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RESEARCH ARTICLE

Reflections of linearly polarized electromagnetic waves from a multilayer periodic mirror

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Abstract

Objectives. The purpose of the article is to carry out a theoretical and experimental study of the angular reflection spectrum of linearly polarized electromagnetic waves from a multilayer periodic mirror on a transparent substrate to exact analytical expressions for reflection and transmission coefficients generalizing the cases of incidence of plane transverse electric (TE) and transverse magnetic (TM) modes on limited periodically structured media with a stepped refractive index profile.

Methods. The theoretical analysis of the reflection problem is based on the search for exact analytical solutions in the form of Floquet–Bloch waves presented in the form of inhomogeneous waves in the domain of periodically structured media. On the basis of the possible existence of a single Floquet–Bloch wave in a limited one-dimensional photonic crystal, it is proposed to search for exact solutions of the wave equation in the form of a linear combination of inhomogeneous waves propagating in different directions. By using the canonical forms of the considered periodic structures, it is possible to carry out the simple transition from the case of TE polarization to TM type in dispersion relations and expressions for the angular reflection spectrum.

Results. Cases of reflection of linearly polarized radiation are considered for the following cases: a flat boundary of two dielectrics, a thin plane-parallel plate, and a multilayer dielectric mirror. Exact analytical expressions for the reflection and transmission coefficients generalizing the cases of incidence of TE and TM polarizations waves on a limited one-dimensional photonic crystal are obtained. The transmission coefficients of a plane TE wave from a multilayer dielectric mirror sputtered on thin glass were experimentally measured.

Conclusions. A quantitative and qualitative agreement of experimental measurements of the transmission coefficient of a plane wave incident from a half-space on a confined photonic crystal with theoretical calculations is obtained. The obtained expressions for the transmission coefficient of a confined one-dimensional photonic crystal, which are shown to be determined by the interference of Floquet–Bloch waves presented in the form of inhomogeneous waves, can be reduced to a form analogous to the expression for the value of the transmission coefficient of a traditional Fabry–Pérot interferometer. In the case of TM polarization, when the Brewster condition is fulfilled at the interlayer boundaries, the Floquet–Bloch wave has the form of homogeneous plane waves in the layers of a photonic crystal.

Keywords: electromagnetic waves, periodic medium, multilayer mirror, one-dimensional photonic crystal, Floquet–Bloch waves

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НАУЧНАЯ СТАТЬЯ

Отражение линейно поляризованных электромагнитных волн от многослойного периодического зеркала

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Резюме
Цели. Цель работы – теоретическое и экспериментальное исследование углового спектра отражения линейно поляризованных электромагнитных волн от многослойного периодического зеркала на прозрачной подложке, вывод точных аналитических выражений для коэффициентов отражения и прохождения, обобщающих случаи падения плоских TE-(transverse electric) и TM-мод (transverse magnetic) на ограниченные периодические структуры со ступенчатым профилем показателя преломления.
Методы. Теоретический анализ задачи отражения основан на поиске точных аналитических решений в виде волн Флоке – Блоха, представленных в форме неоднородных волн, в области периодически структурированных сред. На основе того факта, что в ограниченном одномерном фотонном кристалле возможно существование одиночной волны Флоке – Блоха, предлагается искать точные решения волнового уравнения в виде линейной комбинации волн Флоке – Блоха, бегущих в разные стороны. Канонические формы рассматриваемых периодических структур позволяют достаточно просто осуществлять переход от случая TE-поляризации к TM-типу в дисперсионных соотношениях и выражениях для углового спектра отражения.
Результаты. Рассмотрены случаи отражения линейно поляризованного излучения для следующих случаев: плоской границы двух диэлектриков, тонкой плоскопараллельной пластины и многослойного диэлектрического зеркала. Получены точные аналитические выражения для коэффициентов отражения и прохождения, обобщающие случаи падения волн TE- и TM-поляризаций на ограниченный одномерный фотонный кристалл. Экспериментально измерен коэффициент пропускания плоской TE-волны для многослойного диэлектрического зеркала, напыленного на тонкую стеклянную пластину.
Выводы. Получено количественное и качественное согласование экспериментальных измерений коэффициента пропускания плоской волны, падающей из полупространства на ограниченный фотонный кристалл с теоретическими вычислениями. Показано, что полученные выражения для коэффициента пропускания ограниченного одномерного фотонного кристалла определяются интерференцией волн Флоке – Блоха,

представленных в форме неоднородных волн, и могут быть приведены к виду, аналогичному для величины коэффициента прохождения традиционного интерферометра Фабри – Перо. В случае ТМ-поляризации при выполнении условия Брюстера на межслойных границах волна Флоке – Блоха имеет вид однородных плоских волн в слоях фотонного кристалла.

Ключевые слова: электромагнитные волны, периодическая среда, многослойное зеркало, одномерный фотонный кристалл, волны Флоке – Блоха

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INTRODUCTION

The study of the peculiarities of light propagation in layered media whose properties are constant on planes perpendicular to a fixed direction is a well-known problem in optics. As a historical example, we can cite the classical works of Stokes [1] and Rayleigh [2], in which the phenomena arising from the passage of light through crystalline periodic structures were considered. The wave equation in layered periodic media is known to be reducible to Hill's differential equation; in such media, there may exist transmission and non-transmission windows for the passing radiation [2]. In the one-dimensional case, the solution of the wave equation in layered periodic media is written in the Floquet form [3], while in the three-dimensional case it is written in the Bloch wave form [4]. A fairly detailed background is given in the review [5]. The relatively recent and still growing interest in the study of wave propagation through one-dimensional periodic structures in optics is due to the possibility for such structures, with relative simplicity of fabrication, to provide a complete reflection in a given frequency range of frequencies and angles of incidence for different polarization states [6, 7]. In this connection, one-dimensional periodic structures can be considered as one-dimensional photonic crystals (1D-PC); as such, light propagation in them can be described using the Floquet–Bloch approach [8–13]. However, since the elements of the Floquet–Bloch wave (FBW) theory are not currently developed in sufficient detail, numerical methods are usually used to describe the propagation of electromagnetic waves through such structures. Numerical calculations performed using complex transfer matrices do not provide a clear picture of the explicit quantitative dependence of the studied physical processes on the geometrical and material parameters of the periodic structure [14–16]. In [8, 9], for the case of transverse electric (TE) polarized radiation[1], the FBW in the 1D-PC is represented in the form of an inhomogeneous wave. In particular, the functions describing the amplitude and phase profiles of the wave and the reflection coefficient of a plane wave at the boundary of the 1D-PC were obtained.

