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RESEARCH ARTICLE

Distribution of temperature field strength on the surface of graphene inclusions in a matrix composite

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Abstract
Objectives. The study sets out to obtain an analytical expression for the distribution of the temperature field strength on the surfaces of anisotropic graphene inclusions taking the form of thin disks in the matrix composite and to use the obtained expressions to predict the strength of the temperature field on the surface of inclusions from the matrix side.
Methods. An inclusion taking the form of a thin circular disk represents a special limit case of an ellipsoidal inclusion. To obtain the corresponding analytical expressions, the authors use their previously derived more general expression for the operator of the concentration of the electric field strength on the surface of ellipsoidal inclusion. The approach is justified by the mathematical equivalence of problems of finding the electrostatic and temperature field in the stationary case. The operator relates the field strength on the inclusion surface from the matrix side to the average field strength in the composite sample; the corresponding expression is obtained in a generalized singular approximation.
Results. Analytical expressions were obtained for the operator of the concentration of the temperature field strength on the surface of the inclusion taking the form of a thin disk of multilayer graphene in a matrix composite. The expressions take into account inclusion anisotropy, the position of the point on the inclusion surface, the volume fraction of inclusions in the material, and the inclusion orientation. Two types of inclusion orientation distributions were considered: equally oriented inclusions and uniform distribution of inclusion orientations. Model calculations of the value for the temperature field strength at the points of the inclusion disk edge as a function of the angle between the radius vector of this point and the direction of the applied field strength were carried out.
Conclusions. In the case of graphene multilayer inclusions, it is shown that the field strength at points on their edges can exceed the applied field strength by several orders of magnitude.

Keywords: composite, matrix, graphene, inclusion, operators of temperature field strength concentration, generalized singular approximation

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НАУЧНАЯ СТАТЬЯ

Распределение напряженности температурного поля на поверхности включений графена в матричном композите

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Резюме
Цели. Цель работы – получить аналитическое выражение для распределения напряженности температурного поля на поверхностях анизотропных включений в форме тонких дисков в матричном композите и применить полученные выражения для прогнозирования величины напряженности температурного поля на поверхности графеновых включений со стороны матрицы.
Методы. Включение в форме тонкого кругового диска является частным предельным случаем эллипсоидального включения. Для получения требуемых аналитических выражений используется ранее полученное авторами более общее выражение для оператора концентрации напряженности электрического поля на поверхности эллипсоидального включения, поскольку задачи нахождения электростатического и температурного поля в стационарном случае математически эквивалентны. Данный оператор связывает напряженность поля на поверхности включения со стороны матрицы со средней напряженностью поля в образце композита, выражение для него получено в обобщенном сингулярном приближении.
Результаты. Получены аналитические выражения для оператора концентрации напряженности температурного поля на поверхности включения в форме тонкого диска из многослойного графена в матричном композите с учетом анизотропии включения в зависимости от положения точки на поверхности включения, от объемной доли включений в материале, от ориентации включения. Рассмотрены два вида распределения ориентаций включений: одинаково ориентированные включения и равномерное распределение ориентаций включений. Проведены модельные расчеты величины напряженности температурного поля в точках ребра включения-диска в зависимости от угла между радиус-вектором данной точки и направлением напряженности приложенного поля.
Выводы. Показано, что в случае графеновых многослойных включений в точках на их ребрах величина напряженности поля может на несколько порядков превышать напряженность приложенного поля.

