+\Psi_1\right)+\sqrt{3\Psi_1{}^2\frac{1}{\alpha_1^4}+\left(2\Psi_2+\Psi_1\right)^2}}{-\left(2\Psi_2+\Psi_1\right)+\sqrt{3\Psi_1^2+\left(2\Psi_2+\Psi_1\right)^2}}\right)-\frac{3\Psi_1\left(\alpha_1^4-1\right)}{\alpha_1^4\left[\sqrt{3\Psi_1^2+\left(2\Psi_2+\Psi_1\right)^2}+\sqrt{3\Psi_1^2\frac{1}{\alpha_1^4}+\left(2\Psi_2+\Psi_1\right)^2}\right]}\right.,$$

$$\frac{\partial M_3}{\partial \varepsilon_z}=\frac{\alpha_2^2}{\left(\alpha_2^2-1\right)}\sqrt{\frac{1}{3}}\left\{\ln\left(\alpha_3^2\frac{-\left(2+\Psi_1\right)+\sqrt{3\Psi_1^2+\left(2+\Psi_1\right)^2}}{-\left(2+\Psi_1\right)+\sqrt{3\Psi_1^2\alpha_3^4+\left(2+\Psi_1\right)^2}}\right)-\frac{3\Psi_1\left(\alpha_3^4-1\right)}{\left[\sqrt{3\Psi_1^2\alpha_3^4+\left(2+\Psi_1\right)^2}+\sqrt{3\Psi_1^2+\left(2+\Psi_1\right)^2}\right]}\right..$$

Let $x^2=\left[(3/4)\Delta_2^2f_2^2+(1/3)\Delta_1^2f_1^2\right]$. Then, according to (4.2):

$$\frac{f_1^2}{\sqrt{x^2+f_1^2}+\sqrt{x^2\gamma^2+f_1^2}}=\Omega_1\left(\Psi_1,\Psi_2\right), \tag{4.6}$$

where

$$\gamma=\frac{1}{\alpha_2^2}<1;\ \ \Omega_1\left(\Psi_1,\Psi_2\right)=\frac{\alpha_2^2\sqrt{3}}{2\Delta_1\left(\alpha_2^2+1\right)}\left(-\beta_1\frac{\partial M_1}{\partial \mu_1}+2\beta_1\frac{\partial M_1}{\partial \varepsilon_z}-\beta_2\frac{\partial M_3}{\partial \mu_2}+2\beta_2\frac{\partial M_3}{\partial \varepsilon_z}\right). \tag{4.7}$$

Let us denote

$$z=x/f_1. \tag{4.8}$$

Consequently, we have:

$$f_1\left(\sqrt{z^2+1}+\sqrt{z^2\gamma^2+1}\right)=\Omega_1. \tag{4.9}$$

We represent Eq. (2.19) in the form:

$$M_2=P/Y-\beta_1M_1-\beta_2M_3. \tag{4.10}$$

According to (4.7) and (2.23), we have:

$$\frac{\alpha_2^2}{\left(\alpha_2^2-1\right)}\frac{1}{3}\left\{\sqrt{\left[9f_2^2+4f_1^2\left(1+3A\right)^2\right]+12f_1^2B_1^2}-\sqrt{\left[9f_2^2+4f_1^2\left(1+3A\right)^2\right]\frac{1}{\alpha_2^4}+12f_1^2B_1^2}+\right.$$

$$\left.+2\sqrt{3}f_1B_1\ln\left(\alpha_2^2\frac{\sqrt{12}f_1B_1+\sqrt{\left[9f_2^2+4f_1^2\left(1+3A\right)^2\right]\frac{1}{\alpha_2^4}+12f_1^2B_1^2}}{\sqrt{12}f_1B_1+\sqrt{\left[9f_2^2+4f_1^2\left(1+3A\right)^2\right]+12f_1^2B_1^2}}\right)\right\}=\frac{P}{Y}-\beta_1M_1-\beta_2M_3.$$

Taking into account (4.3)–(4.5) and (4.8), we transform this equation to the form:

$$\sqrt{x^2+f_1^2}-\sqrt{x^2\gamma^2+f_1^2}+f_1\ln\left(\frac{1}{\gamma}\frac{f_1+\sqrt{x^2\gamma^2+f_1^2}}{f_1+\sqrt{x^2+f_1^2}}\right)=\frac{\sqrt{3}\left(\alpha_2^2-1\right)\Delta_2}{2\alpha_2^2}\left(\frac{P}{Y}-\beta_1M_1-\beta_2M_3\right). \tag{4.11}$$

