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RESEARCH ARTICLE

Method for designing DC/DC converters based on Zeta topology

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Abstract

Objectives. The work set out to develop a new design method for DC/DC converters based on the Zeta topology to calculate the ratings of inductors and capacitors of the Zeta converter with magnetically coupled inductors and verify the accuracy of the ultimate continuous mathematical model and design method based on it using SPICE simulation in the *Multisim* computer-aided design (CAD) system.

Methods. The proposed method analyzes an ultimate continuous mathematical model to calculate the ratings of coupled inductors and capacitors of the converter.

Results. The simulation of the Zeta converter with coupled inductors was carried out using the *Multisim* CAD system, during which the load and transfer characteristics of the converter were obtained. These characteristics show the dependencies of the currents flowing through the coupled inductors and voltages across the capacitors on the input voltage, as well as the dependence of the output voltage on the load current. The presented design method is shown to be accurate and in full agreement with the simulation results. A correlation between the transfer and load characteristics of currents and voltages obtained by simulation and calculation is established. The differences between the values calculated using the ultimate continuous mathematical model and the results of simulation in the *Multisim* CAD system are comparable to measurement errors.

Conclusions. The proposed design method is used calculate element ratings for the Zeta topology both with and without taking the inductive coupling into account. The method can also be used to calculate the steady-state values and ripple currents of the inductors and voltages across the capacitors. The design method for DC/DC converters presented in the paper can be used for both preliminary evaluation calculations and more detailed calculations, including analysis of the device operation under various input voltages and load resistances.

Keywords: DC/DC converter, buck-boost converter, Zeta topology, ultimate continuous mathematical model, design method, simulation

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НАУЧНАЯ СТАТЬЯ

Метод проектирования DC/DC-преобразователей, построенных по Zeta-топологии

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Резюме

Цели. Цели работы – разработка нового метода проектирования DC/DC-преобразователей, построенных по топологии Zeta с возможным учетом магнитной связи дросселей, проведение расчетов по предложенной методике номиналов дросселей и конденсаторов Zeta-преобразователя с индуктивно связанными дросселями и проверка достоверности предельной непрерывной математической модели и метода проектирования, основанного на ней, с помощью SPICE-моделирования в системе автоматизированного проектирования *Multisim*.

Методы. Поставленные задачи решены при помощи аналитического анализа предельной непрерывной математической модели. Предложенным методом выполнен расчет номиналов связанных дросселей и конденсаторов преобразователя.

Результаты. С помощью системы автоматизированного проектирования *Multisim* проведено моделирование Zeta-преобразователя со связанными дросселями, в ходе которого получены нагрузочные и передаточные характеристики преобразователя, показывающие зависимости токов, протекающих в связанных дросселях, и напряжений на конденсаторах от входного напряжения, а также зависимость выходного напряжения от тока нагрузки. Показано, что представленный метод проектирования достоверен и полностью соответствует результатам моделирования. Установлена корреляция передаточных и нагрузочных характеристик токов и напряжений, полученных моделированием и расчетным путем. Отличия рассчитанных с помощью предельной непрерывной математической модели значений от результатов моделирования в системе автоматизированного проектирования *Multisim* сопоставимы с погрешностью измерений.

Выводы. Предложенный метод проектирования позволяет рассчитать номиналы элементов Zeta-топологии как с учетом индуктивной связи дросселей, так и без него. Кроме того, с помощью данного метода возможно рассчитать постоянные значения и пульсации токов дросселей и напряжений на конденсаторах. Приведенный в статье метод проектирования DC/DC-преобразователей можно использовать как для предварительного оценочного расчета, так и для более детального расчета с анализом работы устройства при различных входных напряжениях и сопротивлениях нагрузки.

Ключевые слова: DC/DC-преобразователь, понижающе-повышающий преобразователь, топология Zeta, предельная непрерывная математическая модель, метод проектирования, моделирование

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INTRODUCTION

DC/DC converters of various topologies [1–3] are used in space [4], aerospace [5], medical equipment [6], and a wide range of mobile devices [7, 8]. The general trend towards miniaturization of DC/DC converters and autonomous design of radio-electronic devices is driving the need for new methods of DC/DC converter design. The main difficulty in designing DC/DC converters for such devices is associated with the typically low voltages and absorbed currents. At currents below 0.5 A, the efficiency of the DC/DC converter starts to drop sharply[1], requiring a rigorous evaluation of the device operation in different modes using more advanced calculation methods [6, 7, 9–11].

The design of DC/DC converters is based on appropriate mathematical models [11], as is the design of any other radio-electronic device. They provide the basis for a uniform methodical approach to developing, designing and investigating devices. The method for designing unipolar DC/DC converters built according to the Zeta topology, taking into account the inductive coupling of the inductors, is also based on such a model [12, 13].