Ключевые слова: композит, матрица, графен, включение, операторы концентрации напряженности температурного поля, обобщенное сингулярное приближение

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INTRODUCTION

Graphene is a very promising material for various applications due to its exceptional electrical, thermal, and mechanical properties [1–4]. For example, the thermal conductivity coefficient of a single layer of graphene is up to 5000 W/(m·K) [4, 5]. In multilayer graphene, a lower thermal conductivity coefficient value is observed, which can be explained by an increase in phonon scattering due to the interactions between the layers [6]. However, even in multilayer graphene, the thermal conductivity in the plane of the layer remains high enough to be used in the development of composite materials to improve their thermal conductivity properties, which, together with their mechanical properties, are of great importance when undergoing intense external influences of different physical natures. For example, tribocomposite materials undergo uneven heating of the surface and bulk layers during operation, which affects diffusion and segregation processes in the material. As a result, the physical and mechanical properties of tribocomposites can change significantly [7, 8]. The use of materials with enhanced thermal conductivity represents one of the options to reduce the magnitude of the temperature field gradient during operation. Therefore, graphene, due to its very high thermal conductivity along the layers, is considered a very promising material to use as a small additive in composites to increase their thermal conductivity without sacrificing high mechanical and strength properties [9].

In inhomogeneous materials, a significant temperature field gradient value can occur at the micro-level close to the interfaces of homogeneous components that differ significantly in their thermal conductivity properties. This can lead to a change in the properties of the component particles of the inhomogeneous material, a weakening of the bond between inclusions and matrix in the composite, and ultimately a deterioration in the material's performance characteristics. In this regard, the ability to predict the local temperature fields at the interface between inclusions and binder (matrix) in the matrix composite is of great relevance.

There are a number of recent theoretical and experimental studies of the effective thermal conductivity properties of composites [9–12]. Some works also focus on predicting local temperature field distribution in composites, e.g. [13]. On the other hand, there are practically no studies on the distribution of the temperature field at the inclusion-matrix interface.

In [14], fundamental equations are derived for estimating the electric field strength distribution at the inclusion interface in a matrix composite. These results can be used to solve the problem of finding the temperature field strength distribution at the inclusion interface in a matrix composite due to the mathematical equivalence of the problem statements in the stationary case for the distribution of the electrostatic potential and the temperature field [15]. In this paper, a matrix composite with an ED-20 type polymer matrix and graphene multilayer inclusions in the form of thin flakes is considered. The shape of the flakes is approximated by thin circular disks. Analytical expressions are obtained for the concentration operator of the temperature field strength and the vector of the temperature field strength on the surface of the graphene inclusions from the matrix side as a function of the point location on the inclusion surface. Two cases of inclusion orientation distribution in the composite are considered: (1) equally oriented inclusions; (2) uniform spatial distribution of inclusion orientations.

PROBLEM STATEMENT. FIELD STRENGTH CONCENTRATION OPERATOR ON THE INCLUSION SURFACE IN A MATRIX COMPOSITE

We consider a sample of volume V of a statistically homogeneous matrix composite having ellipsoidal inclusions of a similar type. The matrix is isotropic with thermal conductivity k^{m}, while the inclusions (particles) are anisotropic with thermal conductivity tensor $\mathbf{k}^{\mathrm{p}}$ and the volume fraction of inclusions is equal to f. It is assumed that the shape of all inclusions is similar and that the principal axes of the thermal conductivity tensors coincide with the axes of the corresponding ellipsoids. All inclusions are assumed to be randomly distributed throughout the sample volume, while their orientations are distributed according to a probability law. It is further assumed that there are no internal heat sources in the material.

The temperature field in the sample is denoted by $T(\mathbf{r}, t)$, where $\mathbf{r}$ is the radius vector of a point in space and t is time as per classical studies of heat conduction theory, e.g., as presented in [16, 17]. The concept of temperature field strength, which denotes a vector quantity opposite to the temperature field gradient, is neglected in a number of relevant works, i.e., the temperature field gradient is used directly in mathematical formulations [16–18]. However, in many studies dealing with the thermophysical properties of inhomogeneous media, a special notation for the intensity vector of the temperature field is introduced for convenience: $\mathbf{H}(\mathbf{r}, t) = -\nabla T(\mathbf{r}, t)$ (e.g., in [9, 19, 20]).

