Mathematical modeling

Математическое моделирование

*UDC 536.2*
*https://doi.org/10.32362/2500-316X-2025-13-1-136-143*
*EDN XIFHYB*



RESEARCH ARTICLE

# A smoothed particle hydrodynamics approach for numerical simulation of tube heat exchangers

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**Abstract**

**Objectives.** In the confined space of heat exchangers, heat transfer rate plays a key role. The cross-sectional shape of the tubes can affect the heat transfer characteristics. Although circular tubes are easier and less expensive to manufacture, heat transfer in heat exchangers with tubes of other cross-sections can take place at higher rates, thus providing economic advantages. This makes the mathematical modeling of hydrodynamics and heat exchange in a tube apparatus relevant and interesting both from the theoretical and applied point of view. The aim of this study is to determine the influence of the shape of the tube cross-section on the heat transfer intensity.

**Methods.** Numerical investigations were carried out using smoothed particle hydrodynamics. The possibilities of the smoothed particle method for resolving industrial heat transfer problems were demonstrated.

**Results.** Heat transfer intensity was analyzed for tubes of circular and rectangular cross-sections. In cases where the cross sections of tubes in the heat exchanger are elongated in a given direction, the influence of the tube position in relation to the oncoming flow was studied. This was performed either with the long side along the flow or across it. The influence of tube surface protrusions on heat exchange was investigated. The flow around tubes with different cross-sectional shapes was also analyzed. The features of the flow around the tubes were established, and the velocity and temperature fields in the heat exchanger volume were defined. The values of the dimensionless heat flux (Nusselt number) for each case were also found.

**Conclusions.** The influence of finned tubes in the laminar flow regime of heated fluid through the bundle of heat transfer tubes is insignificant. The highest value of the heat flux was observed for tubes of rectangular cross section with the long side transverse to the flow, and the difference with the data obtained for standard round tubes was found to be more than 15%.

**Keywords:** heat transfer, heat exchanger, numerical modeling, smoothed particle hydrodynamics, incompressible fluid, periodic boundary conditions

• **Submitted:** 06.03.2024 • **Revised:** 02.09.2024 • **Accepted:** 19.11.2024

**For citation:** Korenchenko A.E., Sukhov A.V. A smoothed particle hydrodynamics approach for numerical simulation of tube heat exchangers. *Russian Technological Journal.* 2025;13(1):136–143. https://doi.org/10.32362/2500-316X-2025-13-1-136-143, https://elibrary.ru/XIFHYB

**Financial disclosure:** The authors have no financial or proprietary interest in any material or method mentioned.

The authors declare no conflicts of interest.

НАУЧНАЯ СТАТЬЯ

# Моделирование работы трубчатых теплообменников методом сглаженных частиц

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**Резюме**

**Цели.** В работе теплообменных аппаратов ключевую роль играет скорость теплопередачи в условиях ограниченного пространства. Форма сечения труб может повлиять на характеристики теплообмена. Хотя производство труб кругового сечения проще и обходится дешевле, теплообмен в аппаратах с трубами других поперечных сечений может происходить с большей скоростью, так, чтобы это давало экономические преимущества. Поэтому проведение математического моделирования гидродинамики и теплообмена в трубчатом теплообменном аппарате актуально и интересно как теоретически, так и с прикладной точки зрения. Цель исследования – определение влияния формы сечения труб на интенсивность теплопередачи.

**Методы.** Численные исследования выполнены методом гидродинамики сглаженных частиц. Продемонстрированы возможности метода сглаженных частиц для решения задач промышленного теплообмена.

**Результаты.** Анализ интенсивности теплопередачи проведен для труб круглых и прямоугольных сечений. В случаях, когда поперечные сечения труб в теплообменнике являются вытянутыми вдоль некоторого направления, исследовано влияние расположения труб по отношению к набегающему потоку: длинной стороной вдоль потока или поперек его. Исследовано влияние на теплообмен выступов на поверхности труб. Проведен анализ обтекания труб с различными формами поперечных сечений. Выявлены особенности обтекания, найдены поля скоростей и температуры в объеме теплообменника. Найдены значения безразмерного теплового потока (числа Нуссельта) для каждого случая.