1. DETERMINING THE RATINGS OF ZETA CONVERTER RADIO ELEMENTS WITH ALLOWANCE FOR MAGNETIC COUPLING OF INDUCTORS

The circuit diagram of a DC/DC converter based on the Zeta topology is shown in Fig. 1. It has two inductors, two capacitors, an electronic key, typically in the form of a field effect transistor, and a control unit (CU) that determines the transistor mode [2]. The inductors L1 and L2 perform the function of energy storage and transfer by electromagnetic induction. The capacitor C1 is present in the circuit to decouple the input and output of the converter. The separating capacitor C1, which is sometimes referred to as a "flying capacitor", not only separates but also stores and transfers energy. Capacitor C2 is the output smoothing capacitor.

The method for calculating the Zeta converter taking into account the magnetic coupling of the inductors is based on the corresponding mathematical model [11–13]. The design of the Zeta converter on coupled inductors significantly reduces the current ripple [14]. The mathematical model is used to obtain expressions for the constant and variable values of the inverter currents and voltages. By analyzing the obtained expressions, equations are derived to determine the rated electrical values of the electric radio elements, ERE. The following parameters are required for calculating the ERE ratings of the DC/DC converter:

- constant output voltage U_{C2};
- constant input voltage range from $U_{in\ min}$ to $U_{in\ max}$;
- switching period of the power switch T;
- maximum ripple amplitudes of voltages u_{C1}, u_{C2} and currents i_{L1}, i_{L2} are Δu_{C1}, Δu_{C2} Δi_{L1}, and Δi_{L2} respectively;
- range of load resistance variation from $R_{l\ min}$ to $R_{l\ max}$ (or load currents from $I_{l\ min}$ to $I_{l\ max}$);
- coupling coefficient of inductors k_c.

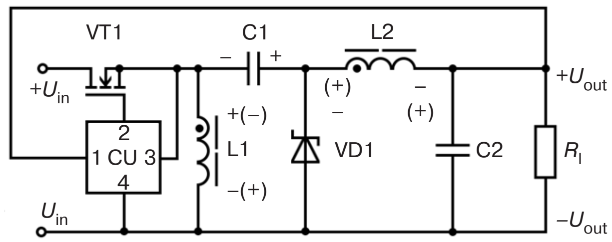

Fig. 1. Circuit diagram of the buck-boost converter based on the Zeta topology. Here and in the following figures, the designations of the circuit elements correspond to those adopted in GOST 2.710-81[2]. R_l is load resistance, U_{in} is input voltage, U_{out} is output voltage

The optional parameters are the active winding resistances of the coupled inductors L1 and L2, which are r_1 and r_2. The resistances r_1 and r_2 are taken into account in the construction of the mathematical model in order to clarify the calculation and to simplify the derivation of the inductor current equations due to the peculiarities in the construction of the limit continuous mathematical models.

For the calculation, it is necessary to determine the input voltage $U_{in\ 0.5}$ with duty factor $D_{0.5}$ equal to 0.5 and the minimum load resistance $R_{l\ min}$ using the following equation:

[1] Datasheet TPS40200 Wide Input Range Non-Synchronous Voltage Mode Controller datasheet (Rev. G). Texas Instruments. SLUS659G – FEBRUARY 2006–REVISED NOVEMBER 2014.

[2] GOST 2.710-81. Interstate Standard. *Unified system for design documentation. Alpha-numerical designations in electrical diagrams*. Moscow: Izd. Standartov; 1985 (in Russ.).

$$U_{\text{in}\,0.5} = \frac{\left(-U_{\text{C2}}D_{0.5}^2 + 2U_{\text{C2}}D_{0.5} - U_{\text{C2}}\right)r_2 - U_{\text{C2}}D_{0.5}^2 r_2 - R_{\text{l av}}U_{\text{C2}}D_{0.5}^2 + 2R_{\text{l av}}U_{\text{C2}}D_{0.5} - R_{\text{l av}}U_{\text{C2}}}{R_{\text{l av}}D_{0.5}^2 - R_{\text{l av}}D_{0.5}}, \qquad (1)$$

where $R_{\text{l av}} = \frac{R_{\text{l min}} + R_{\text{l max}}}{2}$ is the average load resistance.

In addition, the minimum and maximum duty factors $D_{\min}$ and $D_{\max}$ should be calculated using the following equations:

$$D_{\min} = \frac{\sqrt{-\left(4U_{\text{C2}}^2 r_1 r_2\right) + \left(R_{\text{l min}}^2 U_{\text{in max}}^2 - 4R_{\text{l min}}U_{\text{C2}}^2 r_1\right)} - 2U_{\text{C2}}r_2 - \left(2R_{\text{l min}}U_{\text{C2}} + R_{\text{l min}}U_{\text{in max}}\right)}{-2U_{\text{C2}}r_2 - 2U_{\text{C2}}r_1 - \left(2R_{\text{l min}}U_{\text{C2}} + 2R_{\text{l min}}U_{\text{in max}}\right)}, \qquad (2)$$

$$D_{\max} = \frac{\sqrt{-\left(4U_{\text{C2}}^2 r_1 r_2\right) + \left(R_{\text{l min}}^2 U_{\text{in min}}^2 - 4R_{\text{l min}}U_{\text{C2}}^2 r_1\right)} - 2U_{\text{C2}}r_2 - \left(2R_{\text{l min}}U_{\text{C2}} + R_{\text{l min}}U_{\text{in min}}\right)}{-2U_{\text{C2}}r_2 - 2U_{\text{C2}}r_1 - \left(2R_{\text{l min}}U_{\text{C2}} + 2R_{\text{l min}}U_{\text{in min}}\right)}. \qquad (3)$$