Let a uniform temperature field $T_0(\mathbf{r})$ with intensity $\mathbf{H}_0$ = const (a uniform temperature field is a field with constant intensity, analogous to a uniform electrostatic field) be applied to the interface S of a given sample. A stationary temperature field $T(\mathbf{r})$ with intensity $\mathbf{H}(\mathbf{r})$ is then established. The task is to find the temperature field distribution at the S_p interface of any matrix-side inclusion in a given composite sample.

In [14], a similar problem of finding the electric field distribution at the inclusion interface in a composite is considered. Using the full mathematical analogy of the problems of electrostatic and temperature field determination in the stationary case, the expression for the temperature field strength at point $\mathbf{r}$ of the surface S_p of an ellipsoidal inclusion on the matrix side can be written as follows:

$$\mathbf{H}^m(\mathbf{r}) = \mathbf{K}^\text{H}(\mathbf{r})\langle\mathbf{H}\rangle, \quad \mathbf{r} \in S_\text{p}, \tag{1}$$

where $\langle\mathbf{H}\rangle$ is the average strength of the temperature field in the sample, which is equal to the applied field strength under the given boundary conditions of the problem [21]: $\langle\mathbf{H}\rangle = \mathbf{H}_0$; $\mathbf{K}^\text{H}(\mathbf{r})$ is the full concentration operator of the temperature field strength on the inclusion surface on the matrix side.

In turn, $\mathbf{K}^\text{H}(\mathbf{r})$ can be expressed in the following form [14]:

$$\mathbf{K}^\text{H}(\mathbf{r}) = \mathbf{K}^\text{sH}(\mathbf{r})\mathbf{K}^\text{vH}, \quad \mathbf{r} \in S_\text{p}, \tag{2}$$

where $\mathbf{K}^\text{sH}(\mathbf{r})$ is the surface field concentration operator relating the field strength at a given point of the inclusion surface on the matrix side to the average field strength in the matrix; $\mathbf{K}^\text{vH}$ is the volume field concentration operator relating the average field strength in the matrix to the average field strength in the sample.

In the generalized Maxwell–Garnett approximation, these operators have the following form [14]:

$$\mathbf{K}^\text{sH}(\mathbf{r}) = \left(\mathbf{I} + \mathbf{A}(\mathbf{r})(\mathbf{k}^\text{p} - k^\text{m}\mathbf{I})\right) \times \times \left[\mathbf{I} - \mathbf{g}(\mathbf{k}^\text{p} - k^\text{m}\mathbf{I})\right]^{-1}, \quad \mathbf{r} \in S_\text{p}, \tag{3}$$

$$\mathbf{K}^\text{vH} == \left[(1-f)\mathbf{I} + f\left\langle(\mathbf{I} - \mathbf{g}(\mathbf{k}^\text{p} - k^\text{m}\mathbf{I}))^{-1}\right\rangle\right]^{-1}, \tag{4}$$

where $\mathbf{I}$ is a unit tensor of rank 2; $\mathbf{A}(\mathbf{r})$ is a rank 2 tensor defined by the expression

$$\mathbf{A}(\mathbf{r}) = \frac{\mathbf{n}(\mathbf{r}) \otimes \mathbf{n}(\mathbf{r})}{\mathbf{n}(\mathbf{r}) \cdot (\mathbf{k}^\text{m}\mathbf{n}(\mathbf{r}))}, \ \mathbf{r} \in S_\text{p},$$

where $\mathbf{n}(\mathbf{r})$ is the external unit normal to the surface S_p at point $\mathbf{r}$; $\mathbf{k}^\text{m}$ is the heat conduction tensor of the matrix.