**Выводы.** Сделан вывод о малом влиянии оребрения труб при ламинарном режиме протекания нагреваемой жидкости через пучок труб-теплоносителей. Наибольшее значение теплового потока наблюдалось для труб прямоугольного сечения, расположенных длинной стороной поперек потока, причем различие с данными, полученными для стандартных круглых труб, составило более 15%.

**Ключевые слова:** теплопередача, теплообменные аппараты, численное моделирование, гидродинамика сглаженных частиц, несжимаемая жидкость, периодические граничные условия

• **Поступила:** 06.03.2024 • **Доработана:** 02.09.2024 • **Принята к опубликованию:** 19.11.2024

**Для цитирования:** Коренченко А.Е., Сухов А.В. Моделирование работы трубчатых теплообменников методом сглаженных частиц. *Russian Technological Journal.* 2025;13(1):136–143. https://doi.org/10.32362/2500-316X-2025-13-1-136-143, https://elibrary.ru/XIFHYB

**Прозрачность финансовой деятельности:** Авторы не имеют финансовой заинтересованности в представленных материалах или методах.

Авторы заявляют об отсутствии конфликта интересов.

## INTRODUCTION

In the modern technological society, there is an extremely great need for heat exchangers. Heat exchangers are used at enterprises in the petrochemical, metallurgical, and food industries, as well as in shipbuilding and in housing and communal services. In particular, communal heating, hot water supply, and air conditioning systems are built on the basis of heat exchangers. The majority of such heat-exchange equipment involves water-water and steam-water tube heat exchangers. The heat exchanger consists of a block of tubes immersed in the fluid flow. A fluid or gas is passed through the tubes, wherein the fluid media in the tubes and flow possesses differing initial temperatures. The heat exchanger characteristics are of considerable practical interest and have therefore been the subject of numerous experimental and theoretical studies [1–5]. Methods have been developed for heat transfer intensification involving complicating the shape of heat exchanger tubes by finning the surface [6, 7], installing turbulators [8], and rotating the heat exchanger tubes [8]. In [9], the melting characteristics of gallium in a shell-and-tube type heat exchanger with tubes of circular, rectangular, or elliptical cross-section are numerically considered. Here, the solid-liquid transition rate of gallium depends on the heat transfer intensity from tubes. The findings show that the shortest melting time was achieved when using a heat exchanger with tubes of rectangular cross-section, while using tubes of circular cross-section yielded the lowest heat transfer intensity. In [10], the energy feasibility of using elliptical cross-section tubes in thermal energy storage systems is shown.

However, the literature review shows that the possibility of increasing heat transfer by changing the cross-section shape of tubes with heat-transfer medium has not been considered sufficiently and should be investigated additionally.

Smoothed particles hydrodynamics (SPH) is a meshless Lagrangian method for resolving hydrodynamic and aerodynamic problems. The method consists of approximating the fields of physical quantities by a discrete system of particles [11–13]. The method properly describes heat transfer processes in liquid and gas media. It shows good performance and can be used for resolving heat transfer problems in industrial production.

The paper aims to model heat transfer in a tubular heat exchanger and to analyze the heat transfer intensity depending on the cross-section shape of tubes with heat-transfer medium.

## 1. MATHEMATICAL MODEL

This paper models a scheme (Fig. 1) in which a flow of cold water ($T_0$ = 283 K), bounded at the top and bottom by flat surfaces, runs into a block of heated parallel tubes the temperature of which is kept equal ($T_H$ = 363 K). The direction of velocity in the flow is perpendicular to the tubes. The fluid in the flow is assumed to be incompressible and Newtonian. The gravity effect is neglected. We consider the problem in a two-dimensional formulation acceptable under the condition that the tube length is much greater than the distance between the bounding planes.