Since the calculation method is based on the limiting continuous mathematical model of the Zeta converter, the largest discrepancy between the calculated values of the ripple amplitudes and the simulation result is observed at the limits of the duty factor $D_{\min}$ and $D_{\max}$ according to [15]. The highest agreement observed in the neighborhood of the duty factor D is equal to 0.5. Therefore, it would be reasonable to calculate the converter ratings exactly at $D = 0.5$. For this, it is necessary to obtain the analytical dependence between the maximum current and voltage ripples of the converter and their ripples at $D = 0.5$. Thus, by taking into account (1)–(3), the equations for the conversion factors k_{L1}, k_{L2}, k_{C1} and k_{C2} can be derived. Obviously, the equations for calculating ripples Δi_{L1} and Δi_{L2} are the same [13], so the coefficients k_{L1} and k_{L2} are also identical. It can therefore be further assumed that $k_{\text{L1}} = k_{\text{L2}} = k_{\text{L}}$.

$$k_{\text{L}} = \frac{-\frac{U_{\text{in max}}D_{\min}T\left(k_{\text{c}}\sqrt{L_1L_2} + L_2\right)}{L_2L_1\left(k_{\text{c}}-1\right)\left(k_{\text{c}}+1\right)} + \frac{U_{\text{in max}}D_{\min}^3 T r_1\left(k_{\text{c}}\sqrt{L_1L_2} + L_2\right)}{L_2L_1\left(k_{\text{c}}-1\right)\left(k_{\text{c}}+1\right)\left(\left(D_{\min}^2 - 2D_{\min} + 1\right)r_2 + D_{\min}^2 r_1 + R_{\text{l max}}D_{\min}^2 - 2R_{\text{l max}}D_{\min} + R_{\text{l max}}\right)}}{-\frac{U_{\text{in }0.5}D_{0.5}T\left(k_{\text{c}}\sqrt{L_1L_2} + L_2\right)}{L_2L_1\left(k_{\text{c}}-1\right)\left(k_{\text{c}}+1\right)} + \frac{U_{\text{in }0.5}D_{0.5}^3 T r_1\left(k_{\text{c}}\sqrt{L_1L_2} + L_2\right)}{L_2L_1\left(k_{\text{c}}-1\right)\left(k_{\text{c}}+1\right)\left(\left(D_{0.5}^2 - 2D_{0.5} + 1\right)r_2 + D_{0.5}^2 r_1 + R_{\text{l max}}D_{0.5}^2 - 2R_{\text{l max}}D_{0.5} + R_{\text{l max}}\right)}}, \qquad (4)$$

$$k_{\text{C1}} = \frac{\dfrac{-U_{\text{in min}}D_{\max}^2 T\left(D_{\max}-1\right)}{C_1\left(\left(D_{\max}^2 - 2D_{\max} + 1\right)r_2 + D_{\max}^2 r_1 + R_{\text{l min}}D_{\max}^2 - 2R_{\text{l min}}D_{\max} + R_{\text{l min}}\right)}}{\dfrac{-U_{\text{in }0.5}D_{0.5}^2 T\left(D_{0.5}-1\right)}{C_1\left(\left(D_{0.5}^2 - 2D_{0.5} + 1\right)r_2 + D_{0.5}^2 r_1 + R_{\text{l min}}D_{0.5}^2 - 2R_{\text{l min}}D_{0.5} + R_{\text{l min}}\right)}}, \qquad (5)$$

$$k_{\text{C2}} = \frac{\frac{U_{\text{in max}}D_{\min}T^2\left(k_{\text{c}}\sqrt{L_1L_2} + L_1\right)}{8C_2L_1L_2\left(k_{\text{c}}-1\right)\left(k_{\text{c}}+1\right)} + \frac{U_{\text{in max}}D_{\min}^3 T^2 r_1\left(k_{\text{c}}\sqrt{L_1L_2} + L_1\right)}{8C_2L_1L_2\left(k_{\text{c}}-1\right)\left(k_{\text{c}}+1\right)\left(\left(D_{\min}^2 - 2D_{\min} + 1\right)r_2 + D_{\min}^2 r_1 + R_{\text{l max}}D_{\min}^2 - 2R_{\text{l max}}D_{\min} + R_{\text{l max}}\right)}}{\frac{U_{\text{in }0.5}D_{0.5}T^2\left(k_{\text{c}}\sqrt{L_1L_2} + L_1\right)}{8C_2L_1L_2\left(k_{\text{c}}-1\right)\left(k_{\text{c}}+1\right)} + \frac{U_{\text{in }0.5}D_{0.5}^3 T^2 r_1\left(k_{\text{c}}\sqrt{L_1L_2} + L_1\right)}{8C_2L_1L_2\left(k_{\text{c}}-1\right)\left(k_{\text{c}}+1\right)\left(\left(D_{0.5}^2 - 2D_{0.5} + 1\right)r_2 + D_{0.5}^2 r_1 + R_{\text{l max}}D_{0.5}^2 - 2R_{\text{l max}}D_{0.5} + R_{\text{l max}}\right)}}. \qquad (6)$$