Since the matrix is isotropic, i.e., $\mathbf{k}^\text{m} = k^\text{m}\mathbf{I}$, the last expression can be rewritten in a simpler form, as follows:

$$\mathbf{A}(\mathbf{r}) = \frac{1}{k^\text{m}}(\mathbf{n}(\mathbf{r}) \otimes \mathbf{n}(\mathbf{r})). \tag{5}$$

The averaging in (4) is carried out over all the inclusions that are immersed in the matrix. The rank 2 tensor $\mathbf{g}$ related to the given inclusion and used in the generalized singular approximation [22] is also used in Eqs. (3) and (4). The components of tensor $\mathbf{g}$ in the coordinate system related to the ellipsoidal inclusion axes are calculated by the following equation [23]:

$$g_{ij} = -\frac{1}{4\pi}\int_0^\pi\int_0^{2\pi} \frac{n_i n_j}{n_\alpha k^\text{m}_{\alpha\beta} n_\beta} \sin\vartheta \, d\vartheta \, d\varphi, \ i, j = 1, 2, 3, \tag{6}$$

where the components of the normal n_i (i = 1, 2, 3) to the inclusion surface are expressed by the spherical angles ϑ, φ; α, β are the component numbers of the vector and tensor quantities.

Since in the case of an isotropic matrix $k^\text{m}_{\alpha\beta} = k^\text{m}\delta_{\alpha\beta}$ ($\delta_{\alpha\beta}$ is the Kronecker symbol), expression (6) can be rewritten as follows:

$$g_{ij} = -\frac{1}{4\pi k^\text{m}}\int_0^\pi\int_0^{2\pi} n_i n_j \sin\vartheta \, d\vartheta \, d\varphi, \ i, j = 1, 2, 3. \tag{7}$$

SPECIAL CASE OF ANISOTROPIC INCLUSIONS TAKING THE FORM OF THIN CIRCULAR DISKS

Let the inclusions in the matrix composite be anisotropic in the form of circular disks of radius a. We consider a particular inclusion occupying the region V_p with surface S_p. Let the plane of this disk form an angle α with the direction of the applied field intensity vector $\mathbf{H}_0$. We introduce the coordinate system $\xi\eta\zeta$ associated with this inclusion as follows. Proceeding from the origin O at the center of the disk, if $\alpha > 0$, we will orient the ξ axis along the projection of vector $\mathbf{H}_0$ on the plane of the disk, the ζ axis along the projection of $\mathbf{H}_0$ on the axis of rotation of the disk, and the η axis perpendicular to the ξ and ζ axes, so that the coordinate system $\xi\eta\zeta$ is right-handed.

If $\alpha = 0$, i.e., vector $\mathbf{H}_0$ lies in the plane of the disk, we will orient the ξ axis along $\mathbf{H}_0$, the η axis perpendicular to the ξ axis in the plane of the disk, and the ζ axis perpendicular to the plane of the disk, so that the system $\xi\eta\zeta$ is right-handed. We will consider two points on the surface S_p of a given disk: a point M on the side surface of the disk and a point Q on the upper bound of the disk. Let the radius vector of point M make an angle θ with the ξ axis, then for the external unit normal to the surface S_p at point M we have: $\mathbf{n}(M) = (\cos\theta \ \sin\theta \ 0)^\text{T}$. For the normal at point Q: $\mathbf{n}(Q) = (0 \ 0 \ 1)^\text{T}$. Then, for the tensor $\mathbf{A}(\mathbf{r})$ at these points in the system $\xi\eta\zeta$, we derive the following by Eq. (5):

$$\mathbf{A}(M) = \frac{1}{k^\text{m}} \begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta & 0 \\ \cos\theta\sin\theta & \sin^2\theta & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (8)$$

$$\mathbf{A}(Q) = \frac{1}{k^\text{m}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (9)$$

For the tensor $\mathbf{g}$ of the disk-shaped inclusion, we derive from Eq. (7) the following:

$$\mathbf{g} = -\frac{1}{k^\text{m}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (10)$$

For the multilayer graphene inclusion, the heat conduction tensor in the $\xi\eta\zeta$ system has the following form:

$$\mathbf{k}^\text{p} = \begin{pmatrix} k_\perp & 0 & 0 \\ 0 & k_\perp & 0 \\ 0 & 0 & k_\| \end{pmatrix}, \quad (11)$$

where $k_\perp$ and $k_\|$ are the main components of the thermal conductivity along and across the graphene layers, respectively.