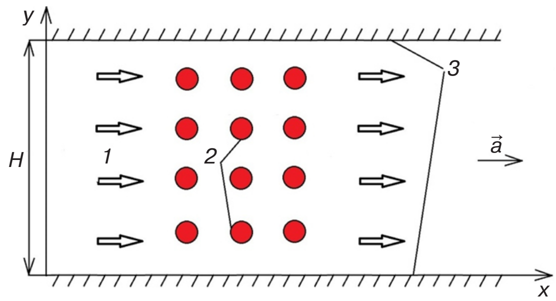



**Fig. 1.** Schematic diagram of the experiment. *H* is the heat exchanger gap; *1* is the fluid flow; *2* is the tubes; *3* is the bounding planes

Conservation equations for the fluid in the flow are written as follows:

$$\frac{\partial \vec{V}}{\partial t} + \left(\vec{V}\vec{\nabla}\right)\vec{V} = -\frac{1}{\rho}\vec{\nabla}P + \nu\vec{\nabla}^2\vec{V} + \vec{a}, \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\left(\rho\vec{V}\right) = 0, \tag{2}$$

$$\frac{\partial T}{\partial t} + \vec{\nabla}\left(T\vec{V}\right) = \frac{\kappa}{\rho c}\vec{\nabla}^2 T. \tag{3}$$

Wherein, (1) is the momentum conservation law, (2) is the continuity equation and (3) is the heat balance equation in neglecting viscous dissipation, where $P$ is pressure in the fluid; $\vec{V} = \{V_x, V_y\}$ and $T$ are velocity and temperature, respectively; $\vec{a}$ is acceleration due to an external force. The thermophysical characteristics of the fluid (water) are denoted as follows: ρ is density, $v$ is kinematic viscosity coefficient, κ is heat conduction coefficient, and $c$ is specific heat capacity. Their values are given in Table 1. The following boundary conditions were chosen: the flat surfaces and tubes are isothermal; while the conditions of non-slip and impermeability for flow particles are fulfilled on solid walls.

**Table 1.** Physical and chemical properties of water

| | |
|---|---|
| ρ | 1000 kg/m$^3$ |
| κ | 0.55 W/(m · K) |
| $v$ | $10^{-6}$ m$^2$/s |
| $c$ | 4200 J/(kg · K) |

## 2. SMOOTHED PARTICLE HYDRODYNAMICS

To calculate system (1)–(3) the smoothed particle hydrodynamics (SPH) is used [11–13]. This method replaces the value for physical variable $f(r)$ at a point in space by the sum of weighted values of this variable for particles located in the neighborhood. The weight is determined by the kernel function, as follows

$$f(\vec{r}) \approx \sum_{j=1}^{N} \frac{m_j}{\rho_j} f_j W(\vec{r}-\vec{r}_j, h). \tag{4}$$

Wherein, $m_j$, $\rho_j$ are mass and density of the $j$th particle, $W(\vec{r}-\vec{r}_j, h)$ is kernel function, and $h$ is smoothing radius. The summation is performed on the particles trapped inside the sphere of radius $h$. The approximations for the gradient, divergence, and Laplace operator are defined as [11, 12]:

$$\begin{gathered}
\vec{\nabla} f(\vec{r}) \approx \sum_{j=1}^{N} \frac{m_j}{\rho_j} f_j \vec{\nabla} W(\vec{r}-\vec{r}_j, h), \\
\vec{\nabla} \vec{F}(\vec{r}) \approx \sum_{j=1}^{N} \frac{m_j}{\rho_j} \vec{F}_j \vec{\nabla} W(\vec{r}-\vec{r}_j, h), \\
\Delta f(\vec{r}) \approx \sum_{j=1}^{N} \frac{m_j}{\rho_j} f_j \Delta W(\vec{r}-\vec{r}_j, h).
\end{gathered} \tag{5}$$

Equations (1–3) written for the $i$th Lagrangian particle have the following form:

$$\begin{cases}
\dfrac{d\vec{V}_i}{dt} = -\dfrac{1}{\rho_i} \displaystyle\sum_{j=1}^{N} \frac{m_j}{\rho_j} P_j \vec{\nabla} W(\vec{r}-\vec{r}_j, h)\Big|_{\vec{r}=\vec{r}_i} + \\
\qquad + \nu \displaystyle\sum_{j=1}^{N} \frac{m_j}{\rho_j} \vec{V}_j \Delta W(\vec{r}-\vec{r}_j, h)\Big|_{\vec{r}=\vec{r}_i} + \vec{a}, \\
\dfrac{d\rho_i}{dt} = -\displaystyle\sum_{j=1}^{N} m_j \vec{V}_j \vec{\nabla} W(\vec{r}-\vec{r}_j, h)\Big|_{\vec{r}=\vec{r}_i}, \\
\dfrac{dT_i}{dt} = \dfrac{\kappa}{\rho_i c} \displaystyle\sum_{j=1}^{N} \frac{m_j}{\rho_j} T_j \Delta W(\vec{r}-\vec{r}_j, h)\Big|_{\vec{r}=\vec{r}_i}.
\end{cases} \tag{6}$$