The above equations include the ERE ratings L1, L2, C1, and C2. Using the *Simplify Mathcad* function[3], the expressions (4)–(6) can be simplified to eliminate variables L_1, L_2, C_1, and C_2, thus obtaining equations that are independent of the inductor and capacitor ratings. However, since the final equations for determining the coefficients k_{L}, k_{C1}, and k_{C2} are quite extensive, they have not been included in the present paper.

[3] https://www.mathcad.com/en. Accessed April 24, 2024.

Using expressions (4)–(6), the equations for the ripple amplitudes of currents and voltages at the duty factor $D = 0.5$ can be written as follows:

$$\Delta i_{\mathrm{L1}\,0.5} = \frac{\Delta i_{\mathrm{L1}}}{k_{\mathrm{L}}},\ \Delta i_{\mathrm{L2}\,0.5} = \frac{\Delta i_{\mathrm{L2}}}{k_{\mathrm{L}}},\ \Delta u_{\mathrm{C1}\,0.5} = \frac{\Delta u_{\mathrm{C1}}}{k_{\mathrm{C1}}},\ \Delta u_{\mathrm{C2}\,0.5} = \frac{\Delta u_{\mathrm{C2}}}{k_{\mathrm{C2}}}. \tag{7}$$

The equations for calculating the ratings of the inductors L1, L2 and capacitors C1, C2 are as follows:

$$L_1 = \frac{\left(\left(D_{0.5}^2 - 2D_{0.5} + 1\right)k_{\mathrm{c}} r_2 + D_{0.5}^2 k_{\mathrm{c}} r_1 + \left(R_{\mathrm{l\,max}} D_{0.5}^2 - 2R_{\mathrm{l\,max}} D_{0.5} + R_{\mathrm{l\,max}}\right)k_{\mathrm{c}}\right)\Delta i_{\mathrm{L2}\,0.5}\sqrt{L_1 L_2}}{\Delta i_{\mathrm{L1}\,0.5}\left(\left(D_{0.5}^2 - 2D_{0.5} + 1\right)r_2 + D_{0.5}^2 r_1 + R_{\mathrm{l\,max}} D_{0.5}^2 - 2R_{\mathrm{l\,max}} D_{0.5} + R_{\mathrm{l\,max}}\right)} +$$
$$+ \frac{\left(U_{\mathrm{in}\,0.5} D_{0.5}^3 - 2U_{\mathrm{in}\,0.5} D_{0.5}^2 + U_{\mathrm{in}\,0.5} D_{0.5}\right)T r_2 + \left(R_{\mathrm{l\,max}} U_{\mathrm{in}\,0.5} D_{0.5}^3 - 2R_{\mathrm{l\,max}} U_{\mathrm{in}\,0.5} D_{0.5}^2 + R_{\mathrm{l\,max}} U_{\mathrm{in}\,0.5} D_{0.5}\right)T}{\Delta i_{\mathrm{L1}\,0.5}\left(\left(D_{0.5}^2 - 2D_{0.5} + 1\right)r_2 + D_{0.5}^2 r_1 + R_{\mathrm{l\,max}} D_{0.5}^2 - 2R_{\mathrm{l\,max}} D_{0.5} + R_{\mathrm{l\,max}}\right)}, \tag{8}$$