At present, various questions on the application of 1D-PC are being actively investigated in the literature. In particular, the transmission spectra of 1D-PC having a complex sequence of superconductor-semiconductor layers are investigated in [17]; the application of photonic crystal as a biosensor based on graphene is considered in [18]. Studies of 1D-PC seem to be particularly relevant due to the possibility of experimental realization of bound or localized states in a continuous spectrum [19]. Thus, the use of birefringent media in combination with 1D-PC was recently proposed, where the existence of such modes is supported at the Brewster angle of incidence.

Therefore, the main objectives of the present work are to extend the developed representation of the FBW in the form of an inhomogeneous wave to the case of transverse magnetic (TM) polarized radiation[2], as well as to experimentally demonstrate the effect of the FBW interference on the magnitude of reflection and transmission coefficients for the case of incidence of a plane linearly polarized wave from a homogeneous medium on a bounded 1D-PC.

[1] Transverse electric, linearly (plane) polarized wave with the electric field intensity vector $\vec{E}$ oriented perpendicular to the plane of incidence.

[2] Transverse magnetic, linearly (plane) polarized wave with the magnetic field intensity vector $\vec{H}$ oriented perpendicular to the plane of incidence.

1. REFLECTION OF A PLANE WAVE AT THE BOUNDARY OF TWO DIELECTRICS AND FROM A THIN PLATE

In the present work, we consider the conditions for the passage of a plane electromagnetic wave through the interface of two media for three characteristic cases (Fig. 1): reflection at the boundary of two homogeneous dielectrics, reflection from a thin dielectric plate, and reflection from a multilayer dielectric mirror (bounded by an 1D-PC).

Let us write the field distribution $E(x, z, t)$ ($H(x, z, t)$) of the waves propagating in the x-z-plane (plane of incidence) as a scalar function $\Psi(x, z, t)$:

$$\left.\begin{matrix} E(x,z,t) \\ H(x,z,t) \end{matrix}\right\} = \Psi(x,z,t) = \Psi(x)e^{i(\omega t - \beta z)}. \qquad (1)$$

When the plane wave $\hat{P}_a$

$$\hat{P}_a = P_a \exp[i(\phi_p + \omega t - \kappa_a x - \beta z)]$$

is incident on the interface of dielectrics with refractive indices n_a, n_b ($n_a{}^2 = \varepsilon_a$, $n_b{}^2 = \varepsilon_b$) and dielectric permittivities ε_a, ε_b, the parameter β is the longitudinal component of the wave vectors ($\beta = k_0 n_a \sin\varphi$) determined by the angle of incidence φ counted from the normal to the interface ($x = 0$) of the media. The amplitude Q_a of the reflected wave $\hat{Q}_a$

$$\hat{Q}_a = Q_a \exp[i(\phi_q + \omega t + \kappa_a x - \beta z)]$$

is determined by the Fresnel reflection coefficient r_a and the amplitude P_a of the incident wave ($Q_a = r_a P_a$), while the amplitude G_b of the refracted wave

$$\hat{G}_b = G_b \exp[i(\phi_g + \omega t - \kappa_b x - \beta z)]$$

is determined by the amplitude transmission coefficient t_b ($G_b = t_b P_a$). Here ϕ_p, ϕ_q, ϕ_g are the initial phases, κ_a, κ_b are the transverse components of wave vectors ($\kappa_a = \sqrt{k_0^2 n_a^2 - \beta^2}$, $\kappa_b = \sqrt{k_0^2 n_b^2 - \beta^2}$), t is time, $k_0 = \omega/c$ is the wave vector of radiation in vacuum, and ω is the frequency of radiation.

Considering the component of the field-vector $\vec{E}$ ($\vec{H}$) perpendicular to the plane of incidence in case of TE- or TM-polarized waves, it is convenient to introduce parameters χ_a, χ_b, which are associated with transverse wave vectors: $\chi_a = \kappa_a / (\varepsilon_a)^\tau$, $\chi_b = \kappa_b / (\varepsilon_b)^\tau$, where τ is the efficient polarization parameter equal to zero (unity) in this case. The Fresnel formulas for the amplitudes of reflected and refracted light can be presented in the same form for the cases of TE- and TM-polarized wave:

$$\begin{gathered} r_a = Q_a / P_a = (\chi_a - \chi_b)/(\chi_a + \chi_b), \\ t_a = G_b / P_a = 2\chi_a / (\chi_a + \chi_b), \\ t_a - r_a = 1. \end{gathered} \qquad (2)$$

If the parameters χ_a, χ_b are real, the amplitude coefficient t_a is always positive, and the phase of the refracted wave at the interface coincides with the phase of the incident wave. At $\chi_a > \chi_b$ and $r_a > 0$, the phase of the reflected wave also coincides with the phase of the incident wave, while at $\chi_a < \chi_b$ and $r_a < 0$, a phase shift equal to π occurs for the reflected wave. It is convenient to equate the initial phases of the numerical values ϕ_p, ϕ_q, ϕ_g, which coincide to the nearest 2π multiplied by an arbitrary integer, to zero. In this case, the coefficients r_a, t_a are real. However, in the general case, when the initial phases are ϕ_p, ϕ_q, ϕ_g we will use the parameter $\hat{r}_a$ ($\hat{r}_a = r_a \exp(i\phi_{ra})$, $\phi_{ra} = \phi_q - \phi_p$) to denote the complex reflection coefficient. The energy coefficients of reflection R_a and transmission T_a are represented as follows:

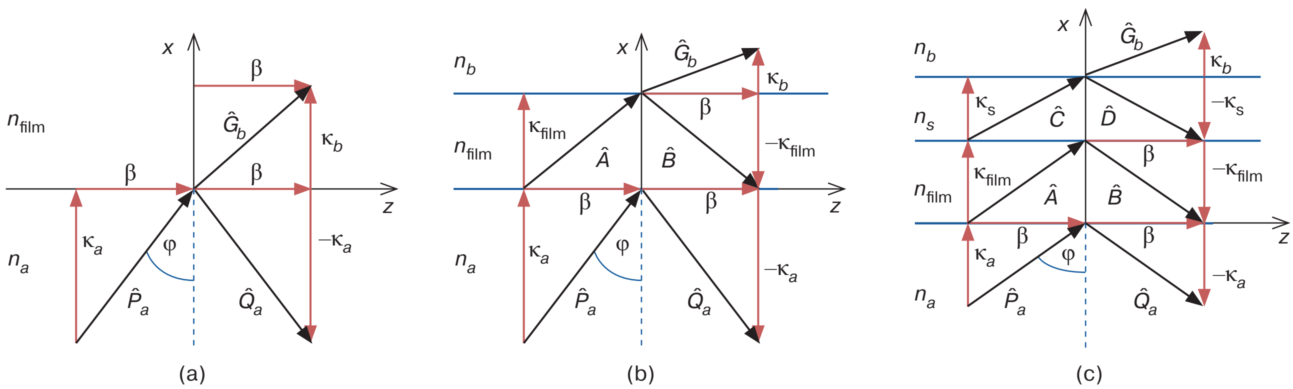

Fig. 1. Directions of the wave vector components $\kappa_a, \kappa_b, \kappa_{film}, \beta$ at the reflection: (a) at the interface of two dielectrics, (b) from a thin dielectric plate, (c) from the 1D-PC

$$R_a = Q_a^2 / P_a^2 = |r_a|^2 = (\chi_a - \chi_b)^2 / (\chi_a + \chi_b)^2;$$
$$T_a = |t_a|^2 \cdot (\chi_b / \chi_a) = 4\chi_a \chi_b / (\chi_a + \chi_b)^2. \quad (3)$$

The energy coefficients R_a and T_a are equal to the ratio of the average energy flux of the reflected and transmitted waves to the average energy flux of the incident wave, respectively. Here, only transverse (i.e., parallel to the *x*-axis) components of the energy fluxes in the medium layers are considered. The condition of continuity of the transverse energy flux is satisfied:

$$\chi_a (P_a^2 - Q_a^2) = \chi_b G_b{}^2. \quad (4)$$

In the case of a plane wave $\hat{P}_a$ incident from a medium with refractive index n_a ($x < 0$) onto a dielectric plate of thickness h and refractive index n_{film} ($n_{\text{film}}^2 = \varepsilon_{\text{film}}$, $\varepsilon_{\text{film}}$ is the dielectric permittivity of the plate material) (Fig. 1b), a reflected wave $\hat{Q}_a$, appears in the area $x < 0$, while in the area $x > h$ the wave $\hat{G}_b$ is refracted into the medium having a refractive index n_b. The field inside the plate can be represented by a forward and an inverse waves $\hat{A}$, $\hat{B}$

$$\hat{A} = A \exp[i(\phi_a + \omega t - \kappa_{\text{film}} x - \beta z)],$$
$$\hat{B} = B \exp[i(\phi_b + \omega t + \kappa_{\text{film}} x - \beta z)],$$

where $\kappa_{\text{film}} = \sqrt{k_0^2 n_{\text{film}}^2 - \beta^2}$. These waves are called partial waves.

For complex amplitudes $\hat{A}, \hat{B}, \hat{G}_b$, the equations $B = \hat{r}_b A$, $G_b = A + B$, where $\hat{r}_b = r_b \exp(i\phi_{rb})$ is the Fresnel reflection coefficient at the $x = h$, are valid:

$$\hat{r}_b = (\chi_{\text{film}} - \chi_b) / (\chi_{\text{film}} + \chi_b),$$
$$\chi_{\text{film}} = \kappa_{\text{film}} / (\varepsilon_{\text{film}})^\tau, \; \chi_b = \kappa_b / (\varepsilon_b)^\tau. \quad (5)$$

Initial phases ϕ_a, ϕ_b, ϕ_g coincide with an accuracy up to the value of 2π multiplied by an arbitrary integer; here it is convenient to select their numerical values equal to $\phi_g = 0$. In this case, the phases of waves $\hat{A}$, $\hat{B}$ at the boundary $x = 0$ will be equal, respectively, to $\pm\phi_h$ (where $\phi_h = \kappa_{\text{film}} h$ is the phase delay arising for a partial wave during the passage of a layer of thickness h).

Now we will write down formulas for the reflection coefficient r_a at the boundary $x = 0$ both reflectance (R_a) and transmittance (T_a) of the plate:

$$r_a = \frac{Q_a}{P_a} = \left(\frac{\left(1 - \frac{\chi_b}{\chi_a}\right)^2 + \left(\frac{\chi_b}{\chi_{\text{film}}} - \frac{\chi_{\text{film}}}{\chi_a}\right)^2 \text{tg}^2\phi_h}{\left(1 + \frac{\chi_b}{\chi_a}\right)^2 + \left(\frac{\chi_b}{\chi_{\text{film}}} + \frac{\chi_{\text{film}}}{\chi_a}\right)^2 \text{tg}^2\phi_h} \right)^{1/2}, \quad (6)$$
$$R_a = \frac{Q_a^2}{P_a^2} = r_a^2, \quad T_a = 1 - R_a.$$

Formulas (6) are in agreement with the well-known Airy formulas obtained when considering multipath wave interference in a transparent plate [20]. Thus, according to (6), at values $2\phi_h = 2\pi m$ and $\pi(2m + 1)$ for R_a, extreme values equal to $(\chi_a - \chi_b)^2 / (\chi_a + \chi_b)^2$ and $(\chi_a \chi_b - \chi_{\text{film}}^2)^2 / (\chi_a \chi_b + \chi_{\text{film}}^2)^2$, respectively, are reached. In particular, in the interference reflection minima that arise, for example, when the simultaneous fulfillment of conditions (1) $2\phi_h = 2\pi m$ and $\chi_a = \chi_b$ or (2) $2\phi_h = \pi(2m+1)$ and $\chi_a \neq \chi_b$, $\chi_{\text{film}}^2 = \chi_a \chi_b$ must be reached, the values of R_a are equal to zero. In the general case of variation of the angle of incidence φ of the incident wave $\hat{P}_a$, alternating maxima and minima of the intensity of the reflected $\hat{Q}_a$ and refracted $\hat{G}_b$ waves should be observed.