For convenience, we introduce a rank 2 tensor $\boldsymbol{\lambda}$ related to a particular inclusion, according to the following equation:

$$\boldsymbol{\lambda} = \left[\mathbf{I} - \mathbf{g}(\mathbf{k}^\text{p} - k^\text{m}\mathbf{I})\right]^{-1}. \quad (12)$$

Given (10) and (11), we obtain the following form for the system $\xi\eta\zeta$:

$$\boldsymbol{\lambda}' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & k^\text{m}/k_\| \end{pmatrix}. \quad (13)$$

Taking into account Eqs. (2)–(4) and (12), the expression for the full concentration operator of the temperature field strength on the inclusion surface then takes the following form:

$$\mathbf{K}^\text{H}(\mathbf{r}) = \left(\mathbf{I} + \mathbf{A}(\mathbf{r})(\mathbf{k}^\text{p} - k^\text{m}\mathbf{I})\right)\boldsymbol{\lambda} \times \\ \times \left[(1-f)\mathbf{I} + f\langle\boldsymbol{\lambda}\rangle\right]^{-1}, \quad \mathbf{r} \in S_\text{p}, \quad (14)$$

where the form of the tensor $\mathbf{A}(\mathbf{r})$ depends on the point on the inclusion surface; for the point M on the edge of the disk, it has the form (8), while for the point Q on the upper bound of the disk, it has the form (9).

The averaging in (14) is performed over all inclusions in the matrix. Since all inclusions are assumed to be identical, this averaging is performed over all inclusion orientations in the xyz coordinate system related to the texture of the composite sample.

For the distribution of inclusion orientations in a composite, we consider two cases: 1) inclusions with equal orientation; 2) uniform distribution of inclusion orientations. In the first case, the composite obtained is anisotropic, the orientations of all the systems $\xi\eta\zeta$ related to the inclusions are identical. Therefore, it is convenient to take a system $\xi\eta\zeta$ as the xyz coordinate system. Then $\langle\boldsymbol{\lambda}\rangle = \boldsymbol{\lambda}'$, and we obtain the following for $\mathbf{K}^\text{H}(\mathbf{r})$:

$$\mathbf{K}^\text{H}(\mathbf{r}) = \left(\mathbf{I} + \mathbf{A}(\mathbf{r})(\mathbf{k}^\text{p} - k^\text{m}\mathbf{I})\right)\boldsymbol{\lambda}' \times \\ \times \left[(1-f)\mathbf{I} + f\boldsymbol{\lambda}'\right]^{-1}, \quad \mathbf{r} \in S_\text{p}.$$

Given the form $\boldsymbol{\lambda}'$ (13), we find:

$$\mathbf{K}^\text{H}(\mathbf{r}) = \left(\mathbf{I} + \mathbf{A}(\mathbf{r})(\mathbf{k}^\text{p} - k^\text{m}\mathbf{I})\right) \times \\ \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \dfrac{k^\text{m}}{(1-f)k_\| + fk^\text{m}} \end{pmatrix}. \quad (15)$$

In the case of a uniform distribution of inclusion orientations [24], we have:

$$\langle\lambda\rangle = \frac{1}{3}(\lambda'_{11} + \lambda'_{22} + \lambda'_{33})\mathbf{I},$$

where $\lambda'_{11}, \lambda'_{22}, \lambda'_{33}$ are the main components of the tensor $\boldsymbol{\lambda}$, i.e., in this case, taking into account (13), we obtain:

$$\langle\lambda\rangle = \frac{1}{3}\left(2 + \frac{k^\text{m}}{k_\|}\right)\mathbf{I},$$

$$\left[(1-f)\mathbf{I}+f\langle\boldsymbol{\lambda}\rangle\right]^{-1}=\frac{3k_{\parallel}}{3k_{\parallel}-f(k_{\parallel}-k^{\mathrm{m}})}\mathbf{I},$$