$$L_2 = \frac{\left(\left(D_{0.5}^2 - 2D_{0.5} + 1\right)k_{\mathrm{c}} r_2 + D_{0.5}^2 k_{\mathrm{c}} r_1 + \left(R_{\mathrm{l\,max}} D_{0.5}^2 - 2R_{\mathrm{l\,max}} D_{0.5} + R_{\mathrm{l\,max}}\right)k_{\mathrm{c}}\right)\Delta i_{\mathrm{L1}\,0.5}\sqrt{L_1 L_2}}{\Delta i_{\mathrm{L2}\,0.5}\left(\left(D_{0.5}^2 - 2D_{0.5} + 1\right)r_2 + D_{0.5}^2 r_1 + R_{\mathrm{l\,max}} D_{0.5}^2 - 2R_{\mathrm{l\,max}} D_{0.5} + R_{\mathrm{l\,max}}\right)} +$$
$$+ \frac{\left(U_{\mathrm{in}\,0.5} D_{0.5}^3 - 2U_{\mathrm{in}\,0.5} D_{0.5}^2 + U_{\mathrm{in}\,0.5} D_{0.5}\right)T r_2 + \left(R_{\mathrm{l\,max}} U_{\mathrm{in}\,0.5} D_{0.5}^3 - 2R_{\mathrm{l\,max}} U_{\mathrm{in}\,0.5} D_{0.5}^2 + R_{\mathrm{l\,max}} U_{\mathrm{in}\,0.5} D_{0.5}\right)T}{\Delta i_{\mathrm{L2}\,0.5}\left(\left(D_{0.5}^2 - 2D_{0.5} + 1\right)r_2 + D_{0.5}^2 r_1 + R_{\mathrm{l\,max}} D_{0.5}^2 - 2R_{\mathrm{l\,max}} D_{0.5} + R_{\mathrm{l\,max}}\right)}, \tag{9}$$

$$C_1 = \frac{-\left(U_{\mathrm{in}\,0.5} D_{0.5}^2 T\left(D_{0.5} - 1\right)\right)}{\Delta u_{\mathrm{C1}\,0.5}\left(\left(D_{0.5}^2 - 2D_{0.5} + 1\right)r_2 + D_{0.5}^2 r_1 + R_{\mathrm{l\,min}} D_{0.5}^2 - 2R_{\mathrm{l\,min}} D_{0.5} + R_{\mathrm{l\,min}}\right)}, \tag{10}$$

$$C_2 = \frac{-\left(U_{\mathrm{in}\,0.5} D_{0.5} T^2\left(D_{0.5} - 1\right)^2\left(r_2 + R_{\mathrm{l\,max}}\right)\left(k_{\mathrm{c}}\sqrt{L_1 L_2} + L_1\right)\right)}{8\Delta u_{\mathrm{C2}\,0.5} L_1 L_2\left(k_{\mathrm{c}} - 1\right)\left(k_{\mathrm{c}} + 1\right)\left(\left(D_{0.5}^2 - 2D_{0.5} + 1\right)r_2 + D_{0.5}^2 r_1 + R_{\mathrm{l\,max}} D_{0.5}^2 - 2R_{\mathrm{l\,max}} D_{0.5} + R_{\mathrm{l\,max}}\right)}. \tag{11}$$

It follows from equations (8) and (9) that the ERE values for coupled inductors are also interdependent. While obtaining the solution to this equation is analytically complicated, it can be taken into account that coupled inductors usually have the same ratings and therefore the same ripple. With this in mind, the equations for determining L_1 and L_2 can be written as follows:

$$L_1 = \frac{\left(-\left(U_{\mathrm{in}\,0.5} D_{0.5}^3\right) + 2U_{\mathrm{in}\,0.5} D_{0.5}^2 - U_{\mathrm{in\,max}} D_{0.5}\right)T r_2}{\left(1 - k_{\mathrm{c}}\right)\left(\left(D_{0.5}^2 - 2D_{0.5} + 1\right)\Delta i_{\mathrm{L1}\,0.5} r_2 + D_{0.5}^2 \Delta i_{\mathrm{L1}\,0.5} r_1 + \left(R_{\mathrm{l\,min}} D_{0.5}^2 - 2R_{\mathrm{l\,min}} D_{0.5} + R_{\mathrm{l\,min}}\right)\Delta i_{\mathrm{L1}\,0.5}\right)} +$$
$$+ \frac{\left(\left(2R_{\mathrm{l\,min}} U_{\mathrm{in}\,0.5} D_{0.5}^2 - R_{\mathrm{l\,min}} U_{\mathrm{in}\,0.5} D_{0.5}^3 - R_{\mathrm{l\,min}} U_{\mathrm{in}\,0.5} D_{0.5}\right)T - 2U_{\mathrm{in}\,0.5} D_{0.5}^3 T r_1\right)}{\left(1 - k_{\mathrm{c}}\right)\left(\left(D_{0.5}^2 - 2D_{0.5} + 1\right)\Delta i_{\mathrm{L1}\,0.5} r_2 + D_{0.5}^2 \Delta i_{\mathrm{L1}\,0.5} r_1 + \left(R_{\mathrm{l\,min}} D_{0.5}^2 - 2R_{\mathrm{l\,min}} D_{0.5} + R_{\mathrm{l\,min}}\right)\Delta i_{\mathrm{L1}\,0.5}\right)}, \tag{12}$$