2. PLANE WAVE REFLECTION FROM A MULTILAYER DIELECTRIC MIRROR

Let us consider a multilayer dielectric mirror (Fig. 1c) consisting of alternating *f*- and *s*-layers having refractive indices n_f and n_s and thicknesses h and s, respectively, which have been placed between homogeneous dielectric media having refractive indices n_a ($x < 0$) и n_b ($x > H$). It is convenient to represent this structure as an 1D-PC framed by two homogeneous dielectric media, which are formed by multiple repetition of a cell composed of two layers (*f*- and *s*-layers) of the size Λ ($\Lambda = h + s$). Figure 1c shows one such cell framed by two homogeneous media. When a plane wave $\hat{P}_a$ is incident on the lower boundary ($x = 0$) of the 1D-PC, reflected $\hat{Q}_a$ and refracted $\hat{G}_b$, respectively, plane waves are generated in the regions $x < 0$ and $x > H$ to excite forward and backward FBW in the region $H > x > 0$. We will give a description of TE-polarized FBW for the case of unbounded 1D-PC in [8, 9]. In this article, we present these waves for the cases of TE- and TM-polarized radiation in a single inhomogeneous wave form $\Psi_u(x, z, t)$:

$$\Psi_u(x,z,t) = \Psi_u(x) \exp\{i[(\omega t + \Phi(x,z)]\}. \quad (7)$$

Here, the functions $\Psi_u(x)$ and $\Phi(x, z)$ define the distribution of the amplitude and phase of the wave, respectively. Surfaces of constant amplitude are planes perpendicular to the *x*-axis. Function $\Psi_u(x)$ is periodic with a period equal to Λ, so that by introducing local coordinates $\xi_f = x - h/2 - \Lambda m$ and $\xi_s = x - s/2 - h - \Lambda m$ (where m is the number of the 1D-PC cell), which are counted from the centers of the corresponding layers, we can write for it:

$$\Psi_u(x) = \begin{cases} \Psi_f(\xi_f) = \left(A^2 + B^2 + 2AB \cdot \cos(2\kappa_f \xi_f)\right)^{1/2}, \\ \qquad -h/2 < \xi_f < h/2, \\ \Psi_s(\xi_s) = \left(C^2 + D^2 + 2CD \cdot \cos(2\kappa_s \xi_s)\right)^{1/2}, \\ \qquad -s/2 < \xi_s < s/2. \end{cases} \quad (8)$$

The phase function $\Phi(x, z)$ depends on two coordinates; in general, the FBW is non-planar:

$$\Phi(x,z) = \phi_0 - K\Lambda(m+\varsigma) - \beta z - \phi_u(x), \quad (9)$$

where ϕ_0 has the meaning of the initial phase of the wave, the phase parameter $\varsigma = 0$ in *f*-layers at $\Lambda m < x < \Lambda m + h$ and $\varsigma = 1/2$ in *s*-layers at $\Lambda m + h < x < \Lambda(m+1)$, while $\phi_u(x)$ is the nonlinear component of the phase function $\Phi(x, z)$, which sets the shape of the profile of the FBW wave surfaces. The constant *K* (Bloch wave number) can be found from the dispersion equation [8]:

$$\cos K\Lambda = \cos(\kappa_f h)\cos(\kappa_s s) - \\ -\frac{1}{2}\left(\frac{\chi_s}{\chi_f} + \frac{\chi_f}{\chi_s}\right)\sin(\kappa_f h)\sin(\kappa_s s), \ \beta < k_0 n_s. \quad (10)$$

Distribution (8) of the FBW field in the 1D-PC layers is given by the amplitude coefficients *A*, *B*, *C*, *D* of the partial waves, which are also real at real κ_f, κ_s, and *K,* and depend on the parameters of the medium cell and the Bloch wave number [8, 9]:

$$\begin{cases} A = A_0 \sin\left((\kappa_f h - \kappa_s s + K\Lambda)/2\right) \times \\ \quad \times \sin\left((\kappa_f h + \kappa_s s + K\Lambda)/2\right), \\ B = A_0 \sin\left((\kappa_f h - \kappa_s s + K\Lambda)/2\right) \times \\ \quad \times \sin\left((\kappa_f h - \kappa_s s - K\Lambda)/2\right) \times \\ \quad \times (\chi_s - \chi_f)/(\chi_s + \chi_f), \\ C = A_0 \sin\left((\kappa_f h + \kappa_s s + K\Lambda)/2\right) \times \\ \quad \times \sin(\kappa_f h)(\chi_s + \chi_f)/2\chi_s, \\ D = A_0 \sin\left((\kappa_f h - \kappa_s s + K\Lambda)/2\right) \times \\ \quad \times \sin(\kappa_f h)(\chi_s - \chi_f)/2\chi_s. \end{cases} \quad (11)$$

Formulas (11), which are a generalization of formulas (12) of [8], are used to describe the features of TM-polarized waves, which have not been considered earlier. For example, according to (11) for the case of TM-polarized radiation, if the condition $\chi_s = \chi_f$ is fulfilled, the *B* and *D* amplitudes of the partial waves are equal to zero, while the coefficients *A* and *C* are equal to each other. FBW in *f*- and *s*-layers of the photonic crystal has the form of homogeneous plane waves for which the angles of incidence α_f, α_s on the interlayer boundaries are in agreement with the Brewster condition $\mathrm{tg}\alpha_f = n_s / n_f$, $\mathrm{tg}\alpha_s = n_f / n_s$. The dispersion equation (10) reduces to the following equations: $\cos K\Lambda = (\kappa_f h + \kappa_s s)$.