while for the operator $\mathbf{K}^{\mathrm{H}}(\mathbf{r})$, we get:

$$\mathbf{K}^{\mathrm{H}}(\mathbf{r})=\left(\mathbf{I}+\mathbf{A}(\mathbf{r})(\mathbf{k}^{\mathrm{p}}-k^{\mathrm{m}}\mathbf{I})\right)\boldsymbol{\lambda}'\times \times\frac{3k_{\parallel}}{3k_{\parallel}-f(k_{\parallel}-k^{\mathrm{m}})},\quad \mathbf{r}\in S_{\mathrm{p}}. \tag{16}$$

In both cases of the distribution of the inclusion orientations, $\mathbf{K}^{\mathrm{H}}(\mathbf{r})$ is diagonal.

We consider the special case when the point M on an edge of the disk lies on the ξ axis, i.e., when $\theta = 0$. In this case we have:

$$\mathbf{A}(M)=\frac{1}{k^{\mathrm{m}}}\begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix}. \tag{17}$$

Then for the diagonal components of the operator $\mathbf{K}^{\mathrm{H}}(\mathbf{r})$ in the case of the same orientation of the inclusions from (15), taking into account (11), we have:

$$K_{11}^{\mathrm{H}}(M(0))=\frac{k_{\perp}}{k^{\mathrm{m}}},\quad K_{22}^{\mathrm{H}}(M(0))=1,$$
$$K_{33}^{\mathrm{H}}(M(0))=\frac{k^{\mathrm{m}}}{(1-f)k_{\parallel}+fk^{\mathrm{m}}}. \tag{18}$$

In the case of uniformly distributed inclusion orientations, from (16), (17), and (11) we obtain:

$$K_{11}^{\mathrm{H}}(M(0))=\frac{3k_{\perp}k_{\parallel}}{k^{\mathrm{m}}(3k_{\parallel}-f(k_{\parallel}-k^{\mathrm{m}}))},$$
$$K_{22}^{\mathrm{H}}(M(0))=\frac{3k_{\parallel}}{3k_{\parallel}-f(k_{\parallel}-k^{\mathrm{m}})}, \tag{19}$$
$$K_{33}^{\mathrm{H}}(M(0))=\frac{3k^{\mathrm{m}}}{3k_{\parallel}-f(k_{\parallel}-k^{\mathrm{m}})}.$$

If the thermal conductivity of graphene multilayer inclusions is considered approximately equal to that of high quality graphite, in this case we have the following values of thermal conductivity component (W/(m·K)): $k_{\perp}=2000$, $k_{\parallel}=5.7$ [25], for an ED-20 type epoxy matrix k^{m} = 0.2 [26]. Then formula (18) gives $K_{11}^{\mathrm{H}}(M(0))=10^4$ for equally oriented inclusions, i.e., the temperature field strength component H_1 at the point M of the matrix-side inclusion interface is 10^4 times higher than the corresponding component of the applied field.

We now obtain the expressions for $\mathbf{K}^{\mathrm{H}}(\mathbf{r})$ at the point Q on the inclusion edge. Substituting (9) into (15), we obtain the following for the diagonal components of the operator $\mathbf{K}^{\mathrm{H}}(Q)$ for equally oriented inclusions:

$$K_{11}^{\mathrm{H}}(Q)=1,\quad K_{22}^{\mathrm{H}}(Q)=1,$$
$$K_{33}^{\mathrm{H}}(Q)=\frac{k_{\parallel}}{(1-f)k_{\parallel}+fk^{\mathrm{m}}}. \tag{20}$$

For uniformly oriented inclusions, we have:

$$K_{11}^{\mathrm{H}}(Q)=K_{22}^{\mathrm{H}}(Q)=K_{33}^{\mathrm{H}}(Q)=\frac{3k_{\parallel}}{3k_{\parallel}-f(k_{\parallel}-k^{\mathrm{m}})}. \tag{21}$$

It can be seen from Eq. (20) and (21) that the field strength at point Q on the inclusion upper bound is of the same order of magnitude as the applied field strength.