$$L_2 =$$
$$= \frac{\left(U_{\mathrm{in}\,0.5} D_{0.5}^3 - 2U_{\mathrm{in}\,0.5} D_{0.5}^2 + U_{\mathrm{in\,max}} D_{0.5}\right)T r_2 + \left(R_{\mathrm{l\,min}} U_{\mathrm{in}\,0.5} D_{0.5}^3 - 2R_{\mathrm{l\,min}} U_{\mathrm{in}\,0.5} D_{0.5}^2 + R_{\mathrm{l\,min}} U_{\mathrm{in}\,0.5} D_{0.5}\right)T_1}{\left(1 - k_{\mathrm{c}}\right)\left(\left(D_{0.5}^2 - 2D_{0.5} + 1\right)\Delta i_{\mathrm{L2}\,0.5} r_2 + D_{0.5}^2 \Delta i_{\mathrm{L2}\,0.5} r_1 + \left(R_{\mathrm{l\,min}} D_{0.5}^2 - 2R_{\mathrm{l\,min}} D_{0.5} + R_{\mathrm{l\,min}}\right)\Delta i_{\mathrm{L2}\,0.5}\right)}. \tag{13}$$

Equations (12) and (13) differ only by a summand $-2U_{\text{in }0.5}D_{0.5}^3Tr_1$ in the numerator in (12), which depends on the equivalent resistance of the inductor L1. Therefore, the ratings of L_1 and L_2 are approximately equivalent.

2. EXAMPLE CALCULATION OF ZETA CONVERSION ERE VALUES

The initial parameters for the converter calculation are as follows: $k_c = -0.99$, $U_{C2} = 12$ V, $U_{\text{in max}} = 17.5$ V, $U_{\text{in min}} = 6.5$ V, $\Delta i_{L1} = 330$ mA, $\Delta i_{L2} = 330$ mA, $\Delta u_{C1} = 7$ mV, $\Delta u_{C2} = 1.9$ mV, $R_{\text{l min}} = 50$ Ohm, and $R_{\text{l max}} = 100$ Ohm. By calculating the input voltage $U_{\text{in }0.5}$, the minimum and maximum duty factors D_{min} and D_{max} using equations (1)–(3), we obtain $U_{\text{in }0.5} = 12.005$ V, $D_{\text{min}} = 0.407$, and $D_{\text{max}} = 0.649$. Using the above parameters and taking into account equation (7), we obtain $k_L = 1.185$, $k_{C1} = 1.298$, and $k_{C2} = 1.186$. Accordingly, the ripple amplitudes at duty factor $D = 0.5$ are $\Delta i_{L1\ 0.5} = 278$ mA, $\Delta i_{L2\ 0.5} = 278$ mA, $\Delta u_{C1\ 0.5} = 5$ mV, and $\Delta u_{C2\ 0.5} = 1.6$ mV.

After calculating the ratings of the elements according to equations (10)–(13), we obtain $L_1 \approx 22$ µHn, $L_2 \approx 22$ µHn, $C_1 \approx 44$ µF, and $C_2 \approx 44$ µF.

3. VALIDATION OF CALCULATION RESULTS IN THE *MULTISIM*[4] MODELING ENVIRONMENT

A SPICE simulation of a Zeta converter with inductively coupled inductors is shown in Fig. 2, where T1 is a block of coupled inductors, each rated at 22 µHn. The coupling coefficient of the inductors is 0.99. This means that the inductors are switched on against each other to suppress current ripple. The resistors r_1 and r_2 simulate the active winding resistances of the coupled inductors, while C1 is a "flying" capacitor, C2 is a smoothing output capacitor, VT1 is a power key that allows the power part to be switched at a given frequency, V1 and V2 are sources of constant voltage and pulse width modulation voltage, and VD1 is an ideal diode. In addition, there are also current and voltage probes used to display current and voltage parameters.

Simulation results in transient analysis mode over 4 ms after onset are shown in Fig. 3. This shows the shapes of the currents through the windings of the inductors L1 and L2, as well as the voltages C_1 and C_2 on the capacitors in stationary operation after the transients have passed. It should be noted that maximum and minimum U_{C2} voltages are midway between each energy storage and transmission stage, as stated in [13].

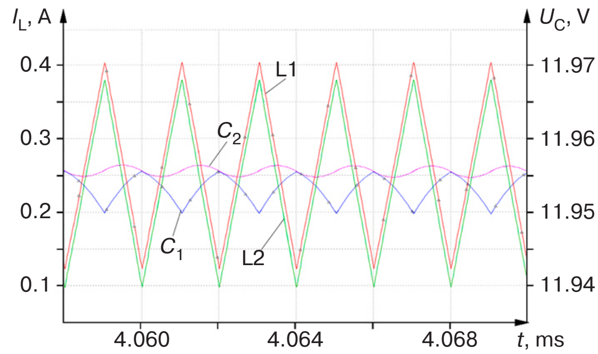

Fig. 3. The graphs of currents and voltages of the Zeta converter in steady state