Parameter A_0 in (11) plays the role of the FBW amplitude. In a confined 1D-PC for forward and backward waves, this parameter may differ. It makes sense to introduce into consideration the amplitude reflection coefficient r_u at the boundary $x = H$ as the ratio of the amplitudes A_{down} and A_{up} (in this case we consider these parameters to be valid) of the backward and forward FBW, respectively. Considering the continuity conditions of the tangential components of the wave fields at the interlayer boundaries inside the 1D-PC and at the boundaries of the 1D-PC with the adjacent media, we can obtain for the coefficient r_u:

$$r_u = \frac{A_{\text{down}}}{A_{\text{up}}} = \frac{1 - F_u}{1 + F_u}, \\ F_u = \frac{\chi_b}{\chi_f}\left(\frac{A - B}{A + B} + \frac{4AB}{A^2 - B^2}\cos^2\phi_h\right). \quad (12)$$

The modulus of the amplitude reflection coefficient r_{au} and the energy coefficients of reflection R_{au} and transmittance T_{au} of the plane wave $\hat{P}_a$, which are obtained for the 1D-PC taking into account the interference of the forward and backward FBW, are found from the following formulas:

$$r_{au} = \frac{Q_a}{P_a} = \\ = \left(\frac{r_a^2 + r_u^2 + 2r_a r_u \cos(2\phi_{\text{p.-q.}} + 2\phi_{\text{p.-d.}})}{1 + r_a^2 r_u^2 + 2r_a r_u \cos(2\phi_{\text{p.-q.}} + 2\phi_{\text{p.-d.}})}\right)^{1/2}, \quad (13) \\ R_{au} = r_{au}{}^2, \ T_{au} = 1 - R_{au},$$

where r_a is the modulus of the amplitude reflection coefficient from the semi-infinite 1D-PC:

$$r_a = \left(\frac{(\chi_a - \chi_f)^2 A^2 + (\chi_a + \chi_f)^2 B^2 + 2AB(\chi_a^2 - \chi_f^2)\cos\kappa_f h}{(\chi_a + \chi_f)^2 A^2 + (\chi_a - \chi_f)^2 B^2 + 2AB(\chi_a^2 - \chi_f^2)\cos\kappa_f h}\right)^{1/2}, \quad (14)$$

$2\phi_{\text{p.-q.}}$, $2\phi_{\text{p.-d.}}$ are the wave phase matching parameters[3] at the boundaries $x = 0$ and $x = h$:

$$2\phi_{\text{p.-q.}} = \\ = \operatorname{arctg}\frac{(\chi_f^2-\chi_a^2)(A^2-B^2)\sin 2\phi_h}{(\chi_f^2-\chi_a^2)(A^2+B^2)-2AB(\chi_f^2+\chi_a^2)\cos 2\phi_h} + \\ + \pi m_{\text{p.-q.}}, \quad (15)$$

$$2\phi_{\text{p.-d.}} = 2K\Lambda N + 2\operatorname{arctg}\left(\operatorname{tg}\phi_h (A-B)/(A+B)\right). \quad (16)$$

Here, parameter $\phi_{\text{p.-d.}}$ with an accuracy to the value 2π multiplied by an integer is equal to the phase delay arising for the FBW at the double passage of the confined 1D-PC; parameter N sets the number of cells of the confined 1D-PC; parameter $\phi_{\text{p.-q.}}$ is equal to the average value of the phases of the incident and reflected waves at the boundary $x = 0$; numerical values of the parameter $m_{\text{p.-q.}}$ variation of the cell parameters of the 1D-PC and FBW can take values 0 ± 1. Analysis of the formulas (13–16) shows that, at variation of the angle of incidence φ of a plane wave on the 1D-PC, interference maxima and minima of intensities for the reflected and refracted waves should occur.

3. EXPERIMENTAL DESIGN AND RESULTS

In the experiments, the multilayer interference dielectric mirror used as the structure under study was produced on a high-vacuum unit for ion-beam deposition of dielectric layers Aspira-150 (Izovac, Belarus)[4]. This structure (sample) was a glass substrate (standard slide glass was used as a substrate material; substrate thickness is 0.7 mm; refractive index is 1.52) with 10 pairs of alternating layers of Nb_2O_5 (niobium oxide (V); thickness is 0.11 μm; refractive index is 2.27) and SiO_2 (quartz glass; thickness is 0.18 μm; refractive index is 1.48) deposited on its surface. Material parameters of the structure layers and substrate were provided by the manufacturer. In the experiments, the dependence of the transmission coefficient T_a of the sample on the angle of incidence φ of the laser light beam was studied. Figure 2 shows the scheme of the experiment. The intensity of helium-neon laser radiation (*1*), which passed through the polarizer (*2*) and the sample (*4*) mounted on the goniometer stage (*3*), was measured by the photodetector (*5*).

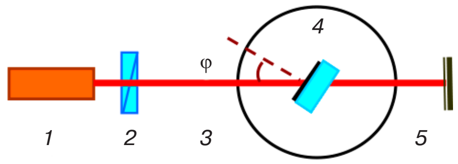

Fig. 2. Experimental design

According to preliminary estimates made by the formulas (6), the angular distance between neighboring maxima of the transmitted light intensity should be the largest at incidence angles close to zero (normal incidence mode) and incidence angles close to 90° (sliding incidence mode). The angular width of interference resonances in these modes is the largest, which should also significantly simplify the confident registration of the maxima and minima of the intensity of transmitted radiation at variation of the angle of incidence φ.

For the case of TE-polarized radiation at variation of the angle of incidence φ within the range from 0° to 8°, Fig. 3 shows the measured intensity of I_{trans} radiation (in arbitrary units, a.u.) that passed through the sample together with the transmittance coefficient of the T_a glass plate calculated by the formulas (6). In full accordance with the preliminary calculations, the angular distance between the resonances and their angular width decrease as the angle of incidence increases from 0° to 8°.