The kernel function $W(\vec{r}-\vec{r}_j, h)$ is chosen in the following form [14]:

$$W(\vec{r}-\vec{r}_j, h) = \frac{7}{4\pi h^2}\left(1-\frac{q}{2}\right)^4 (2q+1),$$
$$q = \left|\vec{r}-\vec{r}_j\right|, \quad 0 \le q \le 2.$$

System (6) is supplemented by the equation of state of water [11], as follows:

$$P = \frac{\rho_0 c_{\text{snd}}^2}{7}\left(\left(\frac{\rho}{\rho_0}\right)^7 - 1\right), \tag{7}$$

wherein $c_{\text{snd}}$ = 1500 m/s is the speed of sound propagation in the fluid, and $\rho_0$ is the density of the undisturbed medium. It is stated in [11] that equation (7) provides a compressibility value not exceeding real water compressibility of about 0.1%.

The mirror particle method proposed in [13] was used to provide slip and impermeability conditions at solid boundaries. The system of ordinary differential equations (6) was resolved using the Runge–Kutta method for the 3rd order of accuracy.

## 3. RESULTS AND DISCUSSION

### 3.1. Poiseuille flow between two parallel planes

In order to verify calculation accuracy, a test problem of viscous fluid flow through a gap between two solid planar surfaces was conducted. Figure 1 shows the schematic diagram of the experiment with correction for the absence of tubes in the computational domain.

At the initial moment of time, the fluid is at rest and fills the gap. The force field with a strength $\vec{a}$ starts affecting it at moment $t = 0$. The fluid and the bounding planes have the same temperature and are assumed to be isothermal. In order to model the flow, a system of equations (6) needs to be resolved, with the exclusion of the heat balance equation. In the flow direction, periodic boundary conditions are set [14, 15]. In two-dimensional formulation and in the absence of gravity, the problem has an analytical solution. Based on this solution, the dependence of fluid velocity components on distance $y$ from the bottom plane is expressed by the following formula:

$$V_x(y) = \frac{a}{2\nu}\left(Hy - y^2\right), \quad V_y = 0. \tag{8}$$

Thus, the velocity of motion in the flow is parallel to bounding planes and is described by the parabolic dependence on transverse coordinate $y$, with the highest value reached at $y = H/2$ and expressed by the following formula:

$$V_{x\max} = aH^2/(8\nu). \tag{9}$$

The solution is performed for $a = 0.1$ m/s$^2$, $H = 2.5$ mm. The Reynolds number $Re = V_{x\max}H/\nu$ in this case does not exceed 1500 corresponding to the laminar flow regime. The calculation results are shown in Figs. 2 and 3. The velocity profiles in the gap at different moments of time are shown in Fig. 2. During the transient time interval of ~10 s, the velocity distribution is established in the gap which differs from the analytical solution (8) by less than 0.1%. The dependence of the highest velocity in the flow on the gap width is shown in Fig. 3. This figure shows that the numerical and analytical results are close to each other, thus demonstrating SPH capabilities for solving hydrodynamics problems.

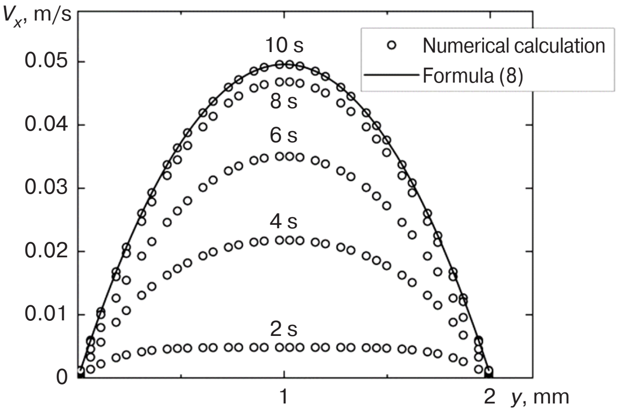



**Fig. 2.** Calculation results for the velocity in the exit section of the gap at different moments of time and analytical calculation of the steady-state velocity profile: $H = 2$ mm, $a = 0.1$ m/s$^2$