We note that according to (4.3), (2.21), and (2.22):

$$M_1=\frac{1}{\sqrt{3}}\frac{1}{\Psi_1\left(\alpha_2^2-1\right)+2\left(\alpha_2^2-\Psi_2\right)}\left\{\left[\sqrt{3{\Psi_1}^2+\left(2\Psi_2+\Psi_1\right)^2}-\sqrt{3\Psi_1^2\frac{1}{\alpha_1^4}+\left(2\Psi_2+\Psi_1\right)^2}\right]-\right.$$

$$\left.-\left(2\Psi_2+\Psi_1\right)\ln\left(\frac{-\left(2\Psi_2+\Psi_1\right)\alpha_1^2+\sqrt{3\Psi_1^2+\alpha_1^4\left(2\Psi_2+\Psi_1\right)^2}}{-\left(2\Psi_2+\Psi_1\right)+\sqrt{3\Psi_1^2+\left(2\Psi_2+\Psi_1\right)^2}}\right)\right\},$$

$$M_3=\sqrt{\frac{1}{3}}\frac{\alpha_2^2}{\Psi_1\left(\alpha_2^2-1\right)+2\left(\alpha_2^2-\Psi_2\right)}\left\{\left[\sqrt{3\Psi_1^2\alpha_3^4+\left(2+\Psi_1\right)^2}-\sqrt{3\Psi_1^2+\left(2+\Psi_1\right)^2}\right]-\right.$$

$$\left.-\left(2+\Psi_1\right)\ln\left(\alpha_3^2\frac{-\left(2+\Psi_1\right)+\sqrt{3\Psi_1^2+\left(2+\Psi_1\right)^2}}{-\left(2+\Psi_1\right)+\sqrt{3\Psi_1^2\alpha_3^4+\left(2+\Psi_1\right)^2}}\right)\right\}.$$

Consequently, Eq. (4.11) can be represented in the form:

$$f_1\left\{\sqrt{z^2+1}-\sqrt{z^2\gamma^2+1}+\ln\frac{1}{\gamma}\frac{1+\sqrt{z^2\gamma^2+1}}{1+\sqrt{z^2+1}}\right\}=\Omega_2\left(\Psi_1,\Psi_2\right), \tag{4.12}$$

where $\Omega_2=\sqrt{3}\left(\alpha_2^2-1\right)\Delta_2/\left(2\alpha_2^2\right)\left(P/Y-\beta_1M_1-\beta_2M_3\right)$.

From Eqs. (4.9) and (4.12) it follows

$$\left(\sqrt{z^2+1}+\sqrt{z^2\gamma^2+1}\right)\left\{\sqrt{z^2+1}-\sqrt{z^2\gamma^2+1}+\ln\left(\frac{1}{\gamma}\frac{1+\sqrt{z^2\gamma^2+1}}{1+\sqrt{z^2+1}}\right)\right\}=\Omega\left(\Psi_1,\Psi_2\right), \tag{4.13}$$

where $\Omega(\Psi_1, \Psi_2) = \Omega_2/\Omega_1$.

Let us consider the function

$$f(z)=\left(\sqrt{z^2+1}+\sqrt{z^2\gamma^2+1}\right)\left\{\sqrt{z^2+1}-\sqrt{z^2\gamma^2+1}+\ln\left(\frac{1}{\gamma}\frac{1+\sqrt{z^2\gamma^2+1}}{1+\sqrt{z^2+1}}\right)\right\}.$$

This function is monotonically increasing at $z > 0$. Consequently, Eq. (4.13) has a single solution $z > 0$ provided that $f(0) < \Omega$.

Let $H^-, H^+; R_2^-, R_2^+; R_3^-, R_3^+$ are the previous and subsequent values of the corresponding parameters. Then ε_z, μ_1, μ_2 can be assumed as:

$$\varepsilon_z \approx 2\left(H^+ - H^-\right)/\left(H^+ + H^-\right),$$
$$\mu_1 \approx 2\left(R_2^+ - R_2^-\right)/\left(R_2^+ + R_2^-\right),$$
$$\mu_2 \approx 2\left(R_3^+ - R_3^-\right)/\left(R_3^+ + R_3\right).$$

Consequently, we have $\Psi_1 \approx \varepsilon_z / \mu_2; \Psi_2 \approx \mu_1 / \mu_2$.

If the value of z is found, than $f_1 = H_1\left(\sqrt{z^2+1} + \sqrt{z^2\gamma^2+1}\right)$. Since $x^2 = z^2 f_1^2$, from the equation $(3/4)\Delta_2^2 f_2^2 + (1/3)\Delta_1^2 f_1^2 = z^2 f_1^2$ we have:

$$f_2 = \frac{2}{\sqrt{3}}\sqrt{\frac{\left(z^2 - \Delta_1^2/3\right)}{\Delta_2^2}}f_1.$$

Below are the results of approximate determination of the functions based on the calculation under the conditions used in Table 1.

Table 2. Initial data for calculation of the functions $f_1(\rho)$, $f_2(\rho)$

Parameter	1 (initial state)	2	3	4
ρ	0.6198	0.6396	0.6594	0.6792
P/Y	0.2544	0.3594	0.4468	0.5272
H	99.2634	98.4711	97.6770	96.8966
R_2	19.9027	19.7805	19.6517	19.5218
R_3	29.7277	29.4550	29.1885	28.9299

The value presented in column 1 of Table 2 was taken as the initial state.