NUMERICAL MODELING RESULTS AND DISCUSSION

Based on the derived expressions for the full concentration operator of the temperature field strength, model calculations are carried out for a composite with an ED-20 type matrix and multilayer graphene inclusions taking the form of circular disks. The ratios of the components and the modulus of the temperature field strength at the point M on the edge of the inclusion disk to the modulus of the applied field strength are calculated as a function of the angle θ between the radius vector of this point and the ξ axis for different inclusion volume fractions, for different values of the angle α between the applied field strength and the inclusion plane. Some results are shown in Figs. 1–3. In all cases, the distribution of inclusion orientations is assumed to be uniform.

The dependencies of the H_i/H_0 ratio of the temperature field strength components at points M on the edge of the graphene inclusion to the applied field strength on the angle between the radius vector to point M and the applied field strength $\mathbf{H}_0$ for the case when $\mathbf{H}_0$ lies in the plane of the inclusion are shown in Fig. 1. An analysis of these dependencies shows that, for a fixed value of the applied field strength, the values of the components H_1 and H_2 at points on the edge of disks on the matrix side depend significantly on the angle θ between the radius vector of this point and the vector of the applied field strength. However, in the vast majority of such points the value of the corresponding strength component is rather high compared to the applied field. At the same time, the H_3 component has a negligible value close to zero.

Similar dependencies of the ratio $H(M)/H_0$ of the absolute magnitude of the temperature field strength to the applied field strength are shown in Fig. 2. It can be seen that the modulus of the field strength at the points on the disk edge in the ranges $\theta \in [0°; 84°]$ and $\theta \in [96°; 180°]$ is more than 10^3 times higher than the applied field strength. As the volume fraction of inclusions in the composite increases with a uniform distribution of their orientations, the absolute values of the components and the modulus of the field strength at the points on the disk edges also increase slightly. In the case of the same inclusion orientations in the composite, the change in the inclusion volume fraction has no effect on the values of the components H_1 and H_2. This follows directly from Eq. (18).

In the general case, with respect to the direction of the vector $\mathbf{H}_0$ of the applied field strength, the inclusion disk planes are oriented differently. The dependencies of the ratio $H(M)/H_0$ on the angle θ between the radius vector of the point M and the projection of the vector $\mathbf{H}_0$ onto the disk plane for different values of the angle α between the vector $\mathbf{H}_0$ and the inclusion plane are shown in Fig. 3. These dependencies show that increasing the angle between the disk plane and $\mathbf{H}_0$ leads to a decrease in the value of the field strength at points at the disk edge. At the same time, this value is still much higher than H_0. For example, for the angle $\alpha = 75°$, the ratio of the surface field strength to the applied field exceeds 10^3 for points in the ranges $\theta \in [0°; 66°]$ and $\theta \in [114°; 180°]$.

From the results, it can be concluded that the physical properties of the binder can be significantly modified by intensifying the diffusion and segregation processes taking place in these regions. This is due to the significant values of the temperature field strength in the regions near the edges of the graphene disks. For small volume fractions of graphene inclusions, these changes have no significant effect on the macroscopic properties of the composite. However, as the inclusion volume fraction increases, the proportion of the binder material regions in which these changes occur also increases. This can lead to a significant degradation of the performance characteristics of the material, which is consistent with the results obtained in [27].