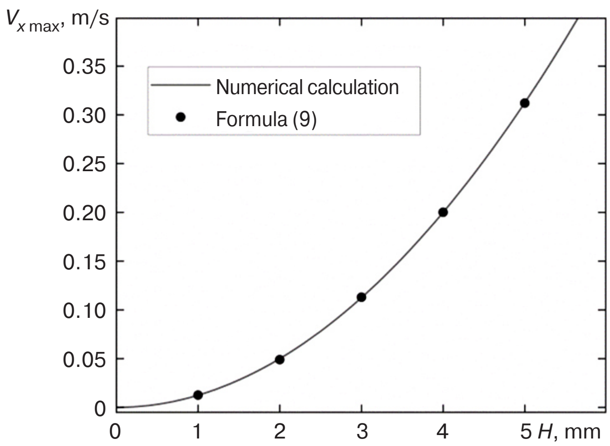
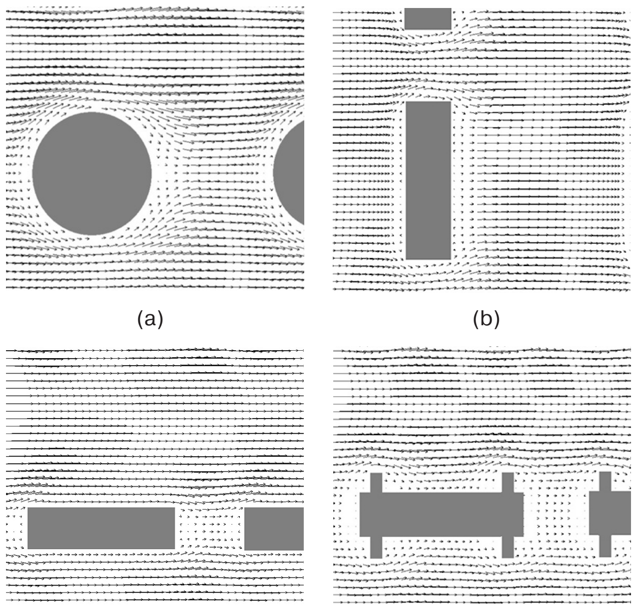



**Fig. 3.** Numerical and analytical dependencies of the highest velocity in the gap on the gap width

### 3.2. Heating water in a tube heat exchanger

Figure 1 shows a schematic representation of a cross-flow heat exchanger. The fluid flows into the heat exchanger from left to right under the influence of a force field and meets a bundle of tubes arranged at right angles to the flow (cross sections are shown in the figure). The distance between centers of the tubes is 4 cm in both vertical and horizontal rows. Numerical calculations were performed for $H = 0.2$ m, and the heat exchanger length is $L_H = 1$ m. The force field strength is chosen to be equal This gives the maximum velocity value estimated "from above" by formula (9), $V_{x\,\max} \approx 2$ m/s and allows it to be stated that the flow will occur in the laminar regime ($Re < 2000$). Tubes with the following different cross sections were considered (Fig. 4): a) circular; b) rectangular with vertical location of the long side (across the flow); c) rectangular with location of the long side along the flow; and d) finned surface model, where rectangular protrusions of 1 mm × 2 mm are located along the pipe surface. At the beginning of calculation, the fluid was filled in the heat exchanger and was at rest at temperature $T_0 = 283$ K. At moment $t = 0$, the force field is switched on with an intensity level $\vec{a}$. The tube temperature is $T_H = 363$ K and is not changed during heat exchange. The problem is resolved in a two-dimensional formulation, and the perimeter of the tube cross-section serves as the measure of the heat source area $S_H$ in this case.



(a) (b)
(c) (d)

**Fig. 4.** Velocity distribution in the flow when flowing around tubes with different cross sections

Figure 4 shows the fields of velocity distributions during flow around tubes. The length of the velocity vector is proportional to its magnitude, and all figures are drawn to the same scale. The cross-section perimeters for tubes in Figs. 4 (a)–(c) are 6.28 cm. The flow around occurs in the laminar regime, with no vortices or breakaway currents formed. Thus, the flow regime, once established, is no longer disturbed. The figures show that in cases (c) and (d), there is practically no fluid motion in gaps between tubes, and velocities in the volume are strictly horizontal, i.e., there is no convective heat propagation in the transverse flow direction.