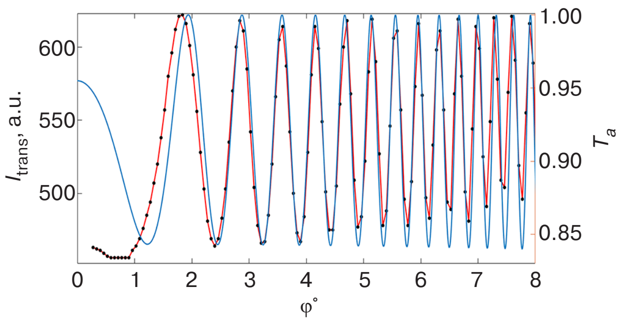

Fig. 3. Measured I_{trans} intensity (red line with dots) and calculated transmittance function T_a of a plate with n_b = 1.52 and d = 0.7 mm (solid blue line) as a function of the angle of incidence φ. Emission wavelength is 0.6328 μm

Figure 4 depicts the measured intensity of I_{trans} radiation (in a.u.), which passed through the sample, and the transmittance T_a of the glass plate calculated by the formulas (6) as a function of the slip angle θ $(\theta = \pi/2 - \varphi)$ for the case of TE-polarized radiation.

[3] Index p.-d. is a phase delay; in p.-q. index the letter p indicates the P_a wave, and the letter q indicates the Q_a wave.
[4] http://izovac.by/ (in Russ.). Accessed September 27, 2024.

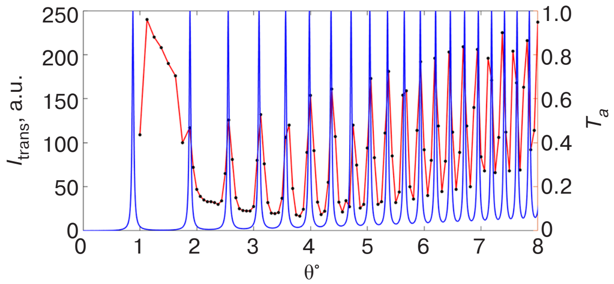

Fig. 4. Measured I_{trans} intensity (red line with dots) and calculated transmission function T_a of a plate with n_b = 1.52 and d = 0.7 mm (solid blue line) as a function of slip angle θ. Emission wavelength is 0.6328 μm

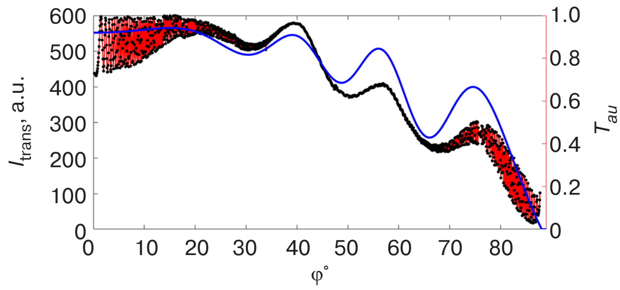

Fig. 5. 1D-PC transmittance function T_{au} (solid blue line) and measured I_{trans} intensity as a function of the angle of incidence φ (red color, black dots). The wavelength is 0.6328 μm

In this slip-fall mode, a marked decrease in the angular width and angular distance between the observed resonances is clearly recorded when increasing the slip angle θ in the presented range from 0° to 8°, which is also in full agreement with the preliminary estimates.

The dependencies presented in Figs. 3 and 4 clearly demonstrate the presence of interference resonances of a thin dielectric plate. At the same time, for the presented I_{trans} dependencies, the presence of a significant increase in transmittance in the intensity maxima and minima both at increasing the angle of incidence φ (in the regime close to normal incidence) and at increasing the slip angle θ (in the regime close to slip incidence) should be noted. These features are explained by the influence of Nb_2O_5 and SiO_2 layers deposited on the glass plate, the interference of waves in which should lead to additional modulations of the intensity of radiation passing through the sample.

Figure 5 shows the measured intensity of I_{trans} TE-polarized He-Ne laser radiation (in a.u.) that passed through the sample (red line, black dots) and the 1D-PC transmittance function of T_{au} with the following cell parameters (13): h = 0.11 μm, s = 0.18 μm, n_f = 2.27, n_s = 1.48 (solid blue line) for the case of variation of the angle of incidence φ in the range from 0° to 90°. The crystal consists of 10 cells framed by two homogeneous media with refractive indices: n_a = 1, n_b = 1.52. According to the calculations, four interference transmission maxima should be observed as a result of FBW interference in the 1D-PC at the variation of the angle of incidence φ in the range from 0° to 90°.

The transmittance of the considered sample is determined by superposition of the interference resonances of the glass plate on the interference resonances of the 1D-PC. Here, the transmittance resonances and their corresponding reflection resonances for the glass plate are clearly registered with the used laser only in a limited range of incidence angles φ and slip angles θ not exceeding 20° (Figs. 6a and 6b).

(a) (b) (c)

Fig. 6. Images of the reflected laser beam at the values of the angle of incidence φ: (a) 12°; (b) 75°; (c) 50°

Within the range of incidence angles φ from 30° to 70°, the angular width of the glass plate resonances is significantly smaller than the angular divergence of the laser beam (Fig. 6c); moreover, these fast intensity modulations are not resolved. At the same time, the slow intensity modulations due to FBW interference are in full agreement with the theory.

The obtained results of the experiments and theoretical simulation agree quite well to demonstrate the presence of interference of FBW in 1D-PC, as a result of which the radiation intensity in the maxima and minima of interference resonances of a thin glass plate can change significantly. Thus, the representation of FBW in the form of inhomogeneous waves can be useful for calculating and optimizing the parameters of optical elements and devices that use interference effects in multilayer periodic structures.