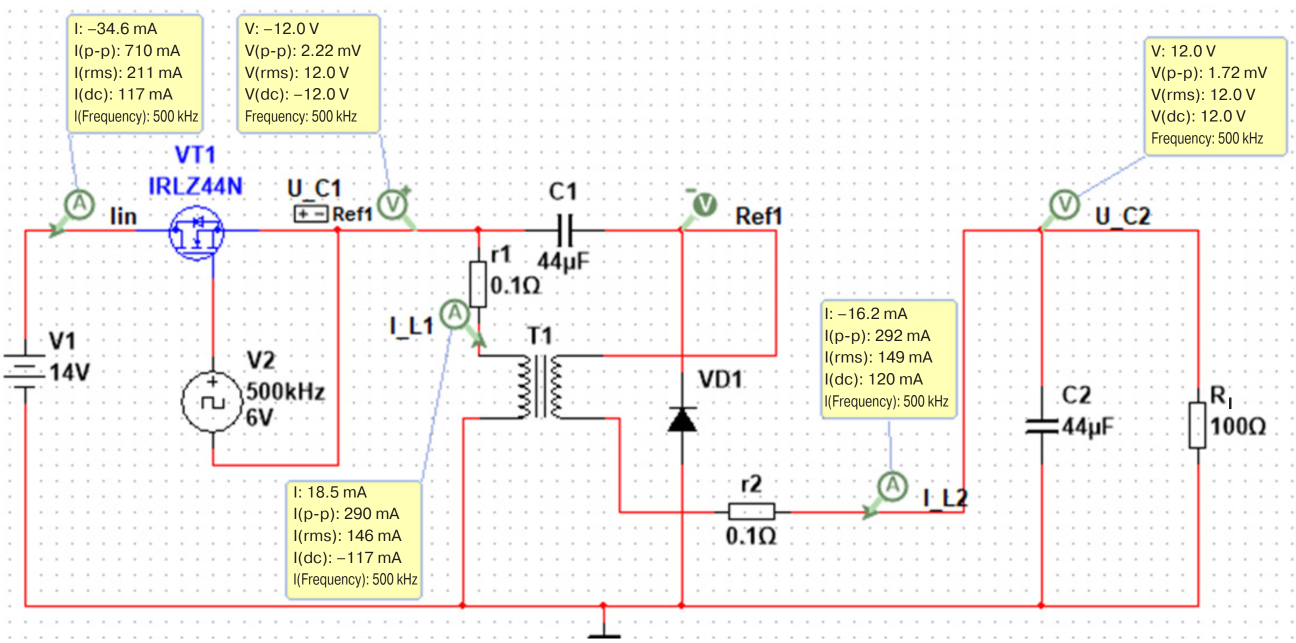

Fig. 2. Simulation scheme for the Zeta converter with inductively coupled inductors

[4] https://www.ni.com/en.html. Accessed April 24, 2024.

The calculated and simulated transfer characteristics of the converter currents and voltages are shown in Figs. 4–7 to demonstrate the accuracy of the calculations.

The transfer characteristic is the dependence of the physical quantity on the converter input voltage. The mathematical model of the converter is used to calculate the values.

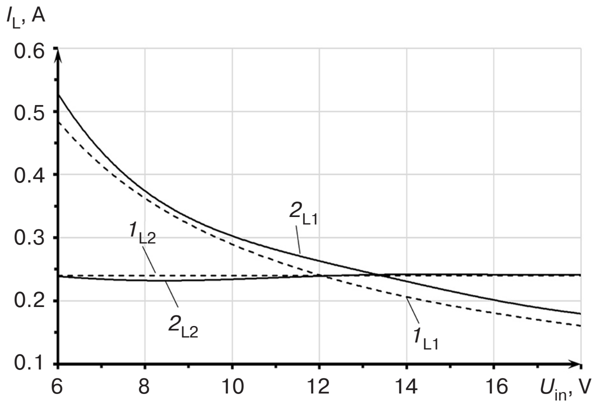

Fig. 4. Transfer characteristics of currents through inductor windings L1 and L2 at a load resistance of 50 Ohm: *1* is the calculated value; *2* is the simulation result

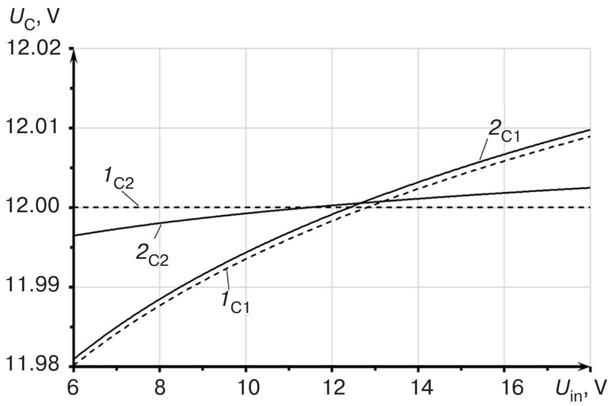

Fig. 5. Transfer characteristics of voltages across capacitors C1 and C2 at a load resistance of 50 Ohm: *1* is the calculated value; *2* is the simulation result