1. The final state is column 2. The relative density increment is 2%. Theoretical values: $f_2 = 0.3324$, $f_1 = 0.3154$. Calculated values: $f_2 = 0.3329$, $f_1 = 0.3180$.
2. The final state is column 3. The relative density increment is 4%. Theoretical values: $f_2 = 0.4184$, $f_1 = 0.3860$. Calculated values: $f_2 = 0.42281$, $f_1 = 0.3646$.
3. The final state is column 4. The relative density increment is 6%. Theoretical values: $f_2 = 0.4977$, $f_1 = 0.4455$. Calculated values: $f_2 = 0.5026$, $f_1 = 0.4079$.

These results show that even at a relative density step of 6%, we obtain quite acceptable agreement between theoretical and calculated data.

Nevertheless, the above method of determining the functions is not applicable if plane deformation is realized (see the second calculation example). In this case, as a rule, one of the capsule walls remains nondeformable. That is, it is in a rigid state. And in this case, the equations for determining the functions are already incorrect, because the minimum is reached at the boundary of the area, and Eqs. (4.1) and (4.10) are obtained under the assumption that the entire system is deformed.

CONCLUSIONS

A model of the HIP process for a powder tube has been developed. To describe the mechanical properties of the powder material, Green's model is adopted; for the capsule material, this involves a model of an ideal plastic body with the condition of incompressibility. However, the application of this model requires knowledge of the following mechanical characteristics of materials: yield strength of capsule materials, yield strength of powder material monolith, two experimentally determined functions of relative density $f_1(\rho)$ and $f_2(\rho)$. The model can be used to analyze different possible variants of the process—complete deformation of the system and the variant of plane deformation with one fixed boundary. The application of this model permits the use of a relatively simple mathematical apparatus.

The possibility in principle of using tubular specimens for experimental determination of functions included in Green's yield condition is confirmed.

Authors' contributions

V.A. Goloveshkin, V.N. Samarov, G. Raisson—problem statement, mathematical model creation, model research, analysis of results.

A.A. Nickolaenko, D.M. Fisunova—model research, calculations, analysis of results.

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About the authors

Vasiliy A. Goloveshkin, Dr. Sci. (Eng.), Professor, Higher Mathematics Department, Institute of Cybersecurity and Digital Technologies, MIREA – Russian Technological University (78, Vernadskogo pr., Moscow, 119454 Russia); Leading Researcher, Institute of Applied Mechanics of Russian Academy of Sciences (7-1, Leningradskii pr., Moscow, 125040 Russia). E-mail: vag-1953@yandex.ru. Scopus Author ID 6602872377, https://orcid.org/0000-0002-5413-8625

Artem A. Nickolaenko, Student, MIREA – Russian Technological University (78, Vernadskogo pr., Moscow, 119454 Russia). E-mail: temanickolaenko2004@yandex.ru. https://orcid.org/0009-0003-2483-4392

Victor N. Samarov, Dr. Sci. (Eng.), Technical Director, Laboratory of New Technologies (15, Simferopol'skii bul., Moscow, 117556 Russia). E-mail: Samarov13@Aol.com. Scopus Author ID 6603606878

Gerard Raisson, Retired Consultant, Clermond Ferrand, France, E-mail: gerard.raisson@gmail.com. Scopus Author ID 6603152593

Daria M. Fisunova, Student, MIREA – Russian Technological University (78, Vernadskogo pr., Moscow, 119454 Russia). E-mail: daria.fisunova@gmail.com. https://orcid.org/0009-0008-2857-3499

Об авторах

Головешкин Василий Адамович, д.т.н., профессор, кафедра высшей математики, Институт кибербезопасности и цифровых технологий, ФГБОУ ВО «МИРЭА – Российский технологический университет (119454, Россия, Москва, пр-т Вернадского, д. 78); ведущий научный сотрудник, ФГБУН «Институт прикладной механики Российской академии наук» (125040, Россия, Москва, Ленинградский пр-т, д. 7, стр. 1). E-mail: vag-1953@yandex.ru. Scopus Author ID 6602872377, https://orcid.org/0000-0002-5413-8625

Николаенко Артем Андреевич, студент, ФГБОУ ВО «МИРЭА – Российский технологический университет (119454, Россия, Москва, пр-т Вернадского, д. 78). E-mail: temanickolaenko2004@yandex.ru. https://orcid.org/0009-0003-2483-4392

Самаров Виктор Наумович, д.т.н., технический директор, ТОО «Лаборатория Новых Технологий» (117556, Россия, Москва, Симферопольский б-р, д. 15). E-mail: Samarov13@Aol.com. Scopus Author ID 6603606878

Рейссон Жерар, консультант, Клермон Ферран, Франция. E-mail: gerard.raisson@gmail.com. Scopus Author ID 6603152593

Фисунова Дарья Михайловна, студент, ФГБОУ ВО «МИРЭА – Российский технологический университет (119454, Россия, Москва, пр-т Вернадского, д. 78). E-mail: daria.fisunova@gmail.com. https://orcid.org/0009-0008-2857-3499

Translated from Russian into English by L. Bychkova
Edited for English language and spelling by Thomas A. Beavitt