CONCLUSIONS

In this article, the exact expressions for the reflection and transmission coefficients for the case of a confined 1D-PC with a stepped refractive index profile are obtained on the basis of wave representation in the form of a linear combination of Floquet and Bloch waves. A qualitative agreement of experimental measurements of the transmission coefficient of a plane TE-wave incident from a half-space on a confined photonic crystal with the theoretical calculations has been established. The graph (Fig. 5) clearly depicts fast intensity modulations, which is in agreement with the well-known Airy functions obtained in the case of multipath wave interference in a transparent plate at a variation of the angle of incidence of the wave. Since the thickness of the glass substrate is many times greater than the thickness of the photonic crystal and the wavelength of the laser radiation, which leads to a large phase delay, rapidly alternating maxima and minima of intensity should be observed, which are difficult to resolve in measurements. Slow intensity modulations, in turn, are due to the interference of the FBW in the photonic crystal, where the phase lag is much smaller at a small Bloch wave vector.

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Authors' contributions

D.Kh. Nurligareev—derivation of exact formulas for reflection and transmission coefficients for the cases of a limited one-dimensional photonic crystal with a stepped refractive index profile and a multilayer dielectric mirror; writing the Introduction, Conclusions, and the 1st and 2nd sections of the article.

I.A. Nedospasov—performing calculations and plotting dependencies of intensities and transmission functions of the studied structures; writing the Abstract and the 3rd section of the article.

K.Yu. Kharitonova—performing measurements and constructing experimental dependencies of radiation intensities; writing the 3rd section of the article.

REFERENCES

1. Lord Rayleigh. On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with periodic structure. *Phyl. Mag. Ser. 5.* 1887;24(147):145–159. https://doi.org/10.1080/14786448708628074
2. Lord Rayleigh. On the remarkable phenomenon of crystalline reflection described by Prof. Stokes. *Phyl. Mag. Ser. 5.* 1888;26(160):256–265. https://doi.org/10.1080/14786448808628259
3. Floquet G. Sur les equations differentielles linearies a coeffients periodiques. (*Annales scientifiques de l'École Normale Supérieure, Serie 2*) *Ann. Ecole Norm. Sup.* 1883;12:47–88. https://doi.org/10.24033/asens.220
4. Bloch F. Uber die Quantenmechanic der elektronen in kristallgittern. *Z. Phys.* 1929;52(85):555–600. https://doi.org/10.1007/BF01339455
5. Elachi C. Waves in active and passive periodical structures: A review. *Proc. IEEE.* 1976;64(12):1666–1698. https://doi.org/10.1109/PROC.1976.10409, available from URL: https://core.ac.uk/download/pdf/216300643.pdf
6. Winn J.N., Fink Y., Fan S., Joannopoulos J.D. Omnidirectional reflection from a one-dimensional photonic crystal. *Opt. Lett.* 1998;23(20):1573–1575. https://doi.org/10.1364/OL.23.001573
7. Chigrin D.N., Lavrinenko A.V., Yarotsky D.A., Gaponenko S.V. Observation of total omnidirectional reflection from a one-dimensional dielectric lattice. *Appl. Phys. A.* 1999;68(1):25–28. https://doi.org/10.1007/s003390050849
8. Nurligareev D.K., Sychugov V.A. Propagation of light in a one-dimensional photonic crystal: analysis by the Floquet–Bloch function method. *Quantum Electron*. 2008;38(5):452–461. https://doi.org/10.1070/QE2008v038n05ABEH013653
   [Original Russian Text: Nurligareev D.K., Sychugov V.A. Propagation of light in a one-dimensional photonic crystal: analysis by the Floquet–Bloch function method. *Kvantovaya elektronika*. 2008;38(5):452–461. Available from URL: https://www.elibrary.ru/tteynf]
9. Nurligareev D.K. Floquet-Bloch waves of a one-dimensional photonic crystal: a general theory. *Naukoemkie tekhnologii.* 2009;10(9):12–23 (in Russ.).
10. Nurligareev D.K., Sychugov V.A. Technique for determining the cell parameters of the confined one-dimensional photonic crystal, based on the Floquet-Bloch formalism. *Bull. Lebedev Phys. Inst.* 2012;39(2):33–37. https://doi.org/10.3103/S1068335612020017
    [Original Russian Text: Nurligareev D.K., Sychugov V.A. Technique for determining the cell parameters of the confined one-dimensional photonic crystal, based on the Floquet-Bloch formalism. *Kratkie soobshcheniya po fizike*. 2012;2:3–10 (in Russ.). Available from URL: https://www.elibrary.ru/ouillp]
11. Caffrey S., Morozov G.V., Sprung D.W., Martorell J. Floquet–Bloch solutions in a sawtooth photonic crystal. *Opt. Quantum Electron.* 2017;49(3):112. https://doi.org/10.1007/s11082-017-0939-1
12. Ibrahim A., Sprung D.W.L., Morozov G.V. Construction and Floquet–Bloch analysis of analytically solvable Hill equations with smooth potentials. *J. Opt. Soc. Am. B.* 2018;35(6):1223–1232. https://doi.org/10.1364/JOSAB.35.001223
13. Shmat'ko A.A., Mizernik V.N., Odarenko E.N. Floquet-Bloch waves in magnetophotonic crystals with transverse magnetic field. *J. Electromagn. Waves Appl.* 2020;34(12):1667–1679. https://doi.org/10.1080/09205071.2020.1780955
14. Yeh P., Yariv A., Hong C.S. Electromagnetic propagation in periodic stratified media. 1. General theory. *J. Opt. Soc. Am.* 1977;67(4):423–438. https://doi.org/10.1364/JOSA.67.000423
15. Walpita L.M. Solutions for planar optical waveguide equations by selecting zero elements in a characteristic matrix. *J. Opt. Soc. Am. A.* 1985;2(4):595–602. https://doi.org/10.1364/JOSAA.2.000595
16. Konopsky V. Design of 1D Photonic Crystals Sustaining Optical Surface Modes. *Coatings.* 2022;12(10):1489. https://doi.org/10.3390/coatings12101489