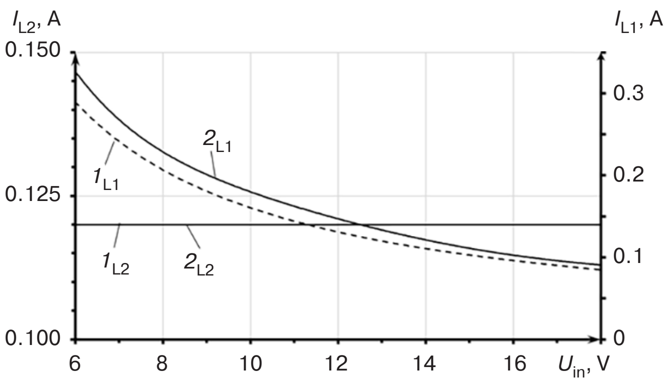

Fig. 6. Transfer characteristics of currents through inductor windings L1 and L2 at a load resistance of 100 Ohm: *1* is the calculated value; *2* is the simulation

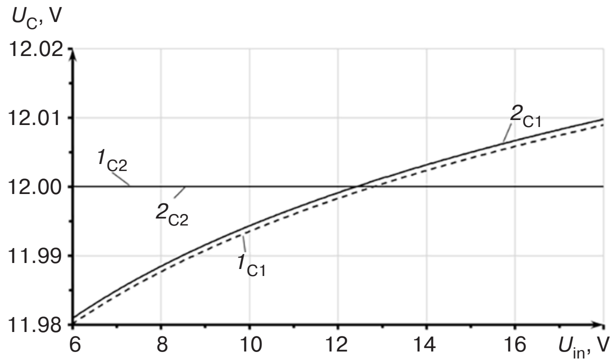

Fig. 7. Transfer characteristics of voltages across capacitors C1 and C2 at a load resistance of 100 Ohm: *1* is the calculated value; *2* is the simulation result

The transfer characteristic graphs show agreement between the calculated values and the simulation results, confirming the reliability of the calculation method. The deviations of the simulated curves from the calculated ones are quite small; for example, the deviation of the current I_{L1} in the simulation from the calculated one ranges from 3.5% to 12%. The deviation ranges similarly from 0% to 3.4% for I_{L2}, from 0% to 0.05% for U_{C1}, and from 0.003% to 0.03% for U_{C2}, demonstrating the reliability of the calculations.

The load characteristics of the converter at input voltages of 6.5, 12, and 17.5 V are shown in Fig. 8. Since the difference between the calculated and simulated output voltages does not exceed one hundredth of a percent, the graphs of the converter load characteristics obtained by calculation and simulation are almost identical. The load characteristics for input voltages of 12 and 17.5 V are plotted in Cartesian coordinates at the bottom and left. The load characteristic at input voltage is plotted in coordinates at the top and right. The information in Fig. 8 shows that the results of calculations using the proposed DC/DC converter design method and mathematical model fully reproduce the load current dependence on the stabilization voltage variation. This demonstrates the reliability of the methods considered in describing the operation of DC/DC converters designed according to the Zeta topology, taking into account the magnetic coupling of the coupled inductors.

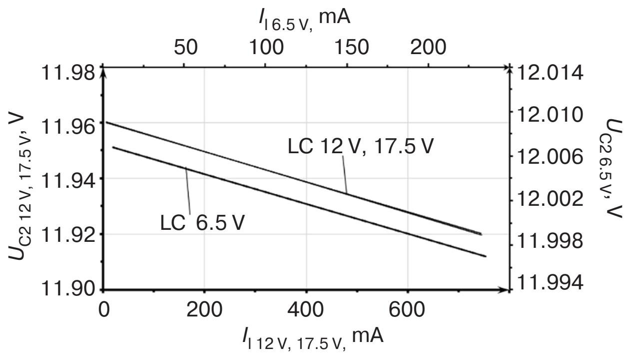

Fig. 8. Load characteristics (LC) at 12 V and 17.5 V, and LC load characteristic at 6.5 V. I_l is load current

CONCLUSIONS

The paper presents a new method for designing DC/DC converters based on the Zeta topology, which allows magnetic coupling of inductors. As well as allowing the capacitor and inductor ratings of the converter power section to be calculated, the presented method takes into account the variable components of the currents flowing through the inductor windings and the voltages across the capacitors.

With the ERE ratings obtained in the design, the simulation is carried out in the *Multisim* computer-aided design system; calculations are performed using the converter limit continuous mathematical model on which the above method is based. The resulting transfer and load characteristics show a high degree of agreement between the mathematical calculations and the *Multisim* simulation results.

Authors' contribution

All authors equally contributed to the research work.

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   [Original Russian Text: Kramm M.N., Bodin O.N., Bodin A.Y., Truong T.L.N., Zhikhareva G.V. Constructional Features of a Multielectrode Electrocardiology Screening System. *Meditsinskaya Tekhnika*. 2022;5(335):37–41 (in Russ.). https://www.elibrary.ru/camqob]
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