Figure 5 shows the dependencies of the dimensionless heat flux that is Nusselt number $Nu = \int_{S_H} \frac{\partial T}{\partial \vec{n}} d\sigma \cdot \frac{L_H}{S_H (T_1 - T_0)}$ on the cross-section perimeter for different tube shapes (Fig. 4). Changes in the perimeter of tubes of circular cross-section result from changes in the radius. For tubes of rectangular cross-section, the length of the short side was fixed at 0.5 cm. The perimeter was changed by modifying the length of the other side or by finning the tube.

The graphs in Fig. 5 allow the following patterns to be established.

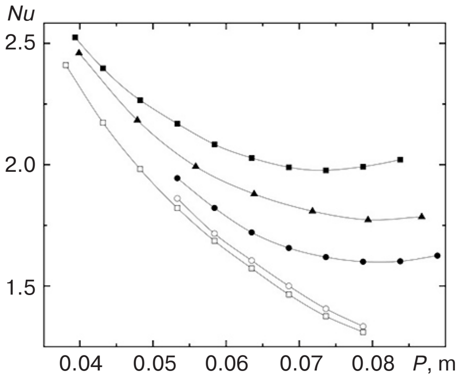



**Fig. 5.** Dependence of dimensionless heat flux on the cross-section perimeter for tubes with different cross-sectional shapes: ▲ is circular cross-section (a); ■ is vertical rectangle without protrusions (b); ● is vertical rectangle with 4 protrusions of 1 mm × 2 mm; □ is horizontal rectangle without protrusions (c); ○ is horizontal rectangle with 4 protrusions of 1 mm × 2 mm (d)

As the tube surface area increases, the Nusselt number decreases. The exception is the flux calculated for round tubes (a) and tubes with a cross-section in the form of a vertical rectangle (b). The figure shows that in these cases, the flow begins to increase for some perimeter value as the perimeter increases. This is apparently due to an increase in the size of tubes in the direction perpendicular to the flow resulting in overlapping of gaps between them, so that the entire flow is directed along the walls of the apparatus, thus intensifying heat transfer insignificantly.

The dependencies of heat flux on tube perimeter converge with decreasing perimeter. This is due to the fact that when reducing the tube size, the cross-section shape becomes an insignificant factor.

Under the considered conditions, finning tubes cannot intensify heat exchange. Protrusions on tubes of horizontally elongated rectangular cross-section (d) result in an insignificant increase of heat flow (~2%) compared to tubes without protrusions. However, for cross-section (b), protrusions result in a noticeable heat flux decrease (~25%), which is apparently due to fluid inhibition and stagnation. In the turbulent regime of flows, the protrusions cause the appearance of breakaway vortex currents, which result in an increase in the heat transfer intensity [16]. However, calculations show that in the laminar regime, this can reduce the heat flux from the heat transfer medium.

The graphs of *Nu* dependence on cross-sectional shape and heat transfer medium size show that in case (c), the molecular thermal conductivity mechanism prevails, while the Nusselt number has the lowest value. The highest Nusselt number value can be observed for tubes with a vertical rectangular cross-section, with the difference between the heat flux and values obtained for tubes with a circular cross-section is ~15%.

## CONCLUSIONS

In this paper, mathematical modeling of heat transfer in a tubular heat exchanger under cross-sectional flow around tubes is performed. Numerical studies were carried out for laminar flow regime and for tubes with different shapes and cross-section perimeters, which allow the following conclusions to be drawn.

1. Finning the outer surface of tubes would not cause a significant increase in the heat flux. In turbulent flow regime, the protrusions generate vortices and breakaway flows resulting in heat transfer intensification. However, this can reduce the heat flux in the laminar regime due to the fluid braking near the pipe when flowing around protrusions.
2. Given equal perimeters, the highest heating intensity occurs in tubes with cross-section stretched across the flow, with an increase in heat flux of ~15% compared to the values obtained for standard-type tubes.
3. Phenomena leading to fluid braking or stagnation cause a decrease in heat transfer intensity.

**Authors' contribution**

All the authors have equally contributed to the scientific work.

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*Translated from Russian into English by K. Nazarov*
*Edited for English language and spelling by Dr. David Mossop*