CONCLUSIONS

The main result of the paper is Eqs. (15) and (16) for the concentration operators of the temperature field strength on the surface of anisotropic disk-shaped inclusions in a matrix composite. These expressions allow the prediction of these values at any point on the surface of the inclusions as a function of the external applied field, the volume fractions and

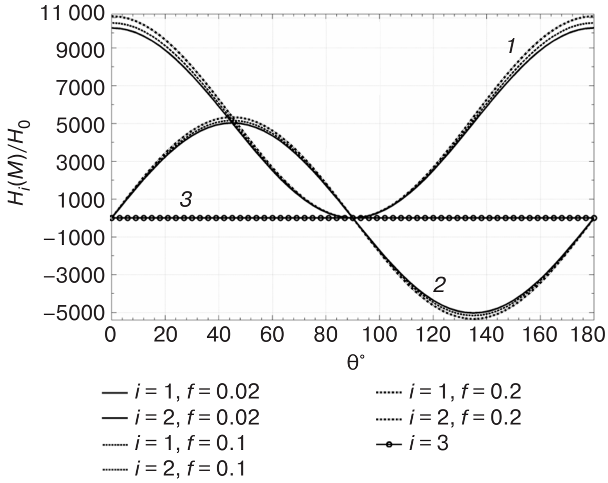

Fig. 1. Dependencies of the H_i/H_0 ratio on the angle between the radius vector to the point M and the applied field strength $\mathbf{H}_0$ for different inclusion volume fractions. The component numbers are given near the corresponding curves

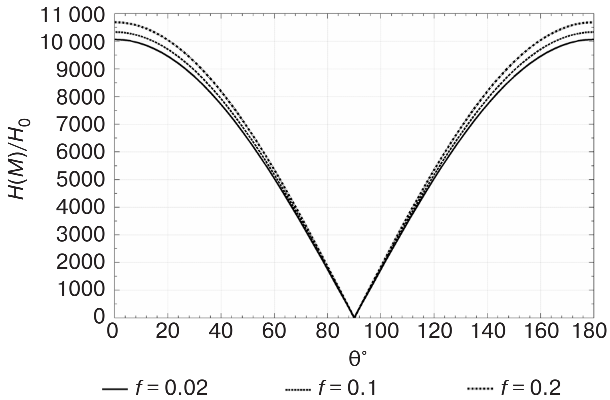

Fig. 2. Dependencies of the $H(M)/H_0$ ratio on the angle between the radius vector to the point M and the vector of the applied field strength $\mathbf{H}_0$ at different inclusion volume fractions

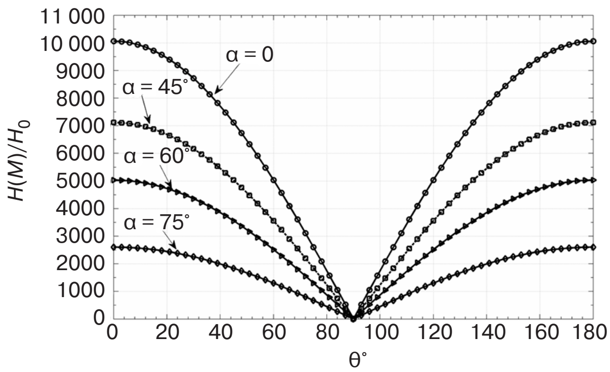

Fig. 3. Dependencies of the $H(M)/H_0$ ratio on the angle between the radius vector to the point M and the projection of the vector $\mathbf{H}_0$ onto the disk plane for different values of the angle α between the vector $\mathbf{H}_0$ and the inclusion plane. Inclusion volume fraction $f = 0.02$

material properties of the composite components, and the orientation of the inclusion with respect to the direction of the applied field strength. Modeling calculations have been carried out for the inclusions of graphene multilayers. It is shown that the temperature field strength can exceed the applied field strength by several orders of magnitude at the surface points on the edges of graphene inclusions.

Authors' contributions

I.V. Lavrov—literature review, deducing the calculation formulas, writing the computer programs, model calculations, plotting, discussion of the results.

V.V. Bardushkin—checking the mathematical correctness of deducing the calculation formulas, proofreading the text of the article, discussion of the results.

V.B. Yakovlev—problem statement, the idea of deducing the calculation formulas, discussion of the results.

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