17. Segovia-Chaves F., Vinck-Posada H., Trabelsi Y., Ali N.B. Transmittance spectrum in a one-dimensional photonic crystal with Fibonacci sequence superconductor–semiconductor. *Optik.* 2020;217:164803. https://doi.org/10.1016/j.ijleo.2020.164803
18. Panda A., Pukhrambam P.D., Wu F., Belhadj W. Graphene-based 1D defective photonic crystal biosensor for real-time detection of cancer cells. *Eur. Phys. J. Plus.* 2021;136(8):809. https://doi.org/10.1140/epjp/s13360-021-01796-z
19. Liu Z., Li X., Chen C., Wang X., Gao W., Ye W., Li L., Liu J. Bound states in the continuum in asymmetric one-dimensional photonic crystal systems guided by anisotropy. *Opt. Express.*2023;31(5):8384–8392. https://doi.org/10.1364/OE.482894
20. Born M., Wolf E. *Osnovy optiki* (*Principles of Optics*). Transl. from Engl. Moscow: Nauka; 1970. 856 p. (in Russ.).
[Born M., Wolf E. *Principles of Optics*. Oxford, London, Edinburg, New York, Paris, Frankfurt: Pergamon Press; 1964, 856 p.]

СПИСОК ЛИТЕРАТУРЫ

1. Lord Rayleigh. On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with periodic structure. *Phyl. Mag. Ser. 5.* 1887;24(147):145–159. https://doi.org/10.1080/14786448708628074
2. Lord Rayleigh. On the remarkable phenomenon of crystalline reflection described by Prof. Stokes. *Phyl. Mag. Ser. 5.* 1888;26(160):256–265. https://doi.org/10.1080/14786448808628259
3. Floquet G. Sur les equations differentielles linearies a coeffients periodiques. (*Annales scientifiques de l'École Normale Supérieure, Serie 2*) *Ann. Ecole Norm. Sup.* 1883;12:47–88. https://doi.org/10.24033/asens.220
4. Bloch F. Uber die Quantenmechanic der elektronen in kristallgittern. *Z. Phys.* 1929;52(85):555–600. https://doi.org/10.1007/BF01339455
5. Elachi C. Waves in active and passive periodical structures: A review. *Proc. IEEE.* 1976;64(12):1666–1698. https://doi.org/10.1109/PROC.1976.10409, URL: https://core.ac.uk/download/pdf/216300643.pdf
6. Winn J.N., Fink Y., Fan S., Joannopoulos J.D. Omnidirectional reflection from a one-dimensional photonic crystal. *Opt. Lett.* 1998;23(20):1573–1575. https://doi.org/10.1364/ol.23.001573
7. Chigrin D.N., Lavrinenko A.V., Yarotsky D.A., Gaponenko S.V. Observation of total omnidirectional reflection from a one-dimensional dielectric lattice. *Appl. Phys. A.* 1999;68(1):25–28. https://doi.org/10.1007/s003390050849
8. Нурлигареев Д.Х., Сычугов В.А. Распространение света в одномерном фотонном кристалле: анализ методом функции Флоке-Блоха. *Квантовая электроника.* 2008;38(5):452–461. URL: https://www.elibrary.ru/tteynf
9. Нурлигареев Д.Х. Волны Флоке-Блоха одномерного фотонного кристалла: общая теория. *Наукоемкие технологии.* 2009;10(9):12–23.
10. Нурлигареев Д.Х., Сычугов В.А. Методика определения параметров ячейки ограниченного одномерного фотонного кристалла, основанная на формализме функций Флоке-Блоха. *Краткие сообщения по физике Физического института им. П.Н. Лебедева РАН*. 2012;2:3–10. URL: https://www.elibrary.ru/ouillp
11. Caffrey S., Morozov G.V., Sprung D.W.L., Martorell J. Floquet–Bloch solutions in a sawtooth photonic crystal. *Opt. Quantum Electron.* 2017;49(3):112. https://doi.org/10.1007/s11082-017-0939-1
12. Ibrahim A., Sprung D.W.L., Morozov G.V. Construction and Floquet–Bloch analysis of analytically solvable Hill equations with smooth potentials. *J. Opt. Soc. Am. B.* 2018;35(6):1223–1232. https://doi.org/10.1364/JOSAB.35.001223
13. Shmat'ko A.A., Mizernik V.N., Odarenko E.N. Floquet-Bloch waves in magnetophotonic crystals with transverse magnetic field. *J. Electromagn. Waves Appl.* 2020;34(12):1667–1679. https://doi.org/10.1080/09205071.2020.1780955
14. Yeh P., Yariv A., Hong C.S. Electromagnetic propagation in periodic stratified media. 1. General theory. *J. Opt. Soc. Am.* 1977;67(4):423–438. https://doi.org/10.1364/JOSA.67.000423
15. Walpita L.M. Solutions for planar optical waveguide equations by selecting zero elements in a characteristic matrix. *J. Opt. Soc. Am. A.* 1985;2(4):595–602. https://doi.org/10.1364/JOSAA.2.000595
16. Konopsky V. Design of 1D Photonic Crystals Sustaining Optical Surface Modes. *Coatings.* 2022;12(10):1489. https://doi.org/10.3390/coatings12101489
17. Segovia-Chaves F., Vinck-Posada H., Trabelsi Y., Ali N.B. Transmittance spectrum in a one-dimensional photonic crystal with Fibonacci sequence superconductor–semiconductor. *Optik.* 2020;217:164803. https://doi.org/10.1016/j.ijleo.2020.164803
18. Panda A., Pukhrambam P.D., Wu F., Belhadj W. Graphene-based 1D defective photonic crystal biosensor for real-time detection of cancer cells. *Eur. Phys. J. Plus.* 2021;136(8):809. https://doi.org/10.1140/epjp/s13360-021-01796-z
19. Liu Z., Li X., Chen C., Wang X., Gao W., Ye W., Li L., Liu J. Bound states in the continuum in asymmetric one-dimensional photonic crystal systems guided by anisotropy. *Opt. Express.*2023;31(5):8384–8392. https://doi.org/10.1364/OE.482894
20. Борн М., Вольф Э. *Основы оптики*: пер с англ. М.: Наука; 1970. 856 